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Essays on Consumer Search

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Proefschrift

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Chapter 1

Introduction

> *"One should hardly have to tell academicians that information is a valuable resource: knowledge is power. And yet it occupies a slum dwelling in the town of economics."* George J. Stigler, 1961.

Consumers typically do not know all available alternatives, prices, and what they offer. Hence, they cannot just buy the alternative they prefer. Instead, consumers first need to gather information to be able to compare alternatives and eventually choose one. Because searching for such information is costly, consumers rarely compare all alternatives and potentially miss out on cheaper or better-suited alternatives.

George J. Stigler introduced this reasoning in economics and his seminal article, leading with the quote above, initiated an extensive literature that studies the resulting search frictions. Common to this literature is that it uses models for how rational consumers decide to search for products or product information. These search models are useful for several reasons. Different models can highlight various aspects of the search process that drive search behavior. They also allow us to analyze how specific changes to the search environment affect how consumers search and which alternative they eventually buy. Moreover, search models can characterize the demand sellers face in settings where consumers have limited information, which, in turn, governs competition in prices or other dimensions (e.g., advertising).

Chapter 2 adds to this literature by developing and solving the "search and discovery problem," a model for the decision process consumers face in various settings. In this problem, a consumer is initially aware of only a few products. For these products, the consumer has some (but not all) information and knows that they exist. Using this information, the consumer can decide in what order to gather detailed information on these alternatives and when to stop searching. Besides, there are also products that the consumer has never heard of. For these alternatives, the consumer cannot just decide to gather more detailed information. Instead, he first needs to discover and become aware of them. Hence, the consumer needs to also decide between searching among alternatives he is already aware of and discovering more products.

The main contribution of Chapter 2 is to prove that optimal search decisions and outcomes in the search and discovery model remain tractable, despite the complexity of the decision problem. Specifically, I show that reservation values fully characterize the optimal policy. Building on this result, I further show that the purchase of a consumer solving the search and discovery problem can be obtained directly from these reservation values. Combined, these two results make it feasible to study settings where consumers have limited awareness without having to consider the multitude of possible choice sequences that consumers could potentially take.

In the search and discovery model, alternatives that consumers discover early in the search process are more likely to be searched and bought. This mechanism explains a commonly observed pattern: "position effects." These position effects occur on product lists that most online retailers or search intermediaries use to present consumers with the alternatives they offer. Each product has a position on this list, where position effects are the effects of a product's position on how many consumers will click on and purchase. It is well-documented that these position effects are substantial such that being moved higher up on the list leads to a measurable increase in demand.

Position effects are important because they create scope for search intermediaries (or online retailers) to increase their revenues simply by changing the ranking of the product list they present to consumers. However, changes in the ranking of alternatives beneficial to a search intermediary may be harmful to consumers. For example, revenues could be increased by moving expensive alternatives to the top of the list, potentially harming price-sensitive consumers. Consequently, there is a potential misalignment of interests between search intermediaries and consumers.

In Chapter 3, I study this potential misalignment of interests and how such "revenue-based" rankings affect consumers. I first show that heterogeneity in position effects determines which alternatives need to be moved higher up on the list. As a result, heterogeneity in position effects also determines how revenue-based rankings affect consumers. Using click-stream data from an online travel agent, I provide descriptive evidence for this heterogeneity that highlights that alternatives with "desirable" attributes (e.g., a low price) have stronger position effects.

To quantify the effects of changes in the ranking, I also develop and implement an estimation approach for the model introduced in Chapter 2. By simulating counterfactual scenarios I show that revenue-based rankings can benefit search intermediaries and consumers relative to various other rankings. Moreover, I find that revenue-based rankings decrease consumer welfare only to a limited extent when compared to utility-based rankings that first show the alternatives that consumers prefer (on average). Combined, these results suggest that, when designing rankings, search intermediaries' and consumers' interests are not strongly misaligned.

In Chapter 2, I treat the different costs consumers incur when inspecting and discovering products as model primitives. This modeling approach is useful because it explains position effects and allows to evaluate the effects of different rankings. Indeed, the position effects and lack of clicks observed in Chapter 3 can only be rationalized if inspecting and discovering products is costly. However, treating costs as model primitives does not explain why searching is costly and how these costs may differ across consumers and settings.

In Chapter 4, co-authored with Yufeng Huang and Ilya Morozov, we use a different modeling approach. We develop a search model where a consumer decides how much time to spend searching in different product categories. In the model, search frictions arise endogenously: because time is limited, spending more time searching in one category requires spending less time searching in other categories or doing some other enjoyable activity. Hence, searching for products incurs the opportunity costs of time in our model.

Our model allows to study two novel aspects of search. First, we can characterize search across different product categories. This allows us to analyze potential cross-category effects and factors that determine when they are important. Second, our model allows us to disentangle the effects of changes to the search technology that determines how much time is required to search an alternative, and changes to the opportunity costs of time. We highlight that the consumer searches more products but may spend less time on search overall when the search technology improves. We also show that when given more time, the consumer does not start searching in additional categories without also spending more time searching in previous ones.

Chapter 2

Optimal Search and Discovery

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Abstract

This paper studies a search problem where a consumer is initially aware of only a few products. At every point in time, the consumer then decides between searching among alternatives he is already aware of and discovering more products. I show that the optimal policy for this search and discovery problem is fully characterized by tractable reservation values. Moreover, I prove that a predetermined index fully specifies the purchase decision of a consumer following the optimal search policy. Finally, a comparison highlights differences to classical random and directed search.

2.1 Introduction

Consumers typically first need to search for product information before being able to compare alternatives. The resulting search frictions have received considerable attention in the literature.[1] Under the rational choice paradigm, the analysis of such limited information settings relies on optimal search policies that describe how a consumer optimally searches among all available alternatives. I add to this literature by developing and solving a sequential search problem that introduces a novel aspect: *limited awareness of available products.*

To fix ideas, consider a consumer looking to buy a mobile phone. Through advertising or recommendations from friends, the consumer initially is aware of a single available phone and has some (but not all) information on what it offers. Given this basic information, the consumer can directly gather more detailed information on this alternative, for example by reading a review online. Besides, there are also phones available that the consumer is initially not aware of. For these alternatives, he knows neither of their existence, nor the features they offer. This precludes the consumer from directly inspecting these phones. Instead, he first needs to discover and become aware of them, for example by getting more recommendations from friends or through a search intermediary. Figure 2.1 depicts a possible choice sequence for this case.

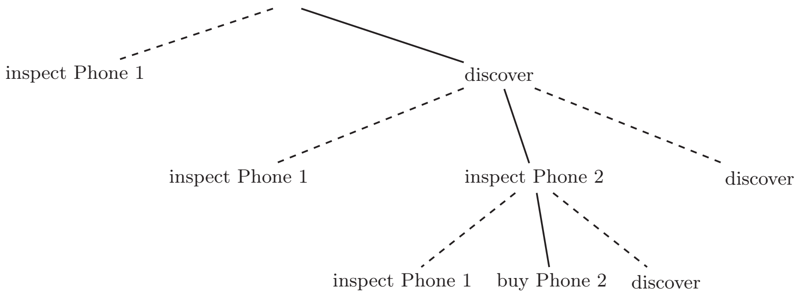

FIGURE 2.1 – Example of a choice sequence in the search and discovery problem.

The "search and discovery problem" introduced in this paper formalizes a consumer's dy-

[1]For example, Stigler (1961); Diamond (1971); Burdett and Judd (1983); Anderson and Renault (1999); Kuksov (2006); Choi et al. (2018); Moraga-González et al. (2017a,b) study search frictions in equilibrium models and Hortaçsu and Syverson (2004); Hong and Shum (2006); De Los Santos et al. (2012); Bronnenberg et al. (2016); Chen and Yao (2017); Zhang et al. (2018); Jolivet and Turon (2019) study implications of search empirically.

namic decision process in this and similar settings. The resulting framework allows to study settings that are difficult to accommodate in existing search problems. In particular, neither random (e.g. McCall, 1970) nor directed search (e.g. Weitzman, 1979) is well suited to study settings where rational consumers remain oblivious to some, while obtaining only partial information on other products. However, such settings are common in practice. For example, online retailers and search intermediaries present an abundance of alternatives on product lists that reveal partial information only for some products. Consumers then decide between clicking on products already discovered on the list to reveal full information, and browsing further to discover more products. More generally, in markets with a large number of alternatives, consumers will remain unaware of many alternatives unless they actively set out to discover more products. Similarly, in markets where rapid technological innovations lead to a constant stream of newly available alternatives, few consumers are aware of new releases without exerting effort to remain informed.

The contribution of this paper is to show that despite its complexity, optimal search decisions and outcomes in the search and discovery problem remain tractable if the consumer has stationary beliefs. First, I prove that the optimal policy is fully characterized by reservation values similar to the well-known reservation prices derived by Weitzman (1979). In each period, a reservation value is assigned to each available action, and it is optimal to always choose the action with the largest value. Each of the reservation values is independent of any other available action and can be calculated without having to consider expectations over a myriad of future periods. Hence, reservation values remain tractable. This allows to determine optimal search behavior under limited awareness without using numerical methods.

Second, I prove that the purchase of a consumer solving the search and discovery problem is equivalent to the same consumer having full information and directly choosing products from a predetermined index. This result generalizes the "eventual purchase theorem" derived independently by Choi et al. (2018), Armstrong (2017) and Kleinberg et al. (2017) to the case of limited awareness.[2] Similar to the eventual purchase theorem, my generalization allows to derive a consumer's expected payoff and market demand without having to consider a multitude of possible choice sequences that otherwise make aggregation difficult.

[2]Choi et al. (2018) introduced the name and noted that "Our eventual purchase theorem was anticipated by Armstrong and Vickers (2015) and has been independently discovered by Armstrong (2017) and Kleinberg et al. (2017)."

This paper also highlights several implications of limited awareness through a comparison of stopping decisions, expected payoffs and market demand with classical random and directed sequential search. A first implication of limited awareness is that it leads to two distinct search actions which posits a novel question: Do consumers benefit more from making it easier to discover more alternatives (e.g. through search intermediaries), or from facilitating inspection by more readily providing detailed product information? For the case where a consumer discovers one product at a time, I show that there exists a (possibly small) threshold for the number of alternatives after which the expected payoff increases more when facilitating discovery instead of facilitating inspection. This highlights the relative importance of discovery costs in settings with many alternatives.

Moreover, limited awareness generates distinct patterns in the resulting market demand. In directed search, more consumers preferring a product based on partial product information increases its market demand. This need not be the case with limited awareness; if consumers remain unaware of a product, its market demand does not increase as it becomes the preferred option. Whereas the same holds with random search, not being able to use partial information to decide whether to inspect a product induces consumers to stop earlier if total costs of revealing full product information remain the same.

The search and discovery problem also provides an intuitive rationalization of ranking effects commonly observed in click-stream data (e.g. Ursu, 2018): as consumers stop search before having discovered all products, products that would be discovered later are less likely to be bought. I show that these ranking effects are independent of the number of available alternatives, and decrease as more products are discovered. This mechanism offers a meaningful interpretation of how advertising that provides partial product information is beneficial for a seller; [3] if a seller's marketing efforts make more consumers aware of a product before search or increase the probability of the product being discovered early on, ranking effects directly imply that they will increase the demand.

Finally, this paper adds to the empirical search literature by discussing implications of limited awareness for the estimation of structural search models. Besides highlighting differences in parameter estimates and counterfactual predictions across the three models, I show that a

[3] This relates to the "informative view" of advertising. See e.g. Bagwell (2007) for a summary and comparison to the "persuasive view".

directed search model will lead to accentuated search cost estimates due to not accounting for limited awareness when rationalizing stopping decisions.

The remainder of this paper is organized as follows. First, I discuss related literature. Section 3 introduces the search and discovery (henceforth SD) problem. Section 4 provides the optimal policy and discusses several extensions as well as limitations. In Section 5, I generalize the eventual purchase theorem of Choi et al. (2018) and use this to derive a consumer's expected payoff as well as market demand. Section 6 compares search problems and discusses empirical implications. Section 7 concludes. Throughout, proofs are deferred to the appendix.

2.2 Related literature

The search and discovery problem introduced in this paper nests both classical random and directed sequential search as special cases. In random search, a searcher has no alternative-specific prior information, hence searches randomly across alternatives and only decides when to end search (e.g. McCall, 1970; Lippman and McCall, 1976). In directed search, the searcher is aware of all available alternatives and uses partial product information to determine an order in which to inspect products and when to end search (e.g. Weitzman, 1979; Chade and Smith, 2006). In contrast, in the search and discovery problem, the consumer is aware of only a few products. Hence, the consumer not only decides in what order to inspect products he is already aware of and when to end search, but also when to try to discover more alternatives.

To prove the optimality I use results from the multi-armed bandit literature to first determine that a Gittins index policy is optimal,[4] and then introduce a monotonicity condition to show that the Gittins index reduces to simple reservation values. Specifically, I use the results of Keller and Oldale (2003) who proved that a Gittins index policy is optimal in their branching bandits framework. This framework differs from the standard multi-armed bandit problem in that taking an action will reveal information on multiple other actions. However, as an action branches off into new actions and reveals information only on those, the state of other available actions is never altered. Hence, the important independence assumption continues to hold.

Similar monotonicity conditions also apply in other multi-armed bandit problems where

[4] Gittins et al. (2011) provide a textbook treatment of multi-armed bandit problems and the Gittins index policy. As purchasing a product ends search, search problems correspond to stoppable superprocesses as introduced by Glazebrook (1979).

they simplify the otherwise difficult calculation of the Gittins index values (see e.g. Section 2.11 in Gittins et al., 2011). The present case differs in that monotonicity is only required for the action of discovering more alternatives, but does not hold when inspecting a product. In a recent working paper, Fershtman and Pavan (2019) independently derived a similar characterization of the optimal policy when applying a monotonicity condition in a general multi-armed bandit problem where a decision maker also extends a set of alternatives.

Moreover, monotonicity conditions also lead to the results in the literature on (random) search problems where a searcher learns about the distribution from which he is sampling (Rothschild, 1974; Rosenfield and Shapiro, 1981; Bikhchandani and Sharma, 1996). These authors determine priors and learning rules that satisfy a similar condition based on which they can derive an optimal policy that is myopic. The SD problem differs in that not all information about a product is revealed when it is discovered such that it entails two distinct search actions. As I show, this makes it difficult to find similar priors or learning rules that would lead to a myopic optimal policy in extensions to the SD problem that incorporate learning.

Several other contributions extend Weitzman's (1979) seminal search problem in different directions. Adam (2001) studies the case where the searcher updates beliefs about groups of alternatives during search and finds a similar reservation value policy to be optimal. Olszewski and Weber (2015) generalize Pandora's rule to search problems where the final payoff depends on all the alternatives that have been inspected, not only the best one. Finally, Doval (2018) analyzes the optimal policy when a searcher can directly choose alternatives without first inspecting them.

This paper also relates to the recent literature studying problems where a consumer gradually reveals more information on products (Branco et al., 2012; Ke et al., 2016; Ke and Villas-Boas, 2019). These problems are formulated in continuous-time and generally do not admit an optimal policy based on an index. The SD problem differs in that it assumes that a consumer cannot purchase a product before having revealed full information. This makes available actions independent such that a tractable reservation value policy is optimal. Furthermore, the SD problem allows that multiple products can be discovered at a time such that with one action, information on multiple products is revealed. Though Ke et al. (2016) also consider correlated payoffs, discovering multiple products differs in that the correlation structure of payoffs changes after the discovery; inspecting one does not reveal information about other products discovered at the same time.

The SD problem also subsumes decision processes considered in the growing empirical literature estimating structural search models (e.g. Honka, 2014; Chen and Yao, 2017; Ursu, 2018). Most closely related are De los Santos and Koulayev (2017) and Choi and Mela (2019), who also model consumers that decide between inspecting and revealing more products. This paper differs in that I provide a tractable optimal policy for the decision problem, whereas these studies use simplifying assumptions and numerical methods to solve their models. The results presented in this paper can serve as a justification for some of these simplifying assumptions: Given that the optimal policy is myopic, the one-step look-ahead approach adopted by De los Santos and Koulayev (2017) yields optimal choices of search actions if monotonicity holds. Moreover, the optimal policy in the SD problem implies that as long as the consumer has not yet revealed the last alternative, it will never be optimal to go back and inspect a product that was discovered earlier if beliefs are stationary. Hence, the simplifying assumption made in Choi and Mela (2019) where consumers cannot go back and inspect a product revealed previously does not affect the estimation as it would not be optimal to do so.

Honka et al. (2017) and Morozov (2019) also consider limited awareness and assume that consumers cannot inspect products they are not aware of. However, in their models consumers cannot discover products beyond those they are initially aware of and the underlying search problem then is equivalent to directed search. Janssen and Non (2009) consider a homogeneous goods market where sellers can advertise and resolve consumers' uncertainty on whether they carry the product and the price they charge. The search problem consumers face in their model differs from the SD problem in that for sellers that advertise, no uncertainty remains. Hence, consumers decide between a known and an unknown payoff, simplifying the characterization of the optimal policy. Koulayev (2014) estimates a search model where consumers also decide whether to reveal more products, but assumes that revealing a product shows all information on that product. Hence, there is no need for inspecting a product as considered in this paper.[5]

Finally, related studies have highlighted other potential biases in search cost estimates. Jindal and Aribarg (2020) show how heterogeneous prior beliefs can lead to an overestimation

[5] Koulayev (2014) solves the dynamic decision problem using numerical backwards induction. For the case where costs are increasing in time (which is the case in his results), the present results suggest that a simple index policy also characterizes the optimal policy for his model.

of search costs, Ursu (2018) argues that that an incomplete search history also accentuates search cost estimates, whereas Yavorsky et al. (2020) discuss the effects of normalizing search benefits.

2.3 The Search and Discovery Problem

A risk-neutral consumer with unit demand faces a market offering a (possibly infinite)[6] number of products gathered in set J. Alternatives are heterogeneous with respect to their characteristics. The consumer has preferences over these characteristics which can be expressed in a utility ranking. To simplify exposition and facilitate a comparison to existing models from the consumer search literature (e.g. Armstrong, 2017; Choi et al., 2018), I assume that the consumer's ex post utility when purchasing alternative j is given by

$$u(x_j, y_j) = x_j + y_j \tag{2.1}$$

where x_j and y_j are valuations derived from two distinct sets of characteristics. Note, however, that the results presented continue to hold for more general specifications that do not rely on linear additive utility.[7] An outside option of aborting search without a purchase offering u_0 is available.

The consumer has limited information on available alternatives. More specifically, in periods $t = 0, 1, \ldots$ the consumer knows both valuations x_j and y_j only for products in a consideration set $C_t \subseteq J$. For products in an awareness set $S_t \subseteq J$, the consumer only knows partial valuations x_j. This captures the notion that if the consumer is aware of a product, he has received some information on the total valuation of the product. Finally, the consumer has no information on any other product $j \in J \setminus (S_t \cup C_t)$.

During search, the consumer gathers information by sequentially deciding which action to take starting from period $t = 0$. If the consumer decides to discover more products, n_d alternatives are added to the awareness set. If less than n_d alternatives have not yet been revealed, only the remaining alternatives are revealed. For each of the n_d alternatives,

[6] The problems with infinitely many arms in a multi-armed bandit problem discussed by Banks and Sundaram (1992) do not arise in the present setting.

[7] Specifically, suppose that when the consumer becomes aware of alternative j, he reveals a signal on the distribution from which the utility of j will be drawn. Appropriately defining the distribution of signals and the distribution of utilities conditional on these signals then yields an equivalent search problem.

the partial valuation x_j is revealed. To reveal the remaining characteristics of a product j, summarized in y_j, the consumer has to inspect the product. This reveals full information on the product and moves it from the awareness into the consideration set. The latter implies $S_t \cap C_t = \emptyset$.

The order in which products are discovered is tracked by positions $h_j \in \{0, 1, \dots\}$, where a smaller position indicates that a product is discovered earlier, and $h_j = 0$ implies either $j \in C_0$ or $j \in S_0$. Without loss of generality, it is assumed that products are discovered in increasing order of their index.[8]

Two precedence constraints on the consumer's actions are imposed. First, the consumer can only buy products from the consideration set. Second, the consumer can only inspect products from the awareness set. Whereas the first constraint is inherent in most search problems and implies that a product cannot be bought before having obtained full information on it,[9] the latter is novel to the proposed search problem. It implies that a product cannot be inspected unless the consumer is aware of it. In an online setting where a consumer browses through a list of products, this constraint holds naturally: Individual product pages are reached by clicking on the respective link on the list. Hence, unless a product has been revealed on the list, it cannot be clicked on. In other environments, this precedence constraint reflects that, unless a consumer knows whether an alternative exists, he will not be able to direct search efforts and inspect the specific alternative. For example, if a consumer is not aware of a newly released phone model, he will not be able to directly acquire detailed information before discovering it.

Given the setting and these constraints, the consumer decides sequentially between the following actions:

1. Purchasing any product from the consideration set C_t and end search.
2. Inspecting any product from the awareness set S_t, thus revealing y_j for that product and adding it to the consideration set.
3. Discovering n_d additional products, thus revealing their partial valuations x_j and adding them to the awareness set.

[8] Note that in equilibrium settings, the order may be determined by sellers' actions, requiring a careful analysis of how these will determine the consumer's beliefs. For example, in online settings it is common for sellers to bid on the position at which their product adverts are shown (see e.g. Athey and Ellison, 2011).

[9] Doval (2018) is a notable exception.

The distinction between *inspecting* and *discovering* products is novel in the SD problem. The two actions differ in three important ways. First, whereas the consumer can use product-specific information to decide the order in which to inspect products from the awareness set, the decision whether to discover more products is based solely on beliefs over products that may be discovered. Second, if $n_d > 1$, discovering products reveals information on multiple products. Finally, discovering products adds them into the awareness set, whereas inspecting a product moves it into the consideration set. In combination with the precedence constraints this implies that the actions that are available in the next period differ.[10]

These actions are gathered in the *set of available actions*, $A_t = C_t \cup S_t \cup \{d\}$, where d indicates discovery. If a consumer chooses an action $a = j \in C_t$, he buys product j, whereas if he chooses an action $a = j \in S_t$, he inspects product j. To clearly differentiate between the different types of actions, this set can also be written as $A_t = \{b0, b3, s4, \ldots, d\}$, where bj indicates purchasing and sj inspecting product j.

Both inspecting a product and discovering more products is costly. Inspection and discovery costs are denoted by $c_s > 0$ and $c_d > 0$ respectively. These costs can be interpreted as the cost of mental effort necessary to evaluate the newly revealed information, or an opportunity cost of the time spent evaluating the new information. In line with this interpretation, I assume that there is free recall: Purchasing any of the products from the consideration set does not incur costs, and c_s is the same for inspecting any of the products in the awareness set.

The consumer has beliefs over the products that he will discover, as well as the valuation he will reveal when inspecting a product j. In particular, x_j and y_j are independent (across j) realizations from random variables X and Y, where the consumer has beliefs over their joint distribution. This implies that the consumer believes that in expectation, products are equivalent. A generalization where the distribution of X depends on index j is discussed in Section 2.4. Note that throughout, capital letters are used for random variables, lower case letters are used for the respective realizations and bold letters indicate vectors.

The consumer also has beliefs over the total number of available alternatives. I assume that the consumer believes that with constant probability $q \in [0, 1]$, the next discovery will

[10] Note that the latter two points also imply that products that the consumer is not aware of cannot be modeled as a set of ex ante homogeneous products that differ in terms of beliefs and associated costs from the products in the awareness set.

be the last.[11] As shown in the next section, the optimal policy is independent of the number of remaining discoveries that may be available in the future. Note, however, that this belief specification implicitly assumes that the consumer always knows whether he can reveal n_d more alternatives. An extension presented in Section 2.4 covers the case where the consumer does not know how many alternatives will be revealed.

All information the consumer has in period t is summarized in the information tuple $\Omega_t = \langle \bar{\Omega}, \omega_t \rangle$. The tuple $\bar{\Omega} = \langle u(x,y), n_d, c_d, c_s, G_X(x), F_{Y|X=x}(y), q \rangle$ represents the consumer's knowledge and beliefs on the setting. It contains the utility function, how many products are discovered, and the different costs. It also contains the consumer's beliefs summarized in the probability q and the cumulative densities $G_X(x)$ and $F_{Y|X=x}(y)$. The latter specifies the cumulative density of Y, conditional on the realization of X, which is observed by the consumer before choosing to inspect a product. As a short-hand notation, I use $G(x)$ and $F(y)$ for these distributions. As a regularity condition, it is assumed that both $G(x)$ and $F(y) \forall x$ have finite mean and variance.

During search, the consumer reveals valuations x_j and y_j for the various products. This information is tracked in the set ω_t, containing realizations x_j for $j \in S_t \cup C_t$ and y_j for $j \in C_t$. The set of available actions A_t and the information tuple Ω_t capture the state in t. The consumer's initial information on the alternatives are captured in ω_0 which will contain (partial) valuations of products in the initial awareness and consideration set. Figure 2.2 shows their transitions starting from period $t = 0$. The depicted example assumes that there are only two alternatives available and that products are discovered one at a time. If the consumer initially chooses the outside option ($b0$), no new information is revealed, and no further actions remain. If the consumer instead reveals the first alternative, he can inspect it in $t = 1$.

2.3.1 The consumer's dynamic decision problem

The setting above describes a dynamic Markov decision process, where the consumer's choice of action determines the immediate rewards, as well as the state transitions. The state in t is given by Ω_t and A_t. As the valuations x_j and y_j can take on any (finite) real values, the state

[11] Note that one can translate beliefs over a specific number of available alternatives to this probability by assuming it varies during search. For example, if $n_d = 1$ and the consumer believes that there are 3 alternatives in total, then $q_t = 0$ when the consumer has not yet discovered the second alternative and $q_t = 1$ otherwise. A specification like this (and any specification where $q_t \leq q_{t+1} \forall t$) also satisfies the monotonicity condition (2.31) presented in Appendix 2.C. Consequently, if it is assumed that the consumer knows $|J|$, monotonicity continues to hold.

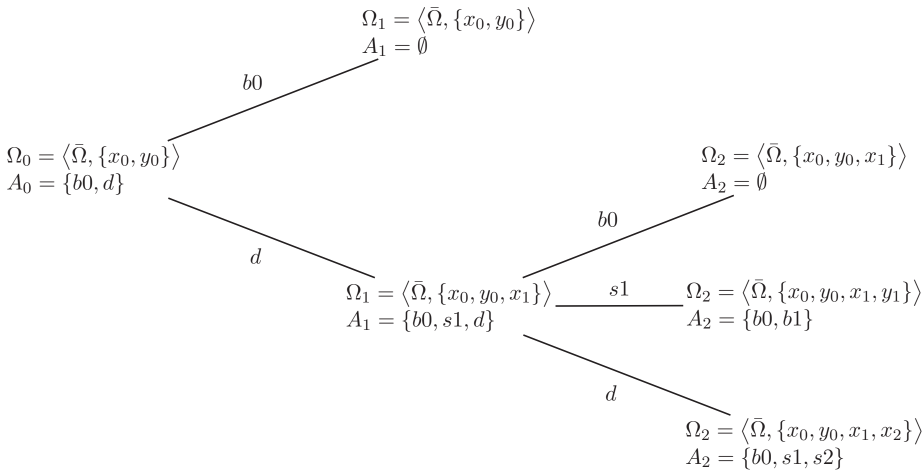

FIGURE 2.2 – Transition of state variables Ω_t (information tuple) and A_t (set of available actions) for $n_d = 1$ and $|J| = 2$.

space in general is infinite.[12] Time t itself is not included in the state; given A_t and Ω_t, it is irrelevant to the agent's choice, because beliefs (over valuations and termination of discovery) are time invariant.

The consumer's problem consists of finding a feasible sequential policy, which maximizes the expected payoff of the whole decision process. A feasible sequential policy selects an action $a_t \in A_t$ given information in Ω_t in each period t. Let Π denote the set containing all feasible policies. Formally, the consumer solves the following dynamic programming problem

$$\max_{\pi \in \Pi} V(\Omega_0, A_0; \pi) \tag{2.2}$$

where $V(\Omega_t, A_t; \pi)$ is the value function defined as the expected total payoff of following policy π starting from the state in t. Let

$$[B_a V](\Omega_t, A_t; \pi) = R(a) + \mathbb{E}_t [V(\Omega_{t+1}, A_{t+1}; \pi) | a] \tag{2.3}$$

denote the Bellman operator, where the immediate rewards $R(a)$ either are inspection costs, discovery costs, or the total valuation of a product j if it is bought. Immediate rewards $R(a)$

[12] An exception is when x_j and y_j are drawn from discrete distributions, which limits the number of possible valuations that can be observed.

therefore are known for all available actions. $\mathbb{E}_t\left[V(\Omega_{t+1}, A_{t+1}; \pi)|a\right]$ denotes the expected total payoff over the whole future, conditional on policy π and having chosen action a.[13] The expectations operator integrates over the respective distributions of X and Y. A purchase in t ends search such that $A_{t+1} = \emptyset$ and $\mathbb{E}_t\left[V(\Omega_{t+1}, \emptyset; \pi)|a\right] = 0$ whenever $a \in C_t$. The corresponding Bellman equation is given by

$$V(\Omega_t, A_t; \pi) = \max_{a \in A_t} \left[B_a V\right](\Omega_t, A_t; \pi) \tag{2.4}$$

2.4 Optimal policy

The optimal policy for the SD problem is fully characterized by three reservation values. In what follows, I first define these reservation values, before stating the main result. At the end of this section, I discuss possible extensions based on a monotonicity condition, as well as limitations.

As in Weitzman (1979), suppose there is a *hypothetical* outside option offering utility z. Furthermore, suppose the consumer faces the following comparison of actions: Immediately take the outside option, or inspect a product with known x_j and end search thereafter. In this decision, the consumer will choose to inspect alternative j whenever the following holds:

$$Q_s(x_j, c_s, z) \equiv \mathbb{E}_Y\left[\max\{0, x_j + Y - z\}\right] - c_s \geq 0 \tag{2.5}$$

$Q_s(x_j, c_s, z)$ defines the expected *myopic* net gain of inspecting product j over immediately taking the outside option. If the realization of Y is such that $x_j + y_j \leq z$, the consumer takes the hypothetical outside option after inspecting j and the gain is zero. When $x_j + y_j > z$, the gain over immediately taking the hypothetical outside option is $x_j + y_j - z$. The expectation operator $\mathbb{E}_Y[\cdot]$ integrates over these realizations.

The *search value* of product j, denoted by z_j^s, then is defined as the value offered by a hypothetical outside option that makes the consumer indifferent in the above decision problem.

[13] In this formulation of the problem, the consumer does not discount future payoffs. This is in line with the consumer search literature, which usually assumes a finite number of alternatives without discounting. However, it is straightforward to show that the results continue to hold if a discount factor $\beta < 1$ is introduced. In this case, the search and discovery values defined in the next section need to be adjusted accordingly.

Formally, z_j^s satisfies

$$Q_s(x_j, c_s, z_j^s) = 0 \tag{2.6}$$

which has a unique solution (see Lemma 1 in Adam, 2001). The search value can be calculated as

$$z_j^s = x_j + \xi \tag{2.7}$$

where ξ solves $\int_\xi^\infty [1 - F(y)]\,\mathrm{d}y - c_s = 0$ (see Appendix 2.B).

The *purchase value* of product j, denoted by z_j^b, is defined as the utility obtained when buying product j:

$$z_j^b = u(x_j, y_j) \tag{2.8}$$

Based on reservation values given by (2.6) and (2.8), Weitzman (1979) proved that in any period, it cannot be optimal to inspect a product j where $z_j^s < \max_{k \in S_t} z_k^s$, or to end search if $\max_{k \in S_t} z_k^s > \max_{k \in C_t} z_k^b$. Hence, given C_t and S_t, it is optimal buy $j \in C_t$ with the largest purchase value among products in C_t whenever $\max_{k \in C_t} z_k^b \geq \max_{k \in S_t} z_j^s$, and else search j with the largest search value among products in S_t. However, this rule does not fully characterize an optimal policy in the SD problem, as the consumer can additionally discover more alternatives.

For this additional action, a third reservation value based on a similar myopic comparison is introduced. Suppose the consumer faces the following comparison of actions: Take a hypothetical outside option offering z immediately, or discover more products and then search among the newly revealed products. The consumer will choose the latter whenever the following holds:

$$Q_d(c_d, c_s, z) \equiv \mathbb{E}_{\boldsymbol{X}} \left[V \left(\left\langle \bar{\Omega}, \omega(\boldsymbol{X}, z) \right\rangle, \{b0, s1, \ldots, sn_d\}\,; \tilde{\pi} \right) \right] - z - c_d \geq 0 \tag{2.9}$$

where $\omega(\boldsymbol{X}, z) = \{z, x_1, \ldots, x_{n_d}\}$ denotes the information the consumer has after revealing the n_d more alternatives and $\tilde{\pi}$ is the policy that optimally inspects the n_d discovered products. Note that with some abuse of notation, product indices were adjusted to the reduced decision problem, such that $j = 0, 1, \ldots, n_d$ indicates the hypothetical outside option and the newly revealed products.

$Q_d(c_d, c_s, z)$ defines the *myopic* net gain of discovering more products and optimally searching among them over immediately taking the outside option. It is myopic in the sense that it ignores the option to continue searching beyond the products that are discovered. In particular, note that $V\left(\left\langle\bar{\Omega},\omega(\boldsymbol{X},z)\right\rangle,\{b0,s1,\ldots,sn_d\};\tilde{\pi}\right)$ is the value function of having an outside option offering z and optimally inspecting alternatives for which partial valuations in $\boldsymbol{X}$ are known. Possible future discoveries and any products in S_t or C_t are excluded from the set of available actions in this value function. This implies that the discovery value does not depend on the consumer's beliefs over whether the next discovery will be the last. Finally, $\mathbb{E}_{\boldsymbol{X}}[\cdot]$ defines the expectation operator integrating over the joint distribution of the partial valuations in $\boldsymbol{X}$. Formal details on the calculation of the expectations and the value function are provided in Appendix 2.B.

As for the search value, let the *discovery value*, denoted by z^d, be defined as the value of the hypothetical outside option that makes the consumer indifferent in the above decision. Formally, z^d is such that

$$Q_d(c_d,c_s,z^d)=0 \tag{2.10}$$

which has a unique solution. In the case where Y is independent of X, the discovery value can be calculated as

$$z^d=\mu_X+\Xi(c_s,c_d) \tag{2.11}$$

where μ_X denotes the mean of X and $\Xi(c_s,c_d)$ solves (2.10) for an alternative random variable $\tilde{X}=X-\mu_X$. Further details for the calculation are provided in Appendix 2.B.

Theorem 1 provides the first main result. It states that the optimal policy for the search problem reduces to three simple rules based on a comparison of the search, purchase and discovery values. In particular, the rules imply that in each period t, it is optimal to take the action with the largest reservation value defined in (2.6), (2.8), and (2.10). Hence, despite being fully characterized by myopic comparisons to a hypothetical outside option, these reservation values rank the expected payoffs of actions over all future periods.

Theorem 1. *Let $\tilde{z}^b(t)=\max_{k\in C_t}u(x_k,y_k)$ and $\tilde{z}^s(t)=\max_{k\in S_t}z^s_k$ denote the largest search and purchase values in period t. An optimal policy for the search and discovery problem is characterized by the following three rules:*

- Stopping rule: *Purchase $j\in C_t$ and end search whenever $z^b_j=\tilde{z}^b(t)\geq\max\left\{\tilde{z}^s(t),z^d\right\}$.*

- Inspection rule: *Inspect $j \in S_t$ whenever $z_j^s = \tilde{z}^s(t) \geq \max\left\{\tilde{z}^b(t), z^d\right\}$.*

- Discovery rule: *Discover more products whenever $z^d \geq \max\left\{\tilde{z}^b(t), \tilde{z}^s(t)\right\}$.*

The proof of Theorem 1 relies on results from the literature on multi-armed bandit problems, specifically the branching bandits framework of Keller and Oldale (2003). These authors show that in a multi-armed bandit problem where taking an action branches off into new actions, a Gittins index policy is optimal. Importantly, as an action branches off, it cannot be taken again in its original state. This ensures that available actions are independent in the sense that taking one does not alter the state of any other available action. The imposed precedence constraints combined with the fact that the consumer cannot discover a product for a second time imply the same branching structure in the SD problem, and the results of Keller and Oldale (2003) therefore imply that a Gittins index policy is optimal. Introducing a monotonicity condition I then show that the Gittins index is equivalent to the simple reservation values defined above.

Based on Theorem 1, optimal search behavior can be analyzed using only (2.6), (2.8) and (2.10). Weitzman (1979) showed that search values decrease in inspection costs and increase if larger realizations y_j become more likely through a shift in the probability mass of Y. The same applies to the discovery value. It decreases in discovery costs and increases if probability mass of X is shifted towards larger values. The discovery value also depends on inspection costs and the conditional distribution of Y through the value function; it decreases in inspection costs and increases if larger values of Y are more likely.

To see the latter, consider the case where alternatives are discovered one at a time. In this case, the myopic net gain of discovering more products reduces to

$$Q_d(c_d, c_s, z) = \mathbb{E}_X\left[\max\left\{0, Q_s(X, c_s, z)\right\}\right] - c_d \tag{2.12}$$

For any $c_s' > c_s$, it holds that $Q_s(x, c_s', z) \leq Q_s(x, c_s, z)$ for all finite values of x and z, implying that $Q_d(c_d, c_s', z) \leq Q_d(c_d, c_s, z)$ for all z. As $Q_d(c_d, c_s, z)$ is decreasing in z (see Appendix 2.A), it follows that the respective discovery values satisfy $z^{d\prime} \leq z^d$.

The optimal policy being fully characterized by simple rules leads to straightforward analysis of optimal choices for any given awareness and consideration sets. For example, consider a period t where $\max\left\{z^d, \tilde{z}^s(t)\right\} < \tilde{z}^b(t)$ such that the consumer stops searching. When decreasing inspection costs sufficiently in this case, the inequality reverts and the consumer will instead either first discover more products, or inspect the best product from the awareness set.

2.4.1 Monotonicity and extensions

For the reservation value policy of Theorem 1 to be optimal, the discovery value needs to fully capture the expected net benefits of discovering more products, including the option value of being able to continue discovering products. The monotonicity condition used in the proof of the theorem ensures that this holds. It states that the expected net benefits of discovering more products do not increase during search. Hence, whenever the consumer is indifferent between taking the hypothetical outside option and discovering more products in t, he will either continue to be indifferent or take the outside option in $t+1$. Whether the consumer can continue to discover products in $t+1$ thus does not affect expected net benefits in t, and the discovery value fully captures the expected net benefits.[14]

In the baseline SD problem, several assumptions directly imply that the monotonicity condition holds. Specifically, (i) the consumer believes that product valuations are independent and identically distributed, (ii) q remains constant and (iii) n_d is known. However, these assumptions can be relaxed to capture a wider range of settings. Below, three related extensions are presented. Formal results and further details are presented in Appendix 2.C.

Ranking in distribution: In some settings, the consumer' beliefs are such that the distribution of partial valuations depends on the position at which a product is discovered. Monotonicity will be satisfied if beliefs are such that the mean of X_j decreases in a product's position h_j, or more generally if beliefs are such that X_j first-order stochastically dominates X_k if $h_j \leq h_k$. The optimal policy then continues to be characterized by Theorem 1, the only difference being that the discovery value is based on the position-specific beliefs and decreases during search, making it optimal to recall products in some cases. This could result in a market environment where sellers of differentiated products compete in marketing efforts for

[14] For the search and purchase values, no monotonicity condition is required. This follows from the fact that in the independent comparison to the hypothetical outside option, both actions do not provide the option to continue searching. After buying a product, search ends, and after having inspected a product, the only option that remains is to either buy the product or choose the hypothetical outside option. Consequently, for inspection and purchase, at most one future period needs to be considered to fully capture the respective net benefits over immediately taking the outside option.

consumers to become aware of their products early on. If sellers offering better valuations have a stronger incentive to be discovered first, they will increase marketing efforts.[15] Consumers' beliefs then will reflect this ordering such that monotonicity holds and the simple optimal policy can be used to characterize equilibria. Similarly, online stores often use algorithms to first present products that consumers may like more. This again satisfies monotonicity such that the tractable optimal policy can be used to rationalize search behavior in click-stream data from such stores.

Unknown n_d: In other environments, a consumer may not know how many alternatives he will discover. For example, a consumer may believe that there are still alternatives he is not aware of and thus try to discover them, only to realize that he already is aware of all the available alternatives. In such cases, a belief over how many alternatives are going to be discovered needs to be specified. The reservation value policy continues to be optimal if these beliefs are such that monotonicity is satisfied. This will be the case if beliefs are constant, or if (more realistically) the consumer expects to discover fewer alternatives the more alternatives he already has discovered.[16] The only difference to the baseline is that in $Q_d(c_d, c_s, z)$, expectations are additionally based on beliefs over how many alternatives will be revealed.

Multiple discovery technologies: Consumers may also have multiple discovery technologies at their disposal. In an online setting, for example, each technology may represent a different online shop offering alternatives. Moreover, advertising measures may separate products into different product pools. In such settings, the consumer also decides which technology to use to discover more alternatives. By assigning each of the discovery technologies a different discovery value, the optimal policy can be adjusted to accommodate this case.[17]

[15] See, for example, the discussion on non-price advertising and the related references cited in Armstrong (2017).

[16] This would reflect the case where the consumer expects it to become harder to discover alternatives the fewer alternatives have not yet been discovered. Alternatively, this could be modeled as either q or c_d to increase with each discovery, which also satisfies monotonicity.

[17] An interesting extension for future research is to model the case where a consumer can choose the order in which products are revealed based on a product characteristic such as price. This requires modeling beliefs that reflect this ordering through updating the support of the price distribution; in an ascending order the minimum price that can be discovered needs to increase with every discovery. Chen and Yao (2017) incorporate choices of such search refinements in their empirical model. However, in their model, a consumer simultaneously decides on the refinement and which position to inspect. In contrast, if such choices are modeled as a SD problem the consumer would sequentially decided between a discovery technology and whether to inspect a product. This is more closely done by De los Santos and Koulayev (2017), who also model sequential choice of search refinements and clicks, but use simplifying assumptions and do not derive the optimal policy.

2.4.2 Limitations

Though the optimal policy applies to a broad class of search problems, two limitations exist. The first is that in the dynamic decision process, all available actions need to be independent of each other; performing one action in t should not affect the payoff of any other action that is available in t. This is required to guarantee that the reservation values fully capture the effects of each action. Recall that each reservation value does not depend on the availability of other actions. If independence does not hold, however, the availability of other actions also influences the expected payoff of an action. Choosing actions based only on reservation values that disregard these effects therefore will not be optimal. Alternative search problems that violate this independence assumption are presented in the appendix.

The second limitation is that the monotonicity condition discussed above needs to hold for the discovery value to be based on *myopic* net benefits. If this condition does not hold, then the discovery value does not fully capture the expected net benefits of discovering more products. However, as long as independence of the available actions is satisfied, a Gittins index policy remains optimal (see proof of Theorem 1). Hence, the optimal policy when monotonicity fails consists of comparing the search and purchase values from equations (2.7) and (2.8) with the Gittins index value for discovery that explicitly accounts for future discoveries.

One interesting case where this fails is if the consumer learns about the distribution of X or the number of alternatives he will discover during search. So far, it was assumed that independent of the information the consumer reveals during search, his beliefs remain unchanged. This will be the case if either the consumer has rational expectations and hence knows the underlying distributions, or simply does not update beliefs. With learning, the consumer updates his beliefs based on partial valuations or number of products revealed in a discovery.

Similar learning models have been studied in the context of classic search (and stopping) problems, where the consumer learns about the distribution he is sampling from (e.g. Rothschild, 1974; Rosenfield and Shapiro, 1981; Bikhchandani and Sharma, 1996; Adam, 2001). [18] Whereas these studies determine prior beliefs or learning rules such that the optimal policy is based on myopic reservation values, similar conditions do not guarantee that monotonicity holds in a SD problem where a consumer learns about the distribution of X or the number of

[18] The SD problem is equivalent to these learning problems in the case where $c_s = 0$ and the consumer updates beliefs about the distribution of the random variable $X + Y$.

alternatives he will discover. The reason is that in classic search problems a consumer reveals full information when inspecting a product. Hence, if a product turns out to be a good match, the value of stopping increases along with the value of continuing search, where the learning rule guarantees that this is such that the expected net benefits of continuing search over stopping with the current best option weakly decrease with each inspection.[19] In contrast, in the SD problem, discovering either more or better partial valuations does not necessarily increase the value of the best option in the consideration set.[20] For example, the consumer can discover many products that look very promising based on partial valuations, but after inspection realize that these products are a bad match after all. In this case, the value of stopping remains the same, whereas beliefs are shifted such that the consumer expects to find better or more products in future discoveries.

Extending the SD problem to the case where the consumer learns about the distribution of X or the number of alternatives therefore comes at the cost of losing tractability of the discovery value; a tractable expression for the Gittins index value for the discovery action (henceforth denoted by z_t^L) is difficult to obtain as it is necessary to determine the value function of a dynamic decision process that includes many future periods. Moreover, whereas the discovery value in Theorem 1 remains constant throughout search, z_t^L changes whenever the consumer updates beliefs. Consequently, the optimal policy when the consumer updates beliefs becomes more complex in that the discovery value changes with each discovery and explicitly includes future periods.

Whereas z_t^L is not tractable and computationally expensive to obtain, it is possible to derive bounds on this value that are easier to compute and can serve as an approximation. First, z_t^L can be approximated from below through k-step look-ahead values. The 1-step look-ahead value is defined by (2.10), where the expectation operator is adjusted to account for the consumer's beliefs in t. As k increases, more future discoveries are considered in (2.10), leading to a more precise approximation of z_t^L up to the point where z_t^L is calculated precisely. Second, a result of Kohn and Shavell (1974) can be used to derive an upper bound. These authors show that the expected value of continuing search when the consumer fully resolves

[19] See e.g. Theorem 1 in Rosenfield and Shapiro (1981).

[20] If the consumer learns about the distribution of Y conditional on X, then discovering more alternatives with similar X can increase the value of the best option. Analyzing this mechanism provides an interesting avenue for future research.

uncertainty on the underlying distributions in the next period exceeds the true continuation value in a classic search problem where a consumer samples from an unknown distribution. The same logic directly applies in the extension to the SD problem and the upper bound then can be computed using the results provided in the next section. A formal treatment of these bounds is provided in the appendix.

2.5 Eventual purchases, consumer's Payoff, and demand

In an environment where consumers sequentially inspect products, a consumer's expected payoff and the market demand result from integrating over different possible choice sequences leading to eventual purchases. Conceptually, this poses a major challenge, as the number of possible choice sequences grows extremely fast in the number of available alternatives.[21]

Theorem 2 allows to circumvent this difficulty. It states that the purchase outcome of a consumer solving the search problem is equivalent to a consumer directly buying a product that offers the highest *effective value*. Importantly, a product's effective value does not depend on the various possible choice sequences leading to its purchase.

Theorem 2. *Let*

$$w_j \equiv \begin{cases} u_j & \text{if } u_j < z^d \text{ and } j \in C_0 \\ \tilde{w}_j & \text{if } \tilde{w}_j < z^d \text{ or } j \in S_0 \\ z^d + f(h_j) + \varepsilon \tilde{w}_j & \text{else} \end{cases}$$

be the effective value for product j revealed on position h_j where $\tilde{w}_j \equiv \min\{z_j^s, z_j^b\} = x_j + \min\{\xi, y_j\}$, $f(h_j)$ is a non-negative function and strictly decreasing in h_j and ε is an infinitesimal. The solution to the search and discovery problem with initial consideration set C_0 and awareness set S_0 leads to the eventual purchase of the product with the largest effective value.

This result is based on and generalizes the "eventual purchase theorem" of Choi et al. (2018) (and independently Armstrong, 2017; Kleinberg et al., 2017) to the case where the consumer has limited awareness. The value $\tilde{w}_j$ used in the theorem is equivalent to the effective value

[21] For example, with only one alternative and an outside option, there are four possible choice sequences. With two alternatives, the number of possible choice sequences increases to 20, and with three alternatives, there are already more than 100 possible choice sequences.

defined by Choi et al. (2018), and the proof follows the same logic; as a product (incl. out outside option) is always bought, the proof only needs to establish that the optimal policy never prescribes to buy a product that does not have the largest effective value.

The generalization to the case of limited awareness follows from the following implication of the optimal policy: Whenever both the inspection and the purchase value of a product in the awareness set exceed the discovery value, the consumer will buy the product and end search. Hence, when $\tilde{w}_j \geq z^d$, the consumer never discovers products on positions beyond h_j. This is captured in the effective values by the term $z^d + f(h_j)$, which ranks alternatives based on when during search they are discovered, yielding a larger effective value if a product is discovered earlier. The infinitesimal in the last condition additionally is necessary to rank products that are revealed on the same position. Suppose we have $\tilde{w}_j > \tilde{w}_k \geq z^d$ for two products discovered on the same position. Without the infinitesimal, the effective value would be $w_j = w_k$, implying the consumer would be indifferent between buying either of the two products. This contrasts the optimal policy, which for $\tilde{w}_j > \tilde{w}_k$ will never prescribe to buy k if both j and k are in the awareness set. If $n_d = 1$, the infinitesimal is not required.

The result continues to hold for extensions of the SD problem, as long as the discovery values are predetermined. The only difference then is that in the effective value of an alternative j, the discovery value depends on the position at which j is revealed.

2.5.1 Expected payoff

Based on these results, it is now possible to derive a simple characterization of a consumer's expected payoff, as summarized in Proposition 2.1. In this expression, the expected payoff does not explicitly depend on inspection and discovery costs; they affect the expected payoff only through the discovery and search values. As the proof shows, this follows from the definition of these values, which relate expected payoffs and costs (as in Choi et al., 2018). Based on this characterization, it is only necessary to derive the distribution of the effective values without having to explicitly consider different choice sequences. Note also that as the effective value is adjusted, the expected payoff does not depend on the choice of function $f(h)$ which ranks alternatives based on their position in the effective value.

Proposition 2.1. *A consumer's expected payoff in the SD problem is given by*

$$V(\Omega_0, A_0; \pi) = \mathbb{E}_{\hat{\boldsymbol{W}}}\left[\max_{j\in J} \hat{W}_j\right]$$

where $\mathbb{E}_{\hat{\boldsymbol{W}}}[\cdot]$ *integrates over the distribution of* $\hat{\boldsymbol{W}} = \left[\hat{W}_0, \ldots, \hat{W}_{|J|}\right]'$, *with* $\hat{w}_j$ *being the effective value adjusted with* $\hat{w}_j = u_j \forall j \in C_0, \hat{w}_j = \tilde{w}_j \forall j \in S_0$, *and* $f(h_j) = \varepsilon = 0 \forall h_j$. *If* $|J| = \infty$, $V(\Omega_0, A_0; \pi) = z^d$.

Whereas it is clear that making either inspection or discovery easier leads to an increase in the expected payoff, it is not obvious which of these two changes is more beneficial for a consumer. For the case where $n_d = 1$, Proposition 2.2 shows that if the number of alternatives exceeds some threshold, then the consumer benefits more from facilitating the discovery of additional products.[22]

Proposition 2.2. *If* $n_d = 1$, *there exists a threshold* n^* *such that whenever* $|J| > n^*$, *a consumer benefits more from a decrease in discovery costs than a decrease in inspection costs. This threshold decreases in the value of the alternatives in the initial consideration and awareness set.*

Whereas the proof is more involved, the intuition is that when there are only few alternatives available, the consumer is more likely to first discover all alternatives and then start inspecting alternatives. Hence in expectation, he pays the inspection costs relatively often and a reduction in inspection costs will be more beneficial. Similarly, when the value of the outside option is large, the consumer is likely to inspect fewer of the products he discovers, leading to relatively small benefits of a reduction in inspection costs.

For settings where $n_d > 1$, it becomes difficult to obtain similarly general results. In particular, for some distributions and n_d, it is possible that decreasing inspection costs increases the discovery value z^d by more than decreasing the discovery costs by the same amount. In such cases, the consumer will benefit more from making inspection less costly. Nonetheless, the general intuition remains the same in such settings; a reduction in inspection costs is more beneficial, the more likely it is that the consumer inspects relatively many alternatives.

[22] Note that this threshold can be zero. For example, this is the case when $u_0 = 0$, $c_s = 0.1$ and $c_d = 0.1$, and the valuations are drawn from standard normal distributions.

2.5.2 Market demand

Using Theorem 2, it is straightforward to derive a market demand function when heterogeneous consumers optimally solve the SD problem. In particular, let the effective value w_{ij} for each consumer i be a realization of the random variable W_j and gather the random variables in $\boldsymbol{W} = \left[W_0, \ldots, W_{|J|}\right]'$. For a unit mass of consumers the market demand for a product j then is given by

$$D_j = \mathbb{E}_h \left[\mathbb{P}_{\boldsymbol{W}} \left(W_j \geq W_k \forall k \in J \backslash j\right)\right] \tag{2.13}$$

where the expectations operator $\mathbb{E}_h [\cdot]$ integrates over all permutations of the order in which products are discovered by a consumer.

As the effective value decreases in the position at which a product is discovered, (2.13) reveals that the demand for a product depends on the probability of each position at which it is displayed. Specifically, the demand for a product exhibits ranking effects; products that are more likely to be discovered early are more likely to be bought. As discussed in detail in the next section, this follows from the structure of the SD problem. As search progresses, it becomes less likely that a consumer has not yet settled for an alternative; hence, fewer consumers become aware of products that would be revealed later, leading to a lower demand for such products.

2.6 Comparison of search problems

To highlight implications of limited awareness and how the SD problem differs from existing approaches, I compare it with the two classical sequential search problems; directed search as in Weitzman (1979) and random search as in McCall (1970). Both these search problems are nested within the SD problem. Directed search results if the consumer initially has full awareness (i.e. $S_0 = J$) such that the consumer knows all partial valuations prior to search and does not need to discover products. Random search results if discovering a product reveals full information on this product, hence the consumer always both inspects and discovers a product, precluding him to use partial product information to only inspect promising products.[23]

For clarity, I focus the comparison on the case where products are discovered one at a

[23] Directed search also results if discovery costs are zero such that the consumer first discovers all products and only then starts inspecting, whereas random search also results if inspection costs are zero and the consumer inspects any products he discovers.

time ($n_d = 1$) and where the consumer initially only knows an outside option ($S_0 = \emptyset$). Furthermore, valuations x_j and y_j are assumed to be realizations of mutually independent random variables X and Y, where the consumer has rational expectations such that beliefs are correct. Assumptions specific to each search problem are described below.

Search and Discovery (SD): The consumer searches as described in Section 2.3, incurring inspection costs c_s and discovery costs c_d. Without loss of generality, I assume that the consumer discovers products in increasing order of their index, making subscripts for position h and product j interchangeable.

Random Search (RS): When discovering a product j, the consumer reveals both x_j and y_j; hence does not have to pay a cost to inspect the product. Costs to reveal this information are given by c^{RS}. In this case, the consumer optimally stops and buys product j if $x_j + y_j \geq z^{RS}$. The reservation value is given by $z^{RS} = \mu_X + \mu_Y + \tilde{\xi}$, where $\tilde{\xi}$ is the same as in (2.7) but defined over the joint distribution of demeaned X and Y. Products are discovered in the same order as in SD. Furthermore, I assume $u_0 < z^{RS}$ to ensure a non-trivial case.

Directed Search (DS): The consumer initially observes $x_j \forall j$, based on which he chooses to search among alternatives following Weitzman's (1979) reservation value policy. Costs to inspect product j are given by a function $c_j^{DS} = v_{DS}(c_s, h_j)$, where c_s are baseline costs that are adjusted for the position through a function $v_{DS} : \mathbb{R}_+^2 \to \mathbb{R}_+$ which is assumed to be strictly increasing in a product's position h_j. As costs vary across products, reservation values are given by $z_j^s = x_j + \xi_j$, where ξ_j is the same as in (2.7) with product-specific inspection costs. The assumption on $v_{DS}(c_s, h_j)$ implies that ξ_j decreases in j. I impose this functional form restriction as otherwise the DS problem does not generate similar patterns, as discussed in Section 2.6.2.

2.6.1 Stopping decisions

In search settings, consumers' stopping decisions determine which products consumers consider and buy. Stopping decisions therefore shape how firms compete in prices, quality or for being discovered early during search. Hence, comparing stopping decisions across the different search problems provides important insights on how well existing approaches are able to capture the more general setting where consumers are not aware of all alternatives and use partial information to determine whether to inspect products.

In the SD problem, a consumer always stops search at a product k whenever the product is both promising enough to be inspected and offers a large enough valuation to not make it worthwhile to continue discovering more products. Formally, this is given by the condition $x_k + \min\{y_k, \xi\} \geq z^d$. The probability that a consumer will stop searching before discovering product j therefore is given by

$$\mathbb{P}_{\boldsymbol{X},\boldsymbol{Y}}(X_k + \min\{Y_k, \xi\} \leq z^d \forall k < j) = 1 - \mathbb{P}_{X,Y}(X + \min\{Y, \xi\} \leq z^d)^{j-1} \tag{2.14}$$

Similarly, in the RS problem, a consumer will always stop search at a product k whenever $x_k + y_k \geq z^{RS}$, hence the probability of stopping search before discovering product j is given by

$$\mathbb{P}_{\boldsymbol{X},\boldsymbol{Y}}(X_k + Y_k \leq z^{RS} \forall k < j) = 1 - \mathbb{P}_{X,Y}(X + Y \leq z^{RS})^{j-1} \tag{2.15}$$

In both search problems, a consumer may stop search before discovering a product j. Consequently, stopping decisions in the SD and the RS problem imply the same feature: Products that a consumer initially has no information on may never be discovered and bought, independent of how the consumer values them.

However, as the consumer has the option of not inspecting products with low partial valuations, stopping probabilities differ. In particular, in the case where the total cost to reveal all information about a product are the same, stopping probabilities are smaller in the SD problem. This is highlighted in Proposition 2.3 and follows from the fact that not having to inspect alternatives with small partial valuations allows to save on inspection costs. This increases the expected benefit of discovering more products, which implies a smaller probability of search stopping, and that on average, more products will be discovered in the SD problem.

Proposition 2.3. *If costs in the RS problem are given by $c^{RS} = c_s + c_d$, a consumer on average ends search at earlier positions in the RS than in the SD problem.*

In contrast, stopping decisions are different in the DS problem. As the consumer initially knows of the existence of all products and can order them based on partial information, there is no stopping decision in terms of discovering products. Instead, the consumer directly compares all partial valuations and the different inspection costs, based on which he decides the order in which to inspect products. Hence, he can directly inspect highly valued products even when they are presented at the last position.

This difference arises from the different assumptions on consumers' initial information and is paramount in the analysis of search frictions. Consider an equilibrium setting where horizontally differentiated alternatives are supplied by firms that compete by setting mean partial valuations (e.g. by setting prices as in Choi et al., 2018). If consumers are aware of all alternatives and search as in the DS problem, all firms will compete directly with each other. In contrast, in a SD problem, the firm that is discovered first initially competes only with the option of discovering potentially better products. This difference is further illustrated in Appendix 2.G, and as it determines how firms compete, will lead to different equilibrium dynamics.[24]

2.6.2 Ranking effects

The above analysis already suggests that the demand structure differs across the three search problems. To provide further details, I focus on a particular pattern that is generated by all three search problems: Market demand for a product decreases in its position. Such ranking effects are important as they determine how fiercely sellers compete for their products to be revealed on early positions, for example through informative advertising or position auctions (e.g. Athey and Ellison, 2011). Furthermore, they have received considerable attention in the marketing literature, which has produced ample empirical evidence that suggests their importance in online markets (e.g. Ghose et al., 2014; De los Santos and Koulayev, 2017; Ursu, 2018).

To compare the mechanism producing ranking effects across the search problems, I use the following definition: The ranking effect for a product is the difference in market demand of the product being revealed at position h and at $h+1$, with the corresponding exchange of the product previously revealed at position $h+1$. Formally, this is given by

$$r_k(h) \equiv d_k(h) - d_k(h+1) \tag{2.16}$$

[24] To give an example, Anderson and Renault (1999) and Choi et al. (2018) model a similar environment, with the difference that in the former, consumers initially are not aware of any alternatives, whereas in the latter they observe prices (i.e. sellers' choices) of all alternatives prior to search. Because of this difference, decreasing inspection costs lowers the equilibrium price in a symmetric equilibrium in the former, whereas the opposite holds in the latter environment. Haan et al. (2018) provide a detailed discussion of this difference.

where $d_k(h)$ denotes the market demand for a product when revealed at position h in search problem $k \in \{SD, RS, DS\}$. For clarity, product specific subscripts are either omitted or exchanged with position subscripts in the following. The former is feasible as effective values are assumed to be independent realizations of a random variable W.

To investigate ranking effects, it is first necessary to derive the market demand at a particular position h. For a unit mass of consumers with independent realizations of effective values, it is given by

$$\begin{aligned} d_{SD}(h) = \mathbb{P}_W(W < z^d)^{h-1}\Big[&\mathbb{P}_W(W \geq z^d) \\ &+ \mathbb{P}_W(W < z^d)^{|J|-(h-1)}\mathbb{P}_{\boldsymbol{W}}(W \geq \max_{k \in J} W_k | W_k < z^d \forall j)\Big] \end{aligned} \quad (2.17)$$

The expression follows from Theorem 2 which implies that if a consumer discovers a product with $w_j \geq z^d$, he will stop searching and buy a product j. The consumer will only discover and have the option to buy a product on position h if $w_j < z^d$ for all products on better positions. In contrast, when $w_j < z^d$, the consumer will first discover more products, and only recall j if he discovers all products and j is the best among them.

In the latter case, a product's position does not affect market demand; once all products are discovered, products are equivalent in terms of their inspection costs and the order in which they are inspected is only determined based on partial valuations. This implies that the ranking effect in the SD problem is independent of the number of alternatives and simplifies to

$$r_{SD}(h) = \mathbb{P}_W\left(W \geq z^d\right)\left[\mathbb{P}_W(W < z^d)^{h-1} - \mathbb{P}_W(W < z^d)^h\right] \quad (2.18)$$

This expression reveals that the ranking effect in the SD problem solely results from the difference in the probability of a consumer reaching positions h or $h+1$ respectively. Besides the distribution of valuations and the inspection and discovery costs, Proposition 2.4 shows that the ranking effect is determined by the position h to which the product is moved. When h is large, fewer consumers will not have already stopped searching before reaching h. Hence, the later a product is revealed, the smaller is the increase in demand when moving one position ahead.

The demand in a random search problem is derived similarly. In RS, a consumer will only be able to buy a product if he has not stopped searching before, which requires that $x+y < z^{RS}$ for all products on better positions. Furthermore, a consumer will also only recall a product if he has inspected all alternatives. Similar to the SD problem, this implies that the ranking effect in the RS problem is given by

$$r_{RS}(h) = \mathbb{P}_{X,Y}\left(X+Y \geq z^{RS}\right)\left[\mathbb{P}_{X,Y}\left(X+Y < z^{RS}\right)^{h-1} - \mathbb{P}_{X,Y}\left(X+Y < z^{RS}\right)^{h}\right] \quad (2.19)$$

Comparing (2.18) with (2.19) reveals that ranking effects in the RS problem are produced by the same mechanism as in the SD problem. In both search problems; fewer consumers buy products at later positions due to the increasing the probability of having stopped searching before discovering these products. It follows that in both search problems, ranking effects decrease in the position and are independent of the total number of alternatives.

Though their extent generally differs, Proposition 2.4 additionally shows that at later positions, ranking effects will be larger in the SD problem. The result is a direct implication of Proposition 2.3; as a consumer is more likely to reach a product at a later position in the SD problem, ranking effects at later positions will be larger.

Proposition 2.4. *The ranking effect in both the SD and the RS problem decreases in position h and is independent of the number of alternatives. Furthermore, if $c^{RS} = c_s + c_d$, there exists a threshold h^* such that $r_{SD}(h) \geq r_{RS}(h)$ for all $h > h^*$.*

Given the different stopping decisions, ranking effects in directed search do not result from consumers having stopped searching before reaching products revealed at later positions. Instead, they result from differences in the cost of inspecting products at different positions. To see this, write the ranking effect in the DS problem as[25]

$$r_{DS}(h) = \mathbb{E}_{\tilde{W}_h}\left[\prod_{k\neq h}\mathbb{P}(\tilde{W}_k \leq \tilde{W}_h)\right] - \mathbb{E}_{\tilde{W}_{h+1}}\left[\prod_{k\neq h+1}\mathbb{P}(\tilde{W}_k \leq \tilde{W}_{h+1})\right] \quad (2.20)$$

This expression reveals that the ranking effect results from two sources in the DS problem. First, by moving a product j one position ahead, the product previously on position h is now

[25] Alternatively, ranking effects could be modeled in a DS problem by assuming that the consumer initially has full information on some products. In this case, the model effectively has only 2 positions (full and partial information), and hence would not be able to explain the decrease in demand across all positions resulting from the SD problem.

more costly to inspect, making it more likely that j is bought for any $\tilde{w}_j$. Second, by making it less costly to inspect j, the distribution of $\tilde{w}_j$ shifts such that larger values $\tilde{w}_j$ become more likely.

In contrast to RS and SD, the ranking effect in the DS problem depends on the number of available alternatives. In RS and SD, ranking effects result from the decreasing probability of a consumer having stopped searching before reaching a particular position, which does not depend on how many alternatives there are in total. In DS, however, a consumer directly compares all alternatives based on partial valuations. Adding more alternatives thus will affect the demand on each position.

Specifically, Proposition 2.5 shows that ranking effects in the DS problem will be smaller if there are many alternatives. The reason is that as the number of alternatives increases, each product is less likely to be bought and differences in the position-specific market demand decrease. Note, however, that in cases where the probability of consumers buying products on the last positions is very small or exactly zero (e.g. when inspection costs are large), adding more alternatives will not affect ranking effects in the DS problem.

Proposition 2.5. *The ranking effect in the DS problem is weakly decreasing in the number of alternatives.*

A second difference to the RS and SD problems is that the ranking effect does not necessarily decrease in position. This is possible as there are two counteracting channels through which position affects the ranking effect in a DS problem. First, as there is lower demand for products at later positions, differences between them will be smaller. Second, if $v_{DS}(c_s, h)$ is such that ξ_h decreases in h at an increasing rate, the difference in the purchase probability at h instead of at $h + 1$ increases in the position. When the latter dominates, the ranking effect will first increase in position.

The above comparison highlights that the mechanism producing ranking effects in the DS problem is distinct from the one in the SD and RS problems, leading to a different demand structure. In the former, ranking effects result from differences in inspection costs relative to differences in partial valuations. Hence, a better partial valuation is a substitute for moving positions ahead. In contrast, in a SD or RS problem, a product's large partial valuation does not affect consumers that stop search before discovering it. Hence, offering a larger partial valuation does not substitute for being discovered early in a SD or RS problem.[26]

Moreover, the size of ranking effects determines how important it is for products to be revealed on an early position. As ranking effects are independent of the number of alternatives in SD and RS, so are sellers' incentives to have their products revealed early during search. In contrast, in DS, the demand increase of moving positions ahead becomes smaller when the number of alternatives increases. Hence, sellers can have smaller incentives to be revealed on early positions when there are many, relative to when there are only few alternatives.

Finally, the above comparison between the number of alternatives and ranking effects also suggests the existence of an empirical test to distinguish the search modes in some settings. If data is available that allows to test whether ranking effects depend on the number of alternatives, then it will be possible to empirically determine whether a DS problem, instead of a RS or SD problem provides a framework that better captures ranking effects in a particular setting. Furthermore, if data is available that allows to test whether a product's partial valuation has an effect on whether it is inspected, it will be possible to distinguish between RS and SD.

2.6.3 Expected payoff

If costs are specified such that the total costs of revealing all product information remain the same, then the three search problems differ only in the information the consumer can use during search. A comparison of a consumer's expected payoff based on such a specification therefore provides some insight into whether it is always to the consumer's benefit to provide information that helps to direct search towards some alternatives.

For total costs of revealing full information about a product on position h to be the same in the three search problems, inspection costs in the RS and DS problem are specified as $c^{RS} = c_s + c_d$ and $c_j^{DS} = c_s + h_j c_d$ respectively.

The SD problem extends the RS problem by additionally providing the consumer with the option to not inspect products depending on their partial valuations. This allows the consumer to save on inspection costs by not inspecting products with small partial valuations. As stated in Proposition 2.6, this increases the expected payoff which implies that providing product information across two layers, as done for example by online retailers or search intermediaries,

[26] Note, however, that in an equilibrium setting, offering larger partial valuations may indirectly serve as a substitute for being discovered early by raising consumers' expectations and induce them to search longer.

is beneficial for consumers.

Proposition 2.6. *If $c^{RS} = c_s + c_d$, then a consumer's expected payoff in the SD problem is larger than in the RS problem.*

In contrast to the SD problem, the consumer can use all partial valuations to direct search in the DS problem. Hence, if inspection costs for all products are the same in both problems (i.e. $c_j^{DS} = c_s \forall j$), a consumer will have a larger expected payoff in the DS problem as he can directly inspect products with large partial valuations. However, under the assumption that total costs of revealing full information are the same in both search problems, a more detailed analysis is necessary to determine which search problem offers a larger expected payoff.

Denote a consumer's expected payoff in a search problem k as π_k for $k \in \{SD, DS\}$. Proposition 2.1 implies that expected payoffs are given by

$$p_{SD} = \mathbb{E}_{\hat{\boldsymbol{W}}}\left[\max\{u_0, \max_{j\in J} \hat{W}_j\}\right]$$
$$p_{DS} = \mathbb{E}_{\tilde{\boldsymbol{W}}}\left[\max\{u_0, \max_{j\in J} \tilde{W}_j\}\right]$$

Furthermore, let $H_k(\cdot)$ denote the cumulative density of the respective maximum value over which the expectation operator integrates in problem k. The difference in expected payoffs of the SD and the DS problem then is given by

$$p_{SD} - p_{DS} = \int_{z^d}^{\infty} H_{DS}(w) - 1\mathrm{d}w + \int_{u_0}^{z^d} H_{DS}(w) - H_{SD}(w)\mathrm{d}w \tag{2.21}$$

The first expression in (2.21) is negative, capturing the advantage of observing partial valuations for all products and being able to directly inspect a product at a later position. Given $H_{DS}(w) \leq H_{SD}(w)$ on $w \in \left[u_0, z^d\right]$, the second expression in (2.21) is positive, revealing that directly observing all partial valuations x_j does not only yield benefits.

The latter stems from the difference in how inspection and discovery costs are taken into consideration in the two dynamic decision processes. In DS, the total cost of inspecting a product j at a later position is directly weighed against its benefits given the partial valuations. In contrast, in SD, the consumer first weighs the discovery costs against the expected benefits of discovering a product with a larger partial valuation. Once product j is revealed, the accumulated cost paid to discover j (jc_d) is a sunk cost and does not affect the decision whether to inspect j.

Hence, in cases where products on early positions have below-average partial valuations x_j, the optimal policy in SD tends to less often prescribe to inspect these products compared to the direct cost comparison in DS. In some cases, the former can be more beneficial, leading to a larger expected payoff.[27] Directly revealing all partial valuations therefore does not always improve a consumer's benefit, if the consumer continues to incur the same total costs to reveal the full valuation of any given product. [28]

2.6.4 Empirical implications

Differences in the underlying search problem also have implications for the estimation of structural search models. For example, a structural search model will use price differences across all products to inform parameter estimates if it abstracts from limited awareness and assumes that consumers observe all prices prior to search. Consumers not inspecting low-price products they are unaware of then may be spuriously attributed either to a small price sensitivity or large inspection costs. Whereas there are many applications of structural search models and an ubiquity of settings where consumers remain unaware of some alternatives, the sensitivity of results from structural search models to limited awareness remains unclear.

I therefore investigate the implications of estimating either a random or directed search model in a setting where consumers instead solve the search and discovery problem. I focus on a scenario where preference and cost parameters are estimated using data on consumers' consideration sets and purchases; a common case as consideration sets are observable in click-stream or survey data. Using a simple specification,[29] I first analyze how the different models attribute observed stopping decisions to structural parameters. A numerical exercise then reveals that this can lead to sizable differences in parameter estimates and counterfactual predictions.

Empirical setting: The data consist of consumers' consideration sets and purchases,[30] as

[27] For example, this is the case if $X \sim N\left(0, \frac{1}{3}\right)$, $Y \sim N\left(0, \frac{2}{3}\right)$, $c_s = c_d = 0.05$ and $|J| = 10$.

[28] No threshold result as in Proposition 2.2 applies in this case. The first expression in (2.21) decreases whereas the second expression increases in the number of alternatives.

[29] The empirical literature extends the simple specification for a range of settings, for example by introducing heterogeneous preferences. The main rationale continues to hold in such settings.

[30] The simulated data also contains consumers with an empty consideration set, i.e. those that did not search any alternatives. This corresponds to an ideal setting where the whole population of consumers is observed.

well as a number of characteristics for each of the available products. The utility of purchasing product j is specified as $u_j = \boldsymbol{x}_j'\beta + y_j$, where $\boldsymbol{x}_j$ is a vector containing the observed product characteristics, β is a vector of preference parameters and y_j is an idiosyncratic unobservable taste shock with mean zero. Depending on the model, consumers are assumed to reveal $\boldsymbol{x}_j$ when either discovering j (SD) or inspecting j (RS), or know $\boldsymbol{x}_j$ prior to search (DS). y_j is revealed after inspecting j in all three models.

Given this setting, Table 2.1 shows sufficient or necessary conditions for the purchase of product j across the three models, conditional on j being the best product inspected and (ii) the observed consideration set not coinciding with the set of all available alternatives. The condition for the SD problem shows that a purchase of product j can be independent of realized valuations of products that the consumer is not aware of in the purchase period $\bar{t}$.[31] j only needs to offer "good enough" characteristics relative to the mean and to products the consumer is aware of at the time of purchase. The RS model features the same structure; a consumer will end search and buy product j if its valuation exceeds the reservation value. However, Ξ and $\tilde{\xi}$ depend differently on the underlying costs and distributions of characteristics in $\boldsymbol{x}_j$ and y_j. Through these non-linear functions, a RS model will attribute observed limited consideration sets differently to preference and cost parameters.

In the DS model rationalizing the purchase of j requires that the valuation of the purchased product is larger than the search values of all uninspected products. If, for example, $\boldsymbol{x}_k > \boldsymbol{x}_j$ for an uninspected product, the DS model will require either relatively small preference parameters, or relatively large inspection costs. Hence, depending on the characteristics of the uninspected products, rationalizing limited consideration sets in a DS model will require a combination of large inspection costs and attenuated preference parameters, as the estimation procedure will try to fit an inequality for each uninspected product.

Table 2.1 – Purchase conditions

SD	$(\boldsymbol{x}_j - \mu_{\boldsymbol{X}})'\beta + y_j$	$\geq \Xi$	&	$(\boldsymbol{x}_j - \boldsymbol{x}_k)'\beta + y_j \geq \xi_k \forall k \in A_{\bar{t}}$	(sufficient)
RS	$(\boldsymbol{x}_j - \mu_{\boldsymbol{X}})'\beta + y_j$	$\geq \tilde{\xi}$			(necessary)
DS	$(\boldsymbol{x}_j - \boldsymbol{x}_k)'\beta + y_j$	$\geq \xi_k \forall k \notin C_{\bar{t}}$			(necessary)

Notes: Sufficient or necessary conditions for purchase of product j conditional on $u_j \geq u_k \forall k \in C_{\bar{t}}$ and $J \nsubseteq C_{\bar{t}}$. $\bar{t}$ denotes the purchase period.

[31] The condition is sufficient but not necessary. A lternatively, the consumer can first become aware of all alternatives, before then purchasing j.

To investigate the extent to which this influences results from structural search models, I perform simulations for this setting. First, I simulate consumers solving a SD problem with the given utility specification and under the assumption that consumers initially aware of one product. Using these data, I then estimate structural parameters in search models based on either the RS and DS problem. For the DS problem, two specifications are estimated. DS1 is a baseline where inspection costs are parameterized as $c_j^{DS1} = c_s$. DS2 introduces an additional cost parameter such that inspection costs increase in position h_j with $c_j^{DS2} = c_s + c_d h_j$. This specification additionally uses data on the order in which products are discovered by consumers. For all three models, the estimation fits inequalities based on the conditions of Table 2.1, as well as other inequalities coming from continuation and purchase decisions. Details on the maximum likelihood estimation are provided in the appendix. As comparison, I also present estimates of a full information (FI) model.

Results of such a simulation are presented in Table 2.2. Parameters for this particular simulation are shown in the same table and were chosen to reflect a setting with relatively few searches, as is often the case in click-stream data.[32] To account for the fact that assuming the distribution of y_j is a normalization in the empirical context, estimates are presented as a ratio to the coefficient of the second characteristic. Given its negative coefficient, this characteristic will be interpreted as a product's price.

Table 2.2 – Estimated Coefficients and Search Set Size

	#Searches	Purchases (%)	β_2	$\beta_1/\lvert\beta_2\rvert$	$\beta_3/\lvert\beta_2\rvert$	$c_s/\lvert\beta_2\rvert$	$c_d/\lvert\beta_2\rvert$
SD	1.35	63.70	-1.00	1.00	3.50	0.03	0.06
DS1	1.18	65.48	-0.19	1.01	2.58	1.79	
DS2	1.18	65.22	-0.19	1.01	2.72	1.58	0.01
RS	1.00	72.85	-0.82	1.28	5.21	0.05	
FI		60.54	-0.62	1.00	5.01		

Notes: Estimation from a simulated dataset with 2,000 consumers and 30 products per consumer. Characteristics are independent draws (across consumers and products) from $x_{1j} \sim N(2, 3.0)$, $x_{2j} \sim N(3.5, 1.0)$ and $y_j \sim N(0, 1)$. The third characteristic is an outside dummy. The data is generated based on the *SD* model with $n_d = |A_0| = 1$, with parameters in the estimated models denoted by $c^{RS} = c_s$, $c_j^{DS1} = c_s$ and $c_j^{DS2} = c_s + c_d h_j$. The first two columns are based either on the generated data (SD) or estimated by generating 5,000 search paths for each consumer.

[32] For example, Ursu (2018) reports an average of 1.12 clicks per consumer and two thirds of consumers ending up booking a hotel.Chen and Yao (2017) reports an average of 2.3 clicks per consumer using data only on consumers that ended up booking a hotel.

The results show that both DS specifications are able to match the number of purchases, as well as the relative preference coefficients relatively well, despite the price coefficient being strongly attenuated and the number of searches being underestimated. However, inspection costs are strongly accentuated in both DS models. This offers a novel explanation for the large estimates of baseline costs estimated with some DS models (e.g. Chen and Yao, 2017; Ursu, 2018): By not accounting for consumers not being aware of some alternatives, a DS model spuriously attributes consumers not inspecting products they are not aware of to large inspection costs.[33] This continues to occur in the DS2 model that could rationalize ranking effects produced by the SD model through inspection costs that increase in the position at which a product is discovered. However, the results show that instead the DS2 model estimates only a small increase in inspection costs across positions and also strongly overestimates baseline inspection costs.

The RS model underestimates inspection costs; they are less than the combined inspection and discovery costs. Moreover, the ratio of preference parameters deviates from the true values. The large differences in the estimated coefficient for the outside option result from how the different models interpret consumers not inspecting or not buying. Whereas in the DS problem this occurs from large inspection costs, the RS model attributes the lack of search mainly to a good outside option.

Differences in the structural search models also influence results from counterfactual simulations. Table 2.3 shows the results of two different counterfactuals for each of the models. For each counterfactual scenario, parameters from Table 2.2 are used for each model to simulate consumer surplus (CS) and the demand for the outside option (D_0), as well as for products shown on the first (D_1) and fifth (D_5) position. Throughout, results are expressed in percentage deviations from the baseline scenario.

The first counterfactual consists of removing all search costs, which can be used to gauge the effects of removing the search friction. For both DS models, accentuated baseline inspection costs lead to a larger increase in consumer surplus compared to the SD model with which the data was generated. Moreover, removing costs in the DS models makes consumers more likely to purchase any product, independent of their position. In contrast, demand in the SD and

[33] Other explanations for large search search cost estimates are incomplete search histories (e.g. Ursu, 2018) and heterogeneous prior beliefs (Jindal and Aribarg, 2020).

RS model decreases for both products listed. This stems from the inherent ranking effects where products on early positions are bought more frequently as consumers stop search early. In this case, removing all costs moves demand to later positions.

The second counterfactual scenario analyzes the effects of a one percent price decrease of products discovered on the fifth position. The change in demand for the first product highlights an important difference in the substitution pattern. In the data-generating SD model, the demand for products on the first position decreases by little, as the price decrease of a product on a later position does not affect choices of consumers that stopped search before becoming aware of the product. In contrast, in a DS (or FI) model, consumers who were previously buying products on the first few positions observe the price decrease and can directly substitute to the fifth product. This translates into more substitution from the first few positions as a response to a price decrease of a product on a later position. The predicted changes in the demand for the fifth product further highlight that the different models lead to different predictions for consumers' responses to price changes; whereas the DS1 and RS models underestimate, the DS2 model overestimates the increase in demand in response to the price change.

Table 2.3 – Counterfactuals

	Remove costs			$\Delta p_5 = -1\%$		
	ΔCS	ΔD_1	ΔD_5	ΔCS	ΔD_1	ΔD_5
SD	28.60	-37.35	-2.32	0.02	-0.01	1.81
DS1	85.06	38.04	43.11	0.01	-0.04	1.72
DS2	81.38	15.53	29.19	0.01	-0.03	2.75
RS	18.73	-25.36	-11.78	0.01	-0.02	1.49
FI	0.00	0.00	0.00	0.01	-0.05	1.91

Notes: Results from simulated counterfactuals based on Table 2.2, where (i) all costs are set to zero and (ii) the price for the 5th product is reduced by 1 % for each consumer. All changes are expressed in % relative to the baseline. Demand and consumer surplus are calculated by averaging across 5,000 simulated search paths for each consumer.

Though results from only a single simulation are presented, I obtained qualitatively similar results across a wide range of parameter values.[34] Throughout, DS models overestimate inspection costs and all estimated models can lead to sizable differences in parameters and

[34]These results can be replicated with the supplementary material.

results from counterfactual predictions. Nonetheless, the SD problem will be more similar to the DS problem if consumers are aware of many alternatives when they end search (e.g. due to small discovery costs). Similarly, if consumers inspect most products they discover independent of their characteristics, the SD problem will be more similar to the RS problem. When estimating search models, researchers should therefore carefully consider the degree to which limited awareness plays a role in the specific setting they are studying and which model is appropriate.

To this end, the results of Propositions 2.4 and 2.5 can be used to empirically differentiate the search modes in some settings. If data are available that allow to test whether ranking effects depend on the number of alternatives, it will be possible to empirically determine whether a DS problem, instead of a RS or SD problem provides a framework that better captures ranking effects. Furthermore, if data are available that allow to test whether a product's partial valuation has an effect on whether it is inspected, it will be possible to distinguish between RS and SD.

2.7 Conclusion

This paper introduces a search problem that generalizes existing frameworks to settings where consumers have limited awareness and first need to become aware of alternatives before being able to search among them. The paper's contribution is to provide a tractable solution for optimal search decisions and expected outcomes for this search and discovery problem. Moreover, a comparison with classical random and directed search highlights how limited awareness and the availability of partial product information determine search outcomes and expected payoffs.

A promising avenue for future research is to build on this paper's results and study limited awareness in an equilibrium setting. This could yield novel insights into how consumers' limited information shapes price competition. Furthermore, the search and discovery problem can serve as a framework to analyze how firms compete for consumers' awareness. For example, informative advertising can make it more likely that consumers are aware of a seller's products from the outset. Ranking effects derived in this paper already suggest that it will be in a seller's best interest to make consumers aware of his product, but further research is needed to determine equilibrium dynamics.

Another avenue for future research entails incorporating the search and discovery problem into a structural model that is estimated with click-stream data. The available actions in the search and discovery problem closely match how consumers scroll through product lists (discovery) and click on products (inspection) on websites of search intermediaries and online retailers. By accounting for the fact that consumers initially do not observe entire list pages, such a model could improve the estimation of consumers' preferences, inspection costs and ranking effects relative to models that abstract from consumers not observing the whole product list.

Appendix

2.A Proofs of main theorems and propositions

2.A.1 Theorem 1

Let $\Theta(\Omega_t, A_t, z)$ denote the value function of an alternative decision problem, where in addition to the available actions in A_t, there exists a hypothetical outside option offering value z. As the SD problem satisfies that taking an action does not change the state of another available action and has the same branching structure, Theorem 1 of Keller and Oldale (2003) states that a Gittins index policy is optimal and that the following holds:[35]

$$\Theta(\Omega_t, A_t, z) = b - \int_z^b \Pi_{a\in A_t} \frac{\partial\Theta\left(\Omega_t, \{a\}, w\right)}{\partial w} \mathrm{d}w \tag{2.22}$$

where b is some finite upper bound of the expected immediate rewards.[36] The Gittins index of action d (discovering products) is defined by $g_t^d = \mathbb{E}_{\boldsymbol{X}}\left[\Theta(\Omega_{t+1}, A_{t+1}\backslash A_t, g_t^d)\right]$. Suppose the consumer knows the total number of alternatives $|J|$, and consider a period t in which more discoveries will still be available in $t+1$ with certainty. In this case we have

$$\begin{aligned} g_t^d &= \mathbb{E}_{\boldsymbol{X}}\left[\Theta(\Omega_{t+1}, \{d, s1, \ldots, sn_d\}, g_t^d)\right] - c_d \\ &= \mathbb{E}_{\boldsymbol{X}}\left[b - \int_{g_t^d}^b \frac{\partial\Theta\left(\Omega_{t+1}, \{d\}, w\right)}{\partial w} \prod_{k=1}^{n_d} \frac{\partial\Theta\left(\Omega_{t+1}, \{s_k\}, w\right)}{\partial w} \mathrm{d}w\right] - c_d \end{aligned} \tag{2.23}$$

where $s_k \in S_{t+1}\backslash S_t \forall k$. $\Theta(\Omega_t, \{s_k\}, z)$ is the value of a search problem with an outside option offering z and the option of inspecting product k (with known partial valuation x_k). $\Theta\left(\Omega_{t+1}, \{d\}, w\right)$ is the value of a search problem with an outside option offering z, and the option to discover more products. Finally, $\mathbb{E}_{\boldsymbol{X}}\left[\cdot\right]$ is the expectation operator integrating over the beliefs over the n_d random variables in $\boldsymbol{X} = [X, \ldots, X]$, which does not depend on time.

Optimality of the Gittins index policy then implies that when $z \geq g_{t+1}^d$, the consumer

[35] Compared to the baseline branching framework discussed by Keller and Oldale (2003), the SD problem does not have discounting, and purchasing a product is a "terminal" action. Note also that whereas not explicitly stated by the authors, their framework accommodates the case where it is not known ex ante to how many "children" an available action branches into. This will be the case in the SD problem if the consumer does not know the number of products he will discover.

[36] Expected immediate rewards are in $[-\max\{c_s, c_d\}, \mathbb{E}[X+Y]]$, hence assuming finite mean of X and Y guarantees that they have a finite upper bound.

will choose the outside option in $t+1$. Hence $\Theta\left(\Omega_t, \{d\}, w\right) = w \forall w \geq g^d_{t+1}$ which yields $\frac{\partial\Theta(\Omega_t,\{d\},w)}{\partial w} = 1 \forall w \geq g^d_{t+1}$. This implies that for $g^d_t \geq g^d_{t+1}$, g^d_t does not depend on whether more products can be discovered in the future, and the optimal policy is independent of the beliefs over the number of available alternatives. As a result, as long as the Gittins index is weakly decreasing during search, i.e. $g^d_t \geq g^d_{t+1} \forall t$, it is independent of the availability of future discoveries and beliefs q.

It remains to show that $g^d_t \geq g^d_{t+1} \forall t$ holds in the proposed search problem. When $|J| = \infty$, $g^d_t = g^d_{t+1}$ is immediately given by the fact that in both periods infinitely many products remain to be discovered and that the consumer has stationary beliefs (i.e. q is constant and valuations are independent and identically distributed). For $|J| < \infty$, backwards induction yields that this condition holds: Suppose that in period $t+1$, no discovery action is available as all products have been discovered. In this case, the Gittins index is given by

$$g^d_{t+1} = \mathbb{E}_{\boldsymbol{X}}\left[b - \int_{g^d_{t+1}}^{b} \prod_{k=1}^{n_d} \frac{\partial\Theta\left(\Omega_{t+1}, \{s_k\}, w\right)}{\partial w} \mathrm{d}w\right] - c_d \tag{2.24}$$

As $0 \leq \frac{\partial\Theta(\Omega_t,\{d\},w)}{\partial w} \leq 1$ and $\frac{\partial\Theta(\Omega_t,\{s_k\},w)}{\partial w} \geq 0$, it holds that

$$\begin{aligned}\mathbb{E}_{\boldsymbol{X}}\left[b - \int_{g^d_{t+1}}^{b} \prod_{k=1}^{n_d} \frac{\partial\Theta\left(\Omega_{t+1}, \{s_k\}, w\right)}{\partial w} \mathrm{d}w\right] \leq q\mathbb{E}_{\boldsymbol{X}}\left[b - \int_{g^d_{t}}^{b} \prod_{k=1}^{n_d} \frac{\partial\Theta\left(\Omega_{t+1}, \{s_k\}, w\right)}{\partial w} \mathrm{d}w\right] + \\ (1-q)\mathbb{E}_{\boldsymbol{X}}\left[b - \int_{g^d_{t}}^{b} \frac{\partial\Theta\left(\Omega_{t}, \{d\}, w\right)}{\partial w} \prod_{k=1}^{n_d} \frac{\partial\Theta\left(\Omega_{t}, \{s_k\}, w\right)}{\partial w} \mathrm{d}w\right]\end{aligned} \tag{2.25}$$

which implies $g_t \geq g_{t+1}$.

Finally, $\Theta(\Omega_{t+1}, \{d, s1, \ldots, sn_d\}, g^d_t) = V\left(\left\langle\bar{\Omega}, \omega(x,z)\right\rangle, \{b0, s, \ldots, sn_d\}; \tilde{\pi}\right)$ in (2.9) implies $z^d = g^d_t$. Similarly, the definition of the inspection and purchase values (in (2.6) and (2.10)) are equivalent to the definition of Gittins index values for these actions and it follows that the reservation value policy is the Gittins index policy.

2.A.2 Theorem 2

Proof. As a product always is bought, it suffices to show that the optimal policy never prescribes to buy product j if there exists another product k with $w_k > w_j$. To account for the case where $C_0 \neq \emptyset$, define $z^s_k = \infty \forall k \in C_0$ which implies $\tilde{w}_k \equiv \min\left\{z^s_k, z^b_k\right\} = z^b_k \forall k \in C_0$. First, consider the case where k is revealed before j ($h_0 \leq h_k < h_j$). In this case, $w_k > w_j$ if and only if either (i) $\tilde{w}_k \geq z^d$ or (ii) $z^d > \tilde{w}_k > \tilde{w}_j$. In the former, the optimal policy prescribes to not discover products beyond k, hence not to buy product j. This follows as $z^s_k \geq z^d$ and $z^b_k \geq z^d$ imply that the optimal policy prescribes that search ends with buying k before discovering j. In the latter, $w_j = \tilde{w}_j < w_k = \tilde{w}_k$, and the optimal policy prescribes to continue discovering such that both products are in the awareness set. The eventual purchase theorem of Choi et al. (2018) then applies, and hence the optimal policy does not prescribe to buy product j. Second, consider the case where k is discovered after j ($h_k > h_j$). In this case, note that $w_j > w_k$ if $\tilde{w}_j \geq z^d$. Hence, $w_k > w_j$ if and only if $z^d > \tilde{w}_k > \tilde{w}_j$, which is the same as (ii) above. Finally, consider the case where k is discovered at the same time as j ($h_k = h_j$). Then $w_k > w_j$ if and only if $\tilde{w}_k > \tilde{w}_j$, which follows from the construction of the effective values. This again is the same as (ii) above and hence the optimal policy does not prescribe to buy j. □

2.A.3 Proposition 2.1

Proof. The proof follows a similar structure as the proof of Corollary 1 in Choi et al. (2018). To simplify exposition, the following additional notation is used: Let $\tilde{w}_j \equiv x_j + \min\{y_j, \xi_j\}$ as in Theorem 1, and $\hat{w}_j$ equal to the effective value from Theorem 2, with the adjustment that $f(h_j) = \varepsilon = 0$. Furthermore, let $\bar{w}_r \equiv \max_{k \in J_{0:r-1}} \hat{w}_k \forall r \geq 1$, $\bar{\tilde{w}}_r \equiv \max_{k \in J_r} \tilde{w}_k$ and $\bar{\tilde{w}}_{r,j} \equiv \max_{k \in J_r \setminus j} \tilde{w}_k$ where $J_{a:b}$ denotes the set of products discovered on position $r \in \{a, \dots, b\}$, and J_r is short-hand for $J_{r:r}$. Finally, let $1(\cdot)$ denote the indicator function and $\bar{h}$ the maximum position.

The payoff of a consumer given realizations x_j and y_j for all j is given by

$$\sum_{r=1}^{\bar{h}} 1(\bar{w}_r < z^d) \left[\sum_{j \in J_r} 1(\tilde{w}_j \geq \max\left\{z^d, \bar{\tilde{w}}_{r,j}\right\})(x_j + y_j) - 1(x_j + \xi_j \geq \max\left\{z^d, \bar{\tilde{w}}_{r,j}\right\})c_s \right]$$
$$+ 1(\bar{w}_0 \geq z^d)\nu_0 - \sum_{r=1}^{\bar{h}} 1(\bar{w}_r < z^d)c_d + 1(w_{\bar{h}} < z^d)\nu \quad (2.26)$$

which follows from the optimal policy and Theorem 2: (i) If $\bar{w}_0 \geq z^d$, the stopping rule implies that the consumer does not discover any products beyond the initial awareness set. Conditional on not discovering any additional products, the payoff then is equal to v_0, which denotes the payoff of a directed search problem over products $k \in S_0$ and an outside option offering $\bar{u}_0 = \max_{k \in C_0} u_k$. (ii) If $\bar{w}_r < z^d$, the continuation rule implies that the consumer continues beyond position $r-1$, i.e. discovers products on position r and pays discovery costs c_d. (iii) Conditional on discovering j, when $\tilde{w}_j \geq \max\left\{z^d, \bar{\tilde{w}}_{r,j}\right\}$, the stopping and inspection rules imply that the consumer buys j, gets utility $x_j + y_j$ and does not continue beyond position r. (iv) Conditional on discovering j, when $x_j + \xi_j \geq \max\left\{z^d, \bar{\tilde{w}}_{r,j}\right\}$, the inspection rule implies that the consumer inspects j and incurs costs c_s. (v) If $w_{\bar{h}} < z^d$, the continuation rule implies that the consumer discovers all products, whereas the inspection rule implies that he inspects all products $\left\{j | x_j + \xi_j \geq z^d\right\}$. Conditional having discovered all products, the consumer therefore has the payoff of a directed search problem over products $\left\{j | x_j + \xi_j < z^d\right\}$ with outside option $\tilde{u}_0 = \max\{u_0, \max_{k \in \left\{j | x_j + \xi_j \geq z^d, x_j + y_j \leq \xi_j\right\}} x_k + y_k\}$. This is denoted by ν.

Let $\mathbb{E}[\cdot]$ integrate over the distribution of $X_j, Y_j \forall j \in J$, and substitute inspection and discovery costs by $c_s = \mathbb{E}\left[1(Y_j \geq \xi_j)(Y_j + x_j - z^s_j)\right] = \forall j$ (with $z^s_j = x_j + \xi_j$) and $c_d = \mathbb{E}\left[1(\bar{\tilde{W}}_r \geq z^d)(\bar{\tilde{W}}_r - z^d)\right]$ (see Appendix 2.B). The expected payoff then is given by:

$$
\begin{aligned}
&\sum_{r=1}^{\bar{h}} \mathbb{E}\Bigg[1(\bar{W}_r < z^d)\Bigg(\sum_{j\in J_r} 1(\tilde{W}_j \geq \max\{z^d, \tilde{\bar{W}}_{r,j}\})(X_j + Y_j) \\
&\qquad - 1(X_j + \xi_j \geq \max\{z^d, \tilde{\bar{W}}_{r,j}\})1(Y_j \geq \xi_j)(Y_j - \xi_j)\Bigg)\Bigg] \\
&\qquad - \sum_{r=1}^{\bar{h}} \mathbb{E}\left[1(\bar{W}_r < z^d)1(\tilde{\bar{W}}_r \geq z^d)(\tilde{\bar{W}}_r - z^d)\right] + \mathbb{E}\left[1(\bar{W}_0 \geq z^d)\nu_0 + 1(\bar{W}_{\bar{h}} < z^d)\nu\right] \\
&= \sum_{r=1}^{\bar{h}} \mathbb{E}\left[1(\bar{W}_r < z^d)\left(\sum_{j\in J_r} 1(\tilde{W}_j \geq \max\{z^d, \tilde{\bar{W}}_{r,j}\})(X_j + \min\{\xi_j, Y_j\})\right)\right] \\
&\qquad - \sum_{r=1}^{\bar{h}} \mathbb{E}\left[1(\bar{W}_r < z^d)1(\tilde{\bar{W}}_r \geq z^d)(\tilde{\bar{W}}_r - z^d)\right] + \mathbb{E}\left[1(\bar{W}_0 \geq z^d)\nu_0 + 1(\bar{W}_{\bar{h}} < z^d)\nu\right] \\
&= \sum_{r=1}^{\bar{h}} \mathbb{E}\left[1(\bar{W}_r < z^d)1(\tilde{\bar{W}}_r \geq z^d)\tilde{\bar{W}}_r\right] \\
&\qquad - \sum_{r=1}^{\bar{h}} \mathbb{E}\left[1(\bar{W}_r < z^d)1(\tilde{\bar{W}}_r \geq z^d)(\tilde{\bar{W}}_r - z^d)\right] + \mathbb{E}\left[1(\bar{W}_0 \geq z^d)\nu_0 + 1(\bar{W}_{\bar{h}} < z^d)\nu\right] \\
&= \sum_{r=1}^{\bar{h}} \mathbb{E}\left[1(\bar{W}_r < z^d)1(\tilde{\bar{W}}_r \geq z^d)z^d\right] \\
&\qquad + \mathbb{E}\left[1(\bar{W}_0 \geq z^d)\max\left\{\bar{u}_0, \max_{k\in S_0}\tilde{W}_k\right\} + 1(\bar{W} < z^d)\max\{\tilde{u}_0, \max_{k\in\left\{k|x_k+\xi_k<z^d\right\}}\tilde{W}_k\}\right] \\
&\qquad = \mathbb{E}\left[\max_{j\in J}\hat{W}_j\right]
\end{aligned}
$$

The second-to-last step substitutes $\nu_0 = \mathbb{E}\left[\max\left\{\bar{u}_0, \max_{k\in S_0}\tilde{W}_k\right\}\right]$ and similarly for ν, which directly follows from Corollary 1 in Choi et al. (2018). The last step combines the expressions of the three mutually exclusive cases using the definition of $\hat{w}_j$.

To prove the second claim, note that the definition of z^d requires that $\mathbb{P}(\tilde{W}_j > z^d) > 0$, as otherwise $Q_d(c_d, c_s, z^d) > 0$. Hence with $|J| = \infty$, $\mathbb{P}\left(\max_{j\in J}\tilde{W}_j < z^d\right) = 0$ such that $\mathbb{E}\left[\max_{j\in J}\hat{W}_j\right] = z^d$. □

2.A.4 Proposition 2.2

Proof. Consider a situation where we decrease costs c_s and c_d to either $c_s' = c_s - \Delta$ or $c_d' = c_d - \Delta$, while keeping the other cost constant. Let $H_1(\cdot)$ and $H_2(\cdot)$ denote the cumulative density of $\bar{W} \equiv \max\{\bar{w}_0, \max_{j\in J\setminus C_0\cup S_0}\hat{W}_j\}$ in the former and the latter case respectively, where $\bar{w}_0 \equiv \max\left\{\max_{k\in C_0} u_k, \max_{k\in S_0}\tilde{w}_k\right\}$ is the value of the alternatives in the initial consideration and awareness sets. Similarly, let z_1^d and z_2^d denote the associated discovery values. Given

$n_d = 1$, we have $\frac{\partial Q_d(c_d,c_s,z)}{\partial c_d} < \frac{\partial Q_d(c_d,c_s,z)}{\partial c_s}$; hence $\left|\frac{\partial z^d}{\partial c_d}\right| > \left|\frac{\partial z^d}{\partial c_s}\right|$ and $z_2^d > z_1^d$. Moreover, note that the definition of the adjusted effective value $\hat{w}_j$ implies $H_i(w) = 1 \forall w \geq z_i^d$ and $H_i(w) = 0 \forall w \leq \bar{w}_0$.

Conditional on $\bar{w}_0 < z_1^d$, the difference in a consumer's expected payoff across the two changes therefore can be written as

$$\int_{z_1^d}^{z_2^d} 1 - H_2(w)\mathrm{d}w - \int_{\bar{w}_0}^{z_1^d} H_2(w) - H_1(w)\mathrm{d}w \tag{2.27}$$

Whereas the first part is strictly positive, the second part is negative. The latter follows as for $w \in [\bar{w}_0, z_1^d]$, $\bar{W} = \max_{j \in J \setminus C_0 \cup S_0} X_j + \min\{Y_j, \xi\}$ and $\frac{\partial \xi}{\partial c_s} < 0$ such that $H_1(w) \leq H_2(w)$. As valuations are independent across products, we have $H_k(w) = \mathbb{P}_{X,Y}\left(X + \min\{Y, \xi_k\} \leq w\right)^{|J|}$; hence, as $|J|$ increases, $H_2(w) - H_1(w)$ and $H_2(w)$ decrease for $w \in [\bar{w}_0, z_2^d]$.[37] Consequently, for all $\Delta > 0$ there exists some threshold n^* for $|J|$ such that the difference in the expected payoff conditional on $\bar{w}_0 < z_1^d$ is positive, i.e.

$$\int_{z_1^d}^{z_2^d} 1 - H_2(w)\mathrm{d}w > \int_{\bar{w}_0}^{z_1^d} H_2(w) - H_1(w)\mathrm{d}w \tag{2.28}$$

Conditional on $\bar{w}_0 \geq z_1^d$, having $z_2^d > z_1^d$ immediately implies that the expected payoff increases by at least as much when decreasing discovery costs. Note also that when $z_2^d < \bar{w}_0$, neither change affects the expected payoff. Finally, integrating over the realizations y_k for $k \in S_0$ that determine $\bar{w}_0$ yields the unconditional expected payoff as a combination of these cases, which implies the first result.

Increasing the value of the alternatives in the initial consideration and awareness set then makes larger values of $\bar{w}_0$ more likely. This implies the second result, as it makes both the case $\bar{w}_0 \geq z_1^d$ more likely, as well as decrease the right-hand-side of (2.28). □

2.A.5 Proposition 2.3

Proof. At $c_s = 0$, we have $z^d = z^{RS}$. $\left|\frac{\partial z^{RS}}{\partial c_s}\right| \geq \left|\frac{\partial z^d}{\partial c_s}\right|$ then implies $z^d \geq z^{RS}$. Using this in (2.14) and (2.15) immediately yields the result. □

2.A.6 Proposition 2.4

Proof. The first two statements immediately follow from (2.18) and (2.19). To see the latter, rewrite (2.18) as $\mathbb{P}_W\left(W < z^d\right)^{h-1}\mathbb{P}_W\left(W \geq z^d\right)^2$, and (2.19) in a similar way. $c^{RS} = c_s + c_d$ then implies $z^d \geq z^{RS}$. Hence, $\mathbb{P}_W\left(W < z^d\right) = \mathbb{P}_{X,Y}(X + \min\{Y, \xi\} < z^d) \geq \mathbb{P}_{X,Y}\left(X + Y < z^{RS}\right)$ which directly implies the existence of the threshold. □

[37] Note that if $\mathbb{P}_{X,Y}\left(X + \min\{Y, \xi_k\} \leq w\right)$ is large, then $H_1(w) - H_2(w)$ will first increase in $|J|$, before starting to decrease.

2.A.7 Proposition 2.5

Proof. Write the first expression in (2.20) (demand at position h) as

$$\mathbb{E}_{\tilde{W}_h}\left[\mathbb{P}\left(\tilde{W}_{h+1} \leq \tilde{W}_h\right) \prod_{k \notin \{h,h+1\}} \mathbb{P}\left(\tilde{W}_k \leq \tilde{W}_h\right)\right] \tag{2.29}$$

When $|J|$ decreases, this expression decreases through the product term, which is weighted by the first term $\mathbb{P}\left(\tilde{W}_{h+1} \leq \tilde{W}_h\right)$. As $\mathbb{P}\left(\tilde{W}_{h+1} \leq t\right) \geq \mathbb{P}\left(\tilde{W}_h \leq t\right) \forall t$, the first expression in (2.20) decreases by more than the second one when the number of alternatives increases. □

2.A.8 Proposition 2.6

Proof. The RS problem is equivalent to a policy in the SD problem that commits on inspecting every product that is discovered, conditional on which the consumer chooses to stop optimally. However, as the optimal policy in the SD problem is not this policy, it must yield a (weakly) larger payoff. □

2.A.9 Uniqueness of discovery value

Proposition 2.7. (2.10) *has a unique solution.*

Proof. $Q_d(c_d, c_s, z)$ with respect to z yields (see Appendix 2.B)

$$\frac{\partial Q(c_d, c_s, z)}{\partial z} = \begin{cases} +H(z) - 1 & \text{if } z < 0 \\ -2 + H(z) & \text{else} \end{cases} \tag{2.30}$$

where $H(\cdot)$ denotes the cumulative density of the random variable $\max_{k \in \tilde{J}} \tilde{W}_k$. This implies that $\frac{\partial Q_d(c_d, c_s, z)}{\partial z} \leq 0$, which combined with continuity, $Q_d(c_d, c_s, \infty) = -c_d$ and $Q_d(c_d, c_s, -\infty) = \infty$ guarantee that a solution to (2.10) exists. Finally, uniqueness requires $Q_d(c_d, c_s, z)$ to be strictly decreasing at $z = z^d$. $\frac{\partial Q_d(c_d, c_s, z^d)}{\partial z} = 0$ would require that $H(z^d) = 1$, which contradicts the definition of the discovery value value z^d in (2.10), as it implies $Q_d(c_d, c_s, z^d) \leq -c_d < 0$. □

2.B Further details on search and siscovery values

The search value of a product j is defined by equation (2.6) and sets the myopic net gain of the inspection over immediately taking a hypothetical outside option offering utility z to zero. This myopic net gain can be calculated as follows:[38]

[38] The second steps holds as with a change in the order of integration we get $\int_{z-x_j}^{\infty}[1 - F(y)]\mathrm{d}y = \int_{z-x_j}\int_{y}^{\infty} f_Y(t)\mathrm{d}t\mathrm{d}y = \int_{z-x_j}\int_{z-x_j}^{t} f_Y(t)\mathrm{d}y\mathrm{d}t = \int_{z-x_j}[yf_Y(t)]_{y=z-x_j}^{y=t}\mathrm{d}t$.

$$
\begin{aligned}
Q_s(x_j, c_s, z) &= \mathbb{E}_Y\left[\max\{0, x_j + Y - z\}\right] - c_s \\
&= \int_{z-x_j}^{\infty} (x_j + y - z)\mathrm{d}F(y) - c_s \\
&= \int_{z-x_j}^{\infty} [1 - F(y)]\,\mathrm{d}y - c_s
\end{aligned}
$$

Substituting $\xi_j = z - x_j$ then yields (2.7).

The discovery value is defined by equation (2.10) and sets the expected myopic net gain of discovering more products over immediately taking a hypothetical outside option offering utility z to zero. Corollary 1 in Choi et al. (2018) and similar steps as the above then imply that:

$$
\begin{aligned}
Q_d(c_d, c_s, z) &= \mathbb{E}_{\boldsymbol{X},\boldsymbol{Y}}\left[\max\left\{z, \max_{k\in\{1,\ldots,n_d\}} \tilde{W}_k\right\}\right] - z - c_d \\
&= \mathbb{E}_{\boldsymbol{X},\boldsymbol{Y}}\left[\max\left\{0, \max_{k\in\{1,\ldots,n_d\}} \tilde{W}_k - z\right\}\right] - c_d \\
&= \int_{z}^{\infty} 1 - H(w)\mathrm{d}w - c_d
\end{aligned}
$$

where $H(\cdot)$ denotes the cumulative density of the random variable $\max_{k\in\tilde{J}} \tilde{W}_j$. The above also implies that in the case where Y is independent of X, a change in variables yields that the discovery value is linear in the mean of X, denoted by μ_X:

$$
z^d = \mu_X + \Xi(c_s, c_d)
$$

where $\Xi(c_s, c_d)$ solves (2.10) for an alternative random variable $\tilde{X} = X - \mu_X$.

2.C Monotonicity and extensions

Monotonicity of the Gittins index values ($g_t^d \geq g_{t+1}^d \forall t$) is satisfied whenever the following holds:

$$
\begin{aligned}
0 \leq &\mathbb{E}_{\boldsymbol{X},Y,n_d,q,t}\left[\Theta(\Omega_{t+1}, \tilde{A}_{t+1}, g_t^d)\right] \\
&- \mathbb{E}_{\boldsymbol{X},Y,n_d,q,t+1}\left[\Theta(\Omega_{t+2}, \tilde{A}_{t+2}, g_{t+1}^d)\right]
\end{aligned}
\tag{2.31}
$$

where g_t^d is the Gittins index of discovering products (defined in equation (2.23)), and $\tilde{A}_{t+1} \equiv \{d, s1, \ldots, sn_d\}$ is the set of actions available in $t{+}1$ containing the newly revealed products and (if available) the possible future discoveries. The expectation operator $\mathbb{E}_{\boldsymbol{X},Y,n_d,J,t}[\cdot]$ integrates over the following random realizations, where the respective joint distribution now can be time-dependent: (i) Partial valuations drawn from $\boldsymbol{X} = [X_1, \ldots, X_{n_d}]$; (ii) conditional distributions $F_{Y|X=x}(y)$; (iii) the number of revealed alternatives (n_d); (iv) whether more products can be discovered in future periods determined by the belief q.

It goes beyond the scope of this paper to determine all possible specifications of beliefs which satisfy this condition. However, Proposition 2.8 provides two specifications that can be of interest and for which (2.31) holds (see also Section 2.4).

Proposition 2.8. (2.31) *holds for the below deviations from the baseline model:*

1. *Y is independent of X. Beliefs are such that the revealed partial valuations in $\boldsymbol{X}$ are i.i.d. with time-dependent cumulative density $G_t(x)$ such that $G_t(x) \leq G_{t+1}(x)\, \forall x \geq z^d - \xi$.*

2. *The consumer does not know how many alternatives he will discover. Instead, he has beliefs such that with each discovery, at most the same number of alternatives are revealed as in previous periods ($n_{d,t+1} \leq n_{d,t}$).*

Proof. Each part is proven using slightly different arguments.

1. Let $\tilde{x} \equiv \max_{k \in \{1,\ldots,n_d\}} x_k$. If $\tilde{z}^s = \tilde{x} + \xi \leq z^d$, $\Theta(\Omega_{t+1}, \tilde{A}_{t+1}, z^d) = 1$, whereas for $\tilde{x} > z^d - \xi$, $\frac{\partial \Theta(\Omega_{t+1}, \{e, s1, \ldots, sn_d\}, z^d)}{\partial \tilde{x}} \geq 0$. Independence implies that the cumulative density of the maximum $\tilde{x}$ is $\tilde{G}_t(x) = G_t(x)^{n_d}$. Consequently, whenever the distribution of X shifts such that $G_t(x) \leq G_{t+1}(x) \forall x \geq z^d - \xi$, larger values of $\Theta(\Omega_{t+1}, \tilde{A}_{t+1}, g_t^d)$ become less likely in $t+1$, and hence (2.31) holds.

2. Since $\frac{\partial \Theta(\Omega_{t+1}, \{s_k\}, w)}{\partial w} \leq 1$, we have $\frac{\partial \Theta(\Omega_{t+1}, \tilde{A}_{t+1}, g_t^d)}{\partial n_d} \geq 0$. Hence (2.31) holds given $n_{d,t+1} \leq n_{d,t}$.

□

Based on this monotonicity condition, Proposition 2.9 generalizes Theorem 1. It implies that whenever (2.31) holds, the discovery value can be calculated based on the expected myopic net gain of discovering products over immediately taking the hypothetical outside option. Hence, whenever (2.31) holds, the optimal policy continues to be fully characterized by reservation values that can be obtained without having to consider many future periods.

Proposition 2.9. *Whenever* (2.31) *is satisfied, Theorem 1 continues to hold (with appropriate adjustment of the discovery value's time-dependence).*

Proof. Follows directly from the proof of Theorem 1. □

2.D Violations of independence assumption

Costly recall: Consider a variation to the search problem, where purchasing a product in the consideration set is costly unless it is bought immediately after it is inspected. If in period t product j is inspected, then inspecting another product or discovering more products in $t+1$ will change the payoff of purchasing product j by adding the purchase cost. In the context of a multi-armed bandit problem, this case arises if there are nonzero costs of switching between arms. Banks and Sundaram (1994), for example, provide a more general discussion on switching costs and the nonexistence of optimal index-based strategies. The same reasoning also applies in a search problem where inspecting a product is more costly if the consumer first discovers more products. The exception is if there are infinitely many alternatives. In this case, the optimal policy never prescribes to recall an alternative.

Learning: Independence is also violated for some types of learning. Consider a variation of the search problem, where the consumer updates his beliefs on the distribution of Y. In this case, by inspecting a product k and revealing y_{ik}, the consumer will update his belief about the distribution of Y, thus affecting the expected payoffs of both discovering more and inspecting other products. Independence therefore is violated and the reservation value policy is no longer optimal.[39] Note, however, that as long as learning is such that only payoffs of actions that will be available in the future are affected, independence continues to hold. This is for example the case when the consumer learns about the distribution of X as discussed in Section 3.2.

Purchase without inspection: A final setting where independence does not hold is when a consumer can buy a product without first inspecting it. In this case, the consumer has two actions available for each product he is aware of. He can either inspect a product, or directly purchase it. Clearly, when the consumer first inspects the product, the information revealed changes the payoff of buying the product. Independence therefore is violated and the reservation value policy is not guaranteed to be optimal. Doval (2018) studies this search problem for the case where a consumer is aware of all available alternatives, and characterizes the optimal policy under additional conditions.

2.E Learning

Several studies consider priors or learning rules under which the optimal policy is myopic when searching with recall (Rothschild, 1974; Rosenfield and Shapiro, 1981; Bikhchandani and Sharma, 1996; Adam, 2001). A sufficient condition for the optimal policy to be myopic is given in Theorem 1 of Rosenfield and Shapiro (1981): Once the expected net benefits of continuing search over stopping with the current best option are negative, they remain so. Hence, whenever it is optimal to stop in t, it is also optimal to stop in all future periods. The monotonicity condition used in this paper directly imposes that this is satisfied; expected benefits of discovering more products remain constant or decrease during search. A fairly general assumption underlying learning rules that satisfy this condition is Assumption 1 in Bikhchandani and Sharma (1996). This assumption requires that beliefs are updated such that values above the largest value revealed so far become less likely.[40] Hence, whenever a better value is found than the current best, finding an even better match in the future becomes less likely.

In the SD problem, similar learning rules that satisfy this condition are difficult to find. When the consumer learns about the number of products that are revealed with each discovery, expected benefits of discovering more products increase if many products are revealed, but the value of stopping remains the same if all these products are bad matches. Hence, a learning rule would need to guarantee that beliefs shift such that the expected benefits of discovering more products do not increase, as opposed to only the net benefits over stopping. Similarly, if the consumer learns about the distribution of partial valuations $\boldsymbol{X}$, the value of stopping need not increase even if partial information indicates a good match leading the consumer to shift beliefs towards larger values; after inspecting a promising product, the consumer may still realize that the product is worse than the previously best option.

Though the optimal policy is not myopic with learning, it is still based on the Gittins index, where the search and purchase values are as in the baseline SD problem. The main

[39] Adam (2001) studies a similar case where independence continues to hold across groups of products. However, his results do not extend to the case with limited awareness, as the beliefs of Y also determine the expected benefits of discovering more products.

[40] Note that Bikhchandani and Sharma (1996) consider search for low prices.

difficulty is calculating the index value for discovering more products, denoted by z_t^L. Whereas calculating this value precisely would require accounting for learning in future periods, it is possible to derive bounds on this value that are easier to calculate and can be used to judge how far off a myopic policy is.

To show this, I focus on the case where the consumer learns about the distribution of partial valuations. In particular, consider the following variation of the search and discovery problem: Let the distribution of partial valuations in $\boldsymbol{X}$ be characterized by a parameter vector θ and denote its cumulative density by $G_\theta(\cdot)$. The consumer initially does not know the true parameter vector and Bayesian updates his beliefs in period t given some prior distribution. Denoting the consumers' beliefs on θ with cumulative density $P_t(\cdot)$, the consumers' beliefs about $\boldsymbol{X}$ drawn in the next discovery are characterized by the cumulative density $\tilde{G}_t(\boldsymbol{X}) = \int G_\theta(\boldsymbol{X})\mathrm{d}P_t(\theta)$.[41]

Denote a k-step look-ahead value as $z_t^d(k)$ and define it as the value of a hypothetical outside option that makes the consumer indifferent between stopping immediately, and discovering more products after which at most $k-1$ more discoveries remain. For example, $z_t^d(1)$ satisfies the myopic comparison in (2.10), where expectations are calculated based on period t beliefs $\tilde{G}_t(\cdot)$. The definition of $z_t^d(1)$ then implies that it is equal to the expected value of continuing to discover products if no future discoveries remain. As the consumer can stop and take this hypothetical outside option in $t+1$, allowing for more discoveries after $t+1$ can only increase the expected value, hence $z_t^d(1) \leq z_t^d(2) \cdots \leq z_t^L$. $z_t^d(1)$ therefore provides a lower bound on z_t^L, and z_t^L can be approximated with increasing precision through k-step look-ahead values.

To derive an upper bound, consider the case where the consumer learns the true θ in $t+1$, if he chooses to discover more products in t. The value of discovering more products in t when the true θ is revealed in $t+1$ then is larger compared to the case where the consumer continues to learn. This is formally derived by Kohn and Shavell (1974) for a search problem where a consumer samples from an unknown distribution. Intuitively, when the true θ is revealed, the consumer is able choose the action in $t+1$ that maximizes the expected payoff going forward for each realization of θ. In contrast, if the consumer does not learn the true θ in $t+1$, he cannot choose the maximizing action for each realization of θ, but only the action that maximizes expected payoff on average across possible θ.

An upper bound therefore is given by the value $\bar{z}_t^d$ such that the consumer is indifferent between stopping and taking a hypothetical outside option offering $\bar{z}_t^d$, and discovering more products after which the true θ is revealed. Formally, $\bar{z}_t^d$ satisfies

$$\bar{z}_t^d = \int\int \tilde{V}(\Omega_{t+1}, A_{t+1}, \bar{z}_t; \theta)\mathrm{d}P_{t+1}(\theta)\mathrm{d}\tilde{G}_t(\boldsymbol{X}) \tag{2.32}$$

where $\tilde{V}(\Omega_{t+1}, A_{t+1}, \bar{z}_t^d; \theta)$ denotes the expected value of a search and discovery problem with known θ and an outside option offering $\bar{z}_t^d$. Proposition 2.1 then directly allows to calculate this value without having to consider all the possible search paths.

Proposition 2.10 summarizes these results. A similar result can also be derived for the case

[41] For example, consider the case of sampling from a Normal distribution with unknown mean and known variance, and assume $n_d = 1$. If the consumer believes in t that the mean is distributed normally with $\theta \sim N(\mu_t, \sigma_t^2)$, then $\tilde{G}_t(x) = \Phi(\frac{x-\mu_t}{\sigma_t})$, where $\Phi(\cdot)$ is the standard normal cumulative density (see e.g. Theorem 1 in DeGroot, 1970, Ch. 9.5).

where the consumer learns about a distribution from which the number of products that are discovered is drawn.

Proposition 2.10. *In the search and discovery problem with Bayesian learning about an unknown distribution of partial valuations $\boldsymbol{X}$, it is optimal to:*

1. *continue whenever* $\max_{k \in C_t} u_k \leq z_t^d(1)$
2. *stop whenever* $\max_{k \in C_t} u_k \geq \bar{z}_t^d$

2.F Estimation details

To estimate the three models I use a simulated maximum likelihood approach based on a kernel-smoothed frequency simulator. Using numerical optimization, parameters are found that maximize the simulated likelihood given by:

$$\max_{\gamma} \sum_i \mathcal{L}_i(\gamma) = \sum_i \log \left(\frac{1}{N_d} \sum_{d=1}^{N_d} \frac{1}{1 + \sum_{k=1}^{N_k} \exp(-\lambda \kappa_{kdi})} \right)$$

where γ is the parameter vector, N_d is the number of simulation draws, λ is a smoothing parameter and κ_{kd} is one of N_k inequalities resulting from the optimal policy in the respective model evaluated for draw d. All three models are estimated with $\lambda = 10$ and $N_d = 500$. At these values, parameters are recovered well when data is generated with the same model.

DS conditions These conditions are the same as in Ursu (2018), who provides further details on how they relate the optimal policy in the DS problem. The difference to her specification is that inspection costs are linear, and that in DS1 there are no positions. For observed consideration set C_i for consumer i, a given draw d for the unobserved taste shocks $y_j(d)$ which defines product utilities $u_j(d)$ as well as the utility of the purchased option $u_i^*(d)$, there are multiple purchase, and stopping conditions expressed in inequalities:

Stopping:	$\kappa_{kdi} = \max_{j \in C_i} u_j(d) - z_m \forall m \notin C_i$
Continuation	$\kappa_{kdi} = z_{m+1} - \max_{j \in C_i(m)} u_j(d) \forall m = 1, 2, \ldots, N_{is} - 1$
Purchase:	$\kappa_{kdi} = u_i^*(d) - u_j(d) \forall j \in C_i$

In the continuation conditions, N_{is} denotes the number of observed inspections, z_{m+1} is the search value of the next inspection, $C_i(m)$ is the consideration set of i after m inspections. Note that the last relies on observing the order in which products are inspected; if this order were not observed, the method proposed by Honka and Chintagunta (2017) could be used to integrate over possible search orders. The stopping condition only applies if not all products are inspected, the continuation condition only applies if i inspected at least one product.

RS conditions The conditions in the RS model are similar to the ones in the DS model. However, the stopping and continuation conditions now are based on the reservation value

z^{RS}, which follows directly from the optimal policy:

$$\begin{aligned}
&\text{Stopping:} && \kappa_{kdi} = \max_{j \in C_i} u_j(d) - z^{RS} \\
&\text{Continuation} && \kappa_{kdi} = z^{RS} - \max_{j \in C_i(m)} u_j(d) \forall m = 1, 2, \ldots, N_{is} - 1 \\
&\text{Purchase:} && \kappa_{kdi} = u_i^*(d) - u_j(d) \forall j \in C_i
\end{aligned}$$

FI conditions In the FI model, standard purchase conditions apply:

$$\kappa_{kdi} = u_i^*(d) - u_j(d) \forall j$$

2.G Sellers' decisions

To illustrate the difference in sellers' decision making across the SD and DS problem, we can compare the market demand generated by the SD problem with the one from the DS problem when there are infinitely many alternatives. Given a unit mass of consumers, market demand for a product discovered at position h is given by

$$d_{SD}(h) = \mathbb{P}_{\boldsymbol{W}} \left(W_k < z^d \forall k < h \right) \mathbb{P}_{W_h} \left(W_h \geq z^d \right) \tag{2.33}$$

where W_h is the random effective value of a product on position h. The expression immediately follows from the stopping decision which implies that if a consumer discovers a product with $w_j \geq z^d$, he will stop searching and buy a product j. Hence, the consumer will only discover and have the option to buy a product on position h if $w_h < z^d$ for all products on earlier positions.

For the DS problem, Choi et al. (2018) showed that the market demand is given by

$$d_{DS}(h) = \mathbb{P}_{\boldsymbol{W}} \left(\tilde{W}_h \geq \max_{k \in J} \tilde{W}_k \right) \tag{2.34}$$

where $\tilde{W}_k = X_k + \min \{Y_k, \xi_k\}$.

Now suppose that the seller of a product on position h sets the mean of X_h, for example by choosing a price. In the SD problem, this is equivalent to choosing $\mathbb{P}_{W_h} \left(W_h \geq z^d \right)$; the probability that the consumer inspects and then stops search by buying the seller's product. Importantly, this does not directly depend on partial valuations of both products at earlier, and products at later positions. This results from the stopping decisions, and given the infinite number of products a consumer will never recall a product discovered earlier.

In contrast, in the DS problem, choosing the mean of X_h influences demand through the joint distribution of all products. As consumers are aware of all products, they compare all partial valuations. Hence, each seller's choice of partial valuations affects all other sellers demand, and sellers do not make independent decisions.

Chapter 3

Heterogeneous Position Effects and the Power of Rankings

Abstract

Most online retailers or search intermediaries present products on product lists. By changing the ordering of products on these lists (the "ranking"), these online outlets can increase their revenues at a potential cost to consumer welfare. This paper shows that rankings increase revenues through the differential impact of higher list positions on purchases of heterogeneous products, and provides empirical evidence for heterogeneity in these "position effects." To quantify consumer welfare effects, I develop an estimation procedure for the search and discovery model introduced in Chapter 2. By simulating counterfactual scenarios I show that revenue-based rankings can benefit search intermediaries and consumers relative to various other rankings. Moreover, I find that revenue-based rankings decrease consumer welfare only to a limited extent when compared to utility-based rankings that first show the alternatives that consumers on average prefer.

3.1 Introduction

A growing number of consumers purchase products online, either from online retailers or through a search intermediary like Expedia. These online outlets typically present consumers with ranked product lists. It is well established that the ordering of such lists has a substantial influence on which products consumers inspect and eventually buy: alternatives on top of the list are more likely to be clicked on and bought.[1] This creates scope for an online outlet to increase its own revenues by deploying ranking algorithms, as highlighted by an extensive literature developing such algorithms.[2] A concern with such "revenue-based" rankings is that they could adversely affect consumers by first displaying relatively expensive items, thus harming price-sensitive consumers that would prefer to first discover cheaper alternatives. When designing rankings, a search intermediary's and consumers' interests, therefore, may be misaligned. In this paper I analyze whether and to which extent this is the case by (i) showing that rankings increase revenues through differences in the impact of higher list positions on the demand of heterogeneous products, (ii) providing empirical evidence for heterogeneity in these "position effects," and (iii) estimating the search and discovery model developed in Chapter 2 to quantify the effects of different rankings on revenues and consumer welfare.

A change in ranking requires switching positions of the available alternatives: moving an alternative up on the list requires moving another one down. Such a switch in positions increases revenues as long as the alternatives that are moved higher up on the list gain more revenues than than the ones being moved down lose. As a consequence, which alternatives need to be moved up on the list to increase revenues and how this is going to affect consumers depends on *heterogeneous* position effects. For example, if position effects are such that the expected revenue increase is largest for relatively expensive alternatives, moving these alternatives higher up on the list increases revenues, but harms consumers that now need to spend more time to find cheaper alternatives or, if they leave the website early, may miss out on them altogether. In contrast, if cheaper products gain the most revenues through strong demand increases, a revenue-based ranking will move these alternatives to the top of the product list, thus increasing revenues while benefiting consumers.

Which of these cases applies and how revenue-based rankings affect consumer welfare is an

[1] See, for example, Ghose et al. (2012), Ghose et al. (2014) or Ursu (2018).
[2] Chu et al. (2020) and Derakhshan et al. (2022) are recent examples. Section 3.2 provides further details.

empirical question that I address in this paper. Answering this research question poses two main challenges. First, it requires quantifying consumer welfare and evaluating scenarios that are not observed in the data. To this end, I estimate a structural search model based on the theoretical framework developed in Chapter 2. The underlying decision process generalizes directed search *á la* Weitzman (1979). With directed search, observed position effects are explained through position-specific search costs (see Ursu, 2018). Instead, my model rationalizes position effects through model primitives. When reaching the product list, consumers decide between discovering more products on the list and clicking on specific hotel listings to reveal detailed information. Position effects, therefore, stem from consumers initially not observing the whole list and ending search before discovering alternatives on later positions. This captures better the decision process consumers face when interacting with ranked product lists, and allows to draw meaningful insights for consumer welfare. Once estimated, I can quantify changes in revenues and consumer welfare across different counterfactual rankings.

Second, position effects need to be identified separately from consumers' preferences; consumers may click on and book hotels on higher positions because they prefer them or because of position effects. To do so, I use data from Expedia with randomized position assignments. This exogenous variation allows to disentangle the two and identifies position effects without convoluting the effect of a ranking that assigns bestsellers to top positions (Ursu, 2018).

I provide descriptive evidence highlighting that product attributes and the position on the list complement each other: on average, clicks and bookings for hotels with desirable characteristics are over-proportionally affected by different positions. A key finding is that there is a negative and significant interaction between price and a product's position on the list; conditional on other attributes, cheaper hotels on average have stronger position effects. This result highlights that consumers' search behavior limits how much moving more expensive products to higher positions can increase total revenues; when first presented with relatively expensive alternatives, consumers either continue to browse along the list to find cheaper ones or leave the website.

The complementarity between hotel attributes and the position on the list is also captured in the structural model. Specifically, "high-utility" hotels have a stronger demand increase when being moved to the top. The complementarity also extends to expected revenues. In a counterfactual analysis where I move randomly selected hotels to higher positions on the list, I find that, on average, expected revenues increase more for high-utility hotels: the stronger demand increase for high-utility hotels offsets that cheaper hotels generate less revenue per booking. As a result, maximizing total revenues entails moving high-utility products to higher positions on the list. This, in turn, also benefits consumers. Nonetheless, there is substantial variation around the average revenue increase of different hotels when being moved to higher positions. A ranking that orders by expected revenues, therefore, may still increase total revenues over one that orders by utility.

To quantify the revenue and consumer welfare effects of a revenue-based ranking, I propose a simple ranking algorithm that orders hotels based on the expected revenues on the *same* position. This ranking is motivated by the observation that a hotel's revenue increase when moving it to the top position is directly related to its revenue on the initial position. This revenue-based ranking increases total revenues over a randomized ranking by 9.10%, whereas a utility-based ranking increases revenues only by 6.85%. Importantly, this revenue increase does not necessarily come at a cost to consumers. The proposed ranking actually increases average consumer welfare over a randomized ranking by 0.47$. Compared to a utility-based ranking, the revenue-based ranking decreases consumer welfare only by 0.26$. When focusing only on consumers that book a hotel, the revenue-based ranking increases consumer welfare over a randomized ranking by 2.81$, and decreases it over the utility-based ranking by 1.71$. These results show that a revenue-based ranking is closer to a utility-based than to a randomized ranking,[3] and highlight that both consumers and a search intermediary like Expedia can benefit from a revenue-based ranking. As a consequence, when designing rankings, a search intermediary's and consumers' interests are not strongly misaligned.

Combined, my results also offer two managerial insights. First, a concern for search intermediaries or online retailers is that a revenue-based ranking could lead to a bad search experience for consumers and, as a consequence, increase customer churn. My results suggest that this is not the case. Because products that consumers prefer benefit more from top positions (both in terms of demand and revenues), a revenue-based ranking can actually benefit consumers. Second, by being shown on top positions, sellers of popular products can boost their revenues beyond their inherently high level. Consequently, if they have the option to

[3] The revenue-based ranking is also further from Expedia's own ranking, which increases consumer welfare on average by 0.17$ (all consumers) and 1.15$ (only consumers that booked).

influence whether they are shown on top positions (for example, through "sponsored listings"), these sellers have a strong incentive to use it.

The remainder of this chapter is organized as follows. First, I discuss related literature. Section 3.3 introduces and discusses the main aspects of the search and siscovery model to study the effects of rankings. Section 3.4 summarizes the data and describes the estimation approach. Finally, Section 3.5 discusses the empirical results and the last section concludes by outlining future areas of research.

3.2 Related literature

This paper relates and contributes to the literature on consumer search, position effects and ranked product lists. First, the descriptive evidence for heterogeneity in position effects adds to the empirical literature analyzing position effects in product lists and search advertising. Closely related is the empirical work of Ghose et al. (2014), who find mixed evidence for how characteristics influence position effects. Specifically, they find that, conditional on other attributes, both a higher price and a higher hotel class amplify position effects. Yet, whereas the click-through-rate (CTR) falls in price, it increases in hotel class. Ursu (2018) shows that having experimental variation in positions is important when studying position effects. Using the same data as this paper, Ursu (2018) finds smaller position effects than previous studies that rely on other methods to account for these endogeneity concerns. This paper instead uses the experimental variation in positions to identify *heterogeneous* position effects.

My results contrast the empirical literature studying heterogeneous position effects in search advertising. Whereas several studies find that ads with a larger CTR on any position have weaker position effects (Goldman and Rao, 2014; Athey and Imbens, 2015; Blake et al., 2015; Jeziorski and Segal, 2015; Jeziorski and Moorthy, 2018), I find that hotels with a larger CTR and more bookings on any position also have stronger position effects. The difference stems from the different context. Consumers are more likely use a product list like the one on Expedia to search for and compare alternatives. In contrast, many consumers enter search terms on Google with a specific outcome already in mind. Hence whether an ad appears on the first or second ad slot does not matter for these consumers as they will click it independent of where the ad is shown.[4]

[4]Blake et al. (2015) find that search advertising is not effective for a well-known platform like eBay; traffic to

Second, the paper adds to the literature examining the effects of different rankings on consumer welfare and search behavior. This literature generally finds that a "utility-based" ranking – a ranking that orders products by the utility they offer to consumers at purchase – increases consumer welfare and the number of transactions (Ghose et al., 2012, 2014; De los Santos and Koulayev, 2017; Ursu, 2018). Results for how a utility-based ranking affect the search intermediary's revenues are mixed. Ghose et al. (2014) find that the utility-based ranking yields the largest total revenues across the rankings considered, whereas Ursu (2018) finds that in three out of the four destinations in her sample total revenues decrease. This paper differs in that I focus on the alignment of the search intermediary's interest in maximizing total revenues and consumer welfare, and show how this relates to heterogeneous position effects. Closer to this goal is the recent study by Zhang et al. (2021) who study welfare effects of revenue-based rankings. However, they do not use a micro-founded search model, and instead use a stylized demand model where the position on the list directly enters the demand function through a functional form assumption. Donnelly et al. (2022) consider welfare effects of personalized rankings, and find only minor profit-driven distortions from such personalization. Ursu and Dzyabura (2020) study how to locate products to maximize the number of searches and sales, but do not consider consumer welfare effects in their model. Choi and Mela (2019) estimate a structural search model in a two-sided market and their counterfactual analyses suggest that auctioning off the top positions to sellers and ordering the remaining positions based on expected revenues yields the largest profits for the search intermediary.

Many studies in operations research derive algorithms to maximize revenues by choosing which alternatives to offer, and in which order to display them to consumers. This literature motivates this papers' focus on the potential misalignment of interests, and the empirical evaluation of how revenue-based rankings affect consumers. Earlier studies solving this class of optimization problems, sometimes referred to as "assortment problems," started by using discrete choice models to characterize the demand side (e.g. Van Ryzin and Mahajan, 1999; Talluri and Van Ryzin, 2004; Davis et al., 2014). More recent studies started using different search models to characterize demand. Chu et al. (2020) analyze optimal rankings that balance sellers', consumers' and the platforms' surplus. An important difference is that a price-decreasing order is not revenue-maximizing in my model. This is because consumers

eBay was barely affected by not placing any ads during the experiment.

that do not buy an alternative do not necessarily discover all alternatives. Hence, moving a cheaper product to the top can increase revenues if it gets an over-proportional demand increase. Derakhshan et al. (2022) show how a platform should optimally order alternatives to either maximize market share or consumer welfare when consumers search following their "two-stage sequential search model." This model differs from the search and discovery model in that consumers first decide how many products to discover, before then inspecting *all* of the discovered products.[5] In a recent working paper, Compiani et al. (2021) develop and estimate a "double-index" model based on which they show that optimally assigning the first few slots on the list can already get reasonably close to the optimum, as long as alternatives shown after these slots are unlikely to be bought. The "double-index" model nests several different potential search strategies. Note, however, that my search and discovery model is not nested within their model: any model admissible in the double-index model requires that consumers are aware of all alternatives prior to search, and can directly search an alternative even if it is displayed on the bottom of the list. In contrast, in the search and discovery model, consumers first need to discover an alternative before being able to inspect it. Hence it provides a micro-founded explanation for the observed position effects: because fewer consumers discover the alternatives at the bottom of the list, fewer consumers eventually click on and buy them.

The mechanism for position effects also distinguishes my paper from the growing literature that estimates structural search models based on a directed search model using Weitzman's (1979) seminal result for the optimal policy (e.g. Kim et al., 2010; Chen and Yao, 2017; Honka and Chintagunta, 2017; Honka et al., 2017; Kim et al., 2017; Morozov, 2019; Yavorsky et al., 2020; Moraga-Gonzalez et al., 2022). Note that the search models used in these studies can be reformulated as special cases of the search and discovery model. Hence, the my simulation procedure for the likelihood may be used to obtain a smooth likelihood function for the cases that use similar data. Other studies estimate structural search models based on a "top-down" search model (Chan and Park, 2015; Choi and Mela, 2019). These models also explain position effects through fewer consumers not reaching the bottom of the page. However, these models use the simplifying assumption that consumers cannot go back and click on a product they discovered previously. The model estimated in this paper is more general as consumers can go back and click on previously discovered products; it therefore can accommodate search orders

[5] This is akin to a fixed sample size search strategy for the discovery process.

where consumers first click on an item shown lower on the list.[6]

3.3 Modeling the effects of rankings

The model builds on the theoretical framework I introduced in Chapter 2. The framework extends the directed search process of Weitzman (1979) in that consumers initially are aware of only a few alternatives. Hence, they decide between inspecting products they are already aware of and discovering more alternatives that then can be inspected.

This decision process matches the actions consumers can take and the information they have at each point in time when interacting with product lists as presented used Expedia and many other online outlets. After entering a search query at Expedia, consumers are directed to the product list and observe the first few hotel listings on the top positions. From the product list, consumers already observe some hotel attributes such as price or the number of hotel stars. Based on this information, consumers then decide between clicking on the listings and scrolling down the product list. Whereas a click reveals more information on the hotel by bringing consumers to its detail page, scrolling reveals more hotel listings that then can be clicked on.

The model differs from directed search which explains position effects through position-specific search costs in three important ways. First, it provides a micro-founded explanation for the observed position effects: because fewer consumers discover the hotels at the bottom of the list, fewer consumers click on and eventually book them. Second, it takes into account that at the beginning of search, consumers have no information on hotels that are shown on later positions on the list. As I show in Chapter 2, not accounting for this can lead to biased preference and search costs estimates. Third, substitution patterns and the benefits to being moved to top positions differ. In directed search models, consumers can directly click on a product at the bottom of the product list and will do so as long as its expected utility is large enough to compensate for the larger search cost. In the more general model this is not the case, as consumers first need to scroll down the list to discover additional products. Consequently, in the former, moving to the top leads to more clicks and bookings because it reduces hotel-specific search costs, whereas in the latter it does so because more consumers

[6] Despite the additional complexity, the estimation is computationally more efficient as it does not rely on value function iteration for the optimal policy as the one of Choi and Mela (2019).

discover and are able to click and book it.

3.3.1 The "Search and Discovery" model

When booking hotel j, consumer i receives utility

$$u_{ij} = -\alpha p_j + \boldsymbol{x}_j'\beta + \varepsilon_{ij} \tag{3.1}$$

where p_j is the price of hotel j,[7] $\boldsymbol{x}_j$ is a vector of observable hotel characteristics (e.g. review score), α and β are respective preference weights, and ε_{ij} is an unobserved idiosyncratic taste shock. The utility of not booking any hotel, i.e. the outside option, is given by $u_{i0} = \beta_0 + \varepsilon_{i0}$. The idiosyncratic taste shocks are assumed to be independent across hotels and consumers, and follow a distribution with zero mean and cumulative density F.

Initially, consumers know their preferences and the value of the outside option. Moreover, they know the attributes $(p_j, \boldsymbol{x}_j)$ for hotels j that they observe when arriving on the product list. Denote the set of hotels i observes when arriving on the list by A_{i0}, and the set of hotels i can potentially discover by J_i.

Consumers sequentially decide between clicking on any of the hotel listings they already revealed and scrolling down to discover more hotels. By inspecting j, consumers reveal the idiosyncratic taste shock ε_{ij}. Throughout, I denote the expected utility of a product by $u_j^e = \mathbb{E}\left[u_{ij}\right] = -\alpha p_j + \boldsymbol{x}_j'\beta$. By scrolling down the product list, consumers reveal $(p_j, \boldsymbol{x}_j)$ for the next n_d hotels.

Both clicking and scrolling are costly actions. When scrolling to discover additional hotels, the consumer incurs discovery costs $c_d(h)$. These discovery costs depend on the position $h = 0, 1, \ldots$ the consumer has reached so far. When clicking on a listing, the consumer incurs inspection costs c_{ijs}. Inspection costs are heterogeneous across hotels and consumers and are drawn from a common distribution. Inspection costs for j are revealed when $(p_j, \boldsymbol{x}_j)$ is revealed and are known prior to inspecting j. The heterogeneous inspection cost assumption captures that factors other than the observed attributes influence consumers click decisions, but not their purchase decision. For example, some consumers may find it easier to evaluate certain hotels because of past experience with similar ones.

There is free recall both for inspecting and discovering. Going back and inspecting a

[7] In the empirical application hotel prices do vary across different consumers. See Section 3.4.1.

previously revealed listing does not add extra costs. Similarly, going back and booking a hotel that was inspected previously does not add extra costs.

Consumers have beliefs on the distribution from which ε_{ij} are drawn. Moreover, they have beliefs on the joint distribution from which both hotel attributes and inspection costs are drawn for hotels that are going to be discovered. The cumulative density of this distribution will be denoted by G_h, where the formal definition is given below. This distribution is allowed to depend on the position h the consumer has reached so far.

I further assume that $c_d(h)$ and G_h depend on h so that the monotonicity condition discussed in Chapter 2 holds. In the present setting, this is guaranteed to hold if $c_d(h)$ weakly increases in h, the mean of G_h weakly decreases in h, and the variance of G_h remains constant. This assumption is required for the policy described below to be optimal. However, this assumption is not very restrictive. If consumers anticipate that a ranking first displays alternatives that many consumers like, they expect worse alternatives to be shown further down, therefore satisfying this assumption. Moreover, the empirical specification described in Section 3.4 introduces a parameter that relates to this assumption, and the resulting estimates suggest that it holds. Finally, I impose the regularity condition that all discussed distributions have finite mean and variance.[8]

3.3.2 The optimal policy is based on reservation values

Consumers search optimally: in each period, they take the action that maximizes the expected payoff over all future periods. As shown in Chapter 2, the optimal policy in this case is characterized by reservation values: in each period, taking the action with the largest reservation value maximizes the expected payoff over all future periods. As there are three types of actions – buying, inspecting, discovering – there are three types of reservation values. But the underlying principle is the same. Specifically, the reservation value for an action is the value of a hypothetical outside option that sets the *myopic net benefit* of taking the action over immediately taking the hypothetical outside option to zero. Crucially, the myopic net gain does not depend on any other available alternatives or the availability of future discoveries. Hence, reservation values, and consequently the optimal policy, can be obtained without having to consider myriad future periods or different available alternatives.

[8] This is required to ensure that the dynamic decision problem has an optimal policy.

For the current problem with the utility specification (3.1) and heterogeneous inspection costs, the reservation values are given by (see Appendix 2.B):

1. Purchase value: $z^p_{ij} = u_{ij} = -\alpha p_j + \boldsymbol{x}'_j\beta + \varepsilon_{ij}$

2. Search value: $z^s_{ij} = -\alpha p_j + \boldsymbol{x}'_j\beta + \xi_{ij}$, where ξ_{ij} solves

$$\int_{\xi_{ij}}^{\infty} [1 - F(t)]\,\mathrm{d}t - c_{ijs} = 0 \tag{3.2}$$

3. Discovery value: $z^d(h)$ solves

$$\int_{z^d(h)}^{\infty} [1 - G_h(t)]\,\mathrm{d}t - c_d(h) = 0 \tag{3.3}$$

   where $G_h(t) = \mathbb{P}_h(Z^s_{ij} \leq t)$.

Note that (3.2) uniquely determines ξ_{ij} given c_{ijs} and the cumulative density F. Similarly, (3.3) uniquely determines $z^d(h)$ given $c_d(h)$ and the cumulative density G_h. Conversely, given ξ_{ij}, $z^d(h)$ and the cumulative densities F and G_h, it is possible to back out c_{ijs} and $c_d(h)$ respectively (see Appendix 3.B.1).

3.3.3 Effective values allow to compute demand and consumer welfare

To calculate consumer welfare and demand, define consumer i's effective value of a product j as

$$w_{ij}(h_{ij}) \equiv \min\{z^d(h_{ij} - 1), \min\{z^s_{ij}, z^p_{ij}\}\} \tag{3.4}$$

where h_{ij} tracks the position at which i discovers j by setting $h_{ij} = 0$ for hotels in A_{i0} and $h_{ij} = s$ for hotels revealed with the sth scroll. Additionally, define $c_d(h) = 0$ such that $z^d(h) = \infty$ for $h < 0$, which accounts for $j \in A_{i0}$ being discovered for free. The effective value of the outside option is given by its utility, i.e. $w_{i0} = \beta_0 + \varepsilon_{i0}$.

It is further useful to define another value that is not adjusted through the discovery value. Let

$$\tilde{w}_{ij} \equiv \min\{z^s_{ij}, z^p_{ij}\} \tag{3.5}$$

with $\tilde{w}_{i0} = u_{i0}$ for the outside option. This value is the same as the "effective values" defined by

Armstrong (2017) and Choi et al. (2018). Because $\tilde{w}_{ij} \geq z^d(h_{ij})$ implies that both the search and purchase values of j are larger than the discovery value, it also implies that the consumer will both click and then book j, before going on to discover more alternatives. Moreover, because $\tilde{w}_{ij} \geq z^s_{ik}$ implies that both the search and purchase value of j exceed the search value of k, it follows that k will not be searched if the consumer discovers j earlier or at the same time.

Both values depend on observable product attributes and the position on the list. To evaluate the effects of rankings, I calculate demand and consumer welfare conditional on observable attributes and the positions. Hence, a ranking r_i takes the available hotels for consumer i as given and only changes their positions. To simplify notation, let $\boldsymbol{X}_i(r_i) \equiv \left[(p_1, \boldsymbol{x}_1, h_{i1}), \ldots, \left(p_{|J_i|}, \boldsymbol{x}_{|J_i|}, h_{i|J_i|}\right)\right]$ gather all observable attributes of hotels consumer i can potentially discover and the positions they will be discovered on. Hence, $\boldsymbol{X}_i(r_i)$ depends on the ranking of alternatives.

Based on the effective values, consumer i's demand for product j, conditional on the ranking and observable attributes can be calculated as[9]

$$d_{ij}(r_i) = \mathbb{P}\left(W_{ij}(h_{ij}) \geq \max_{k \in J_i} W_{ik}(h_{ik}) \middle| \boldsymbol{X}_i(r_i)\right) \tag{3.6}$$

where $W_{ij}(h_{ij})$ denotes the random variable for the effective value. The probability then is taken over the joint distribution of all effective values, conditional on the observable attributes and the ranking.

The (expected) consumer welfare of a consumer i then can be calculated as (see Appendix 3.B.1)

$$\begin{aligned} CS_i(r_i) = \mathbb{E}&\left[\sum_{k=0}^{\bar{h}_i} 1\left(\bar{W}_{ik-1} < z^d(k-1)\right) 1(\bar{W}_{ik} > z^d(k))\bar{W}_k \middle| \boldsymbol{X}_i(r_i)\right] \\ &- \mathbb{E}\left[\sum_{k=0}^{\bar{h}_i - 1} 1\left(\bar{W}_{ik} < z^d(k)\right) c_d(k) \middle| \boldsymbol{X}_i(r_i)\right] \end{aligned} \tag{3.7}$$

where $\bar{W}_{ik} = \max\{\tilde{W}_{i0}, \ldots, \tilde{W}_{ik}\}$ denotes the random variable of the maximum of values

[9] This is an application of Theorem 2 in Chapter 2, where $n_d = 1$ and the discovery value depends on positions h with $z^d(h) \geq z^d(h+1)$.

defined by (3.5) discovered up to position k.[10] $\bar{h}_i$ denotes the maximum position that i can discover.[11] The expectation is calculated conditional on the observable attributes and the ranking. The first expression captures welfare derived from the chosen alternative and inspection costs paid up to that point. The second expression reflects the discovery costs the consumer pays in expectation.[12] A change in ranking affects both parts: the consumer potentially chooses a different alternative, and pays a different amount of discovery costs.

3.3.4 Heterogeneous position effects allow rankings to increase revenues

The large operations literature focusing on revenue-maximization suggests that online retailers and search intermediaries focus on increasing revenues. As they take a share of each alternative's revenues, online retailers and search intermediaries increase their own revenues if they increase total revenues across all alternatives (see also 3.4.1.2).[13] Hence, their goal when implementing different rankings often is to increase total revenues generated across the whole list.

To build intuition how a change in ranking affects these revenues, I focus on the simple case where a consumer i initially is aware of only a single alternative ($|A_{i0}| = 1$) and discovers products one at a time ($n_d = 1$).

Consider a switch of the position of two subsequent products $j = A, B$ displayed on the first two positions. Denoting the demand increase resulting from the switch by $\Delta d_{ij} \equiv d_{ij}(h) - d_{ij}(h+1) \geq 0$, the change in (expected) total revenues can be calculated as[14]

$$\Delta ER = (p_B - p_A)\Delta d_{iA} + p_B(\Delta d_{iB} - \Delta d_{iA}) \tag{3.8}$$

This expression directly reveals that this switch in position can increase revenues through two distinct mechanisms. The first part captures that if the product moved higher up is more expensive, the demand it diverts from the product it replaces yields more revenues. This "price effect" is complemented by a change in the overall demand, i.e. the decrease in the share of

[10] To capture alternatives in A_0, I define $\bar{W}_{i,-1} = 0$, which combined with the previously defined $z^d(-1) = \infty$ ensures that $1\left(\bar{W}_{ik-1} < z^d(k-1)\right) = 1$ for $k = 0$.

[11] $\bar{h}$ does not need to equal $|J_i|$ because alternatives in A_0 have the same position.

[12] If the consumer's beliefs are correct and adapt with the ranking, the expression can be further simplified based on proposition 1 in Chapter 2.

[13] If margins are homogeneous across products, revenue-maximization will be equivalent to profit-maximization.

[14] The expression directly follows from $\Delta ER = p_B \Delta d_{iB} - p_A \Delta d_{iA}$.

the outside option. This "demand effect" captures that if the product we are moving higher up gains more demand than the one we are moving down loses, overall demand and, therefore, revenues increase.

Underlying this demand effect are *heterogeneous* position effects. Without such heterogeneity, $\Delta d_{iB} - \Delta d_{iA} = 0$ and only the price effect would prevail. Through its micro-founded explanation for position effects, the search and discovery model offers a clear explanation for how heterogeneity in position effects arises. Specifically, by conditioning on different probability regions for the utility offered by the outside option we get (see Appendix 3.B.3)

$$\begin{aligned}\Delta d_{iB} - \Delta d_{iA} = \mathbb{P}(U_{i0} > z^d(0)) \Big[& \mathbb{P}\left(\tilde{W}_{iB} > U_{i0} | U_{i0} > z^d(0)\right) \\ & - \mathbb{P}\left(\tilde{W}_{iA} > U_{i0} | U_{i0} > z^d(0)\right)\Big] \end{aligned} \tag{3.9}$$

This expression reveals that heterogeneous position effects only arise if consumers that do not buy an alternative, do not discover all alternatives. To see this, note that (3.9) only depends the probability region where $u_{i0} > z^d(0)$. In this case, the draw of the outside option is so large that the second product never is discovered, even if B is a bad match. This result follows from the fact that in the case where $u_{i0} < z^d(0)$, the product being moved to the first position only diverts demand from the other product; one's gain is the other's loss. In contrast, with $u_{i0} > z^d(0)$, different products on the first position only divert demand from the outside option, such that heterogeneity arises.

The relative strength of the price and demand effect determines which switches increase total revenues. Because in this simple example, Δd_{ij} increases in u^e_j and decreases in p_j (see Appendix 3.B.3), the demand and the price effect can go either in the same, or opposite directions. Specifically, if $p_B > p_A$ and $u^e_B > u^e_A$, both effects go in the same direction and it is clear that the switch increases total revenues. If instead $u^e_B < u^e_A$ because of non-price attributes, the demand effect offsets the price effect such that their relative strength determines whether the switch increases revenues.

Whereas this simplified analysis focuses only on the switch of the first two positions, a similar logic applies to other switches. In the general case of switching products on positions $h_u < h_l$, the demand for all products j with $h_j < h_u$ will also change. This is because with some probability, the consumer stops after discovering products on position h_u. In this case, the demand for products j with $h_j < h_u$ will depend on the product that is shown on position h_u. Moreover, for products with $h_j \in (h_u, h_l)$ the probability of stopping after discovering the product on h_l depends on what product is revealed on this position.

3.3.5 Discussion

Albeit not the focus in this paper, it is worth noting that the described mechanisms for how rankings affect revenues and consumer welfare also apply to personalized rankings. The only difference in personalized rankings is that preference weights are heterogeneous across consumers, and that when constructing the ranking, an online retailer takes into account a specific consumer's preference weights and how that leads to heterogeneous position effects.

Whereas the model provides a micro-foundation to position effects and captures the decision process consumers face when interacting with ranked product lists relatively well, there are some factors it does abstract from. Specifically, in line with directed search models based on Weitzman (1979), I do not model learning across alternatives or costly recall. As highlighted in Chapter 2, the advantage of this is that it yields an optimal policy which remains tractable and leads to a simple characterization of how rational consumers will search among alternatives and which alternative they eventually purchase.

In line with previous literature studying the effects of rankings and deriving optimal ranking algorithms (e.g. Ghose et al., 2014; Chen and Yao, 2017; Ursu, 2018; Chu et al., 2020; Derakhshan et al., 2022), prices and the discovery value $z^d(h)$ do not depend on the ranking. This allows to focus on the direct of effects of changes in rankings, but abstracts from indirect effects through consumers updating beliefs or price adjustments of alternatives. This is justified as even with repeat visits, consumers are unlikely to be able to distinguish the different rankings. The alternatives offered and the rankings vary substantially over time on most websites. Hence, inferring the ranking algorithm used to order alternatives would be very difficult for consumers. Similarly, if alternatives are sold by individual sellers, these sellers would need to be very sophisticated to adjust prices based on constantly changing rankings and demand conditions.[15] Nonetheless, considering long-term adjustments of beliefs or prices may provide an interesting avenue for future research.

[15] There is some empirical evidence of pricing frictions. For example, Garcia et al. (2022) finds that hotel managers follow price recommendations of a platform only with some delay. Huang (2022) finds that sellers on Airbnb often use uniform pricing and do not react to demand changes.

3.4 Data and estimation approach

3.4.1 The Expedia dataset

The dataset is from a leading online travel agency, Expedia, and can be obtained from Kaggle.com.[16] It consists of data on clicks and purchases from 166,039 consumers looking for hotel stays on Expedia.com during the period between November 2012 and June 2013.[17] When submitting a query for a hotel stay, Expedia presents a list of available options. Consumers observe a range of hotel characteristics on this list, such as the price per night and the review score. They then can click on a particular item in the list to go to the hotel's individual page that reveals further information and provides the option to book the hotel.

3.4.1.1 Summary

The main feature and advantage of the data is that for around 30% of consumers, the ranking was randomized; in this sample, Expedia randomly assigned hotels fitting a consumers' query to positions on the list. For the other 70% of consumers in the sample, Expedia used their ranking algorithm to assign hotels to positions.[18]

Having this experimental variation in hotels' positions is important to determine heterogeneous position effects without convoluting the effect of more desirable hotels also being displayed higher. If hotels are positioned on top of the list based on unobservable characteristics that also lead to more clicks and purchases, it would be difficult to disentangle correlations with such unobservables, potentially leading to an overestimation of position effects (see Ursu, 2018). Besides, for the external validity of the results it is also necessary that consumers were assigned randomly into the group that observed the randomized positions. This check is already done in online appendix B of Ursu (2018), who uses the same dataset. The comparison shows that consumer characteristics are similar between the two samples. Moreover, her analysis shows that hotel characteristics are distributed evenly across positions.

Table 3.1 summarizes the data on a hotel- and consumer-level for consumers that observed the random ranking. A detailed description of each variable is provided in Table 3.8 in Ap-

[16] `https://www.kaggle.com/c/expedia-personalized-sort/data`.

[17] This is the final dataset after cleaning. Appendix 3.A shows the respective criteria.

[18] It is not possible to get the specifics of the Expedia ranking algorithm used during the sample period. Most likely it used observable hotel and query characteristics to create a ranking that maximized predicted clicks or bookings.

Table 3.1 – Summary statistics (random ranking)

	N	Mean	Median	Std. Dev	Min.	Max.
Hotel-level						
Price (in $)	1,357,106	171.70	141.04	114.03	10.00	1000.00
Star rating	1,333,734	3.34	3	0.89	1	5
Review score	1,354,996	3.81	4.00	0.97	0.00	5.00
No reviews	1,354,996	0.04	0	0.19	0	1
Chain	1,357,106	0.62	1.00	0.48	0.00	1.00
Location score	1,357,106	3.26	3	1.53	0	7
On promotion	1,357,106	0.24	0	0.43	0	1
Consumer-level						
Number of items	51,510	26.35	31	8.46	5	38
Number of clicks	51,510	1.14	1	0.66	1	25
Made booking	51,510	0.08	0	0.27	0	1
Trip length (in days)	51,510	3.07	2	2.42	1	40
Booking window (in days)	51,510	53.67	31	62.49	0	498
Number of adults	51,510	2.08	2	0.94	1	9
Number of children	51,510	0.43	0	0.82	0	9
Number of rooms	51,510	1.14	1	0.46	1	8

Notes: Summary statistics for observations under random ranking. Detailed variable descriptions are provided in Table 3.8 in Appendix 3.A.

pendix 3.A.[19] In total, 51,510 consumers observed the random ranking. On average, consumers clicked on 1.14 hotel listings, and about 8% eventually booked a hotel. Some consumers observed only a few items, with the minimum number of items observed being only five. This does not result from consumers not browsing further, but from these consumers searching for hotels in destinations or on dates where only few hotels had rooms available. Moreover, the maximum of items consumers observed is 38, despite some destinations potentially offering more alternatives than that. This stems from a limitation in the data where I only observe results displayed on the first page. However, this does not affect the reduced-form analysis due to the way heterogeneous position effects are identified from the data. Moreover, the estimation of the structural model does not assume that consumers must have stopped scrolling on the first page and therefore is little affected, apart from there being less information available to pin down the value of discovering more hotels.

[19] The "no reviews" variable is a dummy indicating whether a hotel has no reviews. In the dataset this is coded as a "review score" of zero. However, given that it differs from a "review score" of zero as well as from a missing "review score," I treat this dummy separately.

3.4.1.2 Expedia benefits from increasing total revenues

By offering hotel bookings on its website, Expedia makes revenues through two different models typical for online travel agents.[20] First, some hotel listings are offered through a merchant model. Under this model, Expedia negotiates with individual hotels the rooms made available through its website and each room's respective pricing. Second, some hotels are listed under the agency model. In this case, Expedia takes a commission and only passes on reservations and payments to the individual hotels that set prices themselves. With the data, it is not possible to determine under which revenue model a specific hotel is listed.[21]

Under both models, Expedia makes revenues by taking a share of the hotel price when it is booked. As a result, Expedia increases its own revenues by increasing total revenues across all hotels on the list. Moreover, if commissions are the same for all hotels, maximizing total revenues across the whole list also maximizes Expedia's profits. This motivates the focus of this paper on revenue-based rankings that target total revenues generated from the whole list.

3.4.1.3 Two limiting features

There are two main limiting features: (i) the dataset contains data only on consumers that made at least one click and (ii), queries that led to a purchase were oversampled. Whereas these limitations are relevant for my analysis, I concur with Ursu (2018) that they can be circumvented. First, position effects are identified from differences in click and purchase rates across positions and characteristics. Hence neither limitation affects the identification of position effects. Second, as only part of the consumer population is observed, counterfactual results may not generalize to the whole population. However, for consumers that did not click on any listing under the ranking they observed, a change in the ranking can only induce them to click and book more (clearly not less). This implies that as long as consumers that did not click on any hotel differ only in search costs from the ones that are observed in the sample, my counterfactual results provide lower bounds for the change in both revenues and consumer welfare.

[20] Expedia also offers other services such as car rentals or flights that are not part of the data.

[21] In 2013 (the sample period), Expedia made 70% of its worldwide revenue through the merchant model and 24% through the agency model. The revenue from the merchant model primarily is from hotel bookings, where as the agency model also includes flights and other products. See the annual report https://s27.q4cdn.com/708721433/files/doc_financials/2013/ar/EXPE_2013_Annual_Report.PDF

3.4.1.4 Estimation sample

To estimate the model, I restrict the sample to the largest four destinations. Moreover, following Ursu (2018) I only include queries that had at least 30 hotel listings in the results. This is the most commonly observed case for these four destinations. The resulting estimation sample captures typical search sessions for the most popular destinations, excluding other queries for less popular destinations or with only few results. The resulting sample consists of 2,890 consumers that on average clicked on 1.15 listings and 5.77% of which booked a hotel. Additional sample statistics are provided in Table 3.5. The data does not contain information as to which cities or areas these destinations correspond to. However, all four destinations are in the same country, for which 80% of queries are for domestic travel within the country. This strongly suggests that the four destinations are in the United States (see Ursu, 2018).

3.4.2 Empirical specification

For the estimation, I assume that consumers discover hotels one at a time ($n_d = 1$), and that consumers discover the first three listings for free ($|A_{i0}| = 3 \forall i$). This is motivated by Expedia's website layout; after entering a search query, Expedia's product list initially reveals three hotel listings, after which each row on the product list only reveals a single hotel. However, note that the estimation approach described in the next session directly generalizes to cases where consumers observe multiple alternatives prior to search and discover multiple alternatives at a time.

Because the optimal policy only depends on the reservation values, I directly parameterize the discovery value as follows:

$$z^d(h) = \Xi - \exp(\rho)h \tag{3.10}$$

Ξ is the initial discovery value, whereas the parameter ρ governs the rate by which the discovery value decreases across the different positions. Both Ξ and ρ are parameters that are estimated. Note that the exponential function ensures that the discovery value is decreasing for all values of ρ. This guarantees that the monotonicity condition is satisfied (see 3.3.1).

Because (3.3) is a unique mapping, discovery costs can be backed out from the estimated $z^d(h)$ by additionally specifying assumptions on consumers' beliefs. Specifically, I assume that discovery costs are constant across positions, hence, a potential decrease in $z^d(h)$ only stems from consumers expecting worse alternatives to be discovered later on. Second, I assume that only the mean of G_h depends on the position. Combined with the rational expectations assumption, this allows me to estimate G_h based on the observed hotel attributes. Further details are provided in Appendix 3.B.2.

Appendix 3.B.2 also details the main advantages of this approach. First, it allows to estimate the model and derive demand implications without having to determine consumers' beliefs over alternatives that are going to be discovered. Backing out the discovery costs is only necessary for consumer welfare analysis. Moreover, during estimation it avoids having to re-estimate consumers' beliefs and having to do the computationally costly inversion (3.3) at each step in the estimation.

Search values as parameterized by specifying that

$$\xi_{ij} = \xi + \rho_\xi \log(\tilde{h}_{ij}) + \nu_{ij} \qquad\qquad \nu_{ij} \sim N(\mu_\nu, \sigma_\nu^2) \tag{3.11}$$

With this specification, $-\alpha p_j + \boldsymbol{x}_j'\beta + \xi + \rho_\xi \log(\tilde{h}_{ij})$ is the average expected net benefit of inspecting hotel j. This term depends on product j's position within the listings that the consumer observes when arriving at the list, denoted by $\tilde{h}_{ij}$. Specifically, recall that $pos_{ij} = 1, 2, \ldots$ denotes the position on the product list at which hotel j is displayed to consumer i. I then define $\tilde{h}_{ij} = \min\{pos_{ij}, 3\}$, as the consumer initially observes three listings. This specification implies that for hotels that are observed initially, expected net benefits of searching differ depending on where in the first few positions a hotel is shown. This additionally captures that it can be more costly for consumers to inspect hotels that are shown further down within the listings observed initially. Moreover, the specified functional form implies that the rate by which the expected net benefits decrease also decreases, which is motivated by the same pattern observed in the data (see Figure 3.3).

The additional error term, ν_{ij}, introduces hotel-specific heterogeneity in inspection costs. Specifying the distribution over the expected net benefits ξ_{ij} is equivalent to specifying a distribution over heterogeneous inspection costs. This follows from (3.2) that allows to uniquely determine c_{ijs} from ξ_{ij} and the distributional assumption on ε_{ij}.

This parametrization of the search values differs from previous approaches (e.g. Honka, 2014; Ursu, 2018). Instead of estimating parameters for the inspection cost and how they increase across positions, I directly estimate parameters for the search values and how they depend on the position. This avoids having to apply the non-linear mapping (3.2) during the estimation, making it computationally more efficient.[22] Moreover, consumer welfare and demand predictions continue to be valid under alternative assumptions on why the net benefits of inspecting vary across hotels. For example, instead of heterogeneous inspection costs, consumers may have heterogeneous beliefs on the distribution of ε_{ij}, generating a distribution over ξ_{ij}.[23]

I further assume that the idiosyncratic taste shocks are i.i.d. normal with mean μ_ε and variance σ^2_ε. Putting things together, the model parameters are β, ξ, ρ, μ_ε, σ^2_ε , μ_ν and σ^2_ν. As described in the following sections, the model parameters are either estimated or normalized.

3.4.3 Estimation approach

I estimate the model parameters using simulated maximum likelihood. In general, the fact that consumers always take the action with the largest reservation value implies a set of inequalities that need to hold for any observed search path. The estimation procedure then finds the parameter values that maximize the log-likelihood of these inequalities holding. Formally, the estimation procedure solves

$$\max_\theta \mathcal{L}(\theta) = \max_\theta \sum_{i=1}^{N} \log \mathcal{L}_i(\theta) \tag{3.12}$$

where $\mathcal{L}_i(\theta)$ is consumer i's likelihood contribution given the model-implied inequalities and parameters gathered in θ.

This set of inequalities follows from the following insights: First, the optimal policy implies that the effective value of the purchased product (or the outside option) determines how many products a consumer discovered: whenever $\bar{w}_i \equiv \max_j w_{ij} \geq z^d(h)$, both the search and purchase value of j exceed the discovery value at position h, hence the consumer buys j without ever discovering products beyond position h. Conversely, if $\bar{w}_i < z^d(h)$, the consumer did not stop before discovering the hotel on position h. Second, a product having been

[22] With heterogeneous search costs, the mapping would have to be done many times during each iteration of the optimizer, as it would have to be applied for every simulation draw of the inspection cost.

[23] Compiani et al. (2021) independently applied a similar logic in their “double-index” search model.

searched requires that its search value exceeds $\bar{w}_i$. Third, those products that are searched but not bought additionally require that their utility is smaller than $\bar{w}_i$. Finally, products that are not searched must have a search value that is smaller than the effective value of the purchased product.

The probability of these model-implied inequalities holding then yields the likelihood contribution of a consumer i. There are multiple cases to consider, which depend on whether a consumer made any clicks and whether a search session ended with a booking or by taking the outside option. Here, I only present the two cases that occur in the data. The case where a consumer does not make any clicks is presented in Appendix 3.B.4.

3.4.3.1 Case 1: consumer makes clicks but takes the outside option

Consider a consumer i that clicked on hotel listings and did not book a hotel (the most common case in the data). To simplify notation, I define the *consideration set* S_i as the set containing all listings the consumer clicked on as well as the the outside option. Furthermore, let $z^s_{i0} = \infty$ such that the search value of the outside option is such that $z_{i0} \geq c$ holds for finite c. Observing a click on j implies that i must have discovered listings at least up to its position. This implies an upper bound on $w_{i0} = u_{i0}$, as the consumer stops scrolling at position h where $z^d(h) < u_{i0}$. Consequently, the upper bound on u_{i0} is given by $z^d(\bar{h}_i - 1)$, where $\bar{h}_i \equiv \max_{j\in S_i} h_{ij}$ denotes the last position at which i clicked. Denoting the set of products that the consumer discovered given u_{i0} by $J_i(u_{i0})$ and the cumulative density of u_{i0} by $H(\cdot)$, the likelihood contribution then can be written as

$$\mathcal{L}_i(\theta) = \int_{-\infty}^{z^d(\bar{h}_i-1)} \mathbb{P}\left(\min_{j\in S_i} Z^s_{ij} \geq u_{i0} \cap \max_{j\in S_i} U_{ij} \leq u_{i0} \cap \max_{j\in J_i(w_{i0})\setminus S_i} Z^s_{ij} \leq u_{i0}\right) \mathrm{d}H(u_{i0}) \quad (3.13)$$

$$= \int_{-\infty}^{z^d(\bar{h}_i-1)} \prod_{j\in J_i(u_{i0})\setminus S_i} \mathbb{P}\left(Z^s_{ij} \leq u_{i0}\right) \times \prod_{j\in S_i} \mathbb{P}(Z^s_{ij} \geq u_{i0})\mathbb{P}(U_{ij} \leq u_{i0})\mathrm{d}H(u_{i0}) \quad (3.14)$$

The second expression follows from the independence of the underlying unobserved error terms. The different terms follow directly from the optimal policy discussed above, where by definition $w_{i0} = u_{i0}$. $\prod_{j\in J_i(u_{i0})\setminus S_i} \mathbb{P}\left(Z^s_{ij} \leq u_{i0}\right)$ takes the product over all hotels the consumer did not click on, where the optimal policy implies that their search values need to be smaller than the utility of the outside option. $\prod_{j\in S_i} \mathbb{P}(Z^s_{ij} \geq u_{i0})\mathbb{P}(U_{ij} \leq u_{i0})$ takes the product over all hotels the consumer did click on, where the optimal policy implies that their search values need to be larger than the outside option, whereas their utilities are smaller.

3.4.3.2 Case 2: consumer makes clicks and books a hotel

Consider a consumer i that clicked on hotel listings in set S_i and ends the search session by booking hotel q. As the inequalities are going to depend on whether different hotels in S_i are displayed before or after q, the set is partitioned into a set S_{i-} containing clicked hotels for which $h_{ij} < h_{iq}$, and S_{i+} containing clicked hotels for which $h_{ij} \geq h_{iq}$. As the search session ends with booking q, the effective value of q (w_{iq}) determines how far the consumer scrolled. If q is the hotel with the maximum position of any clicked hotel ($h_{iq} = \bar{h}_i$), there is no upper bound on $\tilde{w}_{iq}$ as the consumer can discover, click and then book q without scrolling beyond h_{iq}. However, if $\bar{h}_i > h_{iq}$, the consumer must have first discovered additional hotels before going back to book q. In this case, $\tilde{w}_{iq}$ is again bounded above by $z^d(\bar{h}_i - 1)$. To gather both cases in one expression, define the upper bound as $b_q = z^d(\bar{h}_i - 1)$ if $h_{iq} < \bar{h}_i$ and else equal to ∞. Denoting the cumulative density of the value $\tilde{w}_{iq}$ by $H_q(\cdot)$, the likelihood contribution of i can be written as

$$\begin{aligned}\mathcal{L}_i(\theta) = \int_{-\infty}^{b_q} \mathbb{P}\Bigg(\min_{j \in S_{i-}} Z_{ij}^s \geq w_{iq} \cap \min_{j \in S_{i+}} Z_{ij}^s \geq \tilde{w}_{iq} \cap \\ \cap \max_{j \in S_i} U_{ij} \leq \tilde{w}_{iq} \cap \max_{j \in J_i(w_{i0}) \setminus S_i} Z_{ij}^s \leq \tilde{w}_{iq} \Bigg) \mathrm{d}H_q(\tilde{w}_{iq}) \qquad (3.15)\end{aligned}$$

$$\begin{aligned}= \int_{-\infty}^{b_q} \prod_{j \in J_i(\tilde{w}_{iq}) \setminus S_i} \mathbb{P}\left(Z_{ij}^s \leq \tilde{w}_{iq}\right) \times \prod_{j \in S_{i-}} \mathbb{P}(Z_{ij}^s \geq w_{iq}) \mathbb{P}(U_{ij} \leq \tilde{w}_{iq}) \\ \times \prod_{j \in S_{i+}} \mathbb{P}(Z_{ij}^s \geq \tilde{w}_{iq}) \mathbb{P}(U_{ij} \leq \tilde{w}_{iq}) \mathrm{d}H_q(\tilde{w}_{iq}) \qquad (3.16)\end{aligned}$$

The second expression again follows from independence of the idiosyncratic error terms.

There are two main differences to the first case. First, the integration now is over the value $\tilde{w}_{ij}$ defined by (3.5) for the booked hotel instead of the utility of the outside option. Second, listings that are clicked on but not bought are partitioned into two sets. For listings that are discovered before q, the search value needs to be larger than the effective value $w_{iq} = \{z^d(h_{iq} - 1), \tilde{w}_{iq}\}$ because consumers may click on $j \in S_{i-}$ before discovering q such that $z_{ij}^s \geq z^d(h_{iq} - 1)$ is sufficient. However, for other listings $j \in S_{i+}$ this is not sufficient because they are discovered only after q, hence $\tilde{w}_{iq} \leq z_{ij}^s$ needs to hold.

3.4.4 Simulation approach

Given that there is no closed-form solution for the above integrals, I use Monte Carlo integration to calculate them. A straightforward approach would be to take draws for $\tilde{w}_{iq}$ or u_{i0}, calculate the corresponding set $J_i(\cdot)$ for draws that satisfy the respective upper bounds, and then calculate the inner expressions based on the cumulative density of the normal distribution. However, this so-called "crude frequency simulator" (see e.g. Train, 2009) has two main disadvantages: (i) it does not lead to a smooth likelihood function due to how, for example, $\mathbb{P}(\max_{k\in J_i(\tilde{w}_{iq})} Z^s_{ik} \leq w_{i0})$ depends on $J_i(\tilde{w}_{iq})$, and (ii) it would require to take many draws to get a reasonably accurate approximation for the integral.

Given these disadvantages, I develop a procedure akin to the well-known GHK simulator for the Probit model,[24] and similar to the recently developed approach of Chung et al. (2019). By taking draws from $H(\cdot)$ truncated so that $J_i(\cdot)$ remains the same for all draws within the given bounds, I circumvent both these issues. To focus on the main idea, I only show the respective procedure for the relatively simple first case. The more involved procedure for the second case is deferred to Appendix 3.B.5, but is based on the same main insight of using subintervals.

To calculate the integral in (3.14), the procedure below is applied, where J_i denotes the set of hotel listings that consumer i potentially can discover.

1. Partition the interval $(-\infty, z^d(\bar{h}_i - 1)]$ into subintervals

$$(-\infty, z^d(|J_i|)], (z^d(|J_i|), z^d(|J_i| - 1)], \ldots, (z^d(\bar{h}_i - 2), z^d(\bar{h}_i - 1)]$$

2. For each subinterval, take N_r draws for u^r_{i0} truncated to be within the specific subinterval

3. For each subinterval and draw u^r_{i0}, determine $J(u^r_{i0})$ and calculate

$$P(u^r_{i0}) \equiv \prod_{j\in J_i(u_{i0})\backslash S_i} \mathbb{P}\left(Z^s_{ij} \leq u^r_{i0}\right) \times \prod_{j\in S_i} \mathbb{P}(Z^s_{ij} \geq u^r_{i0})\mathbb{P}(U_{ij} \leq u^r_{i0})$$

[24] The procedure is named after the authors of the following contributions (see Train, 2009): Geweke (1989, 1991); Hajivassiliou and McFadden (1998); Keane (1990, 1994).

4. Sum up within-interval means across draws, weighting by the probability of u_{i0}^r falling within the subinterval

Putting this into one formula yields

$$\tilde{\mathcal{L}}_i(\theta) = \sum_{s=1}^{N_s} \mathbb{P}\left(u_{i0} \in B(s)\right) \frac{1}{N} \sum_{r=1}^{N_r} P(u_{i0}^r) \tag{3.17}$$

where N_s is the number of subintervals, and $B(s)$ are the respective bounds for subinterval s. Note that as $u_{i0}^r = \beta_0 + \varepsilon_{i0}^d$, it is i.i.d. normal, standard procedures can be used to take draws that satisfy the bounds and calculate the corresponding probability.[25] Similarly, calculating the probabilities in $P(u_{i0}^r)$ only requires calculating the cumulative densities.

3.4.5 Directed search is a special case

As directed search *á la* Weitzman (1979) is a special case of the search and discovery process underlying the present model, it is worth pointing out that a simplified version of the above estimation approach can also be used to estimate a directed search model. Specifically, removing the upper bounds in the integrals and setting $J(\tilde{w}_{iq}) = J_i$ directly yields the likelihood contributions for a directed search model. As $J(\tilde{w}_{iq})$ is fixed to the whole set of products consumer i can discover, it is also not necessary to partition into subintervals. To incorporate position-specific search costs, it is then only necessary to adjust the specification of the search values in (3.11).

Using this simulation approach has two main advantages for estimating a directed search model. First, it yields a smooth likelihood function without having to manually set a smoothing parameter, which is not an innocuous choice in small samples. Second, it is computationally efficient as numerical integration is performed only over one dimension.

3.4.6 Identification and normalizations

Between-consumer variation in the characteristics of the displayed hotels, their positions and the associated changes in clicks and bookings allows to identify model parameters. Specifically, mean utility parameters (β) are identified by the correlation between hotel characteristics and

[25] Specifically, w_{i0}^d follows a normal distribution truncated at $\left[z_{ij}^d(h_{ij} - b - 1), z_{ij}^d(h_{ij} - b)\right]$. Denoting the cdf of a standard normal distribution by $\Phi(\cdot)$, we have $\mathbb{P}\left(w_{i0} \in \left[z_{ij}^d(h_{ij} - b - 1), z_{ij}^d(h_{ij} - b)\right]\right) = \Phi\left(\frac{z_{ij}^d(h_{ij}-b)}{\sigma_\varepsilon}\right) - \Phi\left(\frac{z_{ij}^d(h_{ij}-b-1)}{\sigma_\varepsilon}\right)$, and taking draws from the truncated normal can be done using standard numerical methods.

the associated click and booking frequency. For example, with a smaller price coefficient (in absolute terms), consumers on average click and book more expensive hotels. The baseline parameter for the expected net benefits of clicking (ξ) is identified by the number of clicks of consumers; with a large ξ, consumers on average click on more listings. Moreover, the respective change in net expected benefits of clicking on one of the first few positions (ρ_ξ) is identified by the differences in the click rates across these early positions and the functional form assumption.

Because the data does not contain information on where on the page a consumer stopped scrolling, product discovery is latent and not directly observed. Nonetheless, both parameters governing the discovery value can be identified by the decrease in the click frequency at later positions. This can be seen in the likelihood contribution shown in (3.14). With a large Ξ or small ρ, larger values of u_{i0} become more likely, making it also more likely that $J(u_{i0})$ contains more listings on later positions. Consequently, the probability of a consumer not clicking on listings in $J(u_{i0})$ will be smaller.

However, the number of available alternatives is limited and a substantial share of consumers are observed to click on positions that are on the bottom part of the page. As the last click of a consumer provides an upper bound for $\tilde{w}_{iq}$, this leaves few alternatives beyond the last click that are required to bound the discovery value from above through fewer clicks being observed on positions after the last click. To circumvent this and provide an upper bound on the discovery value, I additionally assume that consumers always first clicked before going on to discover more hotels. Whereas this implies that consumers always clicked from top to bottom, this is a pattern common to click-stream data.[26]

This provides an upper bound for the discovery value as it additionally requires that for an observed click on hotel j by consumer i, the following inequality holds:

$$z_{ij}^s \geq \Xi \tag{3.18}$$

Additionally imposing this inequality only requires changing the probability $\mathbb{P}(Z_{ij}^s \geq u_{i0})$ to $\mathbb{P}(Z_{ij} \geq \Xi)$ in the likelihood contributions (3.14) and (3.16).

[26] Choi and Mela (2019) find that in 98.2% of visits, consumers do not click from bottom-up. The authors use this to justify their “top-down” search assumption.

The means of the idiosyncratic shocks cannot be identified separately from other parameters. Specifically, μ_ν cannot be separately identified from ξ. This directly follows from the click inequality where both μ_ν and ξ enter linearly.[27] As is also standard in discrete choice models, μ_ε is not identified as it shifts the product utilities equally across all hotels. Consequently, in the estimation both parameters are normalized to $\mu_\nu = \mu_\varepsilon = 0$. σ_ν also can not be separately identified from ξ; all else equal, both an increase in ξ or an increase in σ_ν increases average number of clicks. I therefore set it to $\sigma_\nu = 1$ in the estimation.

Similar to Yavorsky et al. (2020), σ_ε can be identified through the parametric form of the search values and position being an exogenous search cost shifter. Whereas these authors argued in the context of a directed search model based on Weitzman (1979), similar arguments apply to its generalization used in this paper. Whereas this works well in simulations (see below), in the estimation σ_ε is not identified well. This is because with the additionally imposed condition (3.18), probabilities for initial clicks scale in σ_ε, making it more difficult to identify it separately from ξ. Hence instead of estimating this variance, I set it to $\sigma_\varepsilon = 0.1$. As shown in Section 3.5.2, this produces a reasonable model fit.[28]

Monte Carlo simulations confirm that parameters can be identified with the present data. I generate datasets by simulating search paths for given parameter values, and then verify whether the estimation procedure is able to recover these parameter values.[29] For observable hotel characteristics and positions, I directly use the ones observed in the estimation sample described in 3.4.1.4. This ensures that the variation in hotel characteristics across positions is the same during the simulation as in the different estimation samples. I repeat this procedure 20 times and present average results.

Results of two such Monte Carlo simulations are presented in Table 3.2. They show that estimation procedure is able to recover all parameters. Specifically, all parameter estimates are very close to the true ones. It is worth pointing out that these results suggest that the estimation approach indeed provides an improvement over approaches that rely on kernel-smoothed frequency simulators to calculate the likelihood function. Studies that employ such

[27] In the likelihood contribution (3.16), the click inequality is given by $\mathbb{P}(Z^s_{ij} \geq \tilde{w}^r_{iq}) = \mathbb{P}(\nu_{ij} \geq \tilde{w}^r_{iq} - \boldsymbol{x}'_j\beta - \xi - \rho h_j)$, hence increasing the mean of ν is equivalent to increasing ξ.

[28] Setting this to larger values leads the model to overpredict bookings.

[29] As the data is generated without condition (3.18), this condition is also not applied when performing the estimation.

Table 3.2 – Monte Carlo simulation

	Simulation 1			Simulation 2		
	True	Estimate	Std. Error	True	Estimate	Std. Error
Price (in 100$)	-0.20	-0.20	(0.01)	-0.20	-0.19	(0.00)
Star rating	0.10	0.10	(0.02)	0.20	0.19	(0.00)
Review score	0.10	0.11	(0.02)	0.05	0.05	(0.01)
No reviews	0.20	0.23	(0.14)	0.10	0.09	(0.03)
Location score	0.05	0.05	(0.01)	-0.10	-0.10	(0.00)
Chain	0.10	0.10	(0.03)	0.20	0.19	(0.01)
Promotion	0.10	0.10	(0.02)	0.10	0.09	(0.00)
Outside option	2.00	2.02	(0.10)	0.50	0.47	(0.02)
ξ	1.70	1.68	(0.06)	1.50	1.49	(0.03)
Ξ	1.50	1.52	(0.09)	1.40	1.36	(0.05)
ρ	-4.00	-4.00	(0.18)	-4.00	-3.99	(0.48)
ρ_ξ	-0.40	-0.40	(0.03)	-0.20	-0.21	(0.02)
σ_ε	1.00	-0.99	(0.03)	0.20	-0.20	(0.01)

Notes: Average across 20 Monte Carlo simulations. Each estimation is performed with 50 simulation draws. Observable data is the same as in the estimation sample.

a smoothed likelihood approach commonly find that parameter estimates in their Monte Carlo simulations are more than two standard deviations away from their true values (e.g. Ursu, 2018; Honka, 2014; Yavorsky et al., 2020).

3.5 Empirical results

As discussed in 3.3.4, how harmful a revenue-maximizing ranking is to consumers depends on whether a potential demand increase outweighs price effects from a change in ranking. Hence, to understand how a revenue-maximizing ranking affects consumers, it is important to first analyze heterogeneity in position effects.

3.5.1 Descriptive evidence of heterogeneous position effects

To provide descriptive evidence of heterogeneity in position effects, I estimate the following linear probability model (LPM):

$$\mathbb{P}(Y_{ij} = 1|\boldsymbol{z}_{ij}) = \boldsymbol{x}_j'\beta_1 + \boldsymbol{w}_i'\beta_2 + pos_{ij}\gamma + pos_{ij}\boldsymbol{x}_j'\theta + \tau_d \tag{3.19}$$

Each observation is a hotel j in destination d shown on position $pos_{ij} = 1, 2, \dots$ in a consumer query i. Note that a larger position implies that the hotel is shown further down on the product list such that a negative coefficient for γ implies that hotels further down the list are less likely to be clicked on. Hotel characteristics (e.g. price) and an intercept are gathered in column vector $\boldsymbol{x}_j$, whereas query characteristics (e.g. trip length) are gathered in column vector $\boldsymbol{w}_i$. Depending on the specification, Y_{ij} is a dummy indicating whether j was clicked on or booked in consumer query i. τ_d indicates fixed effects on a destination-level. Finally, $\boldsymbol{z}_{ij}$ gathers $\boldsymbol{x}_j$, $\boldsymbol{w}_i$, pos_{ij} and the fixed effects τ_d. Throughout, random variables are indicated in capital letters.

The main parameters of interest are γ and θ. These parameters jointly capture how a hotel's position affects its probability of being clicked on or booked. Specifically, given some hotel characteristics $\tilde{\boldsymbol{x}}_j$, I define the *position effect* as the average effect of the position on the CTR or booking probability, conditional on hotel attributes. Because of the randomized ranking, effects on the CTR or booking probability from hotels on other positions are uncorrelated with the position effect. Hence, this effect can be estimated consistently. Formally, the position effect is the marginal effect defined by:

$$\frac{\partial \mathbb{P}(Y_{ij} = 1 | \tilde{\boldsymbol{z}}_{ij})}{\partial pos_{ij}} = \gamma + \tilde{\boldsymbol{x}}_j' \theta \tag{3.20}$$

Table 3.3 presents coefficient estimates and clustered standard errors.[30] Coefficient estimates are scaled to represent changes in terms of percentage points. Columns 1 and 4 show the results of a baseline model that does not include the interaction term and hence excludes heterogeneity in the position effect.[31] Throughout, coefficients of the baseline position effect and hotel characteristics are significant and have the expected sign. For example, more expensive hotels, on average, are less likely to be clicked on and booked, whereas hotels with more stars are more likely to be clicked on and booked.

When interaction terms are included (columns 2 and 5) there is still a significant direct position effect and hotel characteristics continue to have a significant effect on a hotel's probability of being clicked on and booked. However, the direct position effect is considerably smaller and notably, the interaction between price and position is positive and significant. This implies that, on average and conditional on other attributes, cheaper hotels have stronger position

[30] Standard errors are clustered on a consumer-level. This captures that search behavior can induce correlation in the error terms; a large draw in one alternative can mean that consumers are less likely to click on or book another alternative, suggesting potential negative correlation in the error terms.

[31] This replicates parts of Table 2 in Ursu (2018).

Table 3.3 – Coefficient estimates (LPM, random ranking)

	Clicks			Booking		
	(1)	(2)	(3)	(4)	(5)	(6)
Position	-0.1864***	-0.1424***	-0.1427***	-0.0116***	-0.0028	-0.0028
	(0.0019)	(0.0121)	(0.0123)	(0.0005)	(0.0028)	(0.0029)
Price	-0.0125***	-0.0173***	-0.0190***	-0.0011***	-0.0017***	-0.0017***
	(0.0002)	(0.0004)	(0.0004)	(0.0000)	(0.0001)	(0.0001)
Star rating	1.6211***	2.1556***		0.1051***	0.1227***	
	(0.0317)	(0.0657)		(0.0079)	(0.0163)	
Review score	0.0810**	0.0287		0.0366***	0.0794***	
	(0.0351)	(0.0813)		(0.0082)	(0.0193)	
No reviews	-0.2488	-0.7767**		0.0872**	0.1412*	
	(0.1622)	(0.3839)		(0.0345)	(0.0825)	
Chain	0.2227***	0.3105***		0.0248**	0.0641***	
	(0.0459)	(0.0966)		(0.0113)	(0.0240)	
Location score	0.4381***	0.5602***		0.0529***	0.0556***	
	(0.0168)	(0.0321)		(0.0036)	(0.0071)	
On promotion	1.1578***	1.4961***	1.5318***	0.1261***	0.1436***	0.1463***
	(0.0497)	(0.1100)	(0.1229)	(0.0130)	(0.0287)	(0.0332)
Position × Price		0.0003***	0.0003***		0.0000***	0.0000***
		(0.0000)	(0.0000)		(0.0000)	(0.0000)
Position × Star rating		-0.0318***	-0.0270***		-0.0011	-0.0005
		(0.0028)	(0.0029)		(0.0007)	(0.0007)
Position × Review score		0.0028	0.0044		-0.0025***	-0.0024***
		(0.0035)	(0.0035)		(0.0008)	(0.0008)
Position × No reviews		0.0295*	0.0338**		-0.0032	-0.0026
		(0.0164)	(0.0166)		(0.0035)	(0.0036)
Position × Chain		-0.0053	-0.0047		-0.0023**	-0.0017
		(0.0041)	(0.0042)		(0.0010)	(0.0010)
Position × Location score		-0.0071***	-0.0083***		-0.0002	-0.0004
		(0.0013)	(0.0013)		(0.0003)	(0.0003)
Position × On promotion		-0.0201***	-0.0209***		-0.0010	-0.0010
		(0.0048)	(0.0049)		(0.0012)	(0.0013)
Constant	3.2785***	2.5361***	9.8741***	0.2303***	0.0806	0.8939***
	(0.1433)	(0.2909)	(0.1100)	(0.0348)	(0.0680)	(0.0323)
FE Destination	yes	yes	yes	yes	yes	yes
FE Hotel	no	no	yes	no	no	yes
Query Characteristics	yes	yes	yes	yes	yes	yes
N	1,220,917	1,220,917	1,219,253	1,220,917	1,220,917	1,219,253
R2 (adj.)	0.0149	0.0152	0.0267	0.0026	0.0026	-0.0019

Notes: A higher position means being displayed lower on the product list. Coefficients are scaled to represent changes in terms of percentage points. Standard errors are shown in parenthesis and are clustered at the query level. Star ratings are adjusted so that the position effect from the first row is for a hotel with 1 star, and other characteristics equal to the minimum observed value.

effects. Figure 3.1 depicts the position effect for both clicks and bookings at different percentiles of the price distribution and different values of the star rating. Whereas hotels at the first decile of the price distribution on average have a position effect of -0.22 percentage

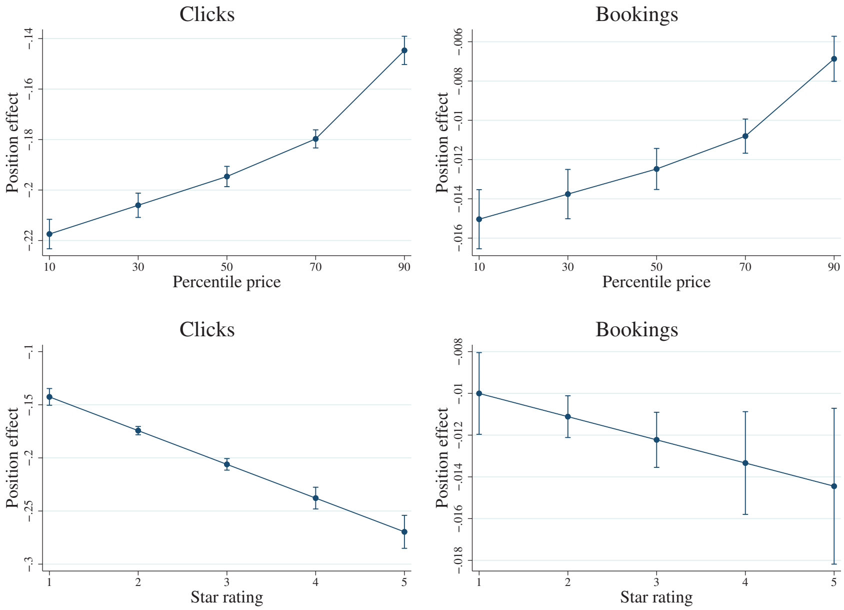

FIGURE 3.1 – Cheaper hotels with more stars have stronger position effects

Notes: Position effects as defined in (3.20) and expressed in percentage points. Values in $\tilde{\boldsymbol{x}}_j$ set to respective percentile of the price distribution or the respective star rating, where values of other characteristics are set to the mean. Bars indicate 95% confidence interval calculated from standard errors clustered on the query level.

points for clicks and -0.015 for bookings, it decreases (in absolute terms) to -0.15 and -0.007 percentage points at the 90% percentile. In contrast, whereas one-star hotels on average have a position effect of -0.14 percentage points for clicks and -0.01 for bookings, it increases (in absolute terms) to -0.26 and -0.014 for five star hotels. For other characteristics such as the review score, the sign of the interaction term also opposes the sign of the direct effect of the characteristics: hotels with characteristics which on average lead to more clicks and bookings on any position overall have stronger position effects.

Figure 3.2 shows combined position effects across different percentiles of the distribution of hotel characteristics $\tilde{\boldsymbol{x}}_j$ in the data. As some characteristics (e.g. price) mute position effects and others (e.g. star rating) amplify it, I construct the different percentiles based on the sign of the respective coefficient in β_1. For example, the first decile in the figure constructs $\tilde{\boldsymbol{x}}_j$ with the price at the 90% percentile of the price distribution, and the star rating of the first decile of the distribution of star ratings. With this construction, the different percentiles order $\tilde{\boldsymbol{x}}_j$ by the predicted click and booking probability given these characteristics, conditional on the position and fixed effects.

Figure 3.2 highlights a clear pattern for position effects in clicks and bookings. Whereas for a hotel at the 10% percentile the position effect is estimated to be around 0.08 percentage points, its magnitude doubles for a hotel at the median and more than triples for a hotel at the 90% percentile. Similarly, the position effect on the probability of being booked increases from close to zero to around 0.02 percentage points.

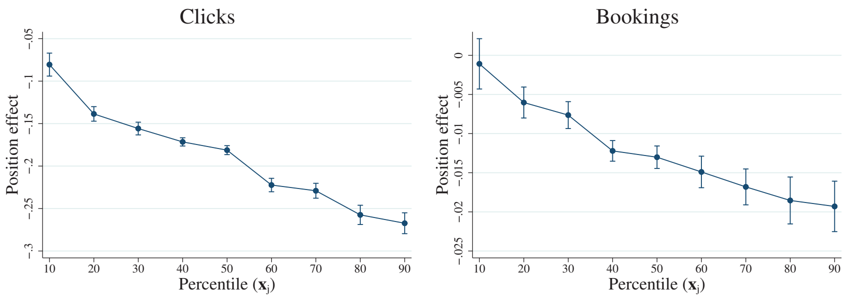

FIGURE 3.2 – Hotel attributes and position effects complement each other

Notes: Position effects as defined in (3.20). Values in $\tilde{\boldsymbol{x}}_j$ set to respective percentile, ordered based on sign of coefficients in columns 2 and 5 in Table 3.3. Bars indicate 95% confidence interval calculated from standard errors clustered on the consumer level.

Though a change of 0.02 percentage points in the booking probability seems small, it is important to note that this change is per position. Compared to a hotel on the tenth position, a hotel with similar characteristics on the first position is estimated to have a 0.2 percentage points larger booking probability. This is a substantial increase compared to no change in the booking probability for products at the low end of the distribution. Moreover, as a search intermediary like Expedia is visited by thousands of consumers every day, even a seemingly small change in the booking probability has substantial effects on revenues.

A hotel's position is exogenous because of the random ranking. However, its price may not be. If there are unobservable hotel characteristics such as quality that are correlated with price and influence the click and booking probability, coefficient estimates for β_1 and θ, and hence the estimated position effects, will be biased. To address this endogeneity concern,

Table 3.3 additionally presents estimates for a specification that adds fixed effects on a hotel level (columns 3 and 6).[32] Only price and whether the hotel is on promotion vary across different search sessions and the coefficients in β_1 cannot be estimated for other characteristics. Nonetheless, the interaction between hotel characteristics and position still can be estimated as it is identified by within-hotel variation of the position. The estimates overall are close to the baseline and confirm these results; cheaper hotels on average have stronger position effects and hotels, conditional on their unobserved characteristics. This is further highlighted in Appendix 3.C.2 which reproduces Figure 3.2 for the specification with hotel-fixed effects.

The similarity of the estimates of the price coefficient alleviates price endogeneity concerns in these data. This is in line with previous findings with data from online travel agencies. Specifically, Ursu (2018) argues that within-hotel price variation in these data is to a large degree explained through the dates of the trip, with the remaining within-hotel price variation occurring due to either experimental price variation or intermediate sellers offering the room on Expedia at different prices. Neither of the latter explanations are demand related, such that conditional on the query (which fixes the trip date), there is little concern for observed price variation being correlated with the error term. Besides, using price instruments in a control function approach, De los Santos and Koulayev (2017) and Chen and Yao (2017) also find that parameter estimates do not change significantly relative to the specification without price instruments.

The above specification also assumes that the change in clicks and bookings is linear in the position. However, the position effect between the very last to the second to last position is likely going to be different from the position effect from the second to the top position. I show in the appendix that the results are robust to a range of alternative specifications that treat position more flexibly. Results from these specifications are shown in Appendix 3.C.2. Specifically, allowing for more flexible position effects by adding squared and cubic position terms and the respective interactions with hotel characteristics continues to show that hotels with desirable characteristics have more pronounced position effects. Moreover, I obtain qualitatively similar results when including position-specific parameters for the first four positions. Besides, estimating a Probit instead of a linear probability model also yields qualitatively similar results.

[32]Several hotels are displayed only to a single consumer and therefore are excluded from this specification.

TABLE 3.4 – Parameter estimates

	Estimate	Std. Error
Price (in 100\$)	-0.11***	(0.01)
Star rating	0.06***	(0.01)
Review score	0.02	(0.01)
No reviews	0.12	(0.06)
Location score	0.06***	(0.01)
Chain	-0.01	(0.01)
Promotion	0.03***	(0.01)
Outside option	0.62***	(0.05)
ξ	-1.03***	(0.03)
Ξ	0.77***	(0.05)
ρ	-5.35***	(0.06)
ρ_ξ	-0.31***	(0.02)
Discovery costs (\$)	0.02	
Click costs (\$)	1,038.80	
Click costs conditional on click (\$)	0.49	
Log likelihood	-15,641.78	
N consumers	2,890	

Notes: Estimation is performed with 50 simulation draws. Standard errors in parentheses.

Overall, these results highlight a strong interaction of position effects with hotel characteristics: hotel attributes and the position on the list complement each other such that hotels with more desirable characteristics have stronger position effects.

3.5.2 Model results

I estimate the model with $N_r = 50$ simulation draws. The resulting parameter estimates are shown in Table 3.4 and have the expected sign. The price coefficient is statistically significant and negative. In contrast, coefficients of the star rating, the location score, and whether the hotel was on promotion are positive and statistically significant. Coefficients of whether a hotel belongs to a larger chain, as well as the two coefficients for the review score are not statistically significant.

Table 3.4 shows that the discovery are estimated to be 2¢. The estimate suggests that consumers would be willing to pay 2¢ to reveal an additional alternative on the list. Discovery costs being small stems from the low booking probability in the data. Because few consumers book a hotel in the data, the estimated model predicts that it is unlikely that a newly discovered

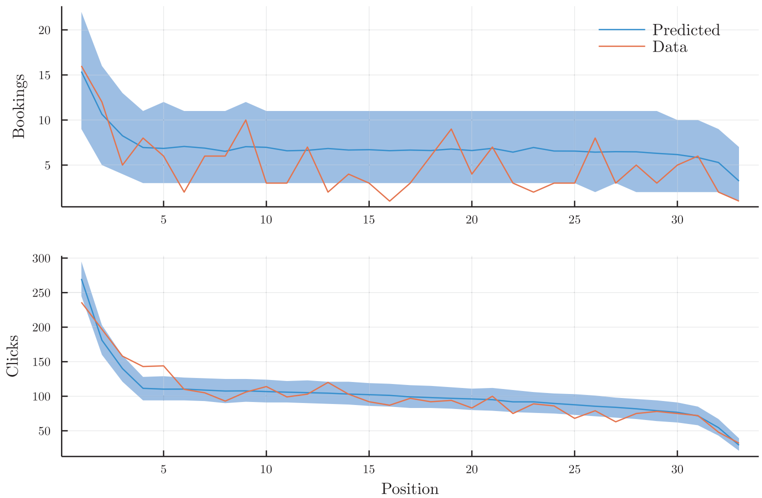

FIGURE 3.3 – The model fits observed clicks an bookings across positions

Notes: Clicks and bookings averaged across 1,000 simulation draws per consumer. The shaded area indicates the 95% percentile of the minimum and maximum number of clicks or bookings across simulation draws and consumers.

alternative will be better than the outside option. Hence, discovery costs need only be small to rationalize the observed position effects.

In contrast, estimates for the mean inspection costs are large: in expectation, clicking on an alternative and learning the yet unknown product information incurs a cost north of 1,000$. However, no consumer is predicted to pay these enormous costs. Instead, mean inspection costs conditional on a click are a more reasonable 50¢.

3.5.2.1 The model fits the data well

Figure 3.3 compares the number of bookings (top panel) and clicks (bottom panel) per position observed in the data with the ones predicted by the model. The figure suggests that the model is able to capture the number of clicks and booking, and where they occur, reasonably well.

Table 3.5 further highlights that the model is able to fit these moments in the data. Specifically, the model matches the number of clicks and where they occur fairly well. The number of bookings and the position at which they occur is somewhat overpredicted. However, as Figure 3.3 reveals, only few bookings occur in the data, with a noisy pattern across positions. This lack of observations makes it more difficult for the model to fit the moments for the

Table 3.5 – The model fits moments in the data

	Bookings: N	Bookings: Position	Clicks: N	Clicks: Position
Data	167.00	15.16	3,316.00	14.94
Predicted	228.85	16.48	3,384.77	15.40

Notes: Simulated search paths generated with 1000 draws for each consumer.

bookings. Nonetheless, the 95% percentile (shaded area) captures the pattern across positions well.

Based on these fit measures, I conclude that the model is able to capture the main patterns in the data. In addition, the model also allows to predict where consumers stopped scrolling within the page. The optimal policy implies that consumers always stop scrolling when the value of the outside option exceeds the discovery value. As a result, the probability that a consumer scrolls *at most* h times is given by

$$\mathbb{P}(U_{i0} > z^d(h)) \tag{3.21}$$

This expression therefore provides a bound for how far consumers scrolled.

Based on this lower bound, the model predicts that at most 79% of consumers discover hotels beyond position 20 and at most 63% of consumers discover the listing on position 30. This explains the decrease in predicted clicks and bookings on positions further down in the list: fewer consumers discover these hotels and are able to click on them.

3.5.2.2 High-utility hotels benefit more from better positions

To analyze how heterogeneous position effects are captured by the estimated model, I analyze counterfactual scenarios where hotels are displayed on different positions than the ones observed in the data. A simple approach to predict the changing booking probabilities would be to draw effective values for each hotel and then predict demand based on the probability of a particular hotel's effective value being the largest.[33] However, most consumers do not book a hotel such that the predicted booking probabilities for each hotel are very small on any position. Analyzing changes in the booking probability for individual hotels, therefore, requires a precise calculation of booking probabilities. To this end, I use a similar simula-

[33] This is the approach taken to predict the fit where this is feasible as search paths are averaged across many consumers.

tion procedure as for the likelihood contributions. Specifically, the procedure takes truncated draws of the effective value of the hotel for which the demand is calculated, and then calculates the probability of this effective value being the largest among hotels that a consumer would discover given this draw. The details for the procedure are provided in Appendix 3.B.6.

To analyze how position effects relate to the utility consumers derive from booking a particular hotel, I predict changes in demand when moving a particular hotel to different positions. In principle, it would be possible to do this for every hotel observed by consumers in the sample. However, this becomes computationally expensive as the demand has to be recomputed thousands of times. Instead, for each consumer, I move a single randomly selected hotel across different positions. Randomized position assignment further ensures that hotels other than the one being moved also are positioned randomly. Relating the resulting change in demand for the focal hotel j to its expected utility $u_j^e = -\alpha p_j + \boldsymbol{x}_j'\beta$ then shows whether high-utility hotels can expect larger demand changes when being moved to different positions.

Figure 3.4 shows predicted changes in demand for four different scenarios. Each dot represents the hotel being moved for one of the consumers in the random sample, where changes in the demand (i.e. the booking probability) are indicated in percentage points. The first panel reveals that there are substantial demand increases when being moved from the middle of the page to the top position. The second and third panel reveal that this demand increase mainly stems from being moved from the third to the very top position, which is in line with the sharp initial decrease in clicks and bookings observed across the first few positions (see Figure 3.3). Nonetheless, the last panel also reveals that there are benefits from being moved from the end to the middle of the product list. This increase comes from a share of consumers being predicted to not scroll all the way to the bottom of the page; being moved to the middle gives the benefit of more consumers discovering the hotel and potentially booking it.

Throughout, Figure 3.4 reveals a strong correlation between a hotel's predicted demand increase and its expected utility. Whereas hotels on the low end of the utility distribution show no change in demand, hotels at the upper end increase the booking probability by up to 2.5 percentage points. This stems from low-utility hotels not being booked even when shown on the top position, whereas being moved to a top position makes more consumers consider the high-utility hotels. Nonetheless, panels one and three reveal that there is some variation, where in various cases hotels with a lower utility have a larger expected increase in demand.

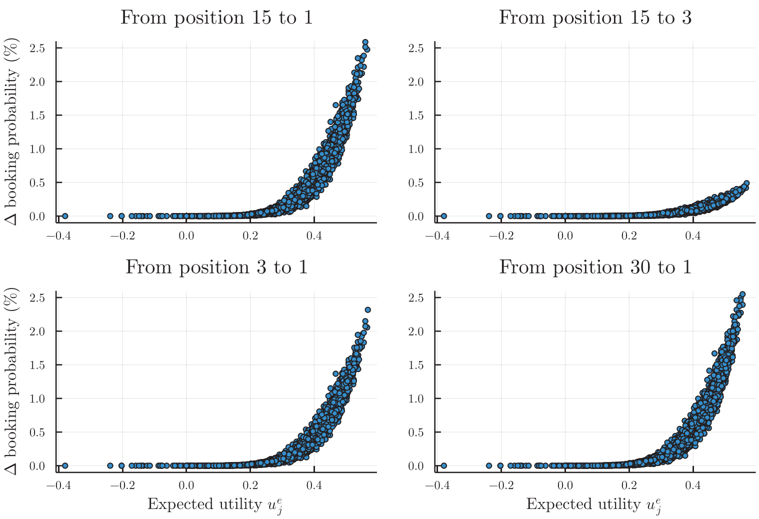

FIGURE 3.4 – High-utility hotels gain more demand

Notes: Changes are indicated in percentage points. Booking probabilities averaged across 5,000 simulation draws.

This stems from differences in the utility offered by other hotels. Suppose many high-utility hotels are displayed on the first few positions. In this case, consumers are not likely to discover hotels beyond the first few positions; they immediately book a high-utility hotel from the first positions. Hence, the demand on later positions will be smaller compared to the case where only low-utility hotels are shown on the first few positions, such that moving such a hotel to the top position increases its demand by more.

Hotels offering larger utility also can expect the largest increase in revenues, at least on average. This is highlighted in Figure 3.5 which depicts predicted changes in expected revenues for hotels being moved to different positions. The substantial variation stems from heterogeneity in hotel prices relative to the strength in their demand increase. Because booking probabilities and their respective changes tend to be small small, the changes in expected revenues for many hotels are also small. For example, the expected revenue increase for a 100$-hotel is only 0.3$ if the booking probability increases by 0.3 percentage points. Nonetheless, with thousands of consumers visiting search intermediaries like Expedia every day, such seemingly small changes lead to substantial gains in the number of transactions and expected revenues.

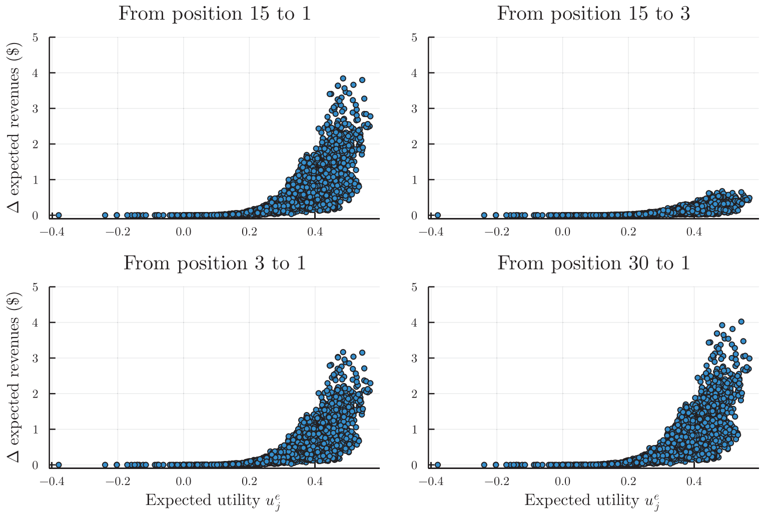

FIGURE 3.5 – High-utility hotels gain more revenues

Notes: Changes in expected revenues are expressed in $, calculated as the change in the booking probability multiplied by the hotel's price. Booking probabilities averaged across 5,000 simulation draws.

The results in Figure 3.5 highlight that it's not the hotels that offer the highest utility that increase revenues by the most from being shown on top. Instead, due to their higher price, more expensive but still desirable hotels gain the most when being shown on top. This suggests that these hotels will have the strongest incentive to influence where on the list they are shown. Hence, if a search intermediary allows sellers to influence the position at which they are revealed to consumers, for example as sponsored listings, the willingness-to-pay of these hotels will be the largest. Moreover, the results imply that a ranking that balances demand increases and prices will increase total revenues over a ranking that's based only on expected utilities.

3.5.3 Comparison of rankings

To analyze how different rankings affect Expedia's revenues and consumer welfare, I compare several different rankings. For this comparison, I use the sample where consumers observed Expedia's ranking and simulate consumers' search paths, eventual purchases and consumer welfare. Though consumers were sampled differently from the estimation sample, using only predicted choices ensures that the comparison is based on the same underlying consumer population. Appendix 3.C.1 shows that the results are qualitatively comparable for the randomized sample.

As a baseline, I construct a randomized ranking where I average results across multiple randomly drawn rankings. Relative to this baseline, I then calculate the effects of five different rankings, as described below. As outlined in Section 3.3.5, consumers do not adjust their beliefs such that the effects of different rankings are short-run effects.

ER Expedia ranking: Expedia's own ranking as given in the sample.

UR Utility-based ranking: This ranking displays hotels in decreasing order of their expected utility $u_j^e = -\alpha p_j + \boldsymbol{x}_j'\beta$. This ranking shows the (on average) best matching alternatives first and aims at increasing consumer welfare. By first showing alternatives that consumers prefer, the ranking can help consumers find better matching alternatives. Moreover, it helps consumers find these alternatives earlier on, thus lowering total search costs consumers incur.

RR Revenue-based ranking: For the ranking that aims at increasing total revenues generated across the whole list, I focus on a simple ranking algorithm that orders hotels based on the expected revenues on the *same* position. As Appendix 3.C.3 shows, the expected increase in revenues for an individual hotel is roughly proportional to its expected revenue at the initial position. Hence, ordering products based on the expected revenue on the same position will also be able to increase total revenues. To implement this ordering, it is only required to calculate expected revenues for each hotel on a particular position, and then reorder listings in decreasing order of these revenues. For the analysis, I again use the position at which consumers purchase on average (position 15, see Table 3.5).

PR Price-decreasing ranking: This ranking displays alternatives in decreasing order of their price, i.e. shows the most expensive alternatives first.

-UR Inverse utility-based ranking: This ranking orders alternatives in increasing order of their expected utility, i.e. it shows the (on average) worst matching alternatives first.

Table 3.6 shows the results of this comparison. Effects are shown as changes relative to the randomized ranking. Note that as Expedia's revenues are roughly proportional to total

TABLE 3.6 – The effects of different rankings

	ER	UR	RR	PR	-UR
Expedia					
Total revenues (%)	1.80	6.85	9.10	-2.44	-4.52
Number of transactions (%)	2.23	8.62	6.24	-3.56	-5.01
Avg. price of booking (%)	-0.42	-1.63	2.69	1.15	0.52
Number of clicks (%)	0.93	2.71	2.25	-2.05	-3.32
Consumers					
Consumer welfare ($, per consumer)	0.17	0.68	0.47	-0.26	-0.34
Consumer welfare ($, per booking)	1.15	4.52	2.81	-1.50	-2.74
Utility of booked hotel ($)	1.11	4.43	2.70	-1.39	-2.56
Search costs ($, per booking)	-0.04	-0.09	-0.11	0.11	0.17

Notes: Predicted changes of Exepdia's ranking (ER), the utility-based ranking (UR), the revenue-based ranking (RR), the price-decreasing ranking (PR) and the inverse utility-based ranking (-UR). All changes are relative to a randomized ranking, obtained by averaging across 20 randomizations. The predicted search paths were generated with 5,000 draws for each consumer.

revenues, the expressed percentage changes for total revenues also translate to percentage changes in Expedia's revenues.

3.5.3.1 Expedia gains the most from the revenue-based ranking

The *ER*, *UR* and *RR* all manage to increase revenues over the randomized ranking. However, the *UR* and *RR* manage to increase revenues substantially more than the *ER*. Moreover, as intended, the *RR* leads to the largest increase in revenues; it roughly adds another three percentage points increase over the *UR*. Whereas the *RR* leads to an increase in the average price of bookings, the *ER* and *UR* lead to a decrease in this price. This difference allows the *RR* to yield the largest increase in revenues, despite the fewer transactions compared to the *UR*.

The results for the *PR* reveal that showing the most expensive alternatives first reduces total revenues. This highlights that the power of rankings in inducing consumers to book more expensive hotels is limited. Instead, the *PR* leads to a substantial decrease in the number of transactions and revenues; consumers do not discover the cheaper alternatives further down in the list page and leave Expedia without making any booking. The substantial decrease in transactions and revenues for the -*UR* further highlights the importance of these demand effects.

3.5.3.2 Revenue-based rankings harm consumers only to a limited extent

The consumer welfare results show that, prior to search, the average consumer is willing to pay 0.17$ to observe the *ER* instead of a randomized ordering. For the *UR* this willingness-to-pay increases to 0.68$ and for the RR to 0.47$. These effects are relatively small, especially when compared to the average price of 154$ (see Table 3.7 in the appendix). This results from the majority of consumers not booking a hotel in any of the rankings. Recall that the optimal policy implies that the consumers that do not buy an alternative discover alternatives until the value of the outside option exceeds the discovery value. As neither is affected by the ranking, consumers that do not book a hotel continue to scroll up to the same point, paying the same amount of discovery costs. When focusing on changes in consumer welfare of consumers that do book a hotel (second row), the three rankings lead to larger increases in consumer welfare of 1.15$, 4.52$ and 2.81$ respectively.

Comparing the increases in consumer welfare between the *UR* and *RR* reveals that Expedia's and consumers' interests are not perfectly aligned. Relative to the randomized ranking, the *UR* adds another 0.21$ consumer welfare for the average consumer, and 1.71$ for consumers that book a hotel. The *RR* leads to an increase in consumer welfare that is about a third smaller than the increase obtained with the *UR*. However, the consumer welfare decrease when moving from the *UR* to the *RR* is substantially smaller than the welfare increase of the *RR* relative to the baseline. Moreover, comparing the *RR* with the *PR* and -*UR* highlights that the *RR* is substantially better for consumers than one might expect. This is because of heterogeneous position effects: as alternatives offering a large utility overproportionally gain demand when being moved to higher positions, a ranking increases both revenues and consumer welfare when moving these alternatives to the top.

The results for the *ER* suggest that Expedia's ranking manages to outperform the randomized ranking in terms of revenues and welfare; Expedia's ranking indeed is working as intended. However, the *ER* is far from achieving consumer welfare or revenue maximization. This is highlighted by the fact that the *UR* and *RR* increase consumer welfare by at least twice as much, while also increasing revenues by more than the *ER*. Hence, there remains considerable scope for improvements for the ranking algorithm employed by Expedia during the sample period.

Table 3.6 also attributes the resulting changes in consumer welfare to either consumers choosing better (or worse) alternatives, or lower incurred costs to discover and inspect alternatives. The results show that the *UR* and *RR* affect consumer welfare to a large extent through helping consumers find better matching alternatives; less than a third of the total welfare change can be attributed to a reduction in search costs.

Combined, these results highlight that, when designing rankings, a search intermediary's and consumers' interests are not severely misaligned. Because of the complementarity of a hotel's position and the utility it offers to consumers, a revenue-based ranking will move high-utility alternatives to higher positions, benefiting both consumers and the search intermediary. Such a ranking therefore manages both to boost short-term revenues, while potentially also long-term revenues through the return of satisfied customers.

3.6 Conclusion & future directions

This paper studies heterogeneous position effects and shows that product attributes and the position on the list complement each other: on average, clicks and bookings for hotels that consumers prefer (e.g. cheap or at a good location) increase more when being displayed on higher positions. I further show that this implies that a revenue-based ranking can benefit both the search intermediary and consumers by moving high-utility products to higher positions.

Whereas the focus of this paper is on heterogeneous position effects and the associated potential of different rankings, related questions remain that I aim to address in future research. First, the model does not feature heterogeneity in either preference parameters or search costs. Introducing such heterogeneity should lead to an improved model fit for the data. Specifically, by allowing some consumers to have a combination of small inspection and large discovery costs, the model would be able to fit the initial sharp decrease in clicks without making initial clicks less costly. Moreover, whereas the proposed heuristic performs well, and I show that heterogeneity in position effects does allow rankings to boost revenues, an open question remains as to how one could design a ranking algorithm that maximizes revenues in the search and discovery model.

Another interesting area for future extensions is the non-trivial joint problem of both ordering products and pricing them, as faced by online retailers. Depending on how each product's price elasticity depends on the position it is displayed on, different rankings can create scope for a retailer to increase prices to further boost revenues. Another interesting topic for future research is to study equilibrium dynamics when sellers of heterogeneous products can bid for being shown on top positions through "sponsored listings." The results in this paper suggest that sellers of high-utility products on average gain the most in terms of revenues when being moved to the top positions. Hence they will have the largest willingness-to-pay to be shown on top positions. However, further research is necessary to determine equilibrium dynamics and how this will affect consumer welfare, as well as the respective revenues of individual sellers and the search intermediary.

Appendix

3.A Data

The original dataset from Kaggle.com contains 9,917,530 observations on a hotel-consumer level. Following Ursu (2018) (see her online Appendix A), I exclude consumers with at least one observation that satisfies any of the following criteria:

1. The implied tax paid per night either exceeds 30% of the listed hotel price (in \$), or is less than 1\$.

2. The listed hotel price is below 10\$ or above 1,000\$.

3. There are less than 50 consumers that were looking for hotels at the same destination throughout the sample period.

4. The consumer observed a hotel on position 5, 11, 17 or 23, i.e. the consumer did not have opaque offers (Ursu (2018) provides a detailed description of this feature in the data).

The final dataset then contains 4,503,128 observations. This number differs from the one in Ursu (2018) by 85 observations. This difference stems from three consumers (IDs 79921, 94604, 373518), where numerical differences resulting from floating point calculations lead to differences in how the criteria are evaluated. For example, for ID 79921 the implied tax of the hotel on position 18 is given by $\frac{167.7}{1.0} - 129.0 = 38.7 = 0.3 \times 129$ and hence the first condition above is not satisfied and the observation is retained. However, floating point calculations can introduce numerical deviations, which leads to Stata evaluating this such that the consumer is dropped.

Completing information on the dataset, Table 3.7 shows summary statistics for consumers that observed the Expedia ranking, and Table 3.8 provides a detailed description of each variable.

Table 3.7 – Summary statistics (Expedia ranking)

	N	Mean	Median	Std. Dev	Min.	Max.
Hotel-level						
Price (in $)	3,146,022	154.54	129.00	96.54	10.00	1000.00
Star rating	3,084,716	3.31	3	0.87	1	5
Review score	3,143,741	3.92	4.00	0.80	0.00	5.00
No reviews	3,143,741	0.02	0	0.13	0	1
Chain	3,146,022	0.68	1.00	0.47	0.00	1.00
Location score	3,146,022	3.13	3	1.50	0	7
On promotion	3,146,022	0.26	0	0.44	0	1
Consumer-level						
Number of items	114,529	27.47	31	7.91	5	38
Number of clicks	114,529	1.11	1	0.59	1	24
Made booking	114,529	0.92	1	0.28	0	1
Trip length (in days)	114,529	2.13	2	1.67	1	38
Booking window (in days)	114,529	32.78	14	48.16	0	482
Number of adults	114,529	1.96	2	0.87	1	9
Number of children	114,529	0.37	0	0.77	0	9
Number of rooms	114,529	1.11	1	0.42	1	8

Notes: Summary statistics as in Table 3.1 for consumers that observed the Expedia ranking.

Table 3.8 – Description Variables

Hotel-level	
Price (in $)	Gross price in USD
Star rating	Number of hotel stars
Review score	User review score, mean over sample period
No reviews	Dummy whether hotels has zero reviews (not missing)
Chain	Dummy whether hotel is part of a chain
Location score	Expedia's score for desirability of hotel's location
On promotion	Dummy whether hotel on promotion
Consumer-level	
Number of items	How many hotels in list for consumer, capped at first page
Number of clicks	Number of clicks by consumer
Made booking	Dummy whether consumer made a booking
Trip length (in days)	Length of stay consumer entered
Booking window (in days)	Number of days in future that trip starts
Number of adults	Number of adults on trip
Number of children	Number of children on trip
Number of rooms	Number of rooms in hotel

3.B Additional derivations

3.B.1 Calculating consumer welfare

The derivation of expected consumer welfare, conditional on observable attributes and the ranking follows almost the same steps as the derivations in 2.A.3. The main adjustment is that the discovery value depends on the position, and that welfare is calculated conditional on observable hotel attributes.

Define $\bar{w}_{ih} \equiv \max\{\tilde{w}_{i0}, \ldots, \tilde{w}_{ih}\}$ as the maximum of values discovered up to position h, with (3.5) defining $\tilde{w}_{ij} = \boldsymbol{x}_j'\beta + \min\{\xi_{ij}, \varepsilon_{ij}\}$. Moreover, let $\bar{w}_{i0} = \max_{j \in A_0} \tilde{w}_{ij}$ denote the maximum value in the initial awareness set, $\bar{h}$ the maximum position to discover, and A_h the set of alternatives that are discovered up to position h. To simplify notation, further define the observed value $y_j \equiv \boldsymbol{x}_j'\beta$ and let $1(\cdot)$ denote the indicator function.

The consumer continues discovering whenever $\bar{w}_{ih} < z^d(h)$. Hence, given realizations ξ_{ij} and ε_{ij} for all hotels, the discovery costs that the consumer pays are given by

$$\sum_{h=0}^{\bar{h}} 1(\bar{w}_{ih} < z^d(h))c_d(h) \tag{3.22}$$

The consumer also stops discovering whenever $\bar{w}_{ih} > z^d(h)$, conditional on which, the consumer searches any alternative that is discovered and satisfies $z_{ij}^s > \bar{w}_{ih}$. Hence, consumer welfare without discoverycosts is given by

$$\sum_{h=0}^{\bar{h}} 1(\bar{w}_{ih-1} < z^d(h-1))1(\bar{w}_{ih} > z^d(h))\times \left(\sum_{j\in A_h} 1(\tilde{w}_{ij} = \bar{w}_{ih})(y_j + \varepsilon_{ij}) - 1(y_j + \xi_{ij} \geq \bar{w}_{ih})c_{sij}\right) \tag{3.23}$$

Using that $c_{sij} = \mathbb{E}\left[1(\varepsilon_{ij} \geq \xi_{ij})1(\varepsilon_{ij} + y_j - \xi_{ij})\right]$ and taking expectations over the whole expression yields

$$\sum_{h=0}^{\bar{h}} \mathbb{E}\left[1(\bar{W}_{ih-1} < z^d(h-1))1(\bar{W}_{ih} > z^d(h))\times \left(\sum_{j\in A_h} 1(\tilde{W}_{ij} \geq \bar{W}_{ih})(y_j + \varepsilon_{ij}) - 1(y_j + \xi_{ij} \geq \bar{W}_{ih})1(\varepsilon_{ij} \geq \xi_{ij})(\varepsilon_{ij} + y_j - \xi_{ij})\right)\right] \tag{3.24}$$

Note that the expectation operator does not integrate over values y_j as we are calculating welfare conditional on observable attributes. Finally, note that $1(y_j + \xi_{ij} \geq \bar{w}_{ih})1(\varepsilon_{ij} \geq \xi_{ij})$ implies $1(\tilde{w}_{ij} \geq \bar{w}_{ih})$ (and is zero otherwise). Hence, the second part simplifies to $1(\tilde{w}_{ij} \geq \bar{w}_{ih})(y_j + \min\{\varepsilon_{ij}, \xi_{ij})$. Combining with (3.22), we get expression (3.7).

3.B.2 Backing out discovery costs

The definition of the discovery value given in (3.3) implies:

$$c_d(h) = \int_{z^d(h)}^{\infty} \left[1 - G_h(t)\right] \mathrm{d}t \tag{3.25}$$

This is a unique mapping from G_h and $c_d(h)$ to $z^d(h)$. Given $c_d(h)$ and G_h, (3.25) can be solved for $z^d(h)$, and given $z^d(h)$ and G_h, it can be solved for $c_d(h)$. However, G_h is based on consumers beliefs' and, hence, needs to be estimated. Recall that $G_h(t) = \mathbb{P}_h(Z_{ij}^s \leq t)$ where search values are given by $z_{ij}^s = -\alpha p_j + \boldsymbol{x}_j'\beta + \xi_{ij}$. The empirical parametrization determines the distribution of ξ_{ij}, but consumers' beliefs over the distribution of observable hotel attributes $-\alpha p_j + \boldsymbol{x}_j'\beta$ are not observed. Importantly, this distribution depends on model parameters that need to be estimated. Hence, it has to be estimated again at each step in the estimation, and then used in the inversion (3.25) to obtain $z^d(h)$. This is computationally costly, as solving (3.25) requires root-finding that does not admit an analytical solution. By directly estimating a function for the discovery values $z^d(h)$, I avoid this step all-together. Moreover, my approach yields demand predictions that do not rely on any additional assumptions on consumers' beliefs, as they only depend on the reservation values, not costs.

To fully solve (3.25) for $c_d(h)$, I assume the following:

1. Discovery costs do not depend on the position, i.e. $c_d(h) = c_d \forall h$
2. The variance of Z^s_{ij} does not depend on the position.
3. The consumer has rational expectations such that beliefs over the distribution of Z^s_{ij} on position $\tilde{h}$ equal the distribution of Z^s_{ij} over all alternatives. In the application, I assume that $\tilde{h}$ is the position at the mean rank, such that in expectation across all positions, beliefs are correct.

Based on these assumptions, $G_{\tilde{h}}(t)$ can be estimated by taking draws $z^s_{ij} = -\alpha p_j + \boldsymbol{x}'_j\beta + \xi_{ij}$, where $(p_j, \boldsymbol{x}_j)$ are drawn from the observed distribution in the data, and ξ_{ij} is drawn from the distribution implied by the empirical specification.

To further simplify calculating (3.25), note that (3.25) can be written as (where $1(\cdot)$ again is the indicator function):

$$c_d(h) = \mathbb{E}_h \left[\max\{0, Z^s_{ij} - z^d(h)\}\right] \tag{3.26}$$

Hence, for given $h = \tilde{h}$, we can directly calculate the expectation by drawing from the distribution of Z^s_{ij}.

3.B.3 Position effects depending on the price

This appendix derives that the size of a product's position effect increases in its expected utility, and therefore decreases in its price. The position effect of being switched from the second to the first position can be calculated as:

$$\begin{aligned}
\Delta d_{iB} =& d_{iB}(1) - d_{iA}(2) \\
=& \mathbb{P}\left(U_{i0} > z^d(0)\right)\mathbb{P}\left(\tilde{W}_{iB} > U_{i0} | U_{i0} > z^d(0)\right) \\
&+ \mathbb{P}\left(\tilde{U}_{i0} \leq z^d(0)\right)\Big[\mathbb{P}(\tilde{W}_{iB} \geq z^d(0))\left(1 - \mathbb{P}(\tilde{W}_{iA} \geq z^d(0)\right) \\
&+ \mathbb{P}(\tilde{W}_{iB} \geq z^d(0)) \cdot 0\Big] \\
=& \mathbb{P}\left(U_{i0} > z^d(0)\right)\mathbb{P}\left(\tilde{W}_{iB} > U_{i0} | U_{i0} > z^d(0)\right) \\
&+ \mathbb{P}\left(\tilde{U}_{i0} \leq z^d(0)\right)\mathbb{P}(\tilde{W}_{iB} \geq z^d(0))\mathbb{P}(\tilde{W}_{iA} \geq z^d(0))
\end{aligned}$$

As $\tilde{W}_{ij} = u^e_j + \min\{\xi_{ij}, \varepsilon_{ij}\}$, the above expression also directly implies that that Δd_{ij} increases in u^e_j, and hence decreases in p_j.

3.B.4 Cases likelihood calculation

3.B.4.1 Case 0: consumer does not click on any hotels

Consider a consumer i that did not make any clicks. In this case, the likelihood contribution of i can be written as

$$\begin{aligned}\mathcal{L}_i(\theta) &= \int_{-\infty}^{\infty} \mathbb{P}\left(\max_{j\in J_i(u_{i0})} Z_{ij}^s \leq u_{i0}\right) \mathrm{d}H(u_{i0}) \\ &= \int_{-\infty}^{\infty} \prod_{j\in J_i(u_{i0})} \mathbb{P}\left(Z_{ij}^s \leq u_{i0}\right) \mathrm{d}H(u_{i0}) \end{aligned} \tag{3.27}$$

3.B.4.2 Case 1: consumer makes clicks but takes the outside option

See main text.

3.B.4.3 Case 2: consumer makes clicks and books a hotel

See main text.

3.B.5 Cases for simulated integral

3.B.5.1 Case 0: consumer does not click on any hotels

This case can be calculated similar to the case where consumer i clicks on some hotels and does not book a hotel. There are only two differences. First, there is no upper bound for u_{i0}. Second, the inner probability is different because there are only products that the consumer did not click on.

The Monte Carlo integration to calculate the integral in (3.27) is as follows (with J_i denoting the set of hotels that consumer i potentially can discover):

1. Partition the interval $(-\infty, \infty)$ into subintervals

$$(-\infty, z^d(|J_i|)], (z^d(|J_i|), z^d(|J_i|-1)], \ldots, (z^d(\bar{h}_i-2), z^d(\bar{h}_i-1)], (z^d(\bar{h}_i-1), \infty)$$

2. For each subinterval, take N_r draws for u_{i0}^r truncated to be within the specific subinterval

3. For each subinterval and draw u_{i0}^r, determine $J(u_{i0}^r)$ and calculate

$$\tilde{P}(u_{i0}^r) \equiv \prod_{j\in J_i(u_{i0}^r)\setminus S_i} \mathbb{P}\left(Z_{ij}^s \leq u_{i0}^r\right)$$

4. Sum up within-interval means across draws, weighting by the probability of u_{i0}^r falling within the subinterval

Putting this into one formula yields

$$\tilde{\mathcal{L}}_i(\theta) = \sum_{s=1}^{N_s} \mathbb{P}\left(u_{i0} \in B(s)\right) \frac{1}{N} \sum_{r=1}^{N_r} \tilde{P}(u_{i0}^r) \tag{3.28}$$

3.B.5.2 Case 1: consumer makes clicks but takes the outside option

See main text.

3.B.5.3 Case 2: consumer makes clicks and books a hotel

The main difference to the case where consumer i does not book a hotel is that the integral now is over $\tilde{w}_{iq} = \boldsymbol{x}_j'\beta + \min\{\xi_{iq} + \nu_{iq}, \varepsilon_{iq}\}$, where $\xi_{ij} \equiv \xi + \rho \log(\tilde{h}_{ij})$. To get a smooth likelihood function nonetheless, draws for $\tilde{w}_{iq}$ are taken with three separate inner cases as outlined below. Throughout, the notation is as in Section 3.4.4.

The Monte Carlo integration procedure to calculate the integral in (3.16) for a consumer i that books hotel q and clicks on hotels in S_i is as follows:

1. Partition the interval $(\infty, b_q]$ into subintervals

$$(\infty, z^d(|J_i|)], (z^d(|J_i|), z^d(|J_i|-1)], \ldots, (z^d(\bar{h}_i - 2), z^d(\bar{h}_i - 1)], (z^d(\bar{h}_i - 1), b_q]$$

2. For each subinterval $B(s)$, and each of the following cases take N_r draws for ε_{iq} and ν_{iq} satisfying the restrictions and calculate the truncation probability:

   (a) $p_1(B(s)) \equiv \mathbb{P}\left(\boldsymbol{x}_{iq}'\beta + \varepsilon_{iq} > B(s)\right) \times \mathbb{P}\left(\boldsymbol{x}_{iq}'\beta + \xi_{iq} + \nu_{iq} \in B(s)\right)$

   (b) $p_2(B(s)) \equiv \mathbb{P}\left(\boldsymbol{x}_{iq}'\beta + \varepsilon_{iq} \in B(s)\right)\mathbb{P}\left(\xi_{iq} + \nu_{iq} \leq \varepsilon_{iq}^r | \varepsilon_{iq}^r\right)$

   (c) $p_3(B(s)) \equiv \mathbb{P}\left(\boldsymbol{x}_{iq}'\beta + \varepsilon_{iq} \in B(s)\right)\mathbb{P}\left(\xi_{iq} + \nu_{iq} > \varepsilon_{iq}^r | \varepsilon_{iq}^r\right)$

   Note that the latter two cases are related in that both probabilities are based on the same draw ε_{iq}^r, resulting in a smooth likelihood function.

3. For each of these cases and resulting draws $\tilde{w}_{iq}^r$ and w_{iq}^r, determine $J(\tilde{w}_{iq}^r)$ and calculate

$$\begin{aligned} P(\tilde{w}_{iq}^r, w_{iq}^r) \equiv & \prod_{j \in J_i(\tilde{w}_{iq}^r) \setminus S_i} \mathbb{P}\left(Z_{ij}^s \leq \tilde{w}_{iq}^r\right) \times \prod_{j \in S_{i-}} \mathbb{P}(Z_{ij}^s \geq w_{iq}^r)\mathbb{P}(U_{ij} \leq \tilde{w}_{iq}^r) \\ & \times \prod_{j \in S_{i+}} \mathbb{P}(Z_{ij}^s \geq \tilde{w}_{iq}^r)\mathbb{P}(U_{ij} \leq \tilde{w}_{iq}^r) \end{aligned} \tag{3.29}$$

4. Sum up within-interval means across draws, weighting by the respective probabilities of the different cases

3.B.6 Calculation of product-specific demand

3.B.6.1 Share outside option

The probability of consumer i taking the outside option given parameters θ is given by

$$D_{i0}(\theta) = \int_{-\infty}^{\infty} \mathbb{P}\left(\max_{j \in J_i(u_{i0})} \tilde{W}_{ij} \leq u_{i0}\right) \mathrm{d}H(u_{i0})$$

This integral can be calculated with the same procedure as described in case 0 in Appendix 3.B.5, with the only difference that the inner probability is calculated as

$$\tilde{P}(u_{i0}^r) = \prod_{j \in J_i(u_{i0}^r)} \mathbb{P}\left(\tilde{W}_{ij} \leq u_{i0}^r\right) = \prod_{j \in J_i(u_{i0}^r)} \mathbb{P}\left(Z_{ij}^s \leq u_{i0}^r\right)\mathbb{P}\left(U_{ij} \leq u_{i0}^r\right)$$

Note also that $J_i(u_{i0})$ always contains all products that the consumer sees when arriving at the page.

3.B.6.2 Share of product q

The probability of consumer i booking hotel q (i.e. buying product q) given parameters θ is given by

$$D_{iq}(\theta) = \int_{-\infty}^{\infty} \mathbb{P}\left(\max_{j \in J_i(\tilde{w}_{iq})} \tilde{W}_{ij} \leq \tilde{w}_{iq}\right) \mathrm{d}H_q(\tilde{w}_{iq})$$

This integral can be calculated with the same procedure as described in case 2 in Appendix 3.B.5, with the only difference that the inner probability is calculated as

$$P(\tilde{w}^r_{iq}, w^r_{iq}) = \prod_{j \in J_i(\tilde{w}^r_{iq})} \mathbb{P}\left(\tilde{W}_{ij} \leq \tilde{w}^r_{iq}\right) = \prod_{j \in J_i(\tilde{w}^r_{iq})} \mathbb{P}\left(Z^s_{ij} \leq \tilde{w}^r_{iq}\right) \mathbb{P}\left(U_{ij} \leq \tilde{w}^r_{iq}\right)$$

Note also that $J_i(\tilde{w}_{iq})$ always contains all products that the consumer discovered before q.

3.C Additional results and robustness

3.C.1 Effects of rankings in randomized ranking sample

This appendix presents the comparison of rankings in the randomized ranking sample. It produces Table 3.6, with the difference that no result for Expedia's ranking is presented. Throughout, the results are in line with the analysis in the main text.

TABLE 3.9 – The effects of different rankings (randomized sample)

	UR	RR	PR	-UR
Expedia				
Total revenues (%)	9.06	12.32	-2.39	-4.93
Number of transactions (%)	10.33	7.60	-3.54	-5.17
Avg. price of booking (%)	-1.15	4.39	1.19	0.25
Number of clicks (%)	3.24	2.78	-2.01	-3.92
Consumers				
Consumer welfare ($, per consumer)	0.40	0.28	-0.13	-0.17
Consumer welfare ($, per booking)	4.49	2.56	-1.13	-2.55
Utility of booked hotel ($)	4.37	2.46	-1.00	-2.38
Search costs ($, per booking)	-0.11	-0.10	0.13	0.17

Notes: Predicted changes the utility-based ranking (UR), the revenue-based ranking (RR), the price-decreasing ranking (PR) and the inverse utility-based ranking (-UR). All changes are relative to a randomized ranking, obtained by averaging across 20 randomizations. The predicted search paths were generated with 5,000 draws for each consumer.

3.C.2 Robust position effects

This appendix presents results for heterogeneous position effects for alternative specifications. Specifically, for each alternative specification the equivalent of Figure 3.2 is produced. The respective specification can be found in each figure's notes. Coefficient estimates for the flexible specification where the position effect is specific to the first four positions are shown in Table 3.10.

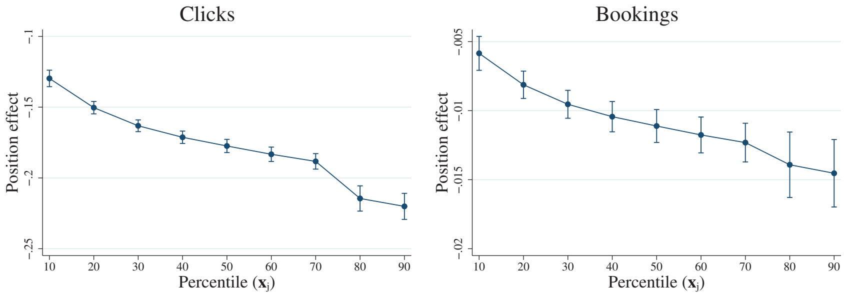

FIGURE 3.6 – Heterogeneous position effects: Hotel FE

Notes: Replicates Figure 3.2 with the specification including hotel fixed effects (columns 3 and 6 in Table 3.3).

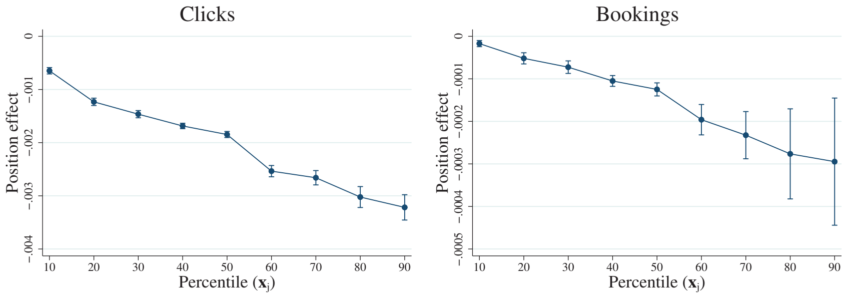

FIGURE 3.7 – Heterogeneous position effects: Probit

Notes: Replicates Figure 3.2 with estimates from a Probit model. The specification is the same as in columns 2 and 5 in (3.3), but estimates are not scaled to reflect percentage point changes.

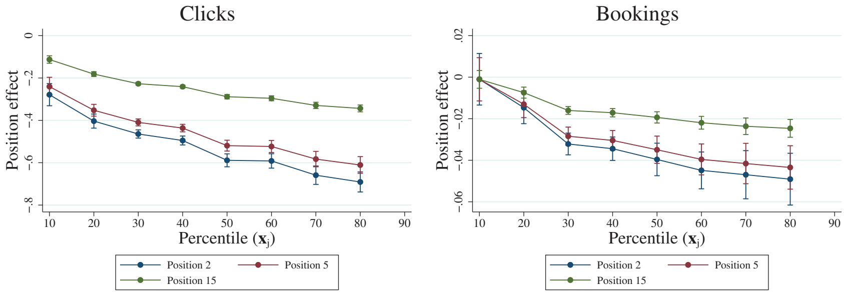

FIGURE 3.8 – Heterogeneous position effects: Flexible position effects I

Notes: Replicates Figure 3.2 with estimates from a linear probability model where pos_{ij}^2 and $pos_{ij}^2\boldsymbol{x}_j$ are added to the specification in columns 2 and 5 of (3.3).

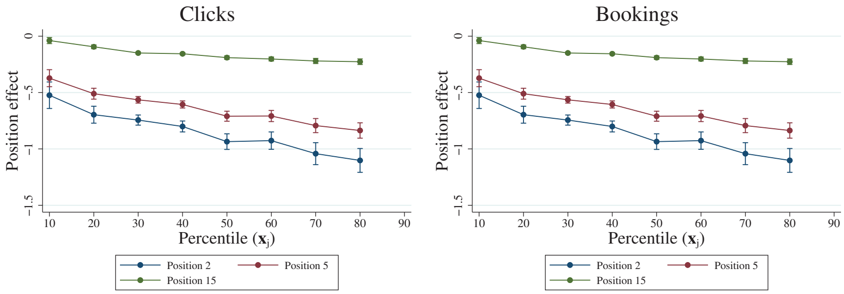

FIGURE 3.9 – Heterogeneous position effects: Flexible position effects II

Notes: Replicates Figure 3.2 with estimates from a linear probability model where pos_i^2, $pos_i^2\boldsymbol{x}_j$, pos_i^3, $pos_i^3\boldsymbol{x}_j$ are additionally included in otherwise the same specification shown in columns 2 and 5 of (3.3).

Table 3.10 – Coefficient estimates (LPM, random ranking)

	Clicks			Booking		
	(1)	(2)	(3)	(4)	(5)	(6)
Position 1	7.8714***	8.2817***	8.0721***	0.3729***	0.2309	0.1888
	(0.1511)	(0.6671)	(0.6580)	(0.0359)	(0.2137)	(0.2152)
Position 2	5.4610***	4.0470***	3.7984***	0.3781***	0.0280	-0.0234
	(0.1358)	(0.5882)	(0.5844)	(0.0360)	(0.2067)	(0.2040)
Position 3	4.1835***	3.0172***	2.7893***	0.2981***	0.0414	0.0390
	(0.1268)	(0.5495)	(0.5474)	(0.0337)	(0.2090)	(0.2117)
Position 4	3.3156***	2.5642***	2.5575***	0.2468***	-0.0027	-0.0162
	(0.1201)	(0.5053)	(0.5063)	(0.0320)	(0.1796)	(0.1815)
Price	-0.0122***	-0.0106***	-0.0126***	-0.0011***	-0.0009***	-0.0010***
	(0.0002)	(0.0002)	(0.0003)	(0.0000)	(0.0000)	(0.0001)
Star rating	1.6408***	1.5449***		0.1064***	0.1012***	
	(0.0317)	(0.0318)		(0.0079)	(0.0078)	
Review score	0.0799**	0.0851**		0.0364***	0.0259***	
	(0.0350)	(0.0347)		(0.0082)	(0.0079)	
No review score	-0.2703*	-0.2873*		0.0855**	0.0641**	
	(0.1619)	(0.1618)		(0.0345)	(0.0327)	
Chain	0.2431***	0.2218***		0.0260**	0.0111	
	(0.0459)	(0.0456)		(0.0113)	(0.0113)	
Location score	0.4413***	0.3942***		0.0530***	0.0522***	
	(0.0168)	(0.0169)		(0.0036)	(0.0036)	
On promotion	1.1645***	1.1058***	1.1346***	0.1267***	0.1206***	0.1235***
	(0.0498)	(0.0503)	(0.0738)	(0.0130)	(0.0129)	(0.0200)
Position 1=1 × Price		-0.0150***	-0.0143***		-0.0012***	-0.0011***
		(0.0013)	(0.0013)		(0.0003)	(0.0003)
Position 2=1 × Price		-0.0071***	-0.0066***		-0.0008**	-0.0007**
		(0.0012)	(0.0012)		(0.0003)	(0.0003)
Position 3=1 × Price		-0.0077***	-0.0073***		-0.0007**	-0.0006**
		(0.0012)	(0.0012)		(0.0003)	(0.0003)
Position 4=1 × Price		-0.0064***	-0.0059***		-0.0010***	-0.0008***
		(0.0010)	(0.0010)		(0.0003)	(0.0003)
Position 1=1 × Star rating		0.6157***	0.5392***		0.0466	0.0211
		(0.2042)	(0.2014)		(0.0500)	(0.0502)
Position 2=1 × Star rating		0.6483***	0.5367***		0.0383	0.0093
		(0.1852)	(0.1850)		(0.0505)	(0.0501)
Position 3=1 × Star rating		0.5109***	0.4574***		-0.0199	-0.0244
		(0.1697)	(0.1699)		(0.0462)	(0.0467)
Position 4=1 × Star rating		0.6251***	0.5218***		0.0471	0.0380
		(0.1640)	(0.1643)		(0.0477)	(0.0480)
Position 1=1 × Review score		-0.1653	-0.2458		0.0561	0.0595
		(0.1645)	(0.1611)		(0.0636)	(0.0640)
Position 2=1 × Review score		-0.0326	-0.0345		0.0812	0.1000*
		(0.1477)	(0.1469)		(0.0620)	(0.0605)
Position 3=1 × Review score		0.0066	-0.0249		0.0979*	0.0927
		(0.1328)	(0.1325)		(0.0582)	(0.0586)
Position 4=1 × Review score		-0.0275	-0.0779		0.0386	0.0361
		(0.1268)	(0.1265)		(0.0550)	(0.0558)
Position 1=1 × Chain		-0.5939*	-0.4256		0.0733	0.0645
		(0.3226)	(0.3203)		(0.0766)	(0.0768)
Position 2=1 × Chain		-0.2539	-0.2020		0.1253	0.1332*
		(0.2910)	(0.2909)		(0.0768)	(0.0777)
Position 3=1 × Chain		0.8958***	0.8946***		0.1355*	0.1227*
		(0.2665)	(0.2668)		(0.0708)	(0.0710)
Position 4=1 × Chain		0.4631*	0.4897*		0.0468	0.0433
		(0.2551)	(0.2565)		(0.0671)	(0.0684)
Position 1=1 × Location score		0.4942***	0.5057***		-0.0057	0.0033
		(0.1060)	(0.1053)		(0.0213)	(0.0214)
Position 2=1 × Location score		0.3808***	0.4000***		-0.0147	-0.0120
		(0.0934)	(0.0937)		(0.0227)	(0.0233)
Position 3=1 × Location score		0.2214**	0.2513***		-0.0072	-0.0051
		(0.0872)	(0.0874)		(0.0213)	(0.0214)
Position 4=1 × Location score		0.0841	0.1138		0.0416**	0.0446**
		(0.0814)	(0.0819)		(0.0190)	(0.0194)
Position 1=1 × On promotion		0.7211**	0.7418**		0.0197	0.0277
		(0.3659)	(0.3640)		(0.0892)	(0.0892)
Position 2=1 × On promotion		0.6246*	0.6655**		0.1877**	0.1962**
		(0.3362)	(0.3367)		(0.0951)	(0.0960)
Position 3=1 × On promotion		0.1221	0.0879		-0.0394	-0.0458
		(0.3118)	(0.3123)		(0.0824)	(0.0820)
Position 4=1 × On promotion		-0.1523	-0.1460		-0.0281	-0.0269
		(0.2963)	(0.2976)		(0.0791)	(0.0801)
Position 1=1 × No review score					0.0295	0.0535
					(0.2795)	(0.2891)
Position 2=1 × No review score					0.2274	0.3185
					(0.2759)	(0.2776)
Position 3=1 × No review score					0.1741	0.1648
					(0.2404)	(0.2395)
Position 4=1 × No review score					0.0926	0.1197
					(0.2412)	(0.2465)
Constant	-0.8893***	-0.7773***	5.0440***	-0.0287	0.0113	0.5519***
	(0.1379)	(0.1372)	(0.0734)	(0.0333)	(0.0327)	(0.0215)
FE Destination	yes	yes	yes	yes	yes	yes
FE Hotel	no	no	yes	no	no	yes
Query Characteristics	yes	yes	yes	yes	yes	yes
N	1,220,917	1,220,917	1,219,253	1,220,917	1,220,917	1,219,253
R2 (adj.)	0.0156	0.0160	0.0276	0.0025	0.0026	-0.0018

Notes: Coefficients are scaled to represent changes in terms of percentage points. Standard errors are shown in parenthesis and are clustered at the query level. Star ratings are adjusted so that baseline position effects are for a hotel with 1 star, and other characteristics equal to the minimum observed value.

3.C.3 Revenue increase depending on initial revenues

Figure 3.10 shows the increase in expected revenues across different positions, depending on a hotel's expected revenues on position 15. Throughout, it shows a strong relationship between the two; the larger the revenues on position 15, the larger the revenue increase.

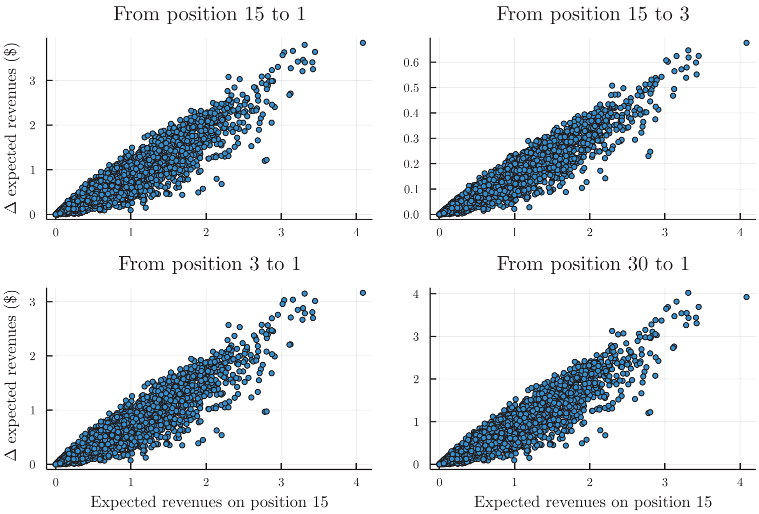

FIGURE 3.10 – Simulated changes in expected revenues

Notes: Changes in expected revenues are expressed in $, calculated as the change in the booking probability multiplied by the hotel's price. Booking probabilities averaged across 5,000 simulation draws.

Chapter 4

Time Allocation and Multi-Category Search

This chapter is co-authored with Y. Huang and I. Morozov.

Abstract

We develop a search model where a consumer decides how much time to spend searching in different product categories. We show that, when given more time, the consumer always first spends it in the same categories before starting to search in additional categories. We also show that the consumer searches more products but may spend less time on search overall when the search technology improves. Finally, we highlight cross-category search effects and derive conditions under which they do not occur and, hence, can be disregarded in search models.

4.1 Introduction

Information is a valuable resource. But information is limited, and so is the time to acquire more. As a result, consumers rarely compare many alternatives and miss out on better-matching products. The resulting frictions have been studied extensively under the premise that searching for alternatives is costly. Whereas Stigler (1961, p. 216) already noted that the "chief cost [of searching] is time", he and many other authors treat search costs as a model primitive. This approach limits the ability to understand how search frictions arise, why they seem to affect some consumers more than others, and how they might change with major shifts in existing search technologies.

In this paper, we show how search frictions endogenously arise in a broader time allocation problem. We introduce a non-sequential search model where a consumer decides how much time to spend searching in different product categories. In our baseline model, categories are independent, and the consumer wants to buy a single alternative in each of them. Spending time searching in a category allows the consumer to discover more alternatives through a stochastic search technology. The search technology reflects a category's specific search environment and determines how many alternatives are revealed within a given amount of time. Besides searching, the consumer can also spend time doing other activities. However, time is limited. Therefore, by spending more time in one category, the consumer forfeits time that could be spent searching in other categories or enjoying other activities.

Our framework allows us to address two novel questions related to search costs. First, we ask whether improving the search technology leads to more search. An improvement in the search technology allows the consumer to reveal more alternatives in the same amount of time. For example, obtaining access to a well-structured shopping website allows consumers to find many products quickly. We prove that the consumer reveals more alternatives following a search technology improvement. In a time allocation model, this result is not obvious. Because there are decreasing marginal benefits of revealing an additional alternative, the consumer may, following an improvement in the search technology, save the time and instead spend it somewhere else. This can indeed occur, but our result implies that the consumer still reveals more alternatives.

Second, we show how opportunity costs of time determine the number of categories a consumer searches in. If opportunity costs of time decrease, the consumer spends more time searching overall. This additional search time can be used to spend more time searching in the same categories or to start searching in new categories. We show that the latter does not occur without the former: a consumer first intensifies search, before expanding it to new categories. If, instead, opportunity costs of time increase, the consumer reduces search time across all categories.

With our framework, we are – to the best of our knowledge – the first to model the decision of how much to search across multiple product categories. We explicitly incorporate the decision whether and how much to search in a specific category, differentiating this paper from prior work that considers consumers deciding how many multi-product stores to visit (e.g. Burdett and Malueg, 1981; McAfee, 1995; Gatti, 1999; Zhou, 2014; Rhodes, 2015).[1] We highlight cross-category search effects where a price increase makes the consumer reallocate time to seemingly unrelated categories. Cross-category search effects, therefore, allow a monopolist retailer to increase profits by pricing even unrelated categories like substitutes. We also derive conditions under which cross-category search effects do not occur. If one of these conditions is satisfied, cross-category effects can be ignored in pricing decisions and in search models.

We further compare our model with standard search models where consumers pay a fixed cost per search (McCall, 1970; Weitzman, 1979; Chade and Smith, 2006). We note that these models are not suited to study cross-category search: because consumers pay a utility cost per search, each category is treated as a separate search problem. Moreover, we argue that assuming a fixed cost per search is restrictive. Marginal search benefits in other categories and marginal benefits of spending time not searching determine the opportunity costs of time. Both these factors rarely are constant. Hence, search costs in most settings will also not be constant. This argument supports Ellison and Wolitzky (2012), who consider search costs that increase in the number of searches based on a similar reasoning.[2]

[1]These authors model the case where consumers search across stores, and each store visit reveals an alternative for every category. Hence, consumers only decide when to stop searching, but not how much to search in each category. Rhodes et al. (2021) consider consumers searching for prices across multiple stores and a multi-product intermediary, but not how many alternatives to sample within a category. Our model also differs from multi-category shopping (e.g. Smith and Thomassen, 2012; Thomassen et al., 2017) where consumers know products ex ante, but pay additional costs if alternatives are not bought at the same store.

[2]Carlin and Ederer (2019) and Ursu et al. (2021) also introduce convex search costs, but attribute them to "consumer fatigue".

Our paper also contributes to the active literature studying the economics of households' time use.[3] Specifically, our model micro-founds the "shopping technology" considered by Aguiar and Hurst (2007a). These authors study households' time allocation across home production and shopping. Building on Becker (1965), they introduce the shopping technology through some general function that allows households to convert time into cheaper prices for the desired basket of goods. In our framework we micro-found this conversion by explicitly modeling how additional search time allows consumers to discover better-matching products and find lower prices.

Related studies have also considered how time constraints affect the types of products consumers buy depending on how "convenient" (i.e. less time-intensive) to consume they are (e.g. Anderson and Shugan, 1991; Bronnenberg et al., 2020), and how much information consumers gather on each alternative (Hauser et al., 1993; Gabaix et al., 2006; Jang et al., 2017; Ursu et al., 2020). Both are conceptually different decision problems.[4]

4.2 The multi-category search model

A consumer decides how much time to spend searching in different product categories. We model the consumer's decision as a static time allocation problem: given a finite time budget, the consumer commits to a time allocation.

By modeling a finite time budget, we explicitly incorporate that time is a limited resource. Hence, searching for alternatives in one category entails the opportunity cost of not being able to spend the time searching in another category or doing something else entirely. The available time is given by the consumers' decision horizon, as well as restrictions on how much time the consumer can freely allocate within this decision horizon. Given our focus on search, we consider relatively short decision horizons in our model. For example, consider a consumer deciding how to spend the 24 hours of the next working day. The consumer has already committed to working 9 hours per working day and has an unavoidable commute of 1 hour each way. In this case, the consumer can decide how to spend the remaining 13 hours of the day on searching or doing other activities (e.g. sleeping, hobbies etc.).

[3] See, for example, Kooreman and Kapteyn (1987); Biddle and Hamermesh (1990); Aguiar and Hurst (2005, 2007b); Aguiar et al. (2013). Aguiar et al. (2012) provide a review of this literature.

[4] In the former, consumers choose different products because the available time influences the consumption value of products. In the latter, consumers decide how much information to gather about substitutes within the same category.

By abstracting from dynamic aspects, we implicitly assume that the consumer plans ahead when deciding how to spend the available time. This often is reasonable because many activities are not possible without planning. For example, meeting friends requires coordination, going for dinner at a good restaurant requires a reservation, and going to an event requires a ticket that may be sold out ahead of time. Hence, within the the decision horizons we consider, assuming that the consumer decides on and commits to a time allocation is appropriate.

Because the decision is static, our model fits into the category of non-sequential search models. However, we emphasize that the above motivation for this modeling choice is not based on whether consumers search sequentially or non-sequentially. Instead, within our time allocation model, search is non-sequential because the consumer plans ahead when deciding how to spend the available time. Hence, the decision aspect of our model is closer to the literature that models households' time allocation and also abstracts from dynamic aspects (e.g. Becker, 1965; Aguiar and Hurst, 2007a).

4.2.1 Setup

There are $\bar{c}$ different categories offering a range of differentiated alternatives. Alternatives within a category are substitutes and we assume that the consumer has unit demand: within a consideration set for a given category (formed through search), the consumer chooses the alternative offering the largest utility. Hence, conditional on category-specific consideration sets $\boldsymbol{S} = \{S_1, \ldots, S_{\bar{c}}\}$, the choice within one category does not influence the choice within another category.

Categories are independent of each other; the value of purchasing a product in one category is independent of the alternatives in another category. In other words, across categories, products are neither substitutes nor complements. Capturing this independent structure while keeping things simple, we assume that utilities from different categories are linear additive. Hence, the consumer's utility given the consideration sets in $\boldsymbol{S}$ is given by

$$\upsilon(\boldsymbol{S}) = \sum_{c=1}^{\bar{c}} \max_{j \in S_c} u_{jc} \tag{4.1}$$

where u_{jc} denotes the utility of purchasing product j from category c. In Section 4.4, we discuss a more general setup that allows for complementarities across categories.

Let u_{0c} denote the consumer's "need" to purchase an alternative from a particular category. A large u_{0c} means that the consumer has little need to buy an alternative from category c; not buying an alternative still yields a large utility. For example, suppose a consumer does not own winter boots and is moving to Chicago in January. Winters in Chicago are freezing, and the consumer prefers not to have cold feet. In this case, the utility of not purchasing any winter boots will be relatively small. If the consumer instead already owns a good pair of winter boots, there is no need for another pair such that u_{0c} will be large.

Prior to searching in a particular category, the consumer only knows the utility of not buying an alternative. Hence, without spending any time searching the consumer gets $v(\boldsymbol{S}) = \sum_{c=1}^{\bar{c}} u_{0c}$. For other alternatives, the consumer initially does not know the utility they offer. Instead, the consumer has beliefs over the distribution of match values. Specifically, we assume that the consumer believes that product utilities are independently drawn from a continuous distribution with cumulative density F_c.[5] To derive demand and cross-category search effects, we assume rational expectations: the consumer's beliefs are correct.

To reveal product utilities and be able to purchase alternatives, the consumer needs to spend time searching in a category. We assume that categories differ in the search technology that reveal alternatives. Hence, for the same amount of time, searching in different categories reveals a different number of alternatives. Moreover, we assume that each search technology is stochastic; prior to search, there is uncertainty over the number of alternatives that will be revealed within a certain amount of time.

By setting up the problem with stochastic search technologies, we do not impose any restrictions as to how consumers search within the allocated time. Instead, we only assume that consumers anticipate their within-category search behavior, and how that determines how many alternatives will be revealed.[6] Besides, in most real-world settings, the exact number of alternatives revealed within a given amount of time is difficult, if not impossible, to anticipate. For example, prior to searching for winter boots, our consumer does not know how many alternatives he will discover within half an hour, let alone within a minute. Uncertainty also

[5] Product utilities being unknown prior to search can also occur with homogeneous products, as long as there is price dispersion. To characterize search, as is the focus of this paper, it does not matter whether the distribution stems from product differentiation or price dispersion.

[6] Burdett and Judd (1983) also introduce uncertainty in their "noisy search" model. Noisy search, however, differs in that it is a sequential search mode where paying a fixed search costs reveals at least one, and with some chance more than one alternative.

stems from not knowing how long it will take to evaluate each alternative. Some alternatives may be obviously unsuitable and the consumer can immediately judge that they are worse than not buying anything, other alternatives will take longer to compare and add to the consideration set.

We represent the stochastic search technology for each category as a discrete random variable N_c with support $\{0, 1, \dots\}$. A realization from N_c is the number of alternatives revealed for category c. By spending time searching in category c, the consumer increases the probability of revealing more alternatives. Hence, we assume that the probability mass function of the distribution of N_c, denoted by $G_c(k, t) \equiv \mathbb{P}_{N_c|t}(N_c = k)$ for $k \in \{0, 1, \dots\}$, depends on the time (t) spent searching in category c.[7] We further assume that the search technology is independent of the utility distribution, i.e. the random variables N_c and U_{jc} are independent. Rational expectations imply that the consumer knows the search technology and hence the distribution of N_c.

Besides spending time searching in categories, the consumer can also spend time enjoying a composite outside activity. This outside activity may be comprised of various different activities. But, to focus on search, we do not model how much time to allocate to each of them. Instead, we denote the value of spending t minutes on the outside activity by $b_0(t)$, and assume that $b_0(t)$ is a twice differentiable and concave function.

The value of spending time on the outside activity does not depend on the products bought in any of the categories. Moreover, we assume that the total utility is also linear additive in the outside activity: given consideration sets $\boldsymbol{S}$ and t_0 minutes spent on the outside activity, the consumer obtains utility $u(\boldsymbol{S}, t_0) = v(\boldsymbol{S}) + b_0(t_0)$. By specifying this function, we implicitly assume that the consumer has well-behaved preferences that can be represented by this utility function. For example, with just a single category, $u_{01} + b_0(t_0) > u_{11} + b_0(t_0 + \Delta)$ implies that the consumer prefers spending t_0 minutes on the outside activity and not buying any product over spending Δ more minutes on the outside activity buying a product offering u_{11}.

[7]Note that assumptions on the number of revealed alternatives are imposed through assumptions on $G_c(k, t)$. For example, assuming $G_c(0, 0) = 0 \forall t$ allows to incorporate that if the consumer does not spend any time searching in c, he does not reveal any alternatives with certainty.

4.2.2 The consumer's formal problem

The consumer maximizes expected utility by choosing a time allocation across categories. Formally, this optimization problem can be stated as

$$\max_{\boldsymbol{t}} EU(\boldsymbol{t}) \text{ s.t. } \sum_{c=0}^{\bar{c}} t_c = \bar{t} \text{ and } t_c \geq 0 \forall c \tag{4.2}$$

where $EU(\boldsymbol{t})$ denotes the expected utility given time allocation $\boldsymbol{t} = [t_0, t_1 \ldots, t_{\bar{c}}]$. Specifically, $EU(\boldsymbol{t}) = \mathbb{E}\left[v(\boldsymbol{S})\right] + b_0(t_0)$ where the expectation operator integrates over the distribution of $U_{jc} \forall c$ and over the distribution of N_c. $\bar{t}$ denotes the time budget available to the consumer.

The utility specification implies that the expected value from revealing k alternatives in category c is given by $\mathbb{E}\left[\max\{u_{0c}, U_{1c}, \ldots, U_{kc}\}\right]$, where the expectation operator integrates over the joint distribution of the random variables $U_{1c}, \ldots, U_{kc}$. The expected benefits of spending t minutes searching in category c then are given by

$$b_c(t) \equiv \sum_{k=0}^{\infty} G_c(k,t) \mathbb{E}\left[\max\{u_{0c}, U_{1c}, \ldots, U_{kc}\}\right] \tag{4.3}$$

To be able to focus on the underlying trade-offs and simplify the analysis, we impose the following assumptions on the probability mass functions $G_c(k,t)$ for all c:

- A1 *Stochastic dominance:* $\mathbb{P}_{N_c|t}(N_c \leq k) \geq \mathbb{P}_{N_c|t'}(N_c \leq k)$ for all k and $t' > t$, and with strictness for at least one k.
- A2 *Concavity:* $G_c(k,t)$ is concave in t for all k, and strictly concave for at least one k.
- A3 *Differentiability:* $G_c(k,t)$ is differentiable in t for all k and $t \geq 0$.

Lemma 4.1 provides several useful properties of the search benefits that follow from these assumptions.

Lemma 4.1. *$b_c(t)$ is weakly increasing, twice differentiable and strictly concave in t if A1-A3 hold.*

Proof. Let $e(k) \equiv \mathbb{E}\left[\max\{u_{0c}, U_{1c}, \ldots, U_{kc}\}\right]$. Because $e(k)$ is increasing in k, A1 implies that an increase in t makes larger values $e(k)$ more likely such that $b_c'(t) = \sum_{k=0}^{\infty} G_c'(k,t) e(k) \geq 0$

must hold. Because $G_c(0,t) = 1 - \sum_{k=1}^{\infty} G_c(k,t)$, we can rewrite $b_c(t) = \sum_{k=1}^{\infty} G_c(k,t)(e(k) - u_{0c})$. As $e(k) - u_{0c} \geq 0 \forall k$ and does not depend on t, A2 implies that $b_c(t)$ is the sum of concave and (at least one) strictly concave functions, hence is strictly concave. Finally, A3 implies that $b_c(t)$ is the sum of differentiable functions, hence is differentiable. $\square$

The first assumption states that increasing the time spent in a category yields a new distribution for N_c that first-order stochastically dominates (FOSD) the original one. The assumption, therefore, guarantees that spending more time searching in a category increases the probability of obtaining more samples from F_c. As a result, it implies that spending more time in a category weakly increases its expected utility.

The second assumption excludes local optima by guaranteeing that $b_c(t)$ is strictly concave. Appendix 4.A.2 shows that the Poisson distribution, a natural choice for modeling the distribution of N_c, satisfies concavity. Similarly, Appendix 4.A.3 shows that the Bernoulli distribution with a success probability that increases in the time spent at a decreasing rate satisfies this assumption.[8] In contrast, a Bernoulli distribution with a success probability that is linear in the time spent searching violates this assumption. In this case, both $G_c(0,1)$ and $G_c(1,k)$ are only weakly concave.[9] Note that A2 is a sufficient, but not necessary, condition for $b_c(t)$ to be strictly concave. $\mathbb{E}\left[\max\{u_{0c}, U_{1c}, \ldots, U_{kc}\}\right]$ increases in k at a decreasing rate, hence even if $G_c(k,t)$ is not concave for all k, $b_c(t)$ can still be strictly concave. As long as the rate by which alternatives are added does not increase so fast that it offsets that at many searches, an additional search has lower expected benefits, $b_c(t)$ is going to be strictly concave.

The third assumption guarantees differentiability and, hence, allows us to use standard techniques to obtain the solution, as well as draw parallels to the budget allocation problem (see Section 4.3.7). However, it excludes the case where the consumer knows exactly how much time each search takes. For example, if the consumer knows that spending one minute reveals one more alternative in a category, $G(0,t)$ jumps from one to zero, and, therefore, is not differentiable at $t = 1$. This restriction is necessary because if the consumer knows exactly how much time each search takes, the consumer ends up facing an optimization problem akin to a problem with integer constraints. In particular, the resulting problem is known in the

[8] For example, $N_c \sim Bernoulli(\min\{\log(1+t), 1\})$.

[9] If $b_0(t)$ is strictly concave, $EU(\boldsymbol{t})$ is still strictly concave and the solution derived in the next section still applies.

operations literature as the “non-linear knapsack problem” and does not admit an analytical solution.

4.2.3 The solution to the search problem

Using (4.3) and the linear additivity assumption, we can rewrite the maximization problem as

$$\max_{\boldsymbol{t}} \sum_{c=0}^{\bar{c}} b_c(t_c) \text{ s.t. } \sum_{c=0}^{\bar{c}} t_c = \bar{t} \text{ and } t_c \geq 0 \tag{4.4}$$

Lemma 4.1 immediately implies that (4.4) is a separable concave optimization problem. This implies that the Karush-Kuhn-Tucker (KKT) sufficient conditions for an optimal time allocation $\boldsymbol{t}^*$ can be written as (see e.g. Ibaraki and Katoh, 1988)

$$b'_c(t^*_c) \leq \lambda \forall c \tag{4.5a}$$

$$t^*_c > 0 \iff b'_c(t^*_c) = \lambda > 0 \tag{4.5b}$$

$$\sum_{c=0}^{\bar{c}} t^*_c - \bar{t} = 0 \tag{4.5c}$$

The solution reveals that the usual intuition can be applied to the multi-category search problem. The conditions require that the marginal search benefits equalize across the categories where it is optimal to spend at least some time searching in. If these conditions do not hold, a marginal improvement exists: the gains from spending more time searching in one category exceed the losses from spending less time searching in another category that offers smaller marginal search benefits.

Condition (4.5a) shows that λ equals the opportunity costs of time. In the optimum, spending more time searching in one category requires that the consumer spends less time searching in another category or enjoying the outside activity. (4.5a) states that, at the margin, the benefit lost from spending less time in other activities is at most λ. Hence, λ equals the opportunity costs of spending less time searching in another category or enjoying the outside activity. This can be best seen in the special case where $b_0(t) = \nu t$ and $\bar{t}$ is sufficiently large so that $t^*_0 > 0$. In this case (4.5a) directly implies that $b'_0(t^*_0) = \nu = \lambda$. Hence, any time spent searching in a category has the opportunity cost of not receiving benefit λ from spending the time on other fun activities.

The opportunity cost of time at the optimal time allocation is fixed. This, however, does not imply that the costs per search are the same for all categories. Categories differ in their search technology: per search, a different amount of time needs to be spent. Hence, the costs per search also depend on the expected number of searches within a given amount of time. As we discuss in detail in Section 4.3.3, this distinction is important when relating our framework to classical search models.

4.3 Comparative statics and implications

We consider two different sets of comparative statics. First, we analyze differences in optimal time allocations across consumers that are independent of the categories they are searching in. Second, we highlight the factors that determine which categories consumers spend more time searching in. Throughout, we define making the outside activity more attractive as a change in its benefit function such that $\Delta b_0'(t) > 0 \forall t \geq 0$. Moreover, we denote the optimal allocation prior to a change with t_c^* for $c \in \{0, \ldots, \bar{c}\}$.

4.3.1 Changes in the opportunity costs of time

The time available to search for products, as well as the attractiveness of doing other activities differ across consumers. For example, without flexible working hours, a consumer will have to spend a fixed number of hours working during the week, and these hours vary across consumers. Consumers with various hobbies will find more value in spending time on those hobbies. To evaluate such differences, we now consider the effects on the optimal time allocation following changes in the available time time, $\bar{t}$, and the value of the outside activity.

Proposition 4.1. *If $t_0^* > 0$ and the outside activity becomes more attractive, the consumer spends more time on the outside activity and searches less in all categories where $t_c^* > 0$. If instead $\bar{t}$ decreases, the consumer spends less time across all categories and the outside activity.*

Proof. Given $\Delta b_0'(t) > 0 \forall t$, the optimality conditions (4.5a) and (4.5c) directly imply the results. □

In line with economic intuition, Proposition 4.1 shows that it is optimal to spend less time searching for products when the outside activity is not attractive or if little time is available. Moreover, the proposition highlights that the time spent searching decreases for *all* categories where it was optimal to spend at least some time searching in prior to the change. Note, however, that the result does not imply that the consumer reduces the time spent in all categories by the same amount. Instead, the time reduction in a particular category depends on the shape of the benefit function.

Whereas Proposition 4.1 suggests that the converse holds as well, it does not address how an increase in the total time spent searching will be allocated across categories. Specifically, the following two cases can happen. First, the consumer can allocate additional search time to searching in "old" categories, categories the consumer was already spending time in prior to the change ($t_c^* > 0$). Second, the consumer can also use the additional time to start searching in "new" categories, i.e. those categories that the consumer previously did not allocate any time to ($t_c^* = 0$). Hence, if the consumer overall adds more time to searching for products, the consumer can intensify search in old categories (intensive margin) or start searching in new categories (extensive margin). To analyze the two cases, we first highlight a result related to marginal changes.

Lemma 4.2. *Marginal changes in either the benefit functions $b_c(t) \forall c \in \{0, \dots, \bar{c}\}$ or the time constraint $\bar{t}$ do not affect the time spent for categories with $t_c^* = 0$. The exception is if $b'_c(0) = b'_{c'}(t^*_{c'})$ for some c' with $t^*_{c'} > 0$.*

Proof. The optimality conditions (4.5b) and (4.5a) imply that $b'_c(t_c^*) < \lambda$ if $t_c^* = 0$, hence marginal changes in λ do not affect t_c^*. The special case arises because then $b'_c(t_c^*) = \lambda$ with $t_c^* = 0$. □

Lemma 4.2 shows that, except for a special case, marginal changes only affect the intensive, but not the extensive margin. Because a marginal change only affects the intensive margin, any substantial change that leads to a change in the extensive margin, must also work through the intensive margin. This logic leads to the next result.

Proposition 4.2. *If $t_0^* > 0$ and the outside activity becomes less attractive, the consumer spends less time in the outside activity and more time in categories with $t_c^* > 0$. If $t_0^* > 0$ and $\bar{t}$ increases, the consumer spends more time in the outside activity and in categories with $t_c^* > 0$. In both cases, the consumer may also start searching in new categories.*

Proof. Follows from Lemma 4.2. □

Proposition 4.2 shows that changes leading to more time being allocated to searching in categories always lead to more time spent searching in old categories. As a result, the consumer always first intensifies search in old categories before expanding search to new categories,

Propositions 4.1 and 4.2 highlight a clear difference between changes to the value of the outside activity and the time constraint. Specifically, we get the opposite effect on the time spent on the outside activity, depending on whether we are looking at the implications of a change in the time constraint or the value of the outside activity.

TABLE 4.1 – Impact of changes to outside activity or time constraint

	More search		Less search	
	Worse outside	More time	Better outside	Less time
Time spent searching	(+)	(+)	(−)	(−)
Time outside activity	(−)	(+)	(+)	(−)

Notes: Changes based on the assumption that $t_0^* > 0$ and $t_c^* > 0$ for at least one category.

Table 4.1 summarizes the different cases. As the four cases are mutually exclusive, it is possible to empirically differentiate the two different changes. Suppose we are able to observe consumers shifting their time allocation following some events. Based on Table 4.1, it is now possible to differentiate between the events shifting the value of the outside activity or changing the time available to the consumer.

4.3.2 Category-specific changes

We now turn to category-specific comparative statics that allow us to predict how consumers allocate search time *between* different categories. In our model, categories differ in three aspects: (i) the need to buy an alternative, (ii) product heterogeneity, and (iii) the search technology. For each of these three we show how respective changes lead to a different optimal time allocation.

Proposition 4.3 shows that if the need to buy an alternative from a category increases, the consumer will spend more time searching in that category. The need to buy an alternative from a particular category enters our model through u_{0c}, the category-specific utility when not buy an alternative. If $\Delta u_c < 0$, the need to buy an alternative increases as the consumer becomes worse off when not buying any alternative from the category. The proposition further shows that the consumer also increases the time spent searching in a category if the mean of the category's utility distribution, denoted by μ_c, increases. Both results are intuitive and proven with similar steps. However, note that the two changes have different implications for consumer welfare: a decrease in u_{0c} leaves the consumer worse off, whereas an increase in μ_c makes the consumer better off.

Proposition 4.3. *If $t_c^* > 0$ and $\Delta u_{0c} < 0$ or $\Delta\mu_c > 0$, the consumer increases the time allocated to searching in c.*

Proof. From (4.3), we get $\Delta b_c'(t) = \sum_{k=0}^{\infty} G'(k,t)\Delta e_c(k)$, where $\Delta e_c(k)$ denotes the change in $\mathbb{E}\left[\max\left\{u_{0c}, U_{1c}, \ldots, U_{kc}\right\}\right]$ due to the change in the utility distribution. Clearly, $\Delta e_c(k) > 0 \forall k > 0$ for $\Delta u_{0c} < 0$ or $\Delta\mu_c > 0$. Because $\Delta e_c(k)$ increases in k for these changes (see Appendix 4.A.1), A1 implies that an increase in t shifts probability mass to larger values of $\Delta e_c(k)$. Hence, $\Delta b_c'(t) > 0 \forall t \geq 0$ holds and the optimality conditions imply the result. □

The case of a mean-shift in the utility distribution is also of special interest because it can reflect a shift in the average price within a category. Specifically, Proposition 4.3 implies that if there is an increase in the average price of a category, a price-sensitive consumer optimally reduces the amount of time spent searching in the category. This result allows us to highlight cross-category search effects in Section 4.3.4. The result also highlights a difference to sequential search models. With non-sequential search, a (positive) mean-shift over-proportionally increases the expected marginal benefit of later searches, i.e. the expected marginal benefit of the 10th search increases by more than the one of the first search.[10] In contrast, with sequential search, the expected number of searches directly results from the probability of not continuing beyond a specific number of searches, which remains unaffected by a mean-shift (see Moraga-González and Sun, 2022).

Proposition 4.3 does not impose additional assumptions on the utility distribution to derive the result. For changes in the utility distribution that capture changes in product heterogeneity, similarly general results are difficult to obtain. Nonetheless, the proof of Proposition 4.3 reveals conditions that guarantee that a change in the utility distribution increases the consumers' search time in the respective category. Let $\Delta e_c(k)$ denote the change in

[10] The proof of Proposition 4.3 relies on the fact that $\Delta e_c(k)$ increases in k. The same fact also leads to more searches in standard non-sequential search models for sufficiently large changes in the utility distribution.

$\mathbb{E}[\max\{u_{0c}, U_{1c}, \ldots, U_{kc}\}]$ due to the change in the utility distribution. Any FOSD shift in the utility distribution satisfies $\Delta e_c(k) \geq 0 \forall k > 0$. The next proposition provides two conditions under which the consumer increases the time spent searching in the category.

Proposition 4.4. *If $t_c^* > 0$ and the utility distribution changes such that $\Delta e_c(k) \geq 0 \forall k > 0$, the consumer increases the time spent searching in c if (i) $\Delta e_c(k)$ increases in k or (ii) $G_c'(k, t_c^*) \geq 0 \forall k > 0$.*

Proof. If $\Delta e_c(k)$ increases in k, A1 immediately implies that $\Delta b_c'(t) > 0 \forall t \geq 0$. Given $\Delta e_c(k) \geq 0 \forall k > 0$ and $\Delta e_c(0) = \Delta u_{0c} = 0$, $G_c'(k, t_c^*) \geq 0 \forall k > 0$ implies $\Delta b_c'(t_c^*) > 0$. □

The first condition is the one used in Proposition 4.3 to prove the result. Whereas the condition holds for FOSD shifts in various utility distributions, showing that it holds for *any* distribution proves difficult. The second condition instead imposes a restriction on the search technology. It requires that, at the optimal time allocation prior to the change, a marginal increase in the time spent searching does not decrease the probability of getting any positive number of alternatives. This condition depends on the original time allocation. If t_c^* is large, an increase in t can reduce the probability of revealing only one alternative such that $G'(1, t_c^*) < 0$, which violates the condition. Besides, it requires that $G'(0, k) < 0$, hence at t_c^*, the probability of revealing zero alternatives cannot be zero.

Changes to search technologies happen regularly. For example, online retailers may make it easier or harder to search in a particular category. To analyze how changes to the search technology of a category affect the time allocation, we define a "search technology improvement" in category c as a FOSD shift in the distribution of N_c: for the same amount of time, it becomes more likely to reveal more alternatives. A simple way to analyze such FOSD shifts is through a category-specific parameter. Specifically, we assume that the consumer maximizes $\sum_{c=1}^{\bar{c}} b_c(r_c t_c) + b_0(t_0)$. This simple specification captures search technology improvements through the parameter r_c. If r_c increases, the same amount of time reveals (in expectation) more alternatives.

Search technology improvements have two opposing effects. We highlight the two effects in the Poisson case with the added category-specific parameter r_c, which is equivalent to assuming $N_c \sim Poisson(r_c t)$. With this specification, r_c is the average arrival rate of alternatives, given the consumer spends t minutes searching in category c.

The derivative of the marginal search benefits with respect to the arrival rate then is given by (see Appendix 4.A.2)

$$\frac{\partial b_c'(r_c t)}{\partial r_c} = \underbrace{\frac{b_c'(r_c t)}{r_c}}_{>0} + \underbrace{\frac{t}{r_c} b_c''(r_c t)}_{<0} \tag{4.6}$$

The first term highlights that each additional minute spent searching in c becomes more valuable with an increase in the arrival rate: each minute gets a larger probability of adding more searches. The second term has the opposite sign. It stems from the decreasing marginal returns to searching an additional alternative. For a given time spent searching in c, an increase in the arrival rate means that for the same amount of time more alternatives are searched. Hence, spending more time to get more searches adds less benefits.

Either of the two effects can outweigh the other and the effect of an improvement in the search technology is ambiguous. As a result, the consumer may either spend more or less time searching in a category following an improvement in the search technology; the consumer either reallocates time from other activities to exploit getting more searches per minute, or instead saves the time to search in other categories.[11]

4.3.3 Relation to per-search cost search models

The ambiguity in how search technology improvements affect the optimal time allocation reveals a difference to standard search problems (McCall, 1970; Weitzman, 1979; Chade and Smith, 2006). In standard search problems, consumers pay a fixed utility cost per search. Hence, an improvement in the search technology will be introduced as lowering the cost per search. This always leads to more search. With a search technology improvement, the effect is not obvious; it can be optimal to reduce the time spent searching, which, in turn, could lead to fewer searches. Our next result shows that this does not happen.

Proposition 4.5. *The expected number of searches for a category increases with a search technology improvement.*

Proof. Let $\tilde{b}_c(\tilde{t}_c^*)$ denote the search benefits at the new optimal allocation, where an improvement in the search technology implies that for any given t, there are more searches (in

[11] As the marginal search benefits determine how much time the consumer allocates to spending in category c, the sign of the change in $b_c(t)$ at $t = t_c^*$ determines whether the consumer in- or decreases t_c^* when the search technology in category c improves. Calculating the optimal time allocation numerically, it is possible to come up with examples for either of the two cases.

expectation) after the change and that $\tilde{b}_c(t) > b(t) \forall t$. For $\tilde{t}_c^* > t_c^*$ we immediately get that there are more searches. With $\tilde{t}_c^* < t_c^*$, fewer searches would imply that $\tilde{b}'_c(\tilde{t}_c^*) > \tilde{b}'_c(t_c^*) > b'_c(t_c^*)$ holds, which contradicts that $\tilde{b}'_c(\tilde{t}_c^*) \leq b'_c(t_c^*)$ must hold at the new optimum. $\square$

Proposition 4.5 shows that even if an improvement in the search technology decreases the time spent searching, the expected number of searches still increases. Hence, improving the search technology is similar to reducing search costs in standard search models; it always helps the consumer reveal more alternatives.

Because it also enters as a search cost reduction, standard search models predict the same effect if the opportunity costs of time decrease. Our framework instead allows for a more nuanced analysis. Specifically, a change in the opportunity costs of time differs from a search technology improvement as it either affects the time constraint, the value of the outside activity, or the value of searching in other categories. As Sections 4.3.1 and 4.3.2 highlight, the effects of these changes are not the same.

With our framework, we can also further analyze what is required for search costs to be fixed, as assumed by standard search models. Clearly, a fixed cost per search requires constant opportunity costs of time. However, once we have more than one category, the opportunity costs of time are not constant.[12] Instead, they increase in the time spent searching in a category because of decreasing marginal search benefits in other categories. Moreover, in many settings, the benefits of doing something other than searching are also not constant in the time spent doing them. For example, the first few minutes of checking social media will be more beneficial than the additional minute after having already spent an hour aimlessly browsing through some curated feed. The first half hour of reading the news on an day gives interesting stories to explore, whereas after some time it becomes less and less likely to see a new and interesting article.

Even if the opportunity costs of time are constant, it is not sufficient to guarantee that the costs *per search* are fixed. For example, if $N_c \sim Poisson(r_c t)$, the expected number of searches is linear in t and increases at a rate of r_c. Hence, the costs per search are constant as long as the opportunity costs of time are. In contrast, if $N_c \sim Bernoulli(\min\{\log(1+t), 1\})$, the expected number of searches increases at a decreasing rate in t. Hence, even if the opportunity

[12] The exception is highlighted in Section 4.3.5.

costs of time are constant, the costs per search are not: in expectation, the time required for an additional search increases in the time spent searching. Hence, the search technology, together with the opportunity costs of time, determine whether the costs per search are independent of the number of searches.

Overall, this highlights that the expected costs per search are only constant if both the opportunity costs of time and the increase in the expected number of searches remain constant. Either with multiple categories, decreasing marginal benefits for the outside activity or increasing difficulty of adding another alternative to the consideration set, the utility costs per search increase in the number of searches.

Besides search costs not being fixed, our framework also deviates from standard search models in that it has a time constraint. In a time allocation model, this constraint is natural as time is limited; a day only has so many hours. Translating this constraint into a more standard search model means restricting the number of searches. Such a constraint is not considered in standard search models. Instead, consumers are able to take as many searches as they like.

Introducing such a constraint has an immediate implication for standard search models: even if we make searching costless, consumers will not get fully informed. Instead, they only search up until they hit the constraint.

This again differs in our model because it allows a more nuanced analysis of search cost reductions. In the case with a single category, making the outside activity worthless implies that the consumer spends all the available time searching for products. As with standard search, the time constraint then implies that the consumer does not get fully informed. In contrast, if we instead increase the arrival rate to infinity, the consumer searches all alternatives in essentially no time. Hence, the consumer gets fully informed despite the time constraint.

4.3.4 Cross-category search effects

Many retailers offer products from multiple categories. For example, Amazon offers products across categories such as “Pet Supplies” and “Beauty & Personal Care”. Often, these categories are priced separately because the categories are not thought of as substitutes or complements. As we now show, consumers allocating time can create cross-category search effects such that pricing categories independently does not maximize profits.

In our framework cross-category effects arise because of the time constraint: time spent searching in one category is time lost in another one. Hence, seemingly independent product categories become related when analyzing them through the lens of a time allocation problem. For example, because our consumer really needs winter boots, he has no time to search for a better squash racket.

This mechanism has implications for cross-category pricing. To highlight the mechanism, we focus on the following simple setup: there is a unit mass of consumers that are searching in two categories, A and B. The two categories differ only in the price of products. To introduce prices, we assume that products are horizontally differentiated: given a consumer's price sensitivity α, purchasing product j in category c yields utility

$$u_{cj} = -\alpha p_c + \varepsilon_{cj} \tag{4.7}$$

Both categories are offered by a monopolist retailer who decides on category-specific prices. Specifically, for each category c, the consumer sets a single category-specific average price p_c. Marginal costs are assumed to be zero.

We assume that consumers have rational expectations such that they correctly anticipate the category-specific average price in each category. However, even correctly anticipating the category-specific average prices, consumers still need to spend time to reveal the idiosyncratic match values ε_{cj}. We assume that these match values are independent draws from a distribution with cumulative densities $F_A = F_B$. Hence, a change in the average price of category A, set by the monopolist, acts like a mean-shift in the distribution of U_{Aj}. We further assume that the other assumptions from Section 4.2.1 are satisfied.

The demand for category A is given by

$$D_A(p_A, p_B) = \sum_k G_A(t_A^*(p_A, p_B), k)\mathbb{P}\left(\max\{U_{A1}, \ldots, U_{Ak}\} - \alpha p_A \geq u_{0c}\right) \tag{4.8}$$

where $t_A^*(p_A, p_B)$ denotes the optimal time consumers allocate to searching in category A, given average prices. The demand for category B can be specified similarly.

The monopolist's profits as a function of category prices then are given by

$$\pi(p_A, p_B) = D_A(p_A, p_B)p_A + D_B(p_A, p_B)p_B \tag{4.9}$$

where the category-specific demand D_c implicitly depends on the optimal time allocation $t_A^*(p_A, p_B)$. The first order condition then reveals the effects of the time allocation, both within and across the categories:

$$\begin{aligned}\frac{\partial \pi(p_A, p_B)}{\partial p_A} = p_A \underbrace{\frac{\partial D_A(p_A, p_B)}{\partial t}}_{>0} \underbrace{\frac{\partial t_A^*}{\partial p_A}}_{\leq 0} + p_B \underbrace{\frac{\partial D_B(p_A, p_B)}{\partial t}}_{>0} \underbrace{\frac{\partial t_B^*}{\partial p_A}}_{\geq 0} \\ + D_A(p_A, p_B) + p_A \frac{\partial D_A(p_A, p_B)}{\partial p_A}\end{aligned} \tag{4.10}$$

Whereas a price increase makes consumers spend less time searching in a category (see Proposition 4.3), more time is freed up to search in the other category. This is reflected in the second term. At the profit-maximizing prices, the above derivative needs to equal zero. Hence, (4.10) immediately implies that the profit-maximizing price increases through the cross-category search effect; because consumers substitute to spending time searching in B and not just to spending time in the outside activity, a price increase in A leads to an increased demand in B, dampening the demand decrease in A.

As a result, pricing the two categories like substitutes will increase the monopolists profits. This suggests that pricing categories independently is not profit maximizing, even if categories are seemingly unrelated.

A similar logic applies when studying different categories as different product markets; lower prices in one market can divert search from another, unrelated market. Nonetheless, the search literature builds on search models that do not consider the decision to search across categories (McCall, 1970; Weitzman, 1979; Chade and Smith, 2006). Instead, these models implicitly abstract from cross-category search effects: because the consumer pays a fixed cost per search, searching in each category would be treated as a separate search problem as long as there is no restriction on the number of searches.[13] Using our framework, we now derive

[13] The search for multiple products as considered by (e.g. McAfee, 1995; Gatti, 1999; Zhou, 2014) does not involve the decision of which categories to search. Instead, consumers only decide when to stop searching, where each search reveals multiple products.

conditions under which disregarding cross-category effects does not impose any limitations.

4.3.5 When can we ignore cross-category effects?

To derive conditions under which cross-category effects can be ignored, we focus on the *marginal* cross category effect, which we define as the effect of a marginal change (e.g. price increase) in one category on the time allocation to another category. Proposition 4.6 provides two separate cases under which there are no marginal cross-category effects.

Proposition 4.6. *There are marginal cross-category effects unless at least one of these cases applies:*

(i) $b_0(t) = \nu t$ *and* $\bar{t}$ *is sufficiently large so that* $b'_c(t^*_c) \leq \nu$ *for all* c

(ii) $t^*_c = 0$ *and* $b'_c(0) < \lambda$ *for all* $c > 0$ *except one*

Proof. Case (i): If $b_0(t)$ is not linear in t, then $b'_c(t^*_c) = \lambda$ is not equal to a constant. Hence, a change in $b'_c(t^*_c)$ for some c implies changes to the optimal time allocation to other categories. Whenever $\bar{t}$ is not sufficiently large such that $b'_c(t^*_c) = \lambda > \nu$, λ again is not constant. Hence, whenever one of the two conditions is not satisfied, there are cross-category effects unless case (ii) applies. When both conditions are satisfied, we have $b'_c(t^*_c) = \nu = \lambda \forall c$ such there are no cross-category effects; only changes to the value of the outside activity and within the same category affect the time allocation to category c. Case (ii): The conditions imply $b_c(t^*_c) < \lambda$ for all c with $t^*_c = 0$. This precludes cross-category effects because a marginal change in λ leaves $t^*_c = 0$ unaffected. If the case does not apply, there are cross-category effects because λ again is not constant, except if case (i) holds. □

The first case requires two conditions: the marginal benefit of the outside activity needs to be constant, and the consumer must have enough time to search in all categories so that increasing the available time will have no effect on how much time is spent searching. Together, these two conditions guarantee that the opportunity costs of time, i.e. the Lagrange multiplier, is a constant and independent of the category specifics at the optimal time allocation. Hence, the time allocation for each category does not depend on the time allocation of any of the other categories. The second case requires that time is allocated only to one category besides the outside activity. Hence, Lemma 4.2 applies, where marginal changes in any of the categories have no effects on the other categories.

Using Proposition 4.6, we can judge whether cross-category effects may be important in some settings. If either of the two cases applies, cross-category effects can be safely ignored without affecting the analysis, at least at the margin. If neither case applies, then there are cross-category effects. To give an example, suppose we model a consumer's search for winter boots as searching across online retailers. If the consumer does not spend any time searching in categories other than winter boots – for example because there is no need to buy a product from another category– case (ii) applies and there are no cross-category effects. If the consumer instead spends time searching in multiple categories and time is limited, neither case applies and there will be cross-category search effects.

Proposition 4.6 only applies to marginal cross-category effects. However, if we additionally assume that the respective case still applies after any relevant change, it generalizes to more substantial changes.

4.3.6 Consumer welfare effects

Consumer welfare effects in our model can be readily obtained by applying the Envelope theorem. For example, the effect on consumer welfare from a change in the need of buying a product in a category, u_{0c}, can be obtained as follows:

$$\frac{\partial u(\boldsymbol{t}^*)}{\partial u_{0c}} = \frac{\partial b_c(t_c^*)}{\partial u_{0c}} \geq 0$$

Hence, decreasing the need to buy a product in any category (increasing u_{0c}), has a positive welfare effect (unless $t_c^* = 0$). Similarly, we can show that a positive FOSD-shift in the utility distribution of, or improving the search technology in any category has positive welfare effects.

4.3.7 Relationship to budget allocation problems

There are various parallels of our time allocation to the well-known budget allocation problem. Both problems are faced by a consumer deciding how to allocate a limited resource, either time or money. By assuming that the consumer has unit demand in each category, we abstract from the budget constraint and focus only on the time and search aspects in consumers' decision making. This is in line with the wider search literature and justified in many settings. Specifically, when choosing from product categories that are relatively inexpensive, budget limitations usually are not important.[14] For example, many consumers will not face a binding budget constraint when deciding whether to buy a 1$ or 2$ chocolate bar. Time constraints, however, can still be important in such cases. If comparing and deciding which one to buy takes some time, the consumer may end up considering only one or none of the two chocolate bars. Hence, the decision which chocolate bar to buy (if any) will be driven by time, as opposed to budget considerations.

Under some restrictions on the utility functions, the mathematical formulations of the problems are also equivalent. Specifically, in both problems the consumer solves a separable concave optimization problem. As a result, we can also apply the Envelope theorem to perform consumer welfare analysis. Moreover, our conditions for the non-existence of cross-category search effects parallel conditions that guarantee the feasibility of partial equilibrium analysis with quasi-linear utility functions.

The budget constraint in standard budget allocation problems is based on prices. With our time allocation problem, we do not have prices for the categories. However, we can introduce variables akin to prices by reformulating the problem. Recall the case with a rate r_c, such that search benefits are given by $b_c(r_c t)$. If we adjust the decision variable to $\tilde{t} = r_c t$, the constraint becomes $\sum_{j=1}^{\bar{c}} p_c \tilde{t}_c + t_0 = \bar{t}$ with $p_c = \frac{1}{r_c}$. With this formulation, improving the search technology of a category is equivalent to decreasing the "price" of searching in the category.

We also obtain a parallel to Giffen goods based on this reformulation. Recall that in Section 4.3.2, we showed that improving a search technology can lead to less time spent searching in a category. For the reformulated problem, this implies that a price decrease can lead to less time spent searching. Hence, categories where this is the case parallel Giffen goods, where a price decrease leads to a lower consumption of the good.

Besides these parallels, there is is also an important difference related to the underlying assumptions. Specifically, the functions that enter the mathematical problem derive their shape from different assumptions. In our framework, we assume unit demand in each category, and impose conditions on the distribution of the number of searches to guarantee a concave shape of search benefits. In a budget allocation problem, however, the consumer decides on

[14]Note that the price sensitivity in (4.7) can be derived from a general model with quasi-linear utility and a sufficiently large budget.

quantities, and the respective concave shape is derived from assumptions on the underlying preferences.

4.4 Generalizations

In the baseline multi-category search model, we exclude any interdependence of categories; within a category, products are substitutes, across categories, they are independent. This restriction has allowed us to focus on comparative statics and implications of formulating search as a time allocation problem. We now extend the analysis in three different directions. First, we show how complementarities across categories influence the consumer's time allocation. Second, we study how the problem can be reformulated to include different search technologies. Finally, we consider shopping costs that we interpret as a minimum amount of time that needs to be spent in a category to be able to buy an already known alternative.

4.4.1 Complementarities across categories

In various settings, alternatives from different categories complement each other. For example, playing squash requires both a racket and a ball, a good wine is more enjoyable with good food, and a nice-looking pair of winter boots looks even better when paired with the right pants. We now show how such complementarities can be introduced, and how they affect a consumer's time allocation.

Complementarities across categories imply that the value of purchasing a product in one, is influenced by the value of purchasing a product in another category. In our framework, we can implement this logic by introducing a utility function that links categories. To highlight the influence of categories being interdependent, we focus on the case with three categories ($\bar{c} = 3$), two of which having complementarities. Formally, we specify the following functional form

$$v(\boldsymbol{S}) = \phi\left(\bar{u}_1^{k_1}, \bar{u}_2^{k_2}, \bar{u}_3^{k_3}\right) = \pi(\bar{u}_1^{k_1}, \bar{u}_2^{k_2}) + \bar{u}_3^{k_3} \tag{4.11}$$

where $\bar{u}_c^{k_c} = \max\left\{u_{0c}, \max_{j\in\{1,\ldots,k_c\}} u_{jc}\right\}$. Hence, we continue to maintain the assumption of unit demand. To capture that better alternatives make the consumer better off, we assume that $\pi()$ is strictly increasing in both arguments. Besides, to focus on the effect of the complementarity we assume that, otherwise, all three categories are identical, i.e. $G_c(k,t) = G(k,t)$

and $\bar{U}_c^k = \bar{U}^k = \max\{u_0, U_1, \ldots, U_k\}$ for all c. Finally, we also keep the other assumptions from the baseline model, allowing us to continue using standard techniques and focus on interior solutions.

In this adjusted model, the consumer continues to maximize expected utility by solving the problem in (4.2). The difference is that the objective function now is given by

$$EU(\boldsymbol{t}) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} G(k_1,t_1)G(k_2,t_2)\mathbb{E}\left[\pi(\bar{U}^{k_1},\bar{U}^{k_2})\right] + \sum_{k=0}^{\infty} G(k,t_3)\mathbb{E}\left[\bar{U}^k\right] + b_0(t_0) \quad (4.12)$$

When categories are independent with $\pi(\bar{u}^{k_1},\bar{u}^{k_2}) = \bar{u}^{k_1} + \bar{u}^{k_2}$ and identical, the optimality conditions (4.5a) - (4.5c) directly imply that it is optimal to spend the same amount of time in each category ($t_1^* = t_2^* = t_3^*$). With complementarities introduced through the function $\pi()$, this is not the case. Instead, the optimal time spent searching in the third category generally differs from the time spent searching in the first category ($t_3^* \neq t_1^* = t_2^*$).[15]

Whether the consumer spends more or less time in the first two categories depends on how the complementarity is shaped. Proposition 4.7 provides results for two cases, where the sign does not depend on the distribution of the number of alternatives that will be discovered.

Proposition 4.7. *In the simple model with complementarities, an interior solution requires $t_1^* = t_2^* > t_3^*$ ($t_1^* = t_2^* < t_3^*$) if $\pi(u_1,u_2) \geq u_1$ ($\pi(u_1,u_2) \leq u_1$) for all realizations (u_1,u_2) in the relevant support, and with strictness for at least one pair.*

Proof. An interior solution requires the following to hold:

$$\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} G(k_2,t_2^*)\left[\mathbb{E}\left[\frac{\partial G(k_1,t_1^*)}{\partial t}\pi(\bar{U}^{k_1},\bar{U}^{k_2}) - \frac{\partial G(k_1,t_3^*)}{\partial t}\bar{U}^{k_1}\right]\right] = 0 \quad (4.13)$$

If $t_1^* = t_3^*$, the conditions imposed on $\pi()$ then directly imply that the left-hand side is strictly larger (smaller) than zero. Hence, for (4.13) to be satisfied, t_1^* and t_2^* need to be smaller (larger) than t_3^*. □

The two cases in Proposition 4.7 impose conditions on the difference $\pi(u_1,u_2) - u_1$.[16] To

[15] The first two categories being identical still means that time is equally allocated to the two of them.
[16] The relevant support is all values in the support of the distribution of $\max\{u_{0c}, U_{jc}\}$.

discuss the conditions, we now interpret the difference $u_{jc} - u_{0c}$ as the consumers' willingness-to-pay for product j from category c, net of the price that is already factored in u_{jc}. This interpretation is valid under the usual partial equilibrium assumptions.[17]

For the first case, two conditions need to apply. First, not buying an alternative in the second category does not decrease the willingness-to-pay for alternatives from the first category. This ensures that $\pi(u_1, u_2) - u_1 \geq 0$ holds independent of u_2. Second, buying a better alternative in the second category increases the willingness-to-pay for an alternative from the first category. This ensures that $\pi(u_1, u_2) - u_1 > 0$ for at least some pair (u_1, u_2). For example, not buying winter boots may not make buying new pants less attractive, but buying a better pair of winter boots would increase the value of buying a (matching) pair of pants.[18]

For the second case, the two conditions are reversed: not buying an alternative in the second category decreases the willingness-to-pay in the first, and buying a better-matched alternative in the second category does not increase the willingness-to-pay for products from the first. For example, buying the best-matching squash racket will have little value unless indoor shoes required to play on the court are also found and bought.[19]

In both cases, the stated conditions ensure that the complementarity either strictly in- or decreases the marginal benefits of searching in either of the first two categories. This allows to determine whether the time spent searching in these categories in- or decreases. For cases where the sign of $\pi(u_1, u_2) - u_1$ depends on the realization u_2, it is difficult to determine the direction of the effect on the marginal search benefits in general. Nonetheless, the two cases provide the intuition that continues to apply: if finding a relatively good match in a complementary category increases the willingness-to-pay, and it is sufficiently likely that such good matches can be found, then the consumer will spend more time searching in complementary categories. In contrast, if not finding a good alternative in a complementary category substantially decreases the willingness-to-pay and it is difficult to find good matches, then it will be optimal to spend less time searching in complementary categories.

[17] The assumptions are that utility is linear additive in a composite outside good that is known prior to search, and that budget is not limited or sufficiently large to guarantee interior solutions.

[18] $\pi(u_1, u_2) = u_1 + u_2 + \lambda u_1 u_2$ with $u_{01} = u_{02} > 0$ and $\lambda > 0$ is an example that guarantees that both conditions hold.

[19] This can be modeled, for example, as two categories being akin to perfect complements with $\pi(u_1, u_2) = \min\{u_1, u_2\}$.

4.4.2 Generalized search technologies

In some settings, consumers not only decide which categories to search in, but also where to search for alternatives. For example, consumers may decide to spend time browsing in different stores that offer alternatives from multiple categories. Settings such as these can be accommodated in our framework by allowing search technologies to reveal alternatives from multiple categories.

Specifically, we can model a joint distribution of the number of revealed alternatives in each category, given the amount of time spent on a search technology. Denote this joint distribution by $\boldsymbol{N}_c = [N_1, \ldots, N_{\bar{c}}]$, with the corresponding probability mass function $G(\boldsymbol{k}, \boldsymbol{t}) = \mathbb{P}_{\boldsymbol{N}_c|\boldsymbol{t}}\,(N_1 = k_1, \ldots, N_{\bar{c}} = k_{\bar{c}})$, with $\boldsymbol{k} = [k_1, \ldots, k_{\bar{c}}]$. An element in the vector $\boldsymbol{t}$ now is the time spent in the respective search technology, that influences the distribution of $\boldsymbol{N}_c$ through the probability mass function $G(\boldsymbol{k}, \boldsymbol{t})$. Note that, with search technologies, the length of the vector $\boldsymbol{t}$ does not need to be equal to $\bar{c}$; there can be fewer or more search technologies available than the number of categories. Hence, unlike in the baseline model, the decision of how much time to spend searching in a category is not equivalent to the decision of how much time to spend on a search technology.

With these generalized search technologies, the consumer continues to solve the optimization problem (4.2), with an adjusted objective function given by

$$EU(\boldsymbol{t}) = \sum_{k_1} \cdots \sum_{k_{\bar{c}}} G(\boldsymbol{k}, \boldsymbol{t}) \mathbb{E}\left[\sum_{c=1}^{\bar{c}} \bar{U}_c^{k_c}\right] + b_0(t_0) \tag{4.14}$$

By imposing additional restrictions, it is then again possible to guarantee that $EU(\boldsymbol{t})$ is well-behaved, so that standard techniques can be applied to characterize the optimal time allocation.

4.4.3 Shopping costs

In our search model, it requires time to search for and evaluate products. However, it does not take any time to buy a product; once an alternative is added into the consideration set through search, no further time is needed to make a decision. When consumers are shopping for products, however, it often takes time to buy a product even if it is known prior to search. In this case, the time costs incurred to buy an alternative from a product category are a combination of shopping and search costs.

To show how shopping costs can be introduced in our framework, we now consider an example of grocery shopping. Suppose a consumer is going to a supermarket and needs to buy one product from each of various categories like yogurt, spices, pasta etc. From past trips to the supermarket, the consumer already knows alternatives in each of the categories. These known alternatives can be modeled through the value of not buying a newly discovered alternative: a large u_{0c} indicates that the consumer already knows a good alternative in the respective category.

To buy a known or unknown alternative from a category, the consumer needs to spend time walking to the respective category in the supermarket. Hence, there is a fixed time cost of buying an alternative from a category. If the consumer buys the known alternative, no additional time is spent. However, evaluating other alternatives from the same category requires additional time. In this grocery store setting, we can also assume that there is no outside activity; the consumer has allocated a fixed amount of time to do shopping, and we focus on the decision to allocate the time across different categories.

As the consumer needs to buy an alternative from every category considered,[20] the fixed time costs are simply subtracted from every category, and enter the time constraint. The resulting search problem is almost equivalent to (4.2). The only differences are the interpretation of u_{0c} and an adjustment to the time constraint. Specifically, denoting the shopping costs for category c by t_c^s, the time constraint becomes $\sum_{c=0}^{\bar{c}} t_c = \bar{t} - \sum_{c=1}^{\bar{c}} t_c^s$.

In this setting, conditions as the ones from Section 4.2.3 continue to characterize the optimal time allocation. Hence, it is straightforward to show that reducing the time required to buy an alternative, i.e. the shopping cost, leads to more time spent comparing alternatives. Note however, that if the consumer does not need to buy an alternative from all categories, the problem becomes more involved and may be difficult to solve in general.[21]

[20] This is equivalent to assuming that the value of not buying any alternative from the categories considered in the model is sufficiently small so that the consumer always pays the fixed time costs, whereas it is sufficiently large for categories outside the model so that the consumer would never pay the fixed shopping cost and buy alternatives from them.

[21] In this case, the consumer not only decides how much time to spend searching in each category, but also which categories to pay the fixed cost for to buy an alternative. We conjecture that in the case where categories differ only in the need to buy an alternative and the options known prior to search, a marginal improvement algorithm as in Chade and Smith (2006) determines the optimal time allocation.

4.5 Conclusion

This paper introduces a search model for settings where consumers search across multiple product categories. By modeling a consumer's decision as a time allocation problem, we obtain a tractable framework that allows to readily derive how much a rational consumer searches across categories, while remaining flexible as to how exactly consumers use the allocated time to search within a category.

Whereas we model consumers' choices under limited information and limited time, we abstract from a third limitation: limited budget. In this sense, we follow the broader search literature that focuses on partial equilibrium analysis. Whereas this focus is justified to study many settings, extending the model to additionally incorporate a budget constraint provides a promising avenue for future research. In such a generalized framework, a consumer would decide (i) how many units to buy for each revealed product, and (ii) how much time to spend searching in each category. Whereas limited budget implies that the number of units is limited, limited time and information restrict which products can be bought in the first place. Combining these limitations then allows to additionally study how cross-category effects arise from budget limitations, and how they relate to the cross-category search effects highlighted in this paper.

Another interesting avenue for future research is to study equilibrium outcomes across different categories when firms influence the utility distribution by setting prices. By characterizing the demand side, our paper provides a first step towards this goal.

Appendix

4.A Additional derivations

4.A.1 $\Delta e(k)$ decreases in k

We first show that for $\Delta u_{0c} = u'_{0c} - u_{0c} < 0$, $\Delta e(k)$ increases in k. Write the expectations as:

$$e(k) = \mathbb{E}\left[\max\{u_{0c}, U_{1c}, \ldots, U_{kc}\}\right] = u_{0c} + \int_{u_{0c}}^{\infty} 1 - F_c(z)^k \mathrm{d}z$$
$$\frac{\partial e(k)}{\partial u_{0c}} = F_c(u_{0c})^k$$

Because $F_c(u_{0c}) \in [0,1]$, $F_c(u_{0c})^k$ decreases in k, which implies $\Delta e(k)$ increases in k if $\Delta u_{0c} < 0$. Similarly, for a change in the mean of the utility distribution, μ_c, we get

$$\frac{\partial \mathbb{E}\left[\mu_c + \max\{u_{0c} - \mu_c, \tilde{U}_{1c}, \ldots, \tilde{U}_{kc}\}\right]}{\partial \mu_c} = 1 - \tilde{F}_c(u_{0c} - \mu_c)^k$$

where $\tilde{F}_c$ is the cumulative density of $\tilde{U}_{jc} = U_{jc} - \mu_c$. Hence, $\Delta e(k)$ increases in k if $\Delta \mu_c > 0$.

4.A.2 Poisson distribution

We show that the Poisson distribution satisfies the three restrictions on the probability mass functions: (i) Stochastic dominance, (ii) concavity, (iii) differentiability. Moreover, we derive the expression for the marginal change of the arrival rate used in Section 4.3.2.

The probability mass function of the Poisson distribution $N_c \sim \mathit{Poisson}(r_c t)$ is given by

$$G_c(k,t) = \exp(-r_c t)\frac{(r_c t)^k}{k!} \tag{4.15}$$

and twice differentiable, hence satisfies (iii). To show stochastic dominance (ii), consider an independent random variable $Z \sim \mathit{Poisson}(r_c\delta)$. Because $\mathbb{P}(Z < 0) = 0$, $N_c + Z$ stochastically dominates N_c. Now note that setting $\delta = t' - t$, we get $N'_c = N_c + Z = \mathit{Poisson}(r_c t')$. Hence, N'_c stochastically dominates N_c.

Finally, to show that the resulting expected search benefits are concave in t, we first simplify notation. Let

$$e(k) \equiv \mathbb{E}\left[\max\{u_{0c}, U_{1c}, \ldots, U_{kc}\}\right] \tag{4.16}$$
$$\Delta e(k) \equiv e(k) - e(k-1) \tag{4.17}$$

denote the expected benefit and marginal benefit of k searches. Combining this with the probability mass function, we can write the expected search benefits , and the first and second order derivatives as

$$b_c(t) = \exp(-r_c t) \sum_{k=0}^{\infty} \frac{(r_c t)^k}{k!} e_c(k) \tag{4.18}$$
$$b_c'(t) = r_c \exp(-r_c t) \left[\sum_{k=0}^{\infty} \frac{(r_c t)^k}{k!} \Delta e_c(k+1) \right] > 0 \tag{4.19}$$
$$b_c''(t) = r_c^2 \sum_{k=0}^{\infty} G_c(k,t)(\Delta e_c(k+2) - \Delta e_c(k+1)) < 0 \tag{4.20}$$

The inequalities show that $b_c(t)$ is strictly concave in t, hence (iii) is also satisfied. The inequalities immediately follow from the fact that additional searches increase the expected benefit, $\Delta e(k) > 0 \forall k$, but that this happens at a decreasing rate, $\Delta e(k+2) - \Delta e(k+1) < 0 \forall k$.

Expression (4.6) can be obtained by taking the derivative of (4.19) with respect to the arrival rate:

$$\begin{aligned} \frac{\partial b_c'(t)}{\partial r_c} &= \sum_{k=0}^{\infty} G_c(k,t) e_c(k) + t \left[r_c \sum_{k=0}^{\infty} G_c(k,t)(\Delta e_c(k+2) - \Delta e_c(k+1)) \right] \\ &= \frac{b_c'(t)}{r_c} + \frac{t}{r_c} b_c''(t) \end{aligned}$$

4.A.3 Bernoulli distribution

We show that the Bernoulli distribution with a success probability that depends on time through a function $\rho(t)$ with $\rho'(t) > 0$, $\rho''(t) < 0$ on the relevant domain, satisfies the three restrictions on the probability mass functions: (i) Stochastic dominance, (ii) concavity, (iii) differentiability.

The probability mass function of the Poisson distribution $N_c \sim \mathit{Bernoulli}(\rho(t))$ is given by

$$G_c(0,t) = 1 - \rho(t) \qquad\qquad G_c(1,t) = \rho(t) \tag{4.21}$$

which is twice differentiable as long as $\rho(t)$ is, hence satisfies (iii). $\rho'(t) > 0$ then immediately implies stochastic dominance (ii). Using again the notation from (4.16), we can write the expected search benefits as

$$b_c(t) = u_0 + \rho(t)(e(1) - u_0) \tag{4.22}$$

Hence, strict concavity (ii) immediately follows from $\rho''(t) < 0$.

References

Adam, K. (2001). Learning while searching for the best alternative. *Journal of Economic Theory 101*(1), 252–280.

Aguiar, M. and E. Hurst (2005). Consumption versus expenditure. *Journal of Political Economy 113*(5), 919–948.

Aguiar, M. and E. Hurst (2007a). Life-cycle prices and production. *American Economic Review 97*(5), 1533–1559.

Aguiar, M. and E. Hurst (2007b). Measuring trends in leisure: The allocation of time over five decades. *Quarterly Journal of Economics 122*(3), 969–1006.

Aguiar, M., E. Hurst, and L. Karabarbounis (2012). Recent developments in the economics of time use. *Annual Review of Economics 4*, 373–397.

Aguiar, M., E. Hurst, and L. Karabarbounis (2013). Time use during the great recession. *American Economic Review 103*(5), 1664–1696.

Anderson, E. W. and S. M. Shugan (1991). Repositioning for changing preferences: The case of beef versus poultry. *Journal of Consumer Research 18*(2), 219.

Anderson, S. P. and R. Renault (1999). Pricing, product diversity, and search costs: A bertrand-chamberlin-diamond model. *The RAND Journal of Economics 30*(4), 719–735.

Armstrong, M. (2017). Ordered consumer search. *Journal of the European Economic Association 15*(5), 989–1024.

Armstrong, M. and J. Vickers (2015). Which demand systems can be generated by discrete choice? *Journal of Economic Theory 158*, 293–307.

Athey, S. and G. Ellison (2011). Position auctions with consumer search. *The Quarterly Journal of Economics 126*(3), 1213–1270.

Athey, S. and G. Imbens (2015). Machine learning methods for estimating heterogeneous causal effects.

Bagwell, K. (2007). The economic analysis of advertising. In *Handbook of Industrial Organization*, Volume 3, Chapter 28, pp. 1701–1844.

Banks, J. S. and R. K. Sundaram (1992). Denumerable-armed bandits. *Econometrica 60*(5), 1071–1096.

Banks, J. S. and R. K. Sundaram (1994). Switching costs and the Gittins index. *Econometrica 62*(3), 687–694.

Becker, G. S. (1965). A theory of the allocation of time. *The Economic Journal 75*(299), 493.

Biddle, J. E. and D. S. Hamermesh (1990). Sleep and the allocation of time. *Journal of Political Economy 98*(5, Part 1), 922–943.

Bikhchandani, S. and S. Sharma (1996, January). Optimal search with learning. *Journal of Economic Dynamics and Control 20*(1), 333–359.

Blake, T., C. Nosko, and S. Tadelis (2015). Consumer heterogeneity and paid search effectiveness: A large-scale field experiment. *Econometrica 83*(1), 155–174.

Branco, F., M. Sun, and J. M. Villas-Boas (2012). Optimal search for product information. *Management Science 58*(11), 2037–2056.

Bronnenberg, B., T. Klein, and Y. Xu (2020). Consumer time budgets and grocery shopping behavior.

Bronnenberg, B. J., J. B. Kim, and C. F. Mela (2016). Zooming in on choice: How do consumers search for cameras online? *Marketing Science 35*(5), 693–712.

Burdett, K. and K. L. Judd (1983). Equilibrium price dispersion. *Econometrica 51*(4), 955–969.

Burdett, K. and D. A. Malueg (1981). The theory of search for several goods. *Journal of Economic Theory 24*(3), 362–376.

Carlin, B. I. and F. Ederer (2019). Search fatigue. *Review of Industrial Organization 54*(3), 485–508.

Chade, H. and L. Smith (2006). Simultaneous search. *Econometrica 74*(5), 1293–1307.

Chan, T. Y. and Y.-H. Park (2015). Consumer search activities and the value of ad positions in sponsored search advertising. *Marketing Science 34*(4), 606–623.

Chen, Y. and S. Yao (2017). Sequential search with refinement: Model and application with click-stream data. *Management Science 63*(12), 4345–4365.

Choi, H. and C. F. Mela (2019). Monetizing online marketplaces. *Marketing Science 38*(6), 913–1084.

Choi, M., A. Y. Dai, and K. Kim (2018). Consumer search and price competition. *Econometrica 86*(4), 1257–1281.

Chu, L. Y., H. Nazerzadeh, and H. Zhang (2020). Position ranking and auctions for online marketplaces. *Management Science 66*(8), 3617–3634.

Chung, J. H., P. K. Chintagunta, and S. Misra (2019). Estimation of sequential search models.

Compiani, G., G. Lewis, S. Peng, and P. Wang (2021). Online search and product rankings: A double index approach.

Davis, J. M., G. Gallego, and H. Topaloglu (2014). Assortment optimization under variants of the nested logit model. *Operations Research 62*(2), 250–273.

De Los Santos, B., A. Hortacsu, M. R. Wildenbeest, and A. Hortaçsu (2012). Testing models of consumer search using data on web browsing and purchasing behavior. *The American Economic Review 102*(6), 2955–2980.

De los Santos, B. and S. Koulayev (2017). Optimizing click-through in online rankings with endogenous search refinement. *Marketing Science 36*(4), 542–564.

DeGroot, M. H. (1970). *Optimal Statistical Decisions* (Wiley Clas ed.). New Jersey: John Wiley & Sons, Inc.

Derakhshan, M., N. Golrezaei, V. Manshadi, and V. Mirrokni (2022). Product ranking on online platforms. *Management Science* (forthcoming).

Diamond, P. A. (1971). A model of price adjustment. *Journal of Economic Theory 3*(2), 156–168.

Donnelly, R., A. Kanodia, and I. Morozov (2022). The long-tail effect of personalized rankings.

Doval, L. (2018). Whether or not to open Pandora's box. *Journal of Economic Theory 175*, 127–158.

Ellison, G. and A. Wolitzky (2012). A search cost model of obfuscation. *The RAND Journal of Economics 43*(3), 417–441.

Fershtman, D. and A. Pavan (2019). Searching for arms.

Gabaix, X., D. Laibson, G. Moloche, and S. Weinberg (2006). Costly information acquisition: Experimental analysis of a boundedly rational model. *The American Economic Review 96*(4), 1043–1068.

Garcia, D., J. Tolvanen, and A. K. Wagner (2022). Demand estimation using managerial responses to automated price recommendations. *Management Science* (forthcoming).

Gatti, J. R. J. (1999). Multi-commodity consumer search. *Journal of Economic Theory 86*(2), 219–244.

Geweke, J. (1989). Bayesian inference in econometric models using Monte Carlo integration. *Econometrica 57*, 1317–1339.

Geweke, J. (1991). Efficient simulation from the multivariate normal and Student-t distributions subject to linear constraints. In *Computing science and statistics: Proceedings of the 23rd symposium on the interface*, Fairfax, Virginia, pp. 571–578. Interface Foundation of North America, Inc.

Ghose, A., P. G. Ipeirotis, and B. Li (2012). Designing ranking systems for hotels on travel search engines by mining user-generated and crowdsourced content. *Marketing Science 31*(3), 493–520.

Ghose, A., P. G. Ipeirotis, and B. Li (2014). Examining the impact of ranking on consumer behavior and search engine revenue. *Management Science 60*(7), 1632–1654.

Gittins, J., K. Glazebrook, and R. Weber (2011). *Multi-armed Bandit Allocation Indices* (2nd ed.). John Wiley & Sons, Inc.

Glazebrook, K. D. (1979). Stoppable families of alternative bandit processes. *Journal of Applied Probability 16*(4), 843–854.

Goldman, M. and J. M. Rao (2014). Experiments as instruments: Heterogeneous position effects in sponsored search auctions.

Haan, M. A., J. L. Moraga-González, and V. Petrikaitė (2018). A model of directed consumer search. *International Journal of Industrial Organization 61*, 223–255.

Hajivassiliou, V. and D. McFadden (1998). The method of simulated scores for the estimation of ldv models. *Econometrica 66*, 863–896.

Hauser, J. R., G. L. Urban, and B. D. Weinberg (1993). How consumers allocate their time when searching for information. *Journal of Marketing Research 30*(4), 452.

Hong, H. and M. Shum (2006). Using price distributions to estimate search costs. *The RAND Journal of Economics 37*(2), 257–275.

Honka, E. (2014). Quantifying search and switching costs in the US auto insurance industry. *The RAND Journal of Economics 45*(4), 847–884.

Honka, E. and P. Chintagunta (2017). Simultaneous or sequential? Search strategies in the U.S. auto insurance industry. *Marketing Science 36*(1), 21–42.

Honka, E., A. Hortaçsu, and M. A. Vitorino (2017). Advertising, consumer awareness, and choice: evidence from the U.S. banking industry. *The RAND Journal of Economics 48*(3), 611–646.

Hortaçsu, A. and C. Syverson (2004). Product differentiation, search costs, and competition in the mutual fund industry: A case study of S & P 500 index funds. *The Quarterly Journal of Economics 119*(2), 403–456.

Huang, Y. (2022). Pricing frictions and platform remedies : The case of airbnb.

Ibaraki, T. and N. Katoh (1988). *Resource allocation problems: algorithmic approaches*. MIT Press.

Jang, S., A. Prasad, and B. T. Ratchford (2017). Consumer search of multiple information sources and its impact on consumer price satisfaction. *Journal of Interactive Marketing 40*, 24–40.

Janssen, M. C. and M. C. Non (2009). Going where the ad leads you: On high advertised prices and searching where to buy. *Marketing Science 28*(1), 87–98.

Jeziorski, P. and S. Moorthy (2018). Advertiser prominence effects in search advertising. *Management Science 64*(3), 1365–1383.

Jeziorski, P. and I. R. Segal (2015). What makes them click: Empirical analysis of consumer demand for search advertising. *American Economic Journal: Microeconomics 7*(3), 24–53.

Jindal, P. and A. Aribarg (2020). The importance of price beliefs in consumer search. *Journal of Marketing Research (forthcoming)*.

Jolivet, G. and H. Turon (2019). Consumer search costs and preferences on the internet. *The Review of Economic Studies 86*(3), 1258–1300.

Ke, T. T., Z.-J. M. Shen, and J. M. Villas-Boas (2016). Search for information on multiple products. *Management Science 62*(12), 3576–3603.

Ke, T. T. and J. M. Villas-Boas (2019). Optimal learning before choice. *Journal of Economic Theory 180*(January), 383–437.

Keane, M. P. (1990). *Four essays in empirical macro and labor economics*. Ph. D. thesis, Brown University.

Keane, M. P. (1994). A computationally practical simulation estimator for panel data. *Econometrica 62*, 95–116.

Keller, G. and A. Oldale (2003). Branching bandits: A sequential search process with correlated pay-offs. *Journal of Economic Theory 113*(2), 302–315.

Kim, J. B., P. Albuquerque, and B. J. Bronnenberg (2010). Online demand under limited consumer search. *Marketing Science 29*(6), 1001–1023.

Kim, J. B., P. Albuquerque, and B. J. Bronnenberg (2017). The probit choice model under sequential search with an application to online retailing. *Management Science 63*(11), 3911–3929.

Kleinberg, R., B. Waggoner, and E. G. Weyl (2017). Descending price optimally coordinates search.

Kohn, M. G. and S. Shavell (1974). The theory of search. *Journal of Economic Theory 9*(2), 93–123.

Kooreman, P. and A. Kapteyn (1987). A disaggregated analysis of the allocation of time within the household. *Journal of Political Economy 95*(2), 223–249.

Koulayev, S. (2014). Search for differentiated products: identification and estimation. *The RAND Journal of Economics 45*(3), 553–575.

Kuksov, D. (2006). Search, common knowledge, and competition. *Journal of Economic Theory 130*(1), 95–108.

Lippman, S. A. and J. J. McCall (1976). The economics of job search: A survey. *Economic Inquiry 14*(2), 155–189.

McAfee, R. P. (1995). Multiproduct equilibrium price dispersion. *Journal of Economic Theory 67*(1), 83–105.

McCall, J. J. (1970). Economics of information and job search. *The Quarterly Journal of Economics 84*(1), 113–126.

Moraga-González, J. L., Z. Sándor, and M. R. Wildenbeest (2017a). Nonsequential search equilibrium with search cost heterogeneity. *International Journal of Industrial Organization 50*, 392–414.

Moraga-González, J. L., Z. Sándor, and M. R. Wildenbeest (2017b). Prices and heterogeneous search costs. *The RAND Journal of Economics 48*(1), 125–146.

Moraga-Gonzalez, J. L., Z. Sándor, and M. R. Wildenbeest (2022). Consumer search and prices in the automobile market. *Review of Economic Studies* (forthcoming).

Moraga-González, J. L. and Y. Sun (2022). Product quality and consumer search. *American Economic Journal: Microeconomics* (forthcoming).

Morozov, I. (2019). Measuring benefits from new products in markets with information frictions.

Olszewski, W. and R. Weber (2015). A more general Pandora rule? *Journal of Economic*

Theory 160, 429–437.

Rhodes, A. (2015). Multiproduct retailing. *The Review of Economic Studies 82*(1), 360–390.

Rhodes, A., M. Watanabe, and J. Zhou (2021, February). Multiproduct intermediaries. *Journal of Political Economy 129*(2), 421–464.

Rosenfield, D. B. and R. D. Shapiro (1981). Optimal adaptive price search. *Journal of Economic Theory 25*(1), 1–20.

Rothschild, M. (1974). Searching for the lowest price when the distribution of prices is unknown. *Journal of Political Economy 82*(4), 689–711.

Smith, H. and Ø. Thomassen (2012). Multi-category demand and supermarket pricing. *International Journal of Industrial Organization 30*(3), 309–314.

Stigler, G. J. (1961). The economics of information. *Journal of Political Economy 69*(3), 213–225.

Talluri, K. and G. Van Ryzin (2004). Revenue management under a general discrete choice model of consumer behavior. *Management Science 50*(1), 15–33.

Thomassen, Ø., H. Smith, S. Seiler, and P. Schiraldi (2017). Multi-category competition and market power: A model of supermarket pricing. *American Economic Review 107*(8), 2308–2351.

Train, K. E. (2009). *Discrete Choice Methods with Simulation*. Cambridge University Press.

Ursu, R. M. (2018). The power of rankings: Quantifying the effect of rankings on online consumer search and purchase decisions. *Marketing Science 37*(4), 530–552.

Ursu, R. M. and D. Dzyabura (2020). Retailers' product location problem with consumer search. *Quantitative Marketing and Economics 18*, 125–154.

Ursu, R. M., Q. Wang, and P. K. Chintagunta (2020). Search duration. *Marketing Science 39*(5), 849–871.

Ursu, R. M., Q. Zhang, and E. Honka (2021). Search gaps and consumer fatigue.

Van Ryzin, G. and S. Mahajan (1999). On the relationship between inventory costs and variety benefits in retail assortments. *Management Science 45*(11), 1496–1509.

Weitzman, M. L. (1979). Optimal search for the best alternative. *Econometrica 47*(3), 641–654.

Yavorsky, D., E. Honka, and K. Chen (2020). Consumer search in the U.S. auto industry: The role of dealership visits. *Quantitative Marketing and Economics 19*(1), 1–52.

Zhang, X., T. Y. Chan, and Y. Xie (2018). Price search and periodic price discounts. *Management Science 64*(2), 495–510.

Zhang, X., P. Ferreira, M. G. De Matos, and R. Belo (2021). Welfare properties of profit maximizing recommender systems: Theory and results from a randomized experiment. *MIS Quarterly: Management Information Systems 45*(1), 1–28.

Zhou, J. (2014). Multiproduct search and the joint search effect. *The American Economic Review 104*(9), 2918–2939.

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This thesis consists of three essays on different topics within the field of consumer search. The first essay proposes and solves a model for how consumers decide between discovering more products and searching among alternatives they are already aware of. The second essay builds on this model and quantifies the effects of different product rankings on consumer surplus and an online search intermediary's revenues. The third and final essay proposes and solves a model for how consumers allocate time to searching across different product categories.

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