



Simplified expressions for reliability assessments in code calibration

Itsaso Arrayago^{a,*}, Hao Zhang^b, Kim J.R. Rasmussen^b

^a Universitat Politècnica de Catalunya, Dept. of Civil and Environmental Engineering, Spain

^b The University of Sydney, School of Civil Engineering, Australia

ARTICLE INFO

Keywords:

Code calibration
Structural reliability
Reliability index
Partial safety factors
Resistance factors
FORM
FOSM

ABSTRACT

First Order Reliability Methods (FORM) have been used by specification committees in the reliability analyses required for the calibration of resistance and safety factors for the past 40 years. However, these methods are iterative, require input information that may not be readily available, and make comparisons between different approaches or design frameworks difficult. This paper presents a set of simplified equations to estimate reliability indices β , resistance factors ϕ and partial safety factors γ_M based on simpler First Order Second Moment (FOSM) considerations for the US and Eurocode frameworks, which are particularized for different load cases, and on the semi-probabilistic approach prescribed in the Eurocode 0. The equations provide direct relationships between the reliability calibration results corresponding to different design frameworks, and can be used to estimate resistance factors as simple cross-checks for the US framework based on the partial safety factors derived for the Eurocode (or vice versa) from basic statistical input information and given target reliability, including when the data available in the literature is insufficient to perform FORM analyses. The accuracy of the proposed equations is assessed against reliability results derived using FORM techniques for an extensive database of steel and stainless steel frames subjected to gravity and combined gravity plus wind load cases collected from the literature, and limitations for their applicability are recommended. The results demonstrate that the set of equations proposed in this paper provides accurate estimations of the reliability index β , resistance factors ϕ and partial safety factors γ_M and can assist specification committees in the process of calibrating suitable ϕ and γ_M -factors.

1. Introduction

New structural verification methods based on direct design approaches are currently being developed for international steel standards as a consequence of the increase of computational power in desktop computers and the better access to advanced finite element software. In these direct design approaches, also referred to as the Direct Design Method, the resistance of structures (or members) can be directly estimated from advanced numerical analyses without requiring further resistance checks, provided the models incorporate all relevant characteristics that affect the strength of the structure, including initial imperfections, nonlinear geometric effects, residual stresses and accurate nonlinear material behaviour. During the last years, significant advances have been made in the development of design recommendations [1–7] and benchmark examples [8] for the direct design of steel structures, the development of new finite elements [9,10], the proposal of strain limits to be used with beam-type finite elements [11,12], and the calibration of system resistance and partial safety factors [13–22]. Since

advanced finite element models are capable of accurately predicting the strength and capturing the failure mode of complex structures, direct design approaches can be applied to whole systems (e.g., frames) to fully exploit the benefits arising from load redistribution, spread of plasticity and strain hardening [11–22]. A few structural steel standards already incorporate versions of system-based direct design approaches, including AS/NZS 4100 [23], AISC 360 [24], AISC 370 [25] and prEN 1993-1-14 [26], which adopt the simple Load and Resistance Factor Design (LRFD) design format given in Eqs. (1) and (2) for the US/Australian and Eurocode design frameworks, respectively. In these equations, R_n and R_k are the nominal and characteristic system resistances, respectively, Q_{ni} and Q_{ki} denote the nominal and characteristic structural loads, ϕ and γ_M are the system resistance and partial safety factors for resistance that account for the uncertainties in the system strength, and γ_i are the load factors corresponding to the load combination rules specified in the load standards.

$$\phi R_n \geq \sum \gamma_i Q_{n,i} \quad (1)$$

* Corresponding author at: Itsaso Arrayago, Jordi Girona 1-3, Building C1, 08034 Barcelona, Spain.

E-mail address: itsaso.arrayago@upc.edu (I. Arrayago).

$$R_k/\gamma_M \geq \sum \gamma_i Q_{k,i} \quad (2)$$

The specifications [23–26] require that a level of reliability equivalent to that obtained from member-based design be achieved when using system-based approaches, but they do not provide system resistance ϕ or partial safety factors γ_M for use in system-based direct design. Thus, recent research efforts have focused on the calibration of system factors to guarantee that the target reliability levels required by the different specifications are met, based on robust reliability frameworks built for different types of structures and materials [14–22]. Nevertheless, the calibration of system factors is generally carried out to one of the existing specifications or design frameworks (e.g., the Australian, US or Eurocode frameworks), using the load combinations and the load statistics specific to that framework. Thus, the extension of design recommendations to other frameworks with different load models and particular load combinations is not direct. It requires the use of advanced techniques and that all the basic input information (i.e., statistical characterization of the system strength and loads) is available.

With the aim of assisting researchers and specification committees use information in the literature to develop new recommendations for different design frameworks, the derivation of simple expressions that estimate the level of reliability associated with a certain resistance or partial safety factor is useful, including expressions that allow performing simple cross-checks on more accurate reliability analyses. Furthermore, having expressions that directly relate the reliability calibration results for different design frameworks through a few basic variables for a rapid comparison, or that estimate suitable resistance (or safety) factors for a target reliability index based on the partial safety (or resistance) factors proposed in a different framework for a different target reliability, is also useful for researchers and specification committees.

This paper presents a set of simple expressions to carry out reliability calculations without requiring the use of more advanced (but iterative) techniques. Although the proposed simplified equations are not intended to replace these advanced reliability analysis procedures for code calibration, they serve different purposes, (i) they provide a simple cross-check on the more accurate reliability calculations, (ii) they allow a direct comparison between codes, notably Eurocodes and US codes, and (iii) they provide a means of calculating partial factors for members and connections whose nominal strength is based on tests, as prescribed in some international specifications such as prEN 1990 [27] and AISI S100 [28]. The derivation of the expressions and their application to different load cases is presented in Section 2, while Section 3 provides an overview of the system reliability studies on steel frames available in the literature, from which a database for the evaluation of the developed expressions has been assembled. Section 4 assesses the accuracy of the derived equations to predict reliability indices and resistance or partial safety factors, while in Section 5 expressions for the direct comparison of reliability levels in the US and Eurocode design frameworks are developed and assessed.

2. Derivation of simplified expressions for β , ϕ and γ_M

2.1. General

The safety or failure of a structure has been traditionally quantified through its probability of failure P_f (i.e., the probability of load effects S exceeding the structural resistance R or reaching a certain limit state), which can be determined as $P_f = P(R \leq S)$. Introducing the limit state function $g = R - S$, then $P_f = P(g \leq 0)$, since $g \leq 0$ defines the failure domain. Often, the reliability index β is used as an indirect measurement of P_f , which is calculated from $\beta = \Phi^{-1}(1 - P_f)$, where Φ^{-1} is the inverse standard normal distribution function. One option to compute the probability of failure is to sample the variables of the limit state function randomly and to obtain the probability using direct Monte Carlo simu-

lation (MCS) techniques. However, this method requires a large number of simulations to accurately capture the lower tail of the density probability function and thus alternative methodologies have been developed over the last decades [29], including First Order Second Moment (FOSM) and First Order Reliability Methods (FORM).

Broadly speaking, code-based reliability traditionally adopts First Order Second Moment (FOSM) reliability procedures, which linearize the limit state function and use only the first two moments (mean and standard deviation) to represent random variables, ignoring higher moments such as the skew and flatness [30]. Although FOSM procedures are simple and useful, refinements have been developed from the original method to overcome some of its drawbacks. These include the extension of the method to nonlinear limit state functions and modifications that allow approximating the actual probability distributions of the random variables using equivalent normal distributions – since the probabilistic distributions of random variables are often known. These new methods are usually referred to as First Order Reliability Methods (FORM) and adopt iterative solution schemes [29,31] which convert non-normal random variables to equivalent normal distributions. Since FORM techniques were shown to provide reliability indices only marginally lower than those obtained using direct Monte Carlo simulations (with deviations of about 2–5%) [14], they were deemed sufficiently accurate to carry out reliability studies for code calibration, and have been systematically used in the calibration of resistance factors, including the system factors derived in [14–22] for the direct design of steel and stainless steel frames.

2.2. Estimation of β , ϕ and γ_M using FOSM methods

The FOSM gives an exact solution of the reliability index when both the resistance R and the load effect S are normal (or lognormal) random variables. For a steel structural member, the resistance R can often be modelled as a lognormal. If the load effect S is also a lognormal, the limit state function can be written as $g = \ln(R/S)$. Note that $\ln(R/S)$ is a normal random variable. In this case, the reliability index can be computed using the mean and standard deviation of $\ln(R/S)$, as per $\beta = \overline{\ln(R/S)} / \sigma_{\ln(R/S)}$. β represents the distance from $\overline{\ln(R/S)}$ to the origin in standard deviation units and the area under $\ln(R/S) \leq 0$ is the probability of failure P_f (see Fig. 1); in other cases (non-normal or non-lognormal distributions), β provides a relative measure of the structural safety. Note that throughout this paper \bar{X} , σ_X and V_X represent the mean value, the standard deviation and the coefficient of variation (COV), respectively, of the random variable X . Using small variance approximations, then $\overline{\ln(R/S)} = \ln(\bar{R}/\bar{S})$ and $\sigma_{\ln(R/S)} = \sqrt{V_R^2 + V_S^2}$, and the reliability index β is approximately given by the relationship shown in Eq. (3), which was the basis for the development of the probability-based LRFD criteria for steel structures in [32].

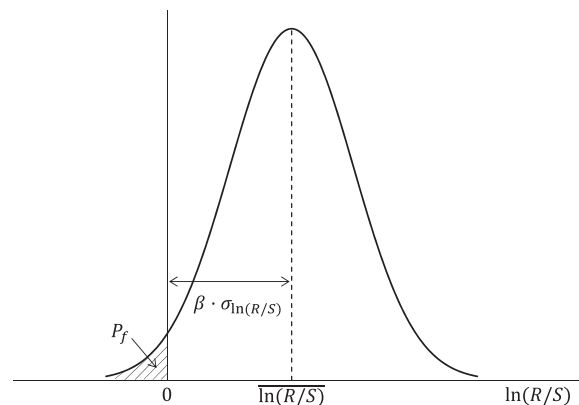


Fig. 1. Graphic definition of the reliability index β and the probability of failure P_f .

$$\beta = \frac{\ln(\bar{R}/\bar{S})}{\sqrt{V_R^2 + V_S^2}} \quad (3)$$

While the values for \bar{R} and V_R can be obtained from theoretical resistance models or extensive numerical simulations that account for all relevant uncertainties, \bar{S} and V_S depend on the type of loading investigated, the load combination and the statistical models (mean and coefficient of variation) assumed for the different loads in the design framework considered. Thus, to develop simplified expressions for reliability calibrations it is necessary to estimate \bar{R}/\bar{S} and $\sqrt{V_R^2 + V_S^2}$ as functions of the basic statistical parameters used in reliability analyses, including stochastic models for loads, suitable load combinations (load factors), stochastic models for resistance, etc. From the general LRFD design equation for the US framework given in Eq. (1), assuming that the structure (or member) is at its limit state and defining a new coefficient C as $C_{US} = \sum \gamma_i Q_{n,i} / \bar{S}$, \bar{R}/\bar{S} can be written as $\bar{R}/\bar{S} = C_{US} / \phi \bar{R}/R_n$, from which a direct relationship can be established between β and ϕ as a function of the coefficient C_{US} , the mean-to-nominal resistance ratio \bar{R}/R_n and the coefficients of variation for the resistance V_R and load effects $V_{S,US}$, as shown in Eq. (4). If this equation is inverted, the expression that provides the required resistance factor ϕ for a certain level of reliability β can be obtained, as given by Eq. (5).

$$\beta = \frac{\ln\left(C_{US} \frac{\bar{R}}{R_n} \frac{1}{\phi}\right)}{\sqrt{V_R^2 + V_{S,US}^2}} \quad (4)$$

$$\phi = C_{US} \frac{\bar{R}}{R_n} \exp\left[-\beta \sqrt{V_R^2 + V_{S,US}^2}\right] \quad (5)$$

For the calculation of C_{US} and $V_{S,US}$, the load combinations prescribed in the ASCE 7 [33] Specification and load statistics specific to the US framework should be considered. Eq. (5) is very similar to the expression prescribed in Chapter K of the AISI S100 [28] Specification, as developed in [32,34] for the calibration of ϕ -factors when the resistance of members is determined through testing, where the mean-to-nominal resistance ratio \bar{R}/R_n is calculated as the product of the mean values of the material factor M_m , the fabrication factor F_m and the professional factor P_m , while V_R depends on the coefficients of variation of the same factors. Despite being originally prescribed for studies based on experimental results, this approach has been systematically used by researchers to carry out reliability calibrations when proposing design expressions for cross-section and member resistance based on hybrid databases comprising experimental and numerical results. In the AISI S100 equation, the C_{US} and $V_{S,US}$ parameters representing uncertainties in the loads are expressed as C_ϕ and V_Q , and are calculated for the gravity load combination with a dead-to-live load ratio of 1/5, the load ratio commonly used for cold-formed steel structures [28,30].

Equivalent relationships to Eqs. (4) and (5) can be derived for the Eurocode framework following the same procedure and considering the Eurocode LRFD equation Eq. (2), as given in Eqs. (6) and (7), in which C_{EN} and $V_{S,EN}$ are to be determined using suitable characteristic load values and load factors according to the different parts of Eurocode 1 [35,36] and the load combinations prescribed in prEN 1990 [27].

$$\beta = \frac{\ln\left(C_{EN} \frac{\bar{R}}{R_k} \gamma_M\right)}{\sqrt{V_R^2 + V_{S,EN}^2}} \quad (6)$$

$$\gamma_M = \frac{1}{C_{EN} \bar{R}/R_k} \exp\left[\beta \sqrt{V_R^2 + V_{S,EN}^2}\right] \quad (7)$$

Eqs. (4) and (6) can be used to predict reliability indices β without performing FORM analyses, while Eqs. (5) and (7) provide a direct estimation of the resistance or partial safety factors required to meet

certain levels of target reliability specific to the design framework under consideration.

2.3. Application to particular load cases

The parameters C_{US} (or C_{EN}) and $V_{S,US}$ (or $V_{S,EN}$) defined in the previous Section depend not only on the design framework considered in the analysis, but also on the load cases investigated. The development of these factors for different load cases is further investigated in this Section, including the gravity load (dead and live loads), wind load and combined gravity plus wind load cases. Since the calibration coefficients C and coefficients of variation of the load effects V_S depend on the load combinations and the statistics assumed for the loads, they typically exhibit different values in the US and Eurocode frameworks; however, the expressions presented herein are generic and can be used in both design frameworks. It should be noted that although the development of the expressions in this Section is based on the nominal load concept, which would correspond to the US design framework, equivalent equations can be obtained if based on the characteristic loads typically used in the Eurocode framework. For simplicity, equations have not been duplicated and only expressions that use nominal loads are provided.

2.3.1. Gravity loads, $G + Q$

When the reliability of structures under gravity loads is evaluated, different dead-to-live load ratios $\zeta = G_n/Q_n$ are customarily considered, where G_n and Q_n are the nominal dead and live load, respectively, since the calibrated reliability indices depend on the load ratio ζ [14–22]. The general form of the load combination under gravity loads is $\sum \gamma_i Q_{n,i} = \gamma_G G_n + \gamma_Q Q_n$, where γ_G and γ_Q are the load factors for the dead and live load, respectively. The mean value of the total load \bar{S} can be re-written as $\bar{S} = \bar{G} + \bar{Q}$, where \bar{G} and \bar{Q} are the mean values of the dead and live loads, from which the C_{GQj} coefficient under gravity loads shown in Eq. (8) can be obtained. Note that the sub-index j refers to the US or the Eurocode design frameworks, and that the C_{GQj} coefficient depends on the load factors (γ_i), the dead-to-live load ratio ζ and the load statistics (\bar{G}/G_n and \bar{Q}/Q_n).

$$C_{GQj} = \frac{\zeta \gamma_G + \gamma_Q}{\zeta \frac{\bar{G}}{G_n} + \frac{\bar{Q}}{Q_n}} \quad (8)$$

Similarly, the coefficient of variation of the total gravity loads $V_{S,GQj}$ can be determined utilising that $V_S = \sigma_S/\bar{S}$ and $\sigma_S^2 = \sigma_G^2 + \sigma_Q^2$ (assuming G and Q are uncorrelated), as per in Eq. (9), which also depends on the ζ -ratio and the stochastic models adopted for the loads (\bar{G}/G_n , \bar{Q}/Q_n , V_G and V_Q). Information about the load models for the different load types and different design frameworks can be found in the literature [21,22,37–39] and is summarized in Table 1, which reports the mean value, coefficient of variation and distribution type for the most common loads for the US and Eurocode frameworks.

$$V_{S,GQj} = \frac{\left[\zeta^2 \left(\frac{\bar{G}}{G_n}\right)^2 V_G^2 + \left(\frac{\bar{Q}}{Q_n}\right)^2 V_Q^2 \right]^{1/2}}{\zeta \left(\frac{\bar{G}}{G_n}\right) + \left(\frac{\bar{Q}}{Q_n}\right)} \quad (9)$$

In particular, the values of the calibration coefficient C_ϕ and the coefficient of variation of the load effects V_Q prescribed in the AISI S100 [28] Specification are 1.52 and 0.21, respectively, which correspond to the C_{GQ} and $V_{S,GQ}$ parameters given in Eqs. (8) and (9) when $\gamma_G = 1.20$, $\gamma_Q = 1.60$, $\zeta = 0.2$ and the load statistics reported in Table 1 for the US framework are adopted. The equations derived herein, together with those presented in the next sub-sections, give a better estimation of the C_ϕ and V_Q coefficients for load cases different to $\zeta = 0.2$ because they are broader and adopt general cases of ζ instead of assuming fixed values of

Table 1
Load statistics adopted in reliability analyses.

Design framework	Load type	Mean	COV	Distr. type	Reference
US framework	Dead load, G	$1.05G_n$	0.10	Normal	[39]
	Live load, Q	$1.00Q_n$	0.25	Extreme Type I	[39]
	Arbitrary-point-in-time live load, Q_{apt}	$0.25Q_n$	0.60	Gamma	[14]
	Wind load, W	$0.47W_n^\dagger$	0.35	Extreme Type I	[37]
Eurocode framework	Dead load, G	$1.00G_k$	0.10	Normal	[38]
	Live load, Q	$0.80Q_k$	0.25	Extreme Type I	[21]
	Arbitrary-point-in-time live load, Q_{apt}	$0.20Q_n$	0.60	Gamma	[21]
	Wind load, W	$0.70W_k^*$	0.35	Extreme Type I	[38]

[†] Wind loads in ASCE 7 [33] are based on a return period of $T = 700$ years.

* Wind loads in EN 1991-1-4 [36] are based on a return period of $T = 50$ years.

the coefficients, and are equivalent to the equations given in [30]. According to AISI S100 [28], these general equations should be adopted in special situations when loading cases deviate from the gravity load case assumed.

2.3.2. Wind loads, W

If only wind loads are considered in the analysis, the derivation of the $C_{W,j}$ and $V_{S,W,j}$ coefficients is simple since the limit state function and the design equation only depend on one load type. These coefficients are as shown in Eqs. (10) and (11), in which γ_W , \bar{W}/W_n and V_W are the load factor, the mean-to-nominal ratio and the coefficient of variation of the wind load, as defined in load standards [27,33] and reported in Table 1.

$$C_{W,j} = \frac{\gamma_W}{\bar{W}/W_n} \quad (10)$$

$$V_{S,W,j} = V_W \quad (11)$$

2.3.3. Combined gravity plus wind loads, $G + Q_{apt} + W$

When load combinations including imposed live loads and wind loads are investigated, it is necessary to consider the arbitrary-point-in-time live load Q_{apt} instead of the 50-year maximum live load Q . As for gravity loads in Section 2.3.1, to derive the $C_{GQ_{apt}W,j}$ and $V_{S,GQ_{apt}W,j}$ coefficients for combined gravity plus wind loads, different ratios of dead-to-live loads $\zeta = G_n/Q_n$ and wind-to-live loads $\delta = W_n/Q_n$ need to be considered. Following similar steps to those adopted for the gravity load case, the expressions shown in Eqs. (12) and (13) can be derived to estimate the $C_{GQ_{apt}W,j}$ and $V_{S,GQ_{apt}W,j}$ coefficients, where \bar{Q}_{apt} is the mean value of the of the arbitrary-point-in-time live load.

$$C_{GQ_{apt}W,j} = \frac{\zeta\gamma_G + \gamma_Q + \delta\gamma_W}{\zeta\frac{\bar{G}}{G_n} + \frac{\bar{Q}_{apt}}{Q_n} + \delta\frac{\bar{W}}{W_n}} \quad (12)$$

$$V_{S,GQ_{apt}W,j} = \frac{\left[\zeta^2 \left(\frac{\bar{G}}{G_n} \right)^2 V_G^2 + \left(\frac{\bar{Q}_{apt}}{Q_n} \right)^2 V_{Q_{apt}}^2 + \delta^2 \left(\frac{\bar{W}}{W_n} \right)^2 V_W^2 \right]^{1/2}}{\zeta\frac{\bar{G}}{G_n} + \frac{\bar{Q}_{apt}}{Q_n} + \delta\frac{\bar{W}}{W_n}} \quad (13)$$

2.4. Eurocode semi-probabilistic approach

According to Annex C of prEN 1990 [27], the calibration of partial safety factors for resistance can be carried out based on First Order Reliability Methods or on full probabilistic methods. Since it is often not possible to use the full probabilistic method due to the lack of statistical data, the approach generally adopted for the calibration of partial safety factors or the safety assessment of new design expressions in the

Eurocode framework is the semi-probabilistic procedure given in Annex D of prEN 1990 [27]. For the case of a large number of tests ($n > 100$), the partial safety factor for resistance γ_M can be determined from Eq. (14),

$$\gamma_M = \frac{R_k}{b \cdot \bar{R} \cdot \exp[-\alpha_R \beta \theta - 0.5\theta^2]} \quad (14)$$

where b is the mean of the model factor, α_R is the First Order Reliability Method sensitivity factor for the resistance and θ is a parameter that accounts for the variability in the design model and the basic random variables, given in Eq. (15). Note that in prEN 1990 [27] the θ parameter is referred to as Q , but a different notation is adopted in this paper to avoid confusion with the live load Q . The θ -parameter depends on the total coefficient of variation of the resistance V_r and can be estimated from Eq. (16) through the coefficients of variation of the model factor and the basic input parameters, V_δ and $V_{r\tau}$.

$$\theta = \sqrt{\ln(1 + V_r^2)} \quad (15)$$

$$(1 + V_r^2) = (1 + V_\delta^2) \cdot (1 + V_{r\tau}^2) \quad (16)$$

One of the main features of the Eurocode semi-probabilistic procedure is that it clearly separates the load and resistance sides of the probabilistic problem, making the calibration of the partial safety factors for the resistance independent of the variability of the loads. This is achieved by allocating constant fractions of the target reliability index to the resistance and the load sides through the factors α_R and α_E , which correspond to the First Order Reliability Method sensitivity factors. For the cases in which the standard deviation of the load effects and the resistance do not deviate much ($0.16 < \sigma_E/\sigma_R < 7.6$), the values adopted for the sensitivity factors are $\alpha_R = 0.8$ and $\alpha_E = -0.7$, where $\sqrt{\alpha_R^2 + \alpha_E^2} \approx 1.0$. The adoption of fixed values for these sensitivity factors results in a non-iterative procedure, in which the value of the partial safety factor for resistance γ_M can be obtained directly. The approach also assumes that the resistance and model factor follow lognormal distributions [40]. Eq. (14) can be further simplified if the total coefficient of variation is small ($V_r < 0.20$), since the $0.5\theta^2$ term in the same equation can be neglected [27], and if model uncertainties are accounted for implicitly in the resistance function, in which case $b = 1$ and $V_r = V_{r\tau}$. Using these simplifications, the equation to estimate the partial safety factor for the resistance γ_M using the Eurocode semi-probabilistic approach is shown in Eq. (17), from which the expression that provides the level of reliability β achieved for a given value of partial safety factor γ_M can be derived, as given by Eq. (18).

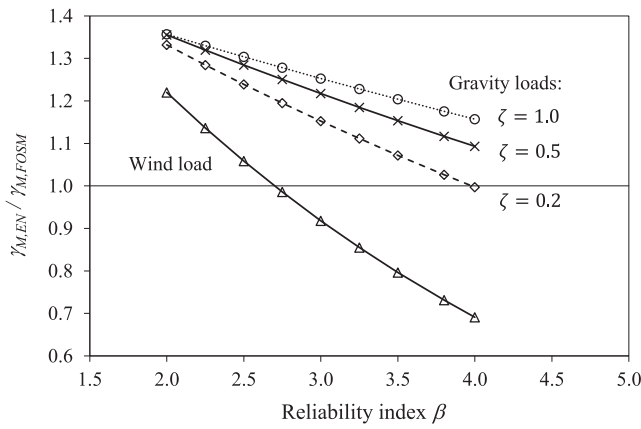


Fig. 2. Comparison between the semi-probabilistic and FOSM approaches for the estimation of partial safety factors γ_M for different load cases.

$$\gamma_M = \frac{1}{\bar{R}/R_k} \exp[\alpha_R \beta \theta] \quad (17)$$

$$\beta = \frac{1}{\alpha_R \theta} \ln \left(\frac{\bar{R}}{R_k} \gamma_M \right) \quad (18)$$

These equations are equivalent to Eqs. (6) and (7) derived in Section 2.2, but are based on the Eurocode semi-probabilistic approach instead of using FOSM methods. The main difference between the Eurocode and other specifications based on FOSM procedures (such as AISI S100-16 [28] or AS/NZS 4600 [41]) is that, unlike the expression given in Eq. (7), the partial safety factor γ_M estimated using Eq. (17) does not depend on the load combination considered (γ_G , γ_Q and γ_W factors) or the variability of the loads (mean values and COVs of the load distributions).

The comparison of the γ_M partial safety factors calculated using the Eurocode semi-probabilistic approach (Eq. (17)) and FOSM procedures (Eq. (7)), in terms of $\gamma_{M,EN}/\gamma_{M,FOSM}$ ratios, is presented in Fig. 2. Different load cases, including gravity load scenarios with dead-to-live load ratios ζ of 1.0, 0.5 and 0.2 and wind load cases are investigated, showing that the $\gamma_{M,EN}/\gamma_{M,FOSM}$ ratios reduce for increasing values of the reliability index β for all load cases. In this comparison, a coefficient of variation equal to $V_r = V_R = 0.10$ was assumed for the variability of the resistance, which is a typical value for steel structures [13]. The results in Fig. 2 also show that for gravity load cases $\gamma_{M,EN}$ factors are higher than their $\gamma_{M,FOSM}$ counterparts, although the difference is reduced as the dead-to-live load ratio ζ decreases, and equivalent results are obtained for $\zeta = 0.2$ at the 3.8–4.0 reliability index range, while the deviations between the two methods are more pronounced for wind loads, with the $\gamma_{M,EN}$ -factors being lower than $\gamma_{M,FOSM}$ in the intermediate-to-high β -range.

3. System reliability calibration for steel frames

To contribute to the widespread and normalization of system-based direct design approaches, recent research efforts have focussed on the statistical characterization of the resistance of steel structural systems and, through comprehensive reliability calibrations, on the recommendation of system resistance and partial safety factors that meet the level of reliability required by the different international specifications [14–22]. Based on the limit state functions $g = R - G - Q$ and $g = R - W$ for the gravity and wind load cases, respectively, these studies have adopted First Order Reliability Method techniques to compute the reliability indices β associated to different values of resistance ϕ and partial safety γ_M factors for steel structures. This process relies on an accurate characterization of the system strength statistics, the implementation of efficient FORM algorithms and the adoption of suitable target reliability indices for the design frameworks under investigation. This Section

provides a summary of the different features in system reliability calibration.

3.1. Statistical characterization of system strength

To provide comprehensive design recommendations for the direct design of steel systems, it is fundamental to choose representative structural types that cover the full range of system behaviour, including different types of failure, regular/irregular system configurations, different connection types, etc. The statistics of the strength of these systems under different loading conditions can be determined from comprehensive experimental programmes or extensive advanced finite element (FE) simulations. Although both tests and numerical data can be considered when determining the strength of members, the use of experimental programmes to characterize system strength distributions is unfeasible and advanced FE simulations need to be used.

While the presence of random properties is inherent in the specimens used for testing, it is necessary to sample randomly the variables that affect the strength of the systems (i.e., material and geometric properties, initial geometric imperfections, residual stresses, and the behaviour of connections) when numerical simulations are used. The basic requirements for this are to build advanced finite element models capable of accounting for all geometric and material nonlinearities and to have knowledge about the variability of each of the random variables involved. In general, the determination of the probabilistic models for system strength entails a large number of simulations, although it can be reduced to typically 250–350 simulations through the adoption of Latin Hypercube Sampling (LHS) techniques [14–22]. Besides, it is necessary to include an additional random variable that accounts for the uncertainties associated with the assumptions and approximations made when building the FE models, i.e., the model factor. Based on the methodology described above, the distribution type, mean value and coefficient of variation of the system strength can be determined, which in combination with the nominal system strength determined following the requirements prescribed in the relevant specifications, constitute the fundamental input information in carrying out reliability calibrations (i.e., \bar{R}/R_n and V_R).

3.2. Reliability calibrations using FORM

Using the system strength statistics determined from experimental results or numerical simulations and the stochastic models for the loads relevant to the design framework and loading types under consideration available in the literature (and summarized in Table 1), different sets of reliability indices β can be computed for a range of resistance ϕ and partial safety γ_M factors using scripts coding the FORM methodology (e.g., see [14–22]) and considering the load factors prescribed in the ASCE 7 [33] and prEN 1990 [27] standards for the US and Eurocode design frameworks, respectively. These sets of $\beta - \phi$ and $\beta - \gamma_M$ relationships established using the FORM method have been utilised in previous studies [14–22] to derive suitable system factors that meet the reliability levels required by international specifications (i.e., prescribed target reliability indices) and to develop design recommendations for the direct design of steel systems. The results have also been used in the subsequent Sections of this paper as benchmark values for reliability indices β and resistance ϕ or partial safety γ_M factors in the assessment of the derived simplified equations.

3.3. Target reliabilities in the US and Eurocode frameworks

Target reliability indices β_0 are prescribed in the international standards for a variety of structure classes and limit states, including ultimate limit states and serviceability limit states. These indices generally correspond to 50-year reference periods (service life for common steel buildings), and thus reliability calibrations are usually

performed using input information that corresponds to this period, including the statistical characterization of the maximum expected loads acting on the structure. prEN 1990 [27] prescribes a minimum value of the target reliability index of $\beta_0 = 3.8$ for ultimate limit states in Consequence Class 2 (CC2) structures and a reference period of 50 years, although for structures dominated by wind loads a lower reliability of $\beta_0 = 2.8$ is usually accepted [22,42]. Likewise, the target reliability indices adopted in the ASCE 7 [33] Specification for Risk Category I and II structures are $\beta_0 = 2.5$ and $\beta_0 = 3.0$, respectively, although for wind load cases the reliability index has been found to be consistently lower than for gravity load cases [15,17,43–45]. However, if the recently updated wind load model proposed in [37] and reported in Table 1 is used when performing reliability calibrations in the US framework, higher reliability indices, equivalent to those obtained for gravity loads, are obtained because these new statistics consider part of the hidden reliability present in the wind load model related to the bias in the wind directionality factor [37]. For code calibration, an intermediate value between $\beta_0 = 2.5$ and $\beta_0 = 3.0$ (e.g., $\beta_0 = 2.8$) can be adopted.

3.4. Previous studies on system factor calibration

To date, reliability studies on structural systems have been conducted on hot-rolled steel [14–17], cold-formed steel [18–20] and cold-formed stainless steel [21–22] structures under different loading conditions with the aim of calibrating system resistance or partial safety factors, featuring planar frames, spatial frames and portal frames. These studies adopted the methodology described in the previous Sections, and conducted extensive numerical simulations to characterize the stochastic models for the system strength of a variety of structures. The system resistance factors calibrated for steel structures and the target reliability indices discussed in the previous Section were around $\phi_s = 0.80–0.85$ for gravity loads [14–16,18,19] and combined gravity plus wind load cases [14,15,17,18,20] in the US design framework. Due to higher overstrength ratios (i.e., mean-to-nominal yield stress ratios), higher system resistance factors of around $\phi_s = 0.90–0.95$ were proposed for stainless steel portal frames in [21–22], while for the Eurocode framework system partial safety factors of $\gamma_{M_s} = 1.15–1.20$ were found to be appropriate.

The \bar{R}/R_n and V_R results derived in these previous studies, which are available in the literature, have been collected herein and used to assess the accuracy of the simplified expressions presented in Section 2. Table 2 presents the database collected from [14–22], summarizing the frame type and load case analysed in each subset of data, in addition to the total number of frames investigated and the ranges of mean-to-nominal resistance ratios \bar{R}/R_n and coefficients of variation for the resistance distributions V_R . Within each subset of data, results

corresponding to different types of frames (frames with and without rigid diaphragm, hinged or rigid beam-to-column connections, compact and slender sections) are included. It should be noticed that since the research works from which the \bar{R}/R_n and V_R values were obtained focused on the development of system-based direct design recommendations for steel and stainless steel structures [14–22], the resistance and partial safety factors considered in this study correspond to system factors. However, the derived simplified equations apply to both member-based and system-based reliability calibrations, as do the conclusions drawn in Section 6. Similarly, the same equations and conclusions would also be valid if strength distributions were determined using experimental data instead of advanced FE simulations.

4. Estimation of reliability indices β or resistance ϕ and partial safety γ_M factors

This Section evaluates the accuracy of the different simplified equations derived in Section 2 for the estimation of reliability indices β and resistance ϕ or partial safety γ_M factors against the corresponding values calculated using the FORM reliability procedures described in Section 3. While for the US framework the equations derived using the FOSM method are considered, for the Eurocode framework expressions based on both the FOSM and semi-probabilistic approaches are analysed.

4.1. Assessment of the estimated reliability indices β

The reliability indices calibrated using the FORM method for the steel and stainless steel frames collected in the database are compared with the β -values predicted from Eqs. (4) and (6) for the US and Eurocode design frameworks, respectively, and with the values calculated using the Eurocode semi-probabilistic approach (Eq. (18)). The resistance and partial safety factors considered were defined based on values of practical interest, ranging approximately between $\phi = 0.80–1.00$ and $\gamma_M = 1.00–1.30$. Since the database included frames under gravity and combined gravity plus wind load cases, suitable C and V_S coefficients were considered and calculated according to the expressions presented in Section 2.3 for the approaches based on the FOSM method. For the gravity load case, different dead-to-live load ratios ζ were assumed, while the analysis for the combined gravity plus wind load case also included a range of gravity-to-wind load ratios. It should be noticed that the combined load cases investigated in this paper and reported in [14,15,17,20,22] correspond to push-over type analyses, in which the uncertainties of the gravity loads are implicitly accounted for in the stochastic models reported for the resistance (i.e., \bar{R}/R_n and V_R values). Thus, only wind load statistics need to be considered in the

Table 2
Summary of assembled database on steel and stainless steel frames for reliability calibrations.

Frame type	Load case	No. of frames	Range of \bar{R}/R_n	Range of V_R	Reference
HR planar sway frames	G+Q	24	1.02–1.11	0.09–0.11	[14,15]
HR planar sway frames	G+Q _{apt} +W	27	1.06–1.36	0.11–0.15	[14,15]
HR planar braced frames	G+Q	57	1.00–1.10	0.07–0.12	[14,15]
HR 3D sway frames	G+Q	40	0.97–1.12	0.09–0.14	[16]
HR 3D sway frames	G+Q _{apt} +W	24	1.07–1.38	0.10–0.16	[17]
HR 3D braced frames	G+Q	22	0.99–1.09	0.09–0.12	[16]
CF HSS 3D sway frames	G+Q	30	0.99–1.08	0.09–0.12	[18]
CF HSS 3D braced frames	G+Q	34	0.97–1.08	0.09–0.11	[18]
CF HSS 3D sway frames	G+Q _{apt} +W	24	1.11–1.39	0.10–0.18	[18]
CF portal frames	G+Q	8	0.99–1.08	0.07–0.10	[19]
CF portal frames	G+W	12	0.93–1.28	0.05–0.18	[20]
CF HSS stainless steel portal frames	G+Q	6	1.18–1.37	0.09–0.12	[21]
CF HSS stainless steel portal frames	G+W	18	1.12–1.32	0.09–0.11	[22]

HR: hot-rolled

CF: cold-formed

HSS: hollow structural section

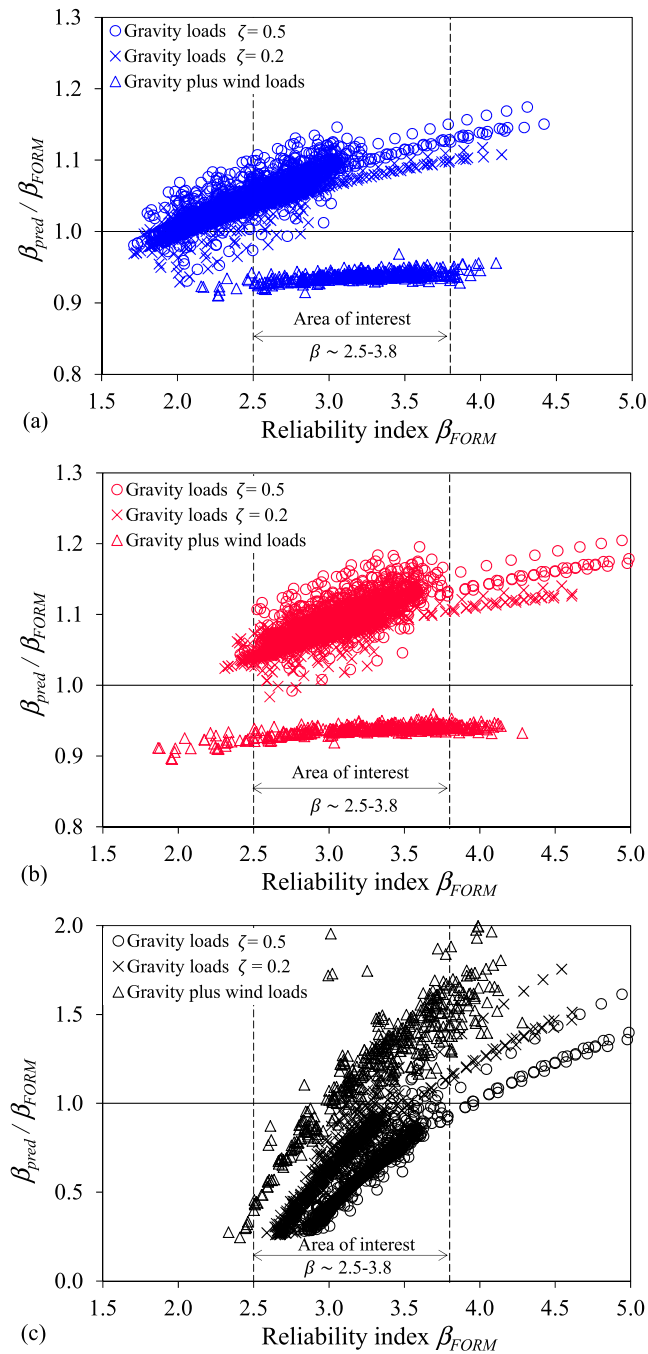


Fig. 3. Assessment of the simplified expressions to calculate reliability indices β for different load cases and design frameworks: a) US-FOSM, b) EN-FOSM and c) EN-semi-probabilistic.

calibration of β , γ_M or ϕ , and the expressions presented in Section 2.3.2 for wind loads only have been considered instead of those reported in Section 2.3.3, which would be suitable for load cases in which all loads are simultaneously applied.

The assessment of the accuracy of Eqs. (4), (6) and (18) to estimate reliability indices is presented in Fig. 3, in which the β -values determined from FORM analyses (β_{FORM}) are compared with the β -values predicted using the proposed method β_{pred} (i.e., computed using Eqs. (4), (6) and (18)) and plotted against the reference β_{FORM} -values. The results are presented separately for the different load cases and the different simplified procedures investigated in this paper, including the FOSM-based equations for the US and Eurocode frameworks, and the semi-

probabilistic approach prescribed in Annex D of prEN 1990 [27] for the Eurocode framework. It should be noticed that although dead-to-live ratios ζ ranging between 0.2–1.0 have been analysed for the gravity load case, for simplicity, Fig. 3 only shows results corresponding to $\zeta = 0.5$ and $\zeta = 0.2$, which are the common load ratios for steel structures [18,21] and cold-formed steel structures [28,30], respectively. According to the results shown in Fig. 3(a) and (b), the estimation of the reliability index for the gravity plus wind load case is slightly more accurate than for the gravity load case for the FOSM-based approaches, and the $\beta_{pred}/\beta_{FORM}$ values for gravity plus wind loads show a considerably lower scatter and more consistent results over the full range of β_{FORM} -values. Conversely, the results for gravity loads are more scattered and the accuracy in the prediction of the reliability index decreases with increasing values of the reliability index.

In general, the results for the US and Eurocode frameworks exhibit the same trend for each of the load cases considered when the reliability index is estimated using FOSM-based approaches. Hence, the accuracy observed for the two design frameworks is similar. However, since the absolute values of the reliability indices calibrated for gravity loads in the Eurocode framework are higher than for the US framework, the accuracy of the β_{pred} -values is somehow lower for the Eurocode. Regarding the Eurocode semi-probabilistic approach, Fig. 3(c) shows a considerably higher scatter in the results, with a significantly lower accuracy in the prediction of the reliability index due to the fact that Eq. (18) does not explicitly consider load factors or the variability of the load effects in the formulation, and that it adopts fixed values of the sensitivity factors α_R and α_E . The values of the sensitivity factors α change significantly depending on the load cases, load ratios and target reliability indices considered in the calculation. In the FORM reliability calibrations carried out in this paper for the Eurocode framework, for example, the sensitivity factors for the resistance α_R ranged between 0.26 and 0.57, while values ranging between -0.80 and -0.96 were observed for the sensitivity factors of the loads α_E . These values are significantly different, for many of the load ratios considered, from the α -values prescribed in prEN 1990 [27] ($\alpha_R = 0.8$ and $\alpha_E = -0.7$), even if the $0.16 < \sigma_E/\sigma_R < 7.6$ condition stated in Section 2.4 is fulfilled. However, when the results corresponding to the β -values relevant to the Eurocode are analysed (i.e., $\beta = 3.8$), it can be appreciated that the $\beta_{pred}/\beta_{FORM}$ ratios for gravity loads are very close to unity.

The analysis of the results also indicates that for the ranges of \bar{R}/R_n and V_R factors investigated in this paper (see Table 2), the accuracy of the estimated reliability indices is uniform for all the load cases and simplification methods, while the influence of the dead-to-live load ratio is found to be more important, with the $\beta_{pred}/\beta_{FORM}$ ratios closest to unity being aligned with the lowest values of G_n/Q_n for the FOSM-based simplified expressions (as demonstrated by the results shown in Fig. 3 (a) and (b)), and with the highest G_n/Q_n ratios for the Eurocode semi-probabilistic approach (see Fig. 3(c)). This is in line with the differences observed between the FORM sensitivity factors and the fixed values of α_R and α_E defined in prEN 1990 [27], since the biggest differences in the α -values correspond to the lowest G_n/Q_n ratios, while the sensitivity factors closest to the prEN 1990 values are observed for the highest G_n/Q_n load ratios. The fact that the accuracy of Eq. (18) improves as the β -values get closer to the target reliability index and for high G_n/Q_n ratios suggests that the α -values prescribed in prEN 1990 [27] were calibrated using these values as reference.

The statistics of the $\beta_{pred}/\beta_{FORM}$ ratios are reported in Table 3 for the different design approaches and load cases analysed, including the mean values, the coefficients of variation and the minimum-to-maximum ratio ranges. Note that the results presented in Table 3 correspond to frames with different dead-to-live and gravity-to-wind load ratios, and that only results lying in the area of interest $\beta_{FORM} = 2.5 - 3.8$ have been considered. From these results, it can be concluded that the reliability indices can be estimated with average differences of about 5% for the US framework and between 5 and 10% for the Eurocode framework using

Table 3

Assessment of the simplified expressions to estimate the reliability index β and the resistance ϕ or partial safety γ_M factors in different design frameworks and load cases (Eqs. (4)–(7), (17), (18)).

Design framework	Load case		$\frac{\beta_{pred}}{\beta_{FORM}}$	$\frac{\phi_{pred}}{\phi_{FORM}}$ or $\frac{\gamma_{M,pred}}{\gamma_{M,FORM}}$
US framework (FOSM)	Gravity loads	Mean	1.05	1.03
		COV	0.034	0.020
		Min-Max	0.91–1.22	0.97–1.12
	Gravity plus wind loads	Mean	0.94	0.93
		COV	0.007	0.006
		Min-Max	0.91–0.97	0.91–0.96
Eurocode framework (FOSM)	Gravity loads	Mean	1.09	0.95
		COV	0.028	0.020
		Min-Max	0.96–1.23	0.87–1.02
	Gravity plus wind loads	Mean	0.94	1.08
		COV	0.009	0.007
		Min-Max	0.90–0.96	1.06–1.12
Eurocode framework (EC-0 [27])	Gravity loads	Mean	0.63	1.13
		COV	0.361	0.056
		Min-Max	0.03–1.76	0.90–1.30
	Gravity plus wind loads	Mean	1.27	0.95
		COV	0.262	0.128
		Min-Max	0.43–2.89	0.74–1.41

the FOSM-based simplified expressions given in Eqs. (4) and (6), with significantly low variations. For the Eurocode semi-probabilistic approach, the accuracy in the predicted β -values is significantly lower, although $\beta_{pred}/\beta_{FORM}$ ratios close to unity are observed in the vicinity of the reliability index relevant to the Eurocode.

4.2. Assessment of the estimated resistance ϕ and partial safety γ_M factors for a given level of reliability

Following a similar approach to that adopted for the assessment of the estimated reliability indices using the simplified expressions reported in Section 2, this Section presents the results corresponding to the estimation of resistance ϕ and partial safety γ_M factors using Eqs. (5), (7) and (17), considering the same load cases and frames investigated in the previous Section. Fig. 4 evaluates the accuracy of Eqs. (5), (7) and (17) by comparing the predicted resistance ϕ_{pred} and partial safety $\gamma_{M,pred}$ factors with the equivalent factors calibrated using FORM techniques, ϕ_{FORM} and $\gamma_{M,FORM}$. Predicted-to-FORM ratios are plotted against the reference β_{FORM} -values, as in Fig. 3, and the results are presented separately for the different load cases and simplification procedures investigated. For simplicity, only gravity load results corresponding to dead-to-live ratios of $\zeta = 0.5$ and $\zeta = 0.2$ and gravity plus wind load cases are shown. Fig. 4 indicates that the accuracy observed for the two FOSM-based simplified expressions (Eqs. (5) and (7)) is very similar, since the results in Fig. 4(a) and (b) are symmetric about the $\phi_{pred}/\phi_{FORM} = 1.0$ (or $\gamma_{M,pred}/\gamma_{M,FORM} = 1.0$) axis. As for the reliability index, the scatter in the prediction of the partial safety factors γ_M is higher for the Eurocode semi-probabilistic method, but the $\gamma_{M,pred}/\gamma_{M,FORM}$ ratios are close to unity as the β -values approach the reference value of 3.8 for gravity load cases. For gravity plus wind load scenarios, the prediction of the γ_M -factors is less accurate.

The results indicate that the FOSM-predicted ϕ -factors for the gravity plus wind load case in the US framework are lower than the corresponding ϕ_{FORM} -values, while the converse is true for gravity loads. This indicates that the prediction of ϕ -factors is conservative for the gravity plus wind load case, and slightly unconservative for gravity loads. It should be emphasized that the results for FOSM and FORM will be same in the particular cases in which R and S are normal, or if R and S are lognormal, for which the FOSM equations are exact. Since for steel structures it is customary to model R as lognormal [13], FOSM and FORM approaches will give similar results if the loads are close to

lognormal distributions, although according to the stochastic models traditionally adopted for gravity and wind loads in the literature, the loads to which structures are subjected will typically be a combination of a normal distribution and an Extreme Type I distribution (see Table 1). These can differ significantly from lognormal distributions, especially for small gravity-to-wind load ratios. Likewise, it is found that the γ_M -factors estimated using Eq. (7) are lower than those calibrated using the FORM method for gravity loads in the Eurocode, while the prediction of γ_M -factors is conservative for gravity plus wind loads. On the contrary, in the case of the Eurocode semi-probabilistic approach the prediction of the partial safety factor is in general unconservative for the gravity plus wind load combinations in the range of reliability levels close to 3.8 and conservative for gravity load cases, although with significant errors. In line with the results reported in the previous Section, the accuracy in the estimation of the ϕ and γ_M -factors was found not to be significantly influenced by the \bar{R}/R_n (or \bar{R}/R_k) and V_R values or the type of material.

Finally, Table 3 reports the mean values, coefficients of variation and minimum-to-maximum ranges of the ϕ_{pred}/ϕ_{FORM} and $\gamma_{M,pred}/\gamma_{M,FORM}$ ratios calculated considering all the frames and load ratios investigated. According to these values, Eqs. (5) and (7) provide accurate estimations of the resistance ϕ and partial safety γ_M factors for the gravity and gravity plus wind load cases, with average differences between 3 and 7% and 5–8% for the US and Eurocode frameworks, respectively, and with low coefficients of variation. On the other hand, the estimation of partial safety factors using the semi-probabilistic approach detailed in Annex D of prEN 1990 [27] provides average differences of 13% and 5% with the FORM-based values for the gravity and gravity plus wind load cases, respectively, in the area of interest: β_{FORM} ranging from 2.5 to 3.8. While the mean accuracy of this approach is reasonable, the range of $\gamma_{M,pred}/\gamma_{M,FORM}$ ratios obtained is 0.74–1.41, as shown in Table 3, too scattered to be used in code calibration, although the accuracy for reliability indices of around 3.8 is notably better according to the results shown in Fig. 4(c). As highlighted in the previous Section, the values of the sensitivity factors α – and therefore the accuracy of the prEN 1990 semi-probabilistic approach – depend on the load ratios and the target reliability indices considered. Providing a fixed value of these sensitivity factors can be useful, but it is necessary to limit the applicability of this expression when used for the calibration of partial safety factors. According to the reliability results analysed in this paper, and considering that an acceptable estimation of γ_M -factors remains in the $\pm 10\%$ range,

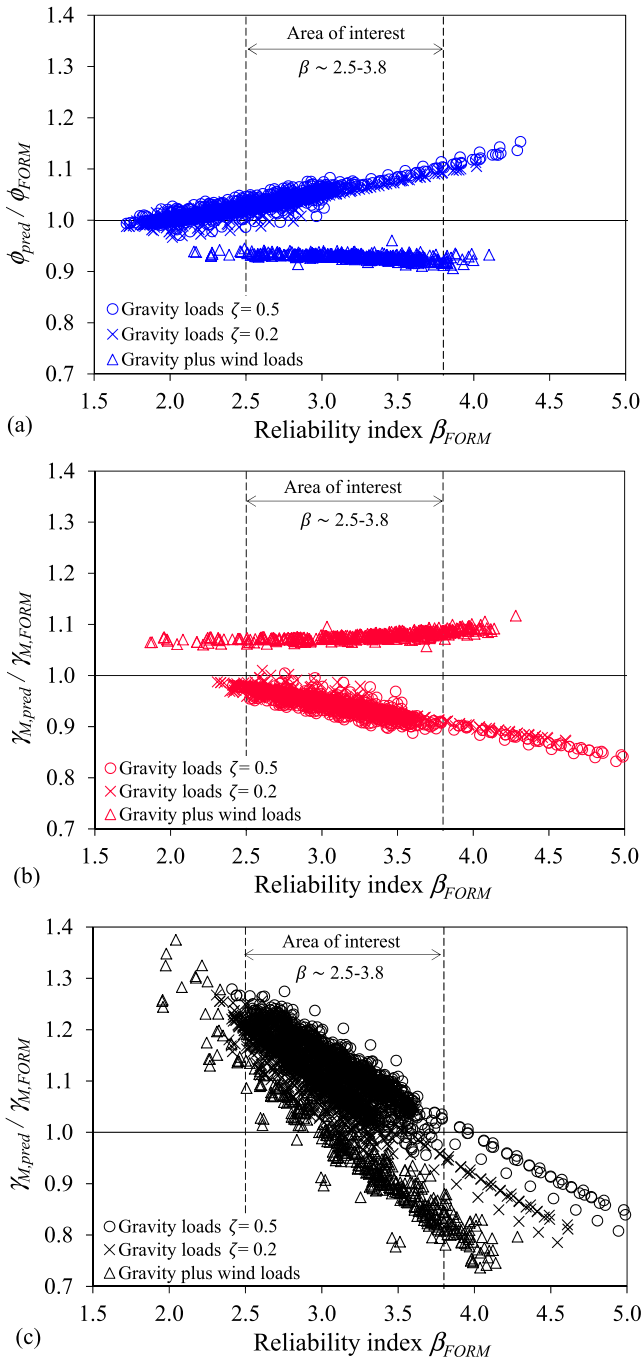


Fig. 4. Assessment of the simplified expressions to calculate resistance ϕ or partial safety γ_M factors for different load cases and design frameworks: a) US-FOSM, b) EN-FOSM and c) EN-semi-probabilistic.

it is recommended to limit the applicability of Eq. (17) to $\beta \in [2.5 - 3.5]$ for wind load cases with G_n/W_n ratios in the range of 0.10–0.33, and to $\beta \in [2.5 - 4.0]$ for gravity load ratios between $\zeta = 0.2$ and $\zeta = 0.5$, ranges that are in the vicinity of the Eurocode target reliability index.

5. Direct comparison of the US and Eurocode frameworks

The simplified expressions to estimate the required resistance ϕ and partial safety γ_M factors corresponding to specific target reliability indices β_0 (Eqs. (5), (7) and (17)) can be used to derive relationships to directly compare the US and Eurocode frameworks. These relationships allow not only to carry out a comparative assessment of the reliability

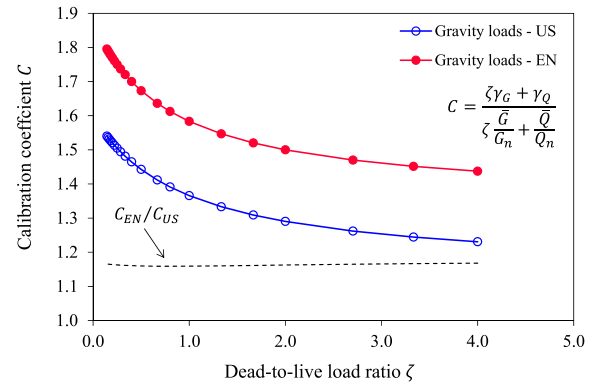


Fig. 5. Comparison of the calibration coefficients C for the US and Eurocode frameworks for gravity load cases.

levels present in the two design frameworks, but can also assist specification committees in developing suitable resistance ϕ factors, corresponding to the particular levels of reliability required by the specification under consideration, using the partial safety factors γ_M prescribed by an independent committee for a different value of target reliability β_0 (or vice versa, to estimate partial safety factors γ_M from existing resistance ϕ factors considering the suitable reliability frameworks and target β_0 values in each case). Expressions for these two scenarios are developed in this Section. Both of the two alternative approaches for the estimation of partial safety factors considered in this paper (equations based on FOSM and semi-probabilistic approaches) are investigated. For each approach, the results are assessed using the resistance and partial safety factors derived from the FORM analyses introduced in Section 3 in order to evaluate how the inaccuracies observed in the previous Section for the estimation of the individual γ_M and ϕ factors compound when Eqs. (5), (7) and (17) are combined.

5.1. Development of direct comparison expressions using FOSM

Combining the expressions to estimate resistance ϕ and partial safety γ_M factors obtained from FOSM approaches (Eqs. (5) and (7), respectively), the relationship given by Eq. (19) can be derived, which directly relates the US and Eurocode design frameworks. The equation can be used for different loading cases through the definition of suitable C_{US} , C_{EN} , $V_{S,US}$ and $V_{S,EN}$ coefficients, as indicated in Section 2.3. Nevertheless, Eq. (19) can be further simplified for different load combinations in some particular cases, as presented in this Section, since Eqs. (5) and (7) have been derived using similar procedures and assumptions.

$$\phi \cdot \gamma_M = \frac{C_{US}(\bar{R}/R_n)_{US} \exp\left(\beta_{EN} \sqrt{V_R^2 + V_{S,EN}^2}\right)}{C_{EN}(\bar{R}/R_k)_{EN} \exp\left(\beta_{US} \sqrt{V_R^2 + V_{S,US}^2}\right)} \quad (19)$$

5.1.1. Calibration coefficients, C

According to the definition of the calibration coefficients C given in Section 2.3.1 for the gravity load case, the C_{US} and C_{EN} terms depend on the G_n/Q_n ratio, the load combination and the stochastic models that characterize the effects of dead and live loads. Since these inputs are different in the US and Eurocode frameworks, the C_{US} and C_{EN} coefficients will typically be different, as illustrated in Fig. 5, where the values of the C_{US} and C_{EN} coefficients are plotted as functions of the dead-to-live load ratio ζ . Nevertheless, Fig. 5 also demonstrates that the C_{EN}/C_{US} ratio shows a nearly constant value of 1.16 in the range of common G_n/Q_n ratios, so it is possible to simplify the C_{US}/C_{EN} term in Eq. (19) for the gravity load case by the constant value of $1/1.16 = 0.86$. On the contrary, the C_{US} and C_{EN} ratios are very similar when the gravity plus wind load case is considered, since from the equations presented in Section 2.3.2, the calibration coefficients for wind loads result in $C_{US} =$

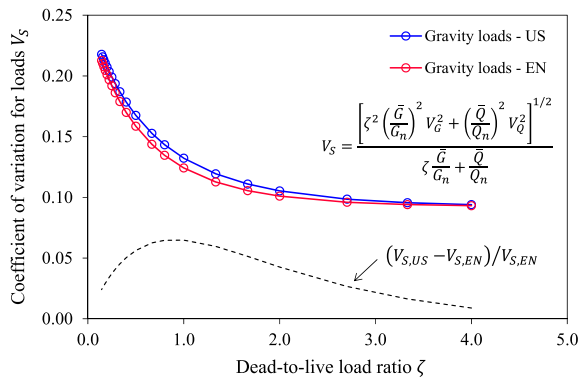


Fig. 6. Comparison of the coefficients of variation for the load effect V_S for the US and Eurocode frameworks for gravity load cases.

2.13 and $C_{EN} = 2.14$. Hence, the C_{US}/C_{EN} term in Eq. (19) can be ignored for this load case.

5.1.2. Coefficient of variation for the load effect, V_S

From the expressions presented in Section 2.3.1 for the calculation of the coefficient of variation for the load effect V_S , it is also evident that V_S depends on the G_n/Q_n load ratio and load statistics, which are different, in principle, for the two design frameworks investigated. Fig. 6 shows the values of the V_S coefficients for the US and Eurocode frameworks for varying G_n/Q_n load ratios, which demonstrate that although the $V_{S,US}$ and $V_{S,EN}$ coefficients are different, they can be reasonably considered as equal. The maximum difference between the two coefficients of variation is about 6% (see the $(V_{S,US} - V_{S,EN})/V_{S,EN}$ curve in Fig. 6), which occurs for a dead-to-live load ratio of $\zeta = 1.0$. For the gravity plus wind load case, the values of the $V_{S,US}$ and $V_{S,EN}$ parameters are equal, since the two design frameworks adopt maximum wind load models with the same coefficient of variation of 0.35 (see Table 1).

5.1.3. Bias and coefficient of variation for the resistance, \bar{R}/R_n , \bar{R}/R_k and V_R

The distribution of the resistance of structures (or members) is generally calculated through an analysis of random samples that account for the most influential uncertainties (i.e., geometric and material properties, imperfections, residual stresses, etc.) following statistical models that are based on measurements on real structures. The random samples of resistance can be the results of either advanced finite element simulations or actual test specimens, from which statistical distributions describing the variability of the resistance can be inferred. Hence, these statistical parameters (mean resistance \bar{R} and coefficient of variation V_R) will typically be the same in the different design frameworks, since they are based on real random structures. On the contrary, the value of nominal resistance R_n will usually differ from one design framework to another, because it is based on a certain structural standard. However, these differences depend on the design approach and material investigated.

When system-based direct design approaches are considered, the nominal R_n or characteristic R_k resistances in the different frameworks are determined from advanced finite element simulations based on specifications that generally prescribe the same (or very similar) nominal characteristics, such as initial imperfection patterns and magnitudes, residual stresses and material properties. Thus, the resulting $R_{n,US}$ and $R_{k,EN}$ resistances will be very similar when calculated to the US and Eurocode frameworks. However, for some particular cases, such as stainless steel structures, the corresponding material specifications prescribe considerably different nominal properties for each framework [21,22]; hence in these situations, the nominal and characteristic resistances can be significantly different. On the other hand, if member-based reliability calibrations are performed, the nominal resistances

will typically be based on analytical expressions incorporated in the relevant specifications (e.g., buckling curves predicting the strength of columns), which can substantially differ depending on the standard considered and produce significantly different $R_{n,US}$ and $R_{k,EN}$ values. Since this paper is concerned with system-based reliability calibrations based on the direct design approach for steel and stainless steel frames, it can be assumed that the $(\bar{R}/R_n)_{US}$ and $(\bar{R}/R_k)_{EN}$ terms in Eq. (19) can be ignored when structural steel frames are analysed, whereas the $R_{k,EN}/R_{n,US}$ coefficient should be kept for stainless steel frame cases owing to the differences in the prescribed nominal material properties for these alloys.

5.2. Direct estimation of resistance ϕ and partial safety γ_M factors for different levels of reliability β

Considering the simplifications discussed in the previous Section, Eq. (19) can be re-written for the particular cases in which the US and Eurocode frameworks are compared for structures subjected to gravity loads (as per in Eq. (20)) and under gravity plus wind loads (as per in Eq. (21)). These equations can be further simplified by adopting $R_{k,EN}/R_{n,US} = 1.0$ when structural steel frames are analysed.

$$\phi \cdot \gamma_M = 0.86 \frac{R_{k,EN}}{R_{n,US}} \exp \left[(\beta_{EN} - \beta_{US}) \sqrt{V_R^2 + V_S^2} \right] \quad (\text{gravity load case}) \quad (20)$$

$$\phi \cdot \gamma_M = \frac{R_{k,EN}}{R_{n,US}} \exp \left[(\beta_{EN} - \beta_{US}) \sqrt{V_R^2 + V_S^2} \right] \quad (\text{gravity plus wind load case}) \quad (21)$$

Eqs. (20) and (21) can be used to propose suitable resistance factors ϕ that meet the reliability requirements indicated in the US framework (i.e., the target reliability indices $\beta_{0,US}$ discussed in Section 2.1) based on the partial safety factors γ_M calibrated for the Eurocode frameworks for a different target reliability $\beta_{0,EN}$ (or vice versa). For the assessment of these equations, this Section presents the comparison of the resistance ϕ and partial safety γ_M factors estimated using Eqs. (20) and (21) with the equivalent values obtained using FORM methods. For example, the predicted resistance factors ϕ_{pred} considered in this Section have been obtained from Eqs. (20) and (21) using $\beta_{0,US}$ and the partial safety factors calibrated from FORM analyses $\gamma_{M,FORM}$ for the typical Eurocode target reliability $\beta_{0,EN}$ and the suitable $R_{k,EN}/R_{n,US}$ relationship, depending on the material considered. In the analysis, the target reliability indices typically assumed in the two design frameworks have been adopted (see Section 2.1): for the gravity load case, $\beta_{0,US} = 2.5$ and $\beta_{0,EN} = 3.8$ values have been assumed; for the gravity plus wind load case, $\beta_{0,US} = \beta_{0,EN} = 2.8$ is adopted.

Fig. 7 presents the assessment of the ϕ_{pred} - and $\gamma_{M,pred}$ -factors estimated using Eqs. (20) and (21) against the values that a proper FORM-based reliability calibration would produce (ϕ_{FORM} and $\gamma_{M,FORM}$) for different load cases. The results are presented separately for the US and Eurocode frameworks, and different markers have been used for the gravity or gravity plus wind load cases, and for steel or stainless steel frames. The figures also show the $\pm 5\%$ and $\pm 10\%$ intervals. According to the results presented in Fig. 7, the prediction of the resistance and partial safety factors is, in general, reasonably accurate, since most of the results lie within the $\pm 5\%$ range for the gravity and gravity plus wind load cases, although the results for the gravity load case are found to be more accurate and less scattered. It is worth mentioning that some of the datapoints corresponding to stainless steel frames included in Fig. 7(a) show resistance factors ϕ higher than unity, which can be explained by the high \bar{R}/R_n ratios resulting from the remarkably low yield stress values prescribed for these alloys in the SEI/ASCE 8 [46] Specification, as discussed in [21,22]. Similar observations have also been reported for timber and masonry structures [29]. The mean values, coefficients of variation and minimum-to-maximum ranges of the $\phi_{pred}/$

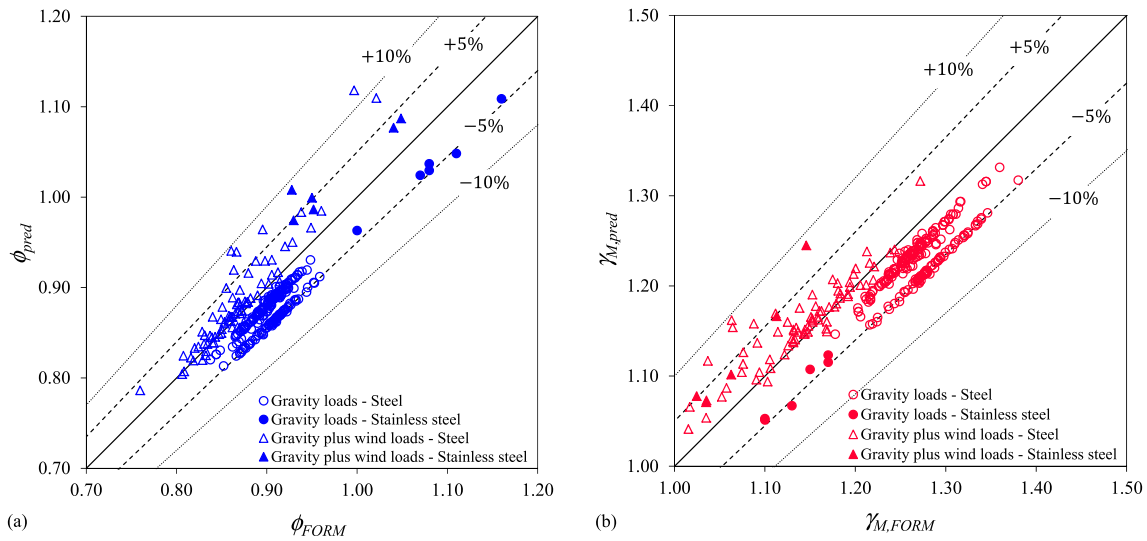


Fig. 7. Assessment of the expressions directly relating the reliability of different design frameworks based on FOSM (Eqs. (20) and (21)).

Table 4

Assessment of the expressions directly comparing the reliability for different frameworks and load cases based on FOSM (Eqs. (20) and (21)).

Design framework	Load case		$\frac{\phi_{pred}}{\phi_{FORM}}$ or $\frac{\gamma_{M,pred}}{\gamma_{M,FORM}}$	
			Mean	Min-Max
US framework	Gravity loads	Mean	0.97	
		COV	0.014	
		Min-Max	0.94–0.99	
	Gravity plus wind loads	Mean	1.03	
		COV	0.031	
		Min-Max	0.98–1.12	
Eurocode framework	Gravity loads	Mean	0.97	
		COV	0.014	
		Min-Max	0.94–0.99	
	Gravity plus wind loads	Mean	1.03	
		COV	0.031	
		Min-Max	0.98–1.12	

ϕ_{FORM} and $\gamma_{M,pred}/\gamma_{M,FORM}$ ratios are reported in Table 4 for the different cases analysed.

5.3. Direct comparison of resistance ϕ and partial safety γ_M factors for the same level of reliability β

The comparison of the resistance ϕ and partial safety γ_M factors calibrated in the US and Eurocode frameworks is not direct, as shown in the previous Section, because the reliability levels required for the two design framework are different (i.e., different target reliabilities are generally adopted). However, Eqs. (20) and (21) can be further elaborated to provide simple expressions that allow a direct comparison of the resistance ϕ and partial safety γ_M factors when the same level of reliability is assumed for the two frameworks, by adopting $\beta_{EN} = \beta_{US}$. In such cases, the exponential terms in Eqs. (20) and (21) are equal to unity and thus ϕ and γ_M -factors are related by the $\phi \cdot \gamma_M = 0.86 R_{k,EN}/R_{n,US}$ and $\phi \cdot \gamma_M = R_{k,EN}/R_{n,US}$ relationships for the gravity and gravity plus wind load cases, respectively. For the particular situations in which $R_{k,EN}/R_{n,US} = 1.0$ (i.e., the steel frames analysed in this paper), these equations can be reduced to $\phi \cdot \gamma_M = 0.86$ and $\phi \cdot \gamma_M = 1.0$. These relationships provide a simple and direct comparison of the equivalent resistance ϕ and partial safety γ_M factors that would be required for the same level of reliability in the two design frameworks. The equation for gravity loads indicates that, in general, the Eurocode framework is less conservative than the US framework for a given level of reliability, since the required

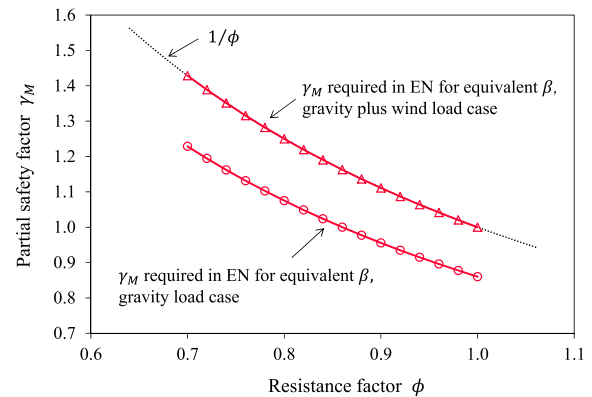


Fig. 8. Comparison of the resistance ϕ and partial safety γ_M factors for the US and Eurocode framework for a constant level of reliability.

γ_M -factors are about 15% lower than the $1/\phi$ factors for equivalent values of β , while for the gravity plus wind load combination the two frameworks are similar in terms of conservatism. This is also illustrated in Fig. 8, which compares the γ_M -factors required in the Eurocode framework for the gravity and gravity plus wind load cases with the $1/\phi$ values, assuming an equivalent level of reliability in all cases.

5.4. Analysis using the Eurocode semi-probabilistic approach

This last Section presents the derivation and assessment of the expression for the direct estimation of resistance and partial safety factors using Eq. (5) and the Eurocode semi-probabilistic approach for γ_M , Eq. (17). The combination of these two equations results in the relationship given by Eq. (22), in which the term $R_{k,EN}/R_{n,US}$ can be ignored when structural steel frames are analysed, as discussed in Section 5.1.3, but should be kept for other materials with different nominal properties prescribed in the relevant US specification and the Eurocode. Further simplifications are not possible in principle because Eqs. (5) and (17) are based on different backgrounds.

$$\phi \cdot \gamma_M = C_{US} \frac{R_{k,EN}}{R_{n,US}} \exp\left(\alpha_R \beta_{EN} \theta - \beta_{US} \sqrt{V_R^2 + V_{S,US}^2}\right) \quad (22)$$

Nevertheless, all parameters in Eq. (22) are basic statistical input information so the relationship can still be used for the direct estimation of resistance and partial safety factors, as per in Section 5.2, the accuracy

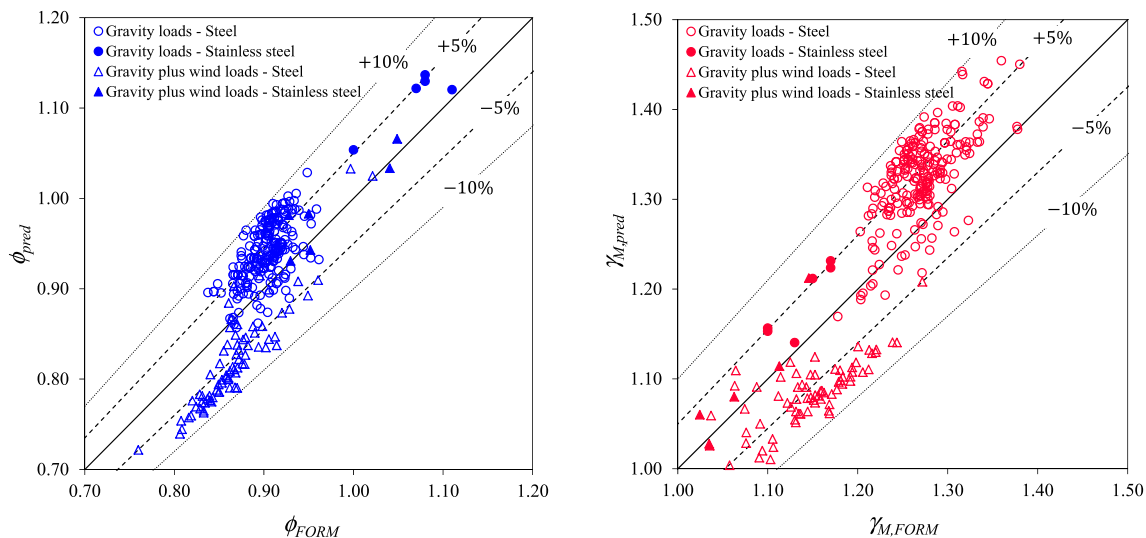


Fig. 9. Assessment of the expressions directly relating the reliability of different design frameworks based on the Eurocode semi-probabilistic approach (Eq. (22)).

Table 5

Assessment of the expressions directly comparing the reliability for different frameworks and load cases based on the Eurocode semi-probabilistic approach (Eq. (22)).

Design framework	Load case		$\frac{\phi_{pred}}{\phi_{FORM}}$ OR $\frac{\gamma_{M,pred}}{\gamma_{M,FORM}}$
US framework	Gravity loads	Mean	1.05
		COV	0.025
		Min-Max	0.98–1.10
	Gravity plus wind loads	Mean	0.96
		COV	0.045
		Min-Max	0.89–1.10
Eurocode framework	Gravity loads	Mean	1.05
		COV	0.025
		Min-Max	0.98–1.10
	Gravity plus wind loads	Mean	0.96
		COV	0.045
		Min-Max	0.89–1.10

of which is assessed herein. Fig. 9 compares the predicted resistance ϕ_{pred} and partial safety $\gamma_{M,pred}$ factors using Eq. (22) with the equivalent values obtained from FORM analyses for different load cases, and also depicts the $\pm 5\%$ and $\pm 10\%$ intervals. The target reliability indices considered in the analysis were those adopted previously, i.e., $\beta_{0,US} = 2.5$ and $\beta_{0,EN} = 3.8$ for the gravity load cases and $\beta_{0,US} = \beta_{0,EN} = 2.8$ for the gravity plus wind load cases. Note that these values are within the applicability ranges of β -values for the use of Eq. (17) proposed in Section 4.2. The results in Fig. 9 indicate that the prediction of the resistance and partial safety factors using Eq. (22) is less accurate than when Eqs. (20) and (21) are used (see Section 5.2 and Fig. 7), although the predictions are still reasonable as they lie within the $\pm 10\%$ range, with no significant differences in accuracy observed for the load cases and design frameworks investigated. Similar results are shown in Table 5, in which the means, coefficients of variation and minimum-to-maximum ranges of the ϕ_{pred}/ϕ_{FORM} and $\gamma_{M,pred}/\gamma_{M,FORM}$ ratios for the different cases analysed are reported.

6. Conclusions

First Order Reliability Methods (FORM), traditionally used by specification committees in the calibration of suitable resistance and safety factors, are advanced and iterative reliability calculation procedures.

Sometimes, these techniques require input information (e.g., statistical characterization of the system strength and load effects) that might not be readily available in the literature. Thus, simplified expressions that require no iteration but are sufficiently accurate in estimating reliability indices β , resistance factors ϕ and partial safety factors γ_M can be of great interest for these committees and the research community in general to provide a simple cross-check on the more accurate reliability calculations or to provide direct relationships between the reliability results for different design frameworks. This paper presents a set of simple equations to estimate reliability indices β , resistance factors ϕ and partial safety factors γ_M for the US and Eurocode frameworks based on basic First Order Second Moment (FOSM) reliability considerations and the semi-probabilistic approach prescribed in Annex D of prEN 1990 [27] for design assisted by testing. The equations based on FOSM are similar to those prescribed in the AISI S100 [28] Specification to derive resistance factors when strengths are determined through testing, but have been extended to different load cases and design frameworks. The paper also proposes simple interrelationships between the reliability calibration results for the US and Eurocode frameworks that allow a direct comparison between the two design frameworks. The assessment of the different equations against reliability results derived using the FORM method for an extensive database on steel and stainless frames subjected to the gravity and gravity plus wind load cases collected from the literature showed that the differences in the predicted values of β , ϕ and γ_M lie within the $\pm 5\%$ and $\pm 10\%$ ranges when compared to the reference FORM-based values for the FOSM-based approaches. The results also indicated that, to guarantee a similar level of accuracy for the prEN 1990 [27] semi-probabilistic approach, some limitations are required due to the additional simplifications made by this method (i.e., the adoption of fixed sensitivity factors for the loads and the resistance).

CRediT authorship contribution statement

Itsaso Arrayago: Software, Data curation, Methodology, Formal analysis, Writing – original draft. **Hao Zhang:** Methodology, Supervision, Writing – review & editing. **Kim J.R. Rasmussen:** Methodology, Supervision, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The project leading to this research has received funding from the European Union's Horizon 2020 Research and Innovation Programme under the Marie Skłodowska-Curie Grant Agreement No. 842395.

References

- [1] White DW, Jeong WY, Toğay O. Comprehensive stability design of planar steel members and framing systems via inelastic buckling analysis. *International Journal of Steel Structures* 2016;16(4):1029–42.
- [2] Surovek A, Camotim D, Hajjar J, Teh L, White D, Ziemian R. Direct second-order analysis for the design of steel structures. *Proceedings of the Structures Congress*, St. Louis, United States 2006.
- [3] Surovek AE, White DW, Ziemian RD, Camotim D, Hajjar JF, Teh LH. In: Recommendations for the use of direct second-order inelastic analysis to design steel frames. United States: New Orleans; 2007. p. 301–16.
- [4] Gonçalves R, Camotim D. A system-based approach for the design of laterally unbraced multi-span steel columns and beams. *Eng Struct* 2017;135:10–20.
- [5] Wang Y, Ziemian RD. Design by advanced elastic analysis: An investigation of beam-columns. *Engineering Journal* 2021;58(2):123–37.
- [6] Ziemian RD, Wang Y. In: Design by advanced analysis - 2016 AISC Specification. Prague: Czechs Republic: SDSS-2019; 2019. p. 55–67.
- [7] Chan SL, Liu YP, Liu SW. In: Research, theory and practice of second order direct analysis for design of steel and composite structures. Timisoara, Romania: SDSS-2016; 2016. p. 3–13.
- [8] Ziemian RD, Abreu JCB. Design by advanced analysis – 3D benchmark problems: Members subjected to major- and minor-axis flexure. *Steel Construction* 2018;11(1):24–9.
- [9] Manta D, Gonçalves R, Camotim D. Combining shell and GBT-based finite elements: Plastic analysis with adaptive mesh refinement. *Thin-Walled Structures* 2021;158:107205. <https://doi.org/10.1016/j.tws.2020.107205>.
- [10] Du Z-L, Ding Z-X, Liu Y-P, Chan SL. Advanced flexibility-based beam-column element allowing for shear deformation and initial imperfection for direct analysis. *Eng Struct* 2019;199:109586. <https://doi.org/10.1016/j.engstruct.2019.109586>.
- [11] Fieber A, Gardner L, Macorini L. Structural steel design using second-order inelastic analysis with strain limits. *J Constr Steel Res* 2020;168:105980. <https://doi.org/10.1016/j.jcsr.2020.105980>.
- [12] Walport F, Kucukler M, Gardner L. Stability design of stainless steel structures. *Journal of Structural Engineering (ASCE)* 2022;148(1). [https://doi.org/10.1061/\(ASCE\)ST.1943-541X.0003165](https://doi.org/10.1061/(ASCE)ST.1943-541X.0003165).
- [13] Rasmussen KJR, Zhang H. Future challenges and developments in the design of steel structures – an Australian perspective. *Proceedings of the 8th European Conference on Steel and Composite Structures (EUROSTEEL 2017)*. Copenhagen, Denmark, 2017.
- [14] Zhang H, Shayan S, Rasmussen KJR, Ellingwood BR. System-based design of planar steel frames, I: Reliability framework. *J Constr Steel Res* 2016;123:135–43.
- [15] Zhang H, Shayan S, Rasmussen KJR, Ellingwood BR. System-based design of planar steel frames, II: Reliability results and design recommendations. *J Constr Steel Res* 2016;123:154–61.
- [16] Liu W, Rasmussen KJR, Zhang H. Systems reliability for 3D steel frames subject to gravity loads. *Structures* 2016;8:170–82.
- [17] Liu W, Zhang H, Rasmussen KJR, Yan S. System-based limit state design criterion for 3D steel frames under wind loads. *J Constr Steel Res* 2019;157:440–9.
- [18] Liu W, Zhang H, Rasmussen KJR. System reliability-based Direct Design Method for space frames with cold-formed steel hollow sections. *Eng Struct* 2018;166:79–92.
- [19] Sena Cardoso F. System reliability-based criteria for designing cold-formed steel structures by advanced analysis. PhD thesis. The University of Sydney, 2016.
- [20] Sena Cardoso F, Zhang H, Rasmussen KJR, Yan S. Reliability calibrations for the design of cold-formed steel portal frames by advanced analysis. *Eng Struct* 2019; 182:164–71.
- [21] Arrayago I, Rasmussen KJR. System-based reliability analysis of stainless steel frames under gravity loads. *Eng Struct* 2021;231:111775. <https://doi.org/10.1016/j.engstruct.2020.111775>.
- [22] Arrayago I, Rasmussen KJR, and Zhang H. System-based reliability analysis of stainless steel frames subjected to gravity and wind loads. Submitted to *Structural Safety*, 2021.
- [23] AS/NZS 4100 Steel Structures. Standards Australia, Sydney, Australia, 2020.
- [24] American Institute of Steel Construction (ANSI/AISC). AISC 360. Specification for Structural Steel Buildings. Illinois, USA, 2016.
- [25] American Institute of Steel Construction (ANSI/AISC). AISC 370. Specification for Structural Stainless Steel Buildings. Illinois, USA, 2021.
- [26] Working Group 22 for Eurocode 3, CEN/TC 250/SC3/WG22. prEN 1993-1-14. Eurocode 3: Design of steel structures – Part 1-14: Design assisted by finite element analysis. Final document. Brussels, Belgium, 2021.
- [27] European Committee for Standardization (CEN). prEN 1990. Eurocode: Basis of Structural Design. Draft Document. Brussels, Belgium, 2020.
- [28] American Iron and Steel Institute (AISI). AISI S100-16. North American Specification for the Design of Cold-Formed Steel Structural Members. Illinois, USA, 2016.
- [29] Melchers RE, Beck AT. *Structural reliability analysis and prediction*. John Wiley & Sons, Third Edition; 2018.
- [30] Meimand VZ, Schafer BW. Impact of load combinations on structural reliability determined from testing cold-formed steel components. *Struct Saf* 2014;48:25–32.
- [31] Ellingwood B.R., Galambos T.V., MacGregor J.G. and Cornell C.A. Development of a Probability Based Load Criterion for American National Standard A58. National Bureau of Standards. Special Publication No. 577. Washington D.C., 1980.
- [32] Ravindra MK, Galambos TV. Load and resistance factor design for steel. *Journal of Structural Division (ASCE)* 1978;104(9):1337–53.
- [33] American Society of Civil Engineers (ASCE). ASCE 7. Minimum design loads and associated criteria for buildings and other structures. Virginia, USA, 2016.
- [34] Hsiao L-E, Yu W-W, Galambos TV. AISI LFRD method for cold-formed steel structural members. *Journal of Structural Engineering (ASCE)* 1990;116(2): 500–17.
- [35] European Committee for Standardization (CEN). prEN 1991-1-1. Eurocode 1: Actions on structures – Part 1-1: General actions. Densities, self-weight, imposed loads for buildings. Draft Document. Brussels, Belgium, 2019.
- [36] European Committee for Standardization (CEN). prEN 1991-1-4. Eurocode 1: Actions on structures – Part 1-4: Wind Actions. Final Document. Brussels, Belgium, 2019.
- [37] McAllister TP, Wang N, Ellingwood BR. Risk-informed mean recurrence intervals for updated wind maps in ASCE 7–16. *Journal of Structural Engineering (ASCE)* 2018;144(5):06018001. [https://doi.org/10.1061/\(ASCE\)ST.1943-541X.0002011](https://doi.org/10.1061/(ASCE)ST.1943-541X.0002011).
- [38] Gulvanessian H, Holicky M. Eurocodes: using reliability analysis to combine action effects. *Proceedings of the Institution of Civil Engineers - Structures and Buildings* 2005;158(4):243–52.
- [39] Galambos TV, Ellingwood BR, MacGregor JG, Cornell CA. Probability based load criteria: Assessment of current design practice. *Journal of the Structural Division (ASCE)* 1982;108(5):959–77.
- [40] Tankova T, Simões da Silva L, Marques L, Rebelo C, Taras A. Towards a standardized procedure for the safety assessment of stability design rules. *J Constr Steel Res* 2014;103:290–302.
- [41] AS/NZS 4600 Cold-formed Structures. Standards Australia, Sydney, Australia, 2018.
- [42] Steenbergen RDJM, Vrouwenvelder ACWM. Safety philosophy for existing structures and partial factors for traffic loads on bridges. *HERON* 2010;55(2): 123–40.
- [43] Ellingwood BR, Tekie PB. Wind load statistics for probability-based structural design. *Journal of Structural Engineering (ASCE)* 1999;125(4):453–63.
- [44] Ellingwood BR. LFRD: implementing structural reliability in professional practice. *Eng Struct* 2000;22(2):106–15.
- [45] Ellingwood BR. Probability-based codified design: past accomplishments and future challenges. *Struct Saf* 1994;13(3):159–76.
- [46] American Society of Civil Engineers (ASCE). SEI/ASCE 8. Specification for the Design of Cold-Formed Stainless Steel Structural Members. Virginia, USA, 2021.