Study of the RF Pulse Heating Phenomenon in High Gradient Accelerating Devices by Means of Analytical Approximations

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***Abstract*—The main objective of this work is to present a simple method, based on analytical expressions, for obtaining a quick approximation of the temperature rise due to the Joule effect inside the metallic walls of an RF accelerating device. This proposal relies on solving the 1D heat-transfer equation for a thick wall, where the heat sources inside the wall are the ohmic losses produced by the RF electromagnetic fields penetrating the metal with finite electrical conductivity. Furthermore, it is discussed how the theoretical expressions of this method can be applied to obtain an approximation to the temperature increase in realistic 3D RF accelerating structures, taking as an example the cavity of an RF electron gun. These theoretical results have been benchmarked with numerical simulations carried out with commercial finite-element method codes, finding good agreement among them.**

***Index Terms*—RF pulse heating, thermal analysis, RF accelerating structures.**

I. INTRODUCTION

RF pulse heating is a phenomenon by which metals are heated by the electric induced currents generated by a pulsed high power RF electromagnetic field. This heating has two different effects. The first one, which happens during the RF pulse, causes a superficial temperature increase within the RF skin depth range in a short time compared to the expansion mechanical time response of the material inducing stresses that might damage the surface. According to recent studies of breakdown rates in high gradient linear accelerators, there is a direct correlation between these rates and RF pulse heating [1]. Because of that, the reliable operating gradient could be potentially limited by the thermal stress suffered by these structures. The second effect occurs after the RF pulse ends and consists in the heat diffusion along the metallic slab and the temperature decrease in the surface until the next RF pulse. As a result, there is an overall temperature rise on the metallic walls of the cavity from one pulse until the arrival of the next. After many RF pulses the device temperature increases, the body expands and the resonant frequency shifts. This frequency shift will detune the cavities and might reflect the RF power.

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This entails that the RF pulse heating effects must be taken into consideration when designing high gradient RF accelerating structures. There are several commercial codes (ANSYS [2], SIMULIA [3], etc.), based on Finite Differences in Time Domain (FDTD) or Finite Elements Method (FEM), which allow a thermal analysis of the accelerating components. However, these software tools usually have a considerable computational cost in terms of time and hardware in order to get accurate results.

In this paper we describe a single procedure for analysing the RF pulse heating within a thick metallic wall exposed to an RF pulsed high power electromagnetic field. First, in Section II, the theoretical model employed for deriving analytical expressions for the thermal heating induced inside the component is discussed. Afterwards, in Section III the feasibility of using the 1D theoretical expressions for analysing the RF pulse heating phenomenon in 3D structures is explored by means of performing the thermal analysis of an RF photoinjector. Finally, in Section IV, the main conclusions of this study are outlined.

II. THEORETICAL MODEL

In this Section we present the theoretical background required to derive the analytical formulas that allow to describe the temperature increase inside the device wall as a function of time, caused by an electromagnetic RF pulsed signal. A typical RF pulsed signal consists in a gated harmonic signal. To analyse this phenomenon, the component wall is modelled as a metallic layer with thickness L (see Fig. 1) that extends infinitely in the y and z dimmensions. The left side of the wall corresponds to the inner side of the device, i.e., where the RF electromagnetic fields are present. The right side of the wall is in contact with the cooling mechanism of the device, modelled as a convective heat exchange, which is characterized by its heat transfer coefficient h and the fluid temperature T_∞. Assuming the metal has a finite electrical conductivity σ, the RF electromagnetic fields inside the cavity (vacuum side in Fig. 1) penetrate into the conductor wall and are attenuated exponentially in a characteristic length δ, known as the skin depth. The time harmonic RF electromagnetic fields inside the metallic wall will decay exponentially according to the skin depth length $\delta = \sqrt{\frac{2}{\sigma\mu_0\omega}}$ [4], where μ_0 is the vacuum permeability, $\omega = 2\pi f$, and f is the RF frequency. As it is well known, the RF electric field inside the metal will induce electric currents which will dissipate power due to the Joule effect [4]. This dissipated power constitutes the heat source that increases the temperature of the component walls.

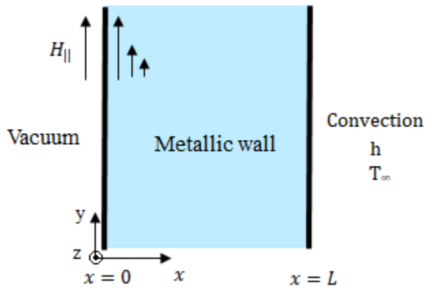

Fig. 1. Scheme of the device metallic wall. The inner surface ($x = 0$) corresponds to the vacuum side inside the component, the outer surface ($x = L$) corresponds to the cooling side modelled as a convective heat exchange. The slab extends infinitely in the y and z directions.

As a consequence of the symmetry of the slab in the y-z plane, the RF electromagnetic fields only depend on the x-coordinate and the thermal problem is reduced to 1-D. The 1D heat transfer equation can be solved analytically as an infinite series in some cases. In the case we are interested in, the heat sources are time-dependent since the RF electromagnetic signal is composed by a succession of pulses with a repetition rate $f_p = 1/T_p$ (T_p being the pulse repetition period). During the pulse (characterized by t_{on}), the electromagnetic fields oscillate harmonically in time, with an harmonic amplitude that also varies with time depending on the characteristic shape of the pulse. When the pulse ends there is a time lapse in which there are no heat sources in the wall until the start of the next one. In this work, two pulse shapes will be taken into consideration, namely, the square pulse, and the pulse with transient, which is the transformation of the square pulse in a Standing Wave (SW) structure.

A. Temperature increase during the pulse

The typical values of t_{on} in pulsed RF accelerators is usually within the range from several hundred nanoseconds to a few microseconds. These time lapses are shorter than the typical times that takes the diffusion mechanism to transfer the heat across the wall from the vacuum side to the cooling boundary. Since a negligible amount of heat is transferred across the wall during the RF pulse, the heat exchange between the convection medium and the right wall boundary will affect the temperature only in the neighbourhood of such boundary. If we are interested in obtaining the temperature increase during the pulse, which only is noticeable in a length of a few skin depths from the vacuum wall, the convection mechanism of the right wall can be dropped from the calculations by assuming a thermally isolated wall. This assumption simplifies the mathematical problem of solving the heat transfer equation, which allows to obtain a simple analytical expression for the temperature increase during the pulse. Next, the solution of the heat transfer differential equation is provided for the square pulse and the pulse with transient cases.

1) Square pulse: The temperature increase ΔT is given by:

$$\Delta T(x,t) = g_0 t + \sum_{n=1}^{\infty} \frac{g_n}{(\frac{\pi D n}{L})^2} \left(1 - e^{-(\frac{\pi D n}{L})^2 t}\right) \cos\left(\frac{\pi n x}{L}\right), \quad 0 \leqslant t \leqslant t_{on} \tag{1a}$$

$$\Delta T(x,t) = g_0 t_{on} + \sum_{n=1}^{\infty} \frac{g_n}{(\frac{\pi D n}{L})^2} \left(1 - e^{-(\frac{\pi D n}{L})^2 t_{on}}\right) e^{-(\frac{\pi D n}{L})^2 (t - t_{on})} \cos\left(\frac{\pi n x}{L}\right), \quad t > t_{on} \tag{1b}$$

with

$$g_0 = \frac{2\alpha}{L}\left(1 - e^{-\frac{2L}{\delta}}\right) \tag{2a}$$

$$g_n = \frac{8\alpha L}{4L^2 + \pi^2\delta^2 n^2}\left(1 - e^{-\frac{2L}{\delta}}(-1)^n\right) \tag{2b}$$

$$\alpha = \frac{R_s |H_{||,0}|^2}{2\rho C_e}; \quad D = \sqrt{\frac{\kappa}{\rho C_e}}$$

where $H_{||,0}$ is the amplitude of the parallel RF magnetic field at the interface, $R_s = 1/(\delta\sigma)$ is the surface resistance, κ is the thermal conductivity, ρ is the density, C_e is the specific heat.

2) Pulse with transient: Let us assume that the RF generator provides a square pulse to the RF circuit. For a SW cavity, such square pulse will be transformed into a transient with a characteristic filling time τ for the cavity. In this case, the temperature increase is given by:

$$\Delta T(x,t) = u_0(t) + \sum_{n=1}^{\infty} u_n(t) \cos\left(\frac{\pi n x}{L}\right), \quad 0 \leqslant t \leqslant t_{on} \tag{3a}$$

$$\Delta T(x,t) = u_0(t_{on}) + \frac{1}{2} v_0 (t - t_{on}) + \sum_{n=1}^{\infty} \left(v_n(t - t_{on}) + u_n(t_{on}) e^{-(\frac{\pi D n}{L})^2 (t - t_{on})}\right) \cos\left(\frac{\pi n x}{L}\right), \quad t > t_{on} \tag{3b}$$

with

$$u_0(t) = \frac{g_0}{2}\left[t + \tau\left(2e^{-\frac{t}{\tau}} - \frac{1}{2}e^{-\frac{2t}{\tau}} - \frac{3}{2}\right)\right]$$

$$u_n(t) = \frac{g_n}{\left(\frac{\pi D n}{L}\right)^2}\left(1 - e^{-\left(\frac{\pi D n}{L}\right)^2 t}\right) + \frac{g_n}{\left(\frac{\pi D n}{L}\right)^2 - \frac{2}{\tau}}\left(e^{-\frac{2t}{\tau}} - e^{-\left(\frac{\pi D n}{L}\right)^2 t}\right) + \frac{2 g_n}{\left(\frac{\pi D n}{L}\right)^2 - \frac{1}{\tau}}\left(e^{-\left(\frac{\pi D n}{L}\right)^2 t} - e^{-\frac{t}{\tau}}\right)$$

$$v_n(t) = \frac{g_n\left(1 - e^{-\frac{t_{on}}{\tau}}\right)^2}{\left(\frac{\pi D n}{L}\right)^2 - \frac{2}{\tau}}\left[e^{-\frac{2t}{\tau}} - e^{-\left(\frac{\pi D n}{L}\right)^2 t}\right]$$

where g_0 and g_n are the same as in eqs. (2a)-(2b).

B. Average temperature increase after many RF pulses

In order to obtain the average temperature increase in the wall after many RF pulses have been driven through the device, two assumptions are taken to simplify the mathematical procedure of solving the 1D heat transfer differential equation. First, the volume heat sources in the wall are replaced by an equivalent superficial heat flux entering through the vacuum wall, which is obtained by integrating the total contribution of volume sources along the wall depth. This assumption gives good results since the skin depth values in RF accelerators is much shorter than the wall depth and hence the volume distribution of the heat sources is focused on the neighbourhood of the vacuum wall. The second assumption considers that the temporal variation of the heat source can be replaced by a time-constant heat source with the time-average value of the heat generated along the RF pulse sequence. For this case, the presence of the cooling system cannot be neglected in the calculations and it is taken into account. Finally, the temperature, assuming as initial conditions a uniform temperature along the wall T_0, is given by

$$T(x,t) = T_\infty + \frac{q_{s,0}}{\kappa}\left(L + \frac{\kappa}{h}\right) - \frac{q_{s,0}}{\kappa}x + \sum_{n=1}^{\infty} C_n \cos(\lambda_n x) e^{-(D\lambda_n)^2 t} \quad (4)$$

$$C_n = \frac{\frac{1}{\lambda_n}\sin(\lambda_n L)[T_0 - T'_\infty] - \frac{q_{s,0}}{\lambda_n^2 \kappa}\left[1 - \left(1 + \frac{hL}{\kappa}\right)\cos(\lambda_n L)\right]}{\frac{L}{2} + \frac{\sin 2\lambda_n L}{4\lambda_n}}$$

where

$$T'_\infty = T_\infty + \frac{q_{s,0}}{\kappa}\left(L + \frac{\kappa}{h}\right) \qquad \lambda_n \tan(\lambda_n L) = \frac{h}{\kappa} \quad (5)$$

where $q_{s,0}$ is the time-averaged total RF power dissipated in the wall in terms of heat. Eq. (5) has to be solved numerically to obtain the eigenvalues λ_n that allow us to get the coefficients C_n. As it can be noticed in eq. (4), the function dependence on time is of the form $e^{-(D\lambda_n)^2 t}$. Thus, for a sufficiently long time, the temperature in the wall will reach a steady state distribution and the summation can be neglected.

III. Thermal analysis of an RF electron gun photoinjector

The purpose of this section is to analyse the suitability of applying the 1D theoretical expressions (derived in Section II) in order to obtain a first approximation of the temperature increase caused by the RF pulse heating phenomenon in 3D cavities which are part of RF accelerating systems. In Section II the temperature increase in a 1D metallic wall was studied by considering separately two effects due to the RF pulse heating.

First, the sharp temperature increase occurring during the RF pulse very close to the vacuum side of the wall. The pulse duration in typical RF accelerating devices is too short to allow effective thermal diffusion across the wall. Therefore, this is a phenomenon depending only on the electromagnetic field intensity at the local point of the surface and thus the 3D details of the device (including the cooling system) can be neglected in the analysis. Hence, the 1D theoretical model presented in Subsection II-A is expected to give good results for realistic 3D devices.

Second, the average temperature increase in the wall after many RF pulses are driven into the structure, which finally evolves to a steady state case with a spatial temperature distribution along the wall. In this case, the 3D details of the geometry of the device might play an important role in the final temperature distribution. As a consequence, further investigation must be carried out for this case in order to evaluate the goodness of using the 1D theoretical expressions to approximate the RF pulse heating phenomenon in realistic 3D devices. To reach this aim, the average temperature increase at the steady state is analysed for an RF electron gun accelerating cavity, comparing the results obtained with the Thermal Transient Ansys (TTA) numerical simulations and the predictions of the 1D theoretical expressions.

The device that we have studied is properly described in [6]. It consists of six accelerating cavities, each of them with length $\lambda/2$ (being λ the wavelength of the RF electromagnetic wave in free space), except the first one which is 0.6 times the length of the others. The present RF gun is designed to operate in SW mode with the π-mode advance per cell operating at a frequency of $f = 11.994\ GHz$. The RF gun is connected to the RF generator external circuit by means of a coaxial coupler. The filling time of the RF gun is $\tau = 112.5\,ns$. The gun is intended to operate in pulsed mode with an RF pulse duration of $t_{on} = 400\ ns$ and $f_p = 400\,Hz$ repetition rate.

We will focus on the thermal analysis of the full cells, in which the magnitude of the RF electric field is almost the same for all of them. Thanks to this, the thermal analysis can be reduced to the study of one single full cell. The scheme of the photoinjector cavity in TTA is depicted in Fig. 2. The cavity radius is $R_c = 11.029\ mm$ and the cavity length is $L_{cav} = 12.498\,mm$. The presence of a water cooling channel has been considered, with square section ($a = 10\,mm$), heat transfer coefficient $h = 1.2 \times 10^4\,Wm^{-2}K^{-1}$, and fluid temperature $T_\infty = 0°C$. The separation between the top of the cavity and the lower side of the cooling channel is $L = 15\,mm$. The material for the bulk is copper. The initial temperature is assumed to be uniform for all the cavity bulk, $T_0 = 0°C$. TTA simulations were carried out for several values of the RF electric field at cathode, E_0.

Now, it remains to estimate the maximum temperature increase at the steady state in the RF gun cavity using the corresponding analytical expression of the 1D model (recall the eq. (4)). Two aspects must be taken into consideration to explore properly this 1D formula to the case of the 3D cavity. First, an equivalent heat transfer coefficient h_{eq} must

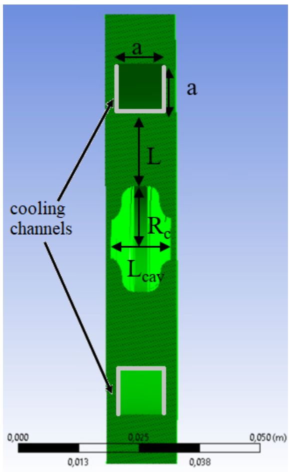

Fig. 2. Scheme of the RF gun cavity with water cooling channel analyzed with TTA.

be chosen for the 1D model in order to compensate the possible difference between the area size of the pillbox inner surface, A_{heat}, where the heat sources are located; and the area of the water cooling channel, $A_{cooling}$, where the heat transfer due to the convection phenomenon occurs. In the 1D model, it is implicitly assumed that the vacuum surface (where the heat sources are) and the cooling system surface have the same area. As a general rule, for 3D devices, it is not expected that both areas are equal. To adjust this mismatch, we propose to take in the 1D model an equivalent heat transfer coefficient defined as $h_{eq} = \frac{A_{cooling}}{A_{heat}} h$, where h represents the heat transfer coefficient of the water cooling channel of the 3D structure. Regarding the area of the water cooling channel, $A_{cooling}$, only the area corresponding to the bottom and the lateral faces (neglecting the top face) is taken into account. According to this, the area of cooling is given by $A_{cooling} = 2\pi(L+R_c)a+2\pi[(L+R_c+a)^2-(L+R_c)^2]$. The physical reason that motivates neglecting the top face is that this surface has no direct line of sight with the heat sources and hence no significant effects in the cooling are expected. This choice might seem a rough approximation but in fact gives satisfactory results as we will see next. For the particular case of this example, $h_{eq} = 5.087h$. The second consideration is related to the fact that in the gun cavity the surface losses are not uniform, whilst the 1D model assumes uniform power losses per unit area, $q_{s,0}$. Consequently, a spatial average power losses per unit area coefficient $q_{s,0,avg}$ has to be defined : $q_{s,0,avg} = \frac{P_{loss}}{A_{heat}}$ where P_{loss} is the total RF power lost by Joule effect in the walls of the cavity. The value of P_{loss} can be calculated using either HFSS or other RF electromagnetic codes like SUPERFISH [5].

In Fig. 3 the maximum temperature increase in the RF gun cavity at the steady state is depicted for different values of the RF electric field amplitude at the photoinjector cathode. The results of both TTA simulations and the 1D model are included for comparison. As it can be observed, there is good agreement between both data.

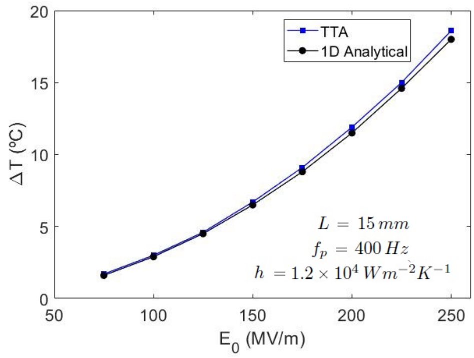

Fig. 3. Maximum temperature increase in the RF gun at the steady state for different values of electric field amplitude at cathode. Comparison between the TTA simulations and the results of the 1D model.

IV. Conclusions

In this paper, the temperature increase due to the RF pulse heating phenomenon has been analysed in RF accelerator structures. For this purpose, the 1D heat transfer differential equation has been solved in a thick metallic slab for certain specific cases, allowing to obtain simple analytic mathematical expressions. The peak temperature increase during the pulse is obtained for the cases of a square RF pulse and an RF pulse with transient. Similarly, the average temperature increase after many RF pulses is also provided. In the latter case, the theoretical model assumes that the device exchanges heat with the environment by means of the convection mechanism. This allows us to consider the interesting case of a cooling system based on a water turbulent flow through a pipe.

The 1D analytical expressions for the temperature increase in the slab presented in this paper are aimed at improving the design procedure of RF accelerating cavities. This is accomplished by reducing the computational time required to perform the thermal analysis with regard to using of 3D FEM codes.

Moreover, a 3D RF photoinjector has been analysed both with the 1D analytical formulas and the Ansys numerical simulations. The results evidence that the 1D analytical formulas provide an useful approximation to the temperature increase in the device. However, it must be remarked that the method described in this manuscript is intended to provide a quick approximation to the temperature increase in the device, but of course it will not be as accurate as proper 3D numerical simulations.

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