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## **Liquidity Risk in Financial Markets**

Driessen, Joost

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## Liquidity Risk in Financial Markets

Rede uitgesproken bij de openbare aanvaarding het ambt van hoogleraar “Financiële Derivaten” aan de Faculteit Economie en Bedrijfswetenschappen van de Universiteit van Tilburg op vrijdag 29 oktober 2010.

Joost Driessen is Professor of Financial Derivatives at Tilburg University. He has published his academic research in journals such as the *Journal of Finance*, *Review of Financial Studies*, *Journal of Econometrics*, and the *Journal of Financial and Quantitative Analysis*. He has presented his research at many universities, financial institutions, and international conferences. His research has been sponsored by a VENI-grant of the Netherlands Organization of Scientific Research (NWO), Inquire, GARP, BSI-Gamma Foundation and Netspar. In November 2009 he received a VIDI grant from NWO. Driessen obtained his Ph.D. at Tilburg University in 2001. He previously held a position at the University of Amsterdam (2001 to 2009), had parttime positions at several financial institutions and is currently research fellow at CEPR and Netspar. His current research interests include liquidity risk, the empirical analysis of derivatives, corporate bonds and credit risk, optimal portfolio choice, and private equity. At Tilburg University, he teaches courses in Derivatives and Asset Pricing.

Mijnheer de rector, dames en heren,

When I was a small boy, a weekly highlight would be the delivery of the Donald Duck magazine on Saturday morning. Of the many characters that played a role in the stories, Scrooge McDuck, or Dagobert Duck in Dutch, was one of the most fascinating characters. Easily the richest person in the world, he kept all his money in a huge money bin, the “geldpakhuis”, and his favorite activity was to swim in his big mountains of money.

This swimming in money of Scrooge McDuck brings me to the key concept of today’s lecture, liquidity. In economics and finance, liquidity is of course not literally swimming in money. Liquidity is broadly defined as the extent to which one can buy or sell financial assets with low costs and without much delay. The costs here consist of direct transaction costs, implicit transaction costs due to the existence of a bid-ask spread, costs due to moving the price by trading, and search costs. To give a practical example, if one changes Euros into another currency at the airport, one will see bid and ask quotes associated with buying or selling the currency. The larger this bid-ask spread, the higher the costs of changing cash into another currency. The timing of transactions is also an aspect of liquidity: the longer it takes to before you can trade, the less liquid a financial asset is.

The most liquid asset is simply cash, and with that I mean cash in your wallet, or, in case of Scrooge McDuck, cash in his big money bin. Typically, stocks or bonds are slightly less liquid than cash. There are costs when buying or selling these assets, and it may take some time before one can sell these assets. For example, for US stocks bid-ask spreads have historically varied from about 0.3% as a fraction of the price for liquid stocks to more than 3% for illiquid stocks (Acharya and Pedersen (2005)). Such a high bid-ask spread substantially lowers the net return on your investment. Assets that are even less liquid are for example investments in real estate, but also investments in hedge funds or private equity funds, since it may take several months or even years before one can liquidate such investments. The concept of liquidity I just talked about is what is often called market liquidity, as it refers to liquidity of assets traded in financial markets. Another aspect of liquidity is

the extent to which financial institutions can raise funding if needed. For example, is a bank able to attract more deposits or borrow money elsewhere when needed? In today's lecture I will focus on market liquidity.

Of course, poor liquidity, or, illiquidity, is not something that investors like. Even worse, the liquidity of financial assets varies over time, and this leads to so-called liquidity risk. An asset that is currently very liquid may become illiquid in the future. What is very important is that assets typically become less liquid during crisis periods. I will now give a few examples to illustrate liquidity patterns in crisis periods. Ferderer (2006) has analyzed the liquidity of US Treasury bonds during the Great Depression period. Illiquidity is measured in two ways in his analysis. First, he uses the quoted bid-ask spread, which captures the differences between the quoted bid price and quoted ask price. His second measure is the effective spread, which measures the extent to which actual transaction prices deviate from the average of the bid and ask prices. His calculations show huge variations in liquidity during the Great Depression, with high bid-ask spreads and several illiquidity spikes around (i) the stock market crash, (ii) changes in the gold standard, and (iii) the introduction of a three-day bank holiday to prevent bank runs.

The second example concerns the stock market crash of 1987, the so-called Black Monday. Naidu and Roseff (1992) analyze the liquidity of the Hong Kong stock market around the crash. In this case, the liquidity measure is given by trading volume divided by the absolute stock return, the inverse of Amihud's (2002) ILLIQ measure. High values of this measure mean that high trading volumes go along with small price changes, and thus correspond to high liquidity. Their results indicate that the liquidity of the Hong Kong stock market was high before the crash, but dropped dramatically after the crash. Interestingly, the market remained illiquid for at least two months after the crash.

The third example is the crisis around the default of Russian government debt and the near-default of the hedge fund Long-Term Capital Management in 1998. A report by the Bank for International

Settlements (1999) shows the quoted bid-ask spread for UK corporate bonds in this period. Again, we see a strong decrease in liquidity in this crisis period, and again this illiquidity persists over time.

Finally, Bongaerts, de Jong and Driessen report quoted bid-ask spreads on credit default swaps written on US corporations. These credit default swaps insure against the default of a firm. They show that bid-ask spreads increase dramatically during the credit crisis, from levels of about 25 basis points to almost 100 basis points. They also report that the price of these credit default swaps, the so-called CDS spread, increases as the crisis unfolds. I will return to this analysis later.

In sum, we see that liquidity decreases in crisis periods. On purpose, each example focuses on a different market and a different country, to illustrate that these liquidity breakdowns typically happen for many markets and countries at the same time.

The fact that assets typically become less liquid in crisis periods is unfortunate, since this is exactly the time when institutions and companies would like to trade. For example, in crisis periods we often observe the so-called flight-to-quality effect: many investors move their portfolio towards assets with low risk such as government bonds, and sell assets with higher risk. Below I will explore the implications of liquidity risk in more detail, but I will first give an example to illustrate the importance of illiquidity in financial markets. In January 2009, the Dutch government bought credit derivative securities from ING in order to restore investor confidence about the financial health of ING. These credit derivatives were so-called mortgage-backed collateralized debt obligations, or, in short, mortgage CDOs. For a total amount of about 28 billion US dollars and using a swap structure, they effectively paid a price equal to 90% of the notional amount for these mortgage CDOs, while these mortgage CDOs traded in the market at about 65% at that time. The government argued that the price of 90% reflected the 'economic value' of the mortgage CDOs, and that the market prices were affected by the lack of trading ('liquidity') in the market for these derivatives. Hence, the

government argued that market prices reflected a large liquidity discount of 25%.<sup>1</sup> This liquidity discount thus equaled about 7 billion USD, roughly 1% of the Dutch GDP.

The above example illustrates how the illiquidity of financial assets can have a dramatic impact on market prices and trading behavior. It is fair to say that, historically, investors and financial institutions have not paid sufficient attention to liquidity risk. Hence, to prevent or minimize the impact of such liquidity crashes in the future, it is necessary that financial institutions incorporate the presence of liquidity risk when pricing financial assets, when choosing portfolios, when regulating banks and doing risk management. This leads me to the main questions that I will focus on in the remainder of this lecture. First, I discuss how the presence of liquidity risk and derivatives affects the optimal portfolio choice of investors, such as pension funds, banks, and hedge funds. Here we see a trade-off between investing in liquid assets and illiquid assets, since illiquid assets are often cheaper than liquid assets and thus may have higher returns. Clearly Scrooge McDuck has an extreme preference for liquid assets, but that may not be optimal from a portfolio perspective. Second, I discuss the pricing of liquidity in financial markets. Here I will distinguish the markets for stocks and bonds, and the market for financial derivatives. I will argue that liquidity effects in financial derivative markets are much more complicated and, therefore, more challenging. By now, the trading volume in financial derivatives exceeds the volume for exchange-traded stocks and bonds, but, surprisingly, we know very little about liquidity in derivative markets. In the coming years I want to fill this gap in the academic literature.

The questions I just mentioned are obviously not only academically relevant, but also of great practical importance. Indeed, the Basel Committee for Banking Supervision has recently updated its guidelines for dealing with liquidity risk, and many banks are currently incorporating liquidity risk in

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<sup>1</sup> See page 2 of 'Beantwoording schriftelijke vragen ING back-up faciliteit', Wouter Bos, 2 februari 2009, [http://www.minfin.nl/Actueel/Kamerstukken/2009/02/Beantwoording\\_schriftelijke\\_vragen\\_ING\\_back\\_up\\_faciliteit](http://www.minfin.nl/Actueel/Kamerstukken/2009/02/Beantwoording_schriftelijke_vragen_ING_back_up_faciliteit).

more detail in their risk management analysis. Similarly, it is currently discussed how the FtK regulation for Dutch pension funds can be adjusted to incorporate illiquidity of financial assets.

## 2 Liquidity risk and portfolio choice

I will now discuss the three questions I mentioned above in more detail. This is based on ongoing research, sponsored by a VIDI grant that I obtained last year from NWO, the Dutch science foundation. NWO funds research on this topic for the coming five years. I have to admit that, when writing down this lecture, I came across a better use of the 800.000 euro granted to me. Inspired by Scrooge McDuck, I have calculated the volume that one gets when converting the 800.000 euro into 80 million coins of 1 eurocent each. A eurocent has a diameter of 16.25 millimeter and height of 1.67 millimeter. If I would make straight towers of these coins, I can fill almost 36 cubic meters with my eurocents. Given the size of my office, which is about 24 square meters, I could transform my office into a eurocent swimming pool with a depth of about one and a half meter. This would still be modest compared to Scrooge McDuck, but nevertheless it is an appealing thought.

After considering this application of the VIDI research money for some time, in the end I decided to stick to the old plan and use the grant to fund PhD students, buy data and fund conference travel.

Let me now turn to the first research direction of this VIDI project.

One of the key topics in finance is the portfolio choice of investors. Until now, both in academia and practice, most of the existing academic work assumes all assets are perfectly liquid or have constant transaction costs, so no liquidity risk. I will now argue that incorporating liquidity risk can have a dramatic impact on the optimal portfolio of many investors. In the presence of liquidity risk, investors have to trade off the expected return on the asset with its liquidity risk, and also take into account any correlations or hedging demands associated with this liquidity risk. This leads to the following questions on liquidity and portfolio choice:

- *How does the presence of liquidity risk affect the optimal portfolio of investors?*

- *Should different types of investors (banks, hedge funds, pension funds) deal differently with liquidity risk?*
- *How do regulatory constraints affect the portfolio choice of investors in the presence of liquidity risk?*
- *How can derivatives be used to hedge liquidity risk?*

The existing literature is to a large extent silent on these issues. This is because the standard framework used both in theory and practice is a one-period model, where investors trade off expected returns and risk, often measured by the variance of returns. These models do not tell us however how much the investor trades at the end of this single period. Obviously, to assess the impact of illiquidity risk, one needs to know when and how much an investor trades. Illiquidity only matters for investors if they trade frequently. Recent academic work has studied multi-period or life-cycle portfolio choice models, but these models typically generate very little trading over short horizons. Hence, if investors would behave according to these theoretical models, liquidity would not matter much. However, in reality investors trade much more than predicted by current theoretical models. This is actually one of the major puzzles in finance: we do not understand why trading volume in financial markets is so high. Hence, to understand the impact of liquidity on portfolio choice, we also need to improve our understanding of trading behavior of investors.

## 2.1 A portfolio choice model with Value-at-Risk constraints and liquidity risk

I will now discuss one first attempt to generate more trading and hence more liquidity effects in a portfolio choice framework, which is inspired by discussions with Theo Nijman and Bart Verschoor. The idea is to take regulatory Value-at-Risk constraints into account. Such constraints restrict the amount of risk investors can take. These constraints are a fact of life for banks, which have to hold sufficient regulatory capital based on the Value-at-Risk of their risky portfolio. Similarly, pension funds are subject to regulatory constraints based on the Solvency-at-Risk, which essentially captures



the probability that the funding ratio is below a certain threshold. Even hedge funds internally use Value-at-Risk constraints to manage their risk.

The presence of an external Value-at-Risk constraint can force the investor to trade even if he would not want to otherwise. This may happen for example when an asset suddenly becomes much more risky, or when large losses on an asset are incurred. A good example is the Greek government debt crisis in the spring of 2010. As the economic problems faced by Greece became evident, government bond prices went down and the default risk increased, as indicated by a lower credit rating. This led many banks to sell part of their holdings of Greek debt, to avoid higher capital requirements for holding these bonds in their portfolio.

I will now present a simple model to illustrate how a combination of Value-at-Risk constraints and liquidity risk affects optimal portfolio choice. The model has two periods and two assets: one perfectly liquid risk-free asset and a risky asset, let's say a stock, subject to liquidity risk. This asset is currently liquid but may turn illiquid after 1 period, in the sense that trading involves positive transaction costs. The expected return on the risky asset is higher than the risk-free return. The risky asset has stochastic volatility, and I incorporate that typically both illiquidity and volatility increase when the asset price goes down, which is what we see in real life. As discussed in detail in the appendix, the model is set up as a trinomial tree. After one period, there are three states of the world, with various levels for the asset price, volatility and transaction costs. I consider two cases for the transaction costs. First, a case where costs are the same across all three states of the world, which I denote the "no liquidity risk" case. Second, a case where transaction costs are highest when asset prices decrease and volatility increases. This latter case thus has liquidity risk and is more in line with empirical evidence.

The investor is risk-averse and maximizes his risk-return tradeoff, formally through maximizing her expected CRRA utility over wealth at the end of the second period, under a Value-at-Risk constraint at period 1. The investor can choose the amount to invest in the risky asset today and after one

period. The Value-at-Risk constraint implies that the investors' current wealth should at least equal four times the 99% Value-at-Risk of the risky asset position. This Value-at-Risk constraint thus becomes tighter as the volatility of the asset increases and when the investors' wealth decreases. The mathematical details of the model setup are available in the appendix.

Given this setup, the key question we can answer is how much the investor invests in the risky asset in each period. We first show what happens without the Value-at-Risk constraint, for different levels of transaction costs, from zero costs to expected costs equal to 5%. In this case, Figure 1 shows that illiquidity has a small impact on the optimal portfolio choice. The figure shows the optimal number of stocks that the investor should buy at time 0, and the number of stocks the investor should hold at time 1, for each of the three possible states of the world, an upward state with a positive stock return, a middle state and downward state with a negative stock return. Without transaction costs, all the investor wants to do is rebalance his portfolio after one period to maintain constant portfolio weights, as it is well known that in this setting CRRA utility leads to constant portfolio weights over time. With positive transaction costs, the investor gives up this desire for rebalancing and does not trade at all after one period. Figure 1 shows that this leads to small effects of transaction costs on optimal portfolio choice.

This changes however substantially when the Value-at-Risk constraint is introduced. Figure 2 shows the optimal amount invested in the risky asset for different levels of transaction costs. In contrast to the previous figure, this figure focuses on the risky investment at time 0 only. We first see that the Value-at-Risk constraint by itself does not lead to a large change in the risky investment. However, when we combine the Value-at-Risk constraint with positive transaction costs, we do find a large decrease in the risky investment. In this case, the investor realizes that in some cases, he will need to substantially change her portfolio after one period in order to satisfy the Value-at-Risk constraint. Specifically, he needs sell risky assets to lower the risk of her portfolio in the downward state of world, where prices went down and volatility went up. Given the transaction costs, such trading is

costly. Hence, to avoid excessive trading costs, the current investment in the risky asset is much lower. We also see that this effect is much larger in case of liquidity risk, due to the correlation between asset prices, volatility and liquidity. If transaction costs are very large, the investor will lower current investment in the risky asset such as to completely avoid trading after one period in the states with positive transaction costs.

## 2.2 Hedging liquidity risk with options

The simple example just discussed illustrates that regulatory constraints can amplify the effects of liquidity risk on portfolio choice. Then, it could be interesting for the investor to hedge this liquidity risk. Currently, derivatives directly written on liquidity measures do not exist. However, options on many asset classes do exist, and given the relation between equity returns, volatility risk and liquidity risk, buying for example a put option on the underlying asset could partially hedge liquidity risk, since a put option pays off more when stock prices are down and volatility is up, which often coincides with low liquidity. Concretely, Figure 3 shows what happens if I allow the investor to invest in a two-period at-the-money put option, without rebalancing of the put at time 1. The figure shows the optimal number of stocks bought at time 0 and 1, and the optimal number of put options bought at time 0, in case of liquidity risk and a Value-at-Risk constraint. The Value-at-Risk calculation incorporates the risk associated with the position in the put option.

Figure 3 shows that, if liquidity risk is small, the investor hardly uses the put option. But, with larger transaction costs, the investor starts buying put options. This can be understood as follows. Without access to options, the investor knows he cannot hold many stocks in the downward state at time 1, because the Value-at-Risk constraint becomes binding. Then, to avoid excessive trading, the investor invests less in the stock at time 0. Now, with access to put options, the investor can increase his exposure to stocks, still satisfy the Value-at-Risk constraint in the downward state, and at the same time avoid trading in the stock at time 1. Indeed, we see that the investment in the stock at time 0 is larger compared to the case without put options. Also note that a Value-at-Risk constraint by itself is

not sufficient to generate demand for put options, only when we allow for liquidity risk we generate a positive demand for put options. The reason is that, without liquidity costs, if anything the investor actually wants to buy additional risky assets to maintain his risk exposure in the downward state of the world. However, he cannot do this because of the Value-at-Risk constraint. Hence, the investor has no preference for buying put options only because of the Value-at-Risk constraint.

These results may help to shed some light on an important puzzle in option markets. Several articles, including some of my own work with Pascal Maenhout, have shown that prices of put options on stock indices seem excessively high. As a result, it has been a challenge to understand why investors would buy options at such high prices. My simple model may help to explain this puzzle. It shows that investors like to buy put options to avoid excessive trading in bad times when markets are illiquid.

The experts in the audience may have noticed a subtle issue. In this trinomial tree with transaction costs, no unique no-arbitrage price exists for the put option. In the analysis above, I picked a specific risk-neutral measure to price the option. Obviously, it would be very interesting to know how illiquidity affects the prices of equity options. I will return to this issue later today.

### 2.3 Portfolio choice: Concluding remarks

In the analysis above I focused on calculating the optimal portfolio for an investor that faces a Value-at-Risk constraint and liquidity risk. A related analysis would be to take the perspective of a regulator who typically monitor the Value-at-Risk of financial institutions. The regulator can assess how the presence of liquidity risk affects the actual risk profile and Value-at-Risk of an investor. Especially when an investor does not incorporate liquidity risk in his portfolio choice the effect of liquidity risk on the Value-at-Risk may be substantial, since this investor does not anticipate the future costs of trading.

The example I just discussed is just one way to improve upon existing work on liquidity and portfolio choice. There are many other reasons why investors trade and rebalance and, as a result, care about liquidity. For example, in delegated portfolio management, such as mutual funds, the specific incentive structures of fund managers or traders may generate nontrivial trading patterns, which would also lead to an important role of liquidity risk. Another important direction in the portfolio choice literature is to improve the modeling of liquidity dynamics across different asset categories, including highly illiquid assets such as private equity.

### 3 Pricing of liquidity in stock and bond markets

For those of you who got lost in the forest of trinomial trees and Value-at-Risk constraints, I will now leave this forest and turn the next topic, the pricing of liquidity in financial markets. I have just argued that investors should care about liquidity and liquidity risk. This logically implies that prices of financial assets should be affected by liquidity. Before I go into the modeling details, I give two simple examples to illustrate the potential effects of liquidity. The first example comes from an article by Longstaff (2004) who studies the market for US Treasury bonds, and compares the prices of these bonds with prices of bonds issued by the Resolution Funding Corporation (Refcorp), a government agency. These Refcorp bonds are guaranteed by the US government and have the same default risk as Treasury bonds. Refcorp bonds are however less liquid than US Treasury bonds.

Focusing on 30-year maturity bonds, Longstaff finds that this difference in liquidity leads to Refcorp bond prices that are about 5% lower than US Treasury bond prices, a considerable liquidity discount.

The second example comes from the stock market. Inspired by the work of Amihud and Mendelson (1986), several articles have studied the relation between the average return on stocks and the liquidity level of these stocks. Figure 4 shows the typical outcome of such an analysis. This figure graphs the average return on 25 US stock portfolios from 1962 to 2008 versus the illiquidity of the stocks, where illiquidity is measured using the price-impact measure of Amihud (2002). The portfolios are constructed by sorting on this illiquidity measure, thus constructing portfolios of

stocks with low liquidity, intermediate liquidity and high liquidity. The figure shows a positive relation between the average return and the illiquidity measure. In other words, more illiquid stock portfolios have higher average returns, or, in other words, relatively lower prices.

I now proceed with a more formal discussion of liquidity effects on stock and bond prices, and thereafter focus on derivative assets. Intuitively, one may think that illiquid assets should always trade at lower prices and give higher returns, since investors dislike illiquidity, but below I will show that this intuition does not always apply.

### 3.1 Liquidity in stock and bond markets

The standard model for pricing of liquidity in financial markets is the Liquidity CAPM of Acharya and Pedersen (2005). They modify the standard capital asset pricing model by incorporating stochastic transaction costs for all assets. These transaction costs are correlated across assets, which leads to systematic liquidity risk, defined as market-wide shocks to transaction costs. Systematic liquidity risk means that a given asset typically becomes illiquid exactly when other assets also become illiquid. In the Acharya-Pedersen model the prices of assets are affected by two types of liquidity: the current liquidity level and systematic liquidity risk. Higher current transaction costs obviously lead to a lower price for the asset and a higher expected return. Also, higher exposure to systematic liquidity risk leads to a lower price and higher return. For example, if the asset becomes illiquid at times when the entire market becomes less liquid, this asset will have a low price and high expected return.

Several articles have examined the empirical performance of this model, and generally find evidence that liquidity levels and/or liquidity risk affect expected returns. The estimates vary substantially across articles however. For example, Acharya and Pedersen (2005) compare illiquid and liquid stocks and find that the current liquidity level leads to expected return differences of about 3% per year, while liquidity risk has a smaller return effect of about 1%. Pastor and Stambaugh (2005) only incorporate liquidity risk and find return differences of 6% to 7% per year. This may be partially

explained by the use of different liquidity measures, but could also be due to several stringent assumptions in the Liquidity CAPM. I will now discuss some of my own work in which we relax two assumptions of the Liquidity CAPM.

### 3.2 Holding periods

One key assumption in the LCAPM is that all investors have the same holding period for their investments, and that this holding period is constant over time. This is in clear contrast with what we see in the real world. Some investors trade very frequently, others not. Moreover, even at the aggregate level holding periods vary over time. Figure 5 shows the aggregate monthly turnover rate for the US stock market, defined as the daily trading volume divided by the total market capitalization, for the crisis years 2007 and 2008. We see that there are periods in which investors trade frequently, and periods with less trading. In addition, there are also cross-sectional differences: some stocks have much higher turnover than others. This has important implications for liquidity. Indeed, I will argue below that liquidity should matter more if investors trade frequently.

Together with Alessandro Beber and PhD student Patrick Tuij, I have developed an extension of the LCAPM that incorporates that holding periods vary across investors, across assets and over time. In this model each investor trades a particular asset with some probability, and these probabilities vary over time, across assets and across investors. I do not have time to go through the equations, but will focus on the two main innovative terms. First, we find that the impact of the liquidity level and liquidity risk is not the same across all assets as in the LCAPM, but depends on the trading frequency of the investors holding the asset. Effectively, the impact of liquidity and liquidity risk is larger when investors have shorter holding periods. Second, the covariance between the trading frequency and liquidity matters. If investors trade more in an asset exactly when that asset becomes illiquid, the price of the asset will be lower. In sum, the model captures that liquidity only matters when investors trade.

I will now present some first preliminary results of taking this model to the data. I focus on the US stock market, specifically on the 25 stock portfolios sorted on liquidity that we discussed earlier. I then basically analyze whether liquidity and liquidity risk affect the average returns on these portfolios. Here I incorporate that investors in these portfolios have different holding periods, and compare the results with the model of Acharya-Pedersen, which has constant holding periods. Our preliminary results show that both expected liquidity and liquidity risk affect the returns on the stock portfolios. The most important result is that, by incorporating the variation in holding periods, we can achieve a better separation of current liquidity levels and liquidity risk. Technically speaking, the Acharya-Pedersen model suffers from severe multicollinearity, while this is much less so in our model. Our current estimates imply that liquidity levels lead to an effect of about 5% per year on expected returns, while liquidity risk leads to a much smaller effect between 0.1% and 1%.

One key assumption in the model I just discussed is that investors trade for exogenous reasons, or, more precisely, that liquidity is not affecting the trading frequency of investors. Intuitively, one may argue that investors will trade less because of an increase in transaction costs. We are currently extending the analysis to deal with this endogeneity issue. However, it is not fully clear how important this endogeneity issue is. Empirically, it seems to be the case that investors actually trade more when markets are less liquid, as shown above for the turnover of the US stock market during the recent crisis. It may well be the case that investors are essentially forced to trade in bad times, irrespective of the liquidity levels. This would be in line with the portfolio choice model presented above, where investors trade more in illiquid times because of regulatory Value-at-Risk constraints.

### 3.3 Short-selling and liquidity in derivative markets

A second stringent assumption in the LCAPM is that investors are homogenous in essentially all dimensions. Together with the assumption of positive net supply of the assets, this implies that all investors hold a long position in the same portfolio, the market portfolio. Together they hold the total positive supply of stocks and bonds. In this case, illiquidity always leads to a lower price.



In practice we see that many investors take short positions in stocks and bonds. In addition, for derivative markets the net supply of derivatives is, by construction, equal to zero. In other words, the total position of buyers in the derivative market equals the total position of sellers, so that short-selling is as important as buying in derivative markets. In recent work with Dion Bongaerts and Frank de Jong, we show that, if some investors are optimally short-selling an asset, the effect of illiquidity on the price of the asset is not necessarily negative, but can have any sign or even be zero.

To see this in a simple example, imagine that I buy an option on a stock from you. Together we have a zero position in this option. Suppose the option has liquidity risk, so that it may be hard to trade this option again in the future. Both you and I are affected by this liquidity risk, since it may be costly for both of us to trade this option again in the future. To get compensated for this risk, I would like to pay a lower price for the option, and you would like to sell at a higher price. What happens in equilibrium? In the model that we have developed we show that the most aggressive investor will get compensated for illiquidity. Here we define aggressiveness as having more wealth, lower risk aversion, or a shorter holding period. The intuition for this result is that more aggressive investors are most strongly affected by liquidity, and thus need to be compensated in order to persuade them to trade.

This has important implications for the way we think about liquidity effects in financial markets, especially given that many of the most-traded assets are derivatives, such as interest-rate swaps, currency swaps, options and credit default swaps. In these markets, it is possible that illiquidity leads to higher prices instead of lower prices. Furthermore, even for stocks and bonds it is possible that illiquidity leads to higher prices, if the short-sellers in this stock are sufficiently more aggressive than the buyers.

I will now present two empirical applications of our theory. First, I discuss how illiquidity matters in the market for credit default swaps. Thereafter, I present some preliminary results on illiquidity effects in the market for equity options.

### 3.4 Credit default swap market

Earlier today I already discussed my own analysis of the bid-ask spread movements for the market for credit default swaps. This analysis uses data on credit default swaps for about 600 US corporations. I find that bid-ask spreads are substantial in this market, and that they increase dramatically in the crisis period. There is also a positive relation between the bid-ask spread and the price of the CDS contracts, which is the insurance premium, also called CDS spread. Figure 6 presents cross-sectional results on liquidity and CDS spreads. It shows the relation between the expected returns on CDS contracts and bid-ask spreads, across CDSs with different ratings. The expected CDS return is essentially equal to the CDS spread minus a correction for the expected payment in case of default. This graph also suggests a positive relation between the expected CDS return and the bid-ask spread. For each rating category, we see that CDS contracts with higher bid-ask spreads have higher expected CDS returns. From these graphs it is tempting to conclude that illiquidity in fact leads to higher CDS prices. However, to draw such conclusions one needs to control for other determinants of CDS spreads, such as credit risk and exposure to market risk. When we do this we still see that illiquidity leads to higher expected CDS returns, and thus to higher CDS spreads. This implies that part of the insurance premium, paid to insure against default of a firm, is not due to default risk but due to the illiquidity of the CDS contract. The average CDS insurance premium is about 1% of the insured amount, and our estimates imply that about a quarter of this premium is due to illiquidity. This has important practical implications, since in many cases investors use the CDS spread as a measure for default risk. This happened for example during the Greek debt crisis in the spring of 2010. Our results thus imply that one cannot interpret the CDS spread as a pure measure for default risk, as is often done practice.

### 3.5 Market for equity options

My second example on liquidity and derivative markets concerns the market for equity options.

Using a large dataset of options on more than 500 US stocks for 9 years of daily data, together with

Frank de Jong and PhD student Sohail Ahmed I am currently studying how liquidity affects the prices of these options. Compared to the CDS market just discussed, an additional interesting feature of the equity option market is that we have two degrees of liquidity that matter for the option price. First, the liquidity of the underlying stock and, second, the liquidity of the option itself. The liquidity of the underlying stock may matter because standard option pricing theory argues that options can, at least partially, be hedged by trading dynamically in the underlying stock. If a stock is illiquid, this dynamic hedging becomes more costly and this may lead to an effect on the option price. As discussed before, the sign of these effects will depend on the liquidity and hedging preferences of the buyers and sellers of the option, and can be positive, zero, or negative.

Figure 7 presents some first results on how the liquidity of the stock and option affect the price of the option. For liquidity we use the quoted bid-ask spread as measure, and the option price is measured in terms of Black-Scholes implied volatility. The figure reports regression coefficients of panel regressions of implied volatilities on the liquidity measures, controlling for many other determinants of option prices. The regressions are done per year, from 2001 to 2008. The figure shows that the option price is higher when the option is less liquid and when the underlying stock is less liquid. Hence, we again see that illiquidity leads to higher prices in derivative markets.

Interestingly, we also see that the impact of liquidity varies over time.

These regressions are only a first step in our analysis of equity option markets. Most importantly, it is important to construct actual returns on option portfolios, so that a formal asset pricing model can be empirically tested. Further, there is much that needs to be done at the theoretical side. Our current theory of liquidity and derivative prices does not incorporate the presence of a market maker. In option markets market makers try to hedge their option positions using the underlying asset, and this is more difficult in the presence of volatility risk, jump risk and illiquidity. A formal model incorporating these issues may generate interesting testable implications and deepen our understanding of liquidity effects in derivative markets.

#### 4 Conclusion

In this lecture, I have discussed various ways in which liquidity affects our financial decision making and prices in financial markets. I have shown that liquidity may have a substantial impact on the (i) portfolio choice of investors, (ii) on prices of stocks and bonds, and (iii) on the prices of financial derivatives. However, I have also indicated that much needs to be done to gain a complete understanding of these issues. This will keep me busy the coming years!

#### 5 Tot slot

Ik wil graag allereerst een dankwoord uitspreken naar de decaan van de FEB en het college van bestuur van de UvT voor het vertrouwen dat mij is gegeven met deze benoeming. Het voelt zeer goed om weer terug te zijn bij mijn alma mater, en ik zal er alles aan doen om het mij gegeven vertrouwen waar te maken.

Hooggeleerde de Jong, beste Frank,

Jij bent een rode draad geworden voor mijn academische carrière. Je hebt me ooit aangenomen als student-assistent in 1994, en sindsdien heb ik ononderbroken met je samengewerkt, als aio in Tilburg, aan de UvA, en nu weer hier in Tilburg. Je bent ook verschillende keren mijn baas geweest, en nu ook weer, en daar ben ik zeer blij om. Ik hoop dat we nog heel lang op allerlei manieren samen ons steentje aan de wetenschap en universiteit kunnen bijdragen.

Hooggeleerde Nijman, beste Theo,

Ik herinner me nog goed het eerste college financiële econometrie dat ik bij volgde. In plaats van meteen te beginnen met modelaannames en formules, wat de meeste andere docenten deden, schreef jij eerst maar eens paar praktijkgerichte vragen op het bord. Vanaf dat moment ben je een inspirator en voorbeeld voor mij geweest, als afstudeerbegeleider, promotor en nu in je rol bij Netspar.

Dear colleagues of the finance department at Tilburg,

It has been a great pleasure to come back in Tilburg and meet so many new and interesting people in the finance department. I highly appreciate the international atmosphere we have, and look forward to working and interacting with you in the future.

Dear co-authors, Dear Antoon, Bertrand, Pascal, Martijn, David, Luc, Pieter, Theo, Frank, Greg, Dion, Enrico, Otto, Ludo, Alessandro, Patrick,

Research is seldom something you do all alone. Working with you has been both inspiring and fun.

Thank you for this. I hope to collaborate with many of you in the future.

Ten slotte wil ik nog een aantal mensen buiten de academische wereld bedanken. Allereerst alle vrienden die mij af en toe goed weten af te leiden van de wetenschap. Ik wil hier Tjeerd in het bijzonder noemen, onder andere vanwege zijn helaas kansloze pogingen om mijn squashniveau omhoog te krijgen.

Dan is er de familie, in het bijzonder bedank ik mijn pap, mam, zus Patricia en broer Pim. Ik ben heel gelukkig met de goede band die wij hebben. Ik hoop dat deze nog lang en in goede gezondheid zal bestaan.

Lieve Jorna, Quinten en Mare,

Wij vormen een hecht team met z'n vieren, een vet cool team zou Quinten zeggen. Jullie brengen me elke dag liefde en knuffels, grappen en verhalen, en zoveel meer.

Ik heb gezegd.

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## Appendix

This appendix describes the portfolio choice analysis of section XX in detail. The model has two periods, and a non-recombining trinomial tree in each period, where each outcome has an actual probability of  $1/3$ . The trinomial tree approximates a normally distributed return on the asset with excess drift equal to  $\mu\sigma_t^2$  and conditional volatility  $\sigma_t$ . The asset price  $P_0$  at time 0 is normalized to 1. At time 1, the price can take three values, according to the following transition equation

$$P_t = P_{t-1} (1 + \mu(\sigma_{t-1})^2 + \sqrt{(3/2)}\sigma_{t-1}u), \quad u=1,0,-1$$

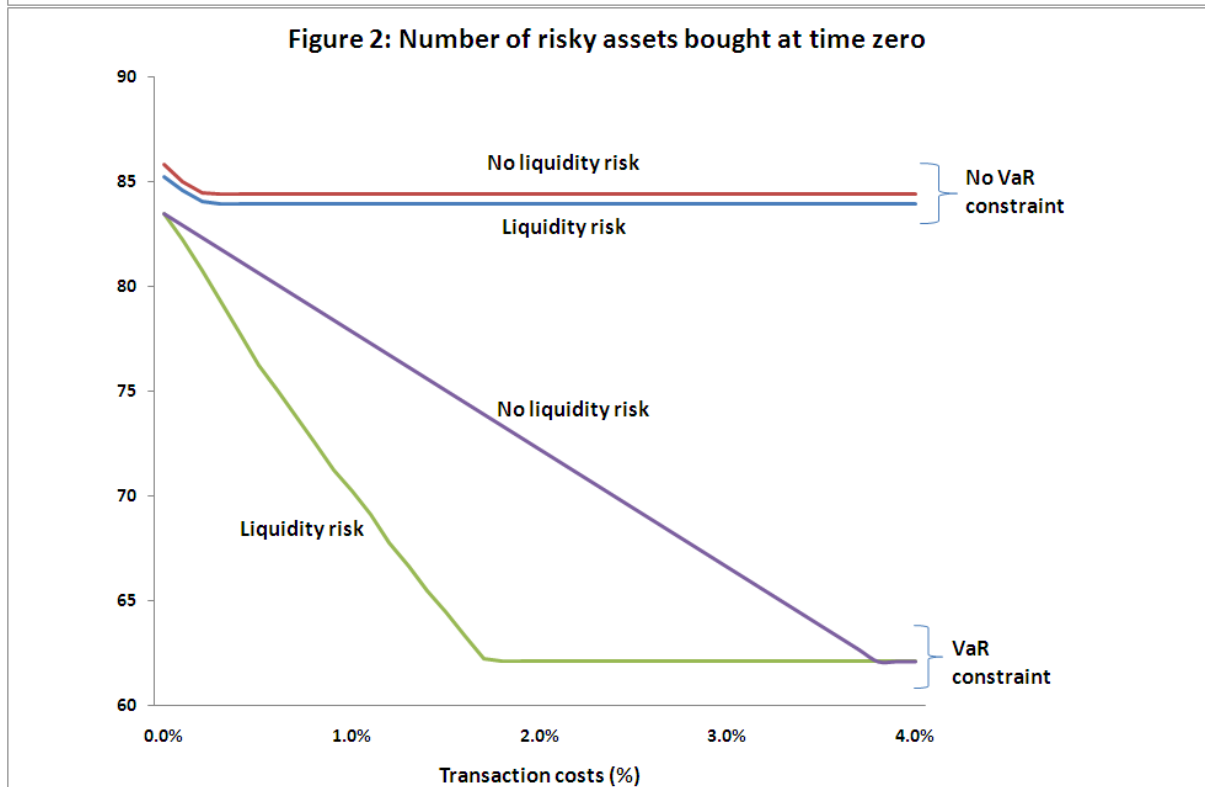
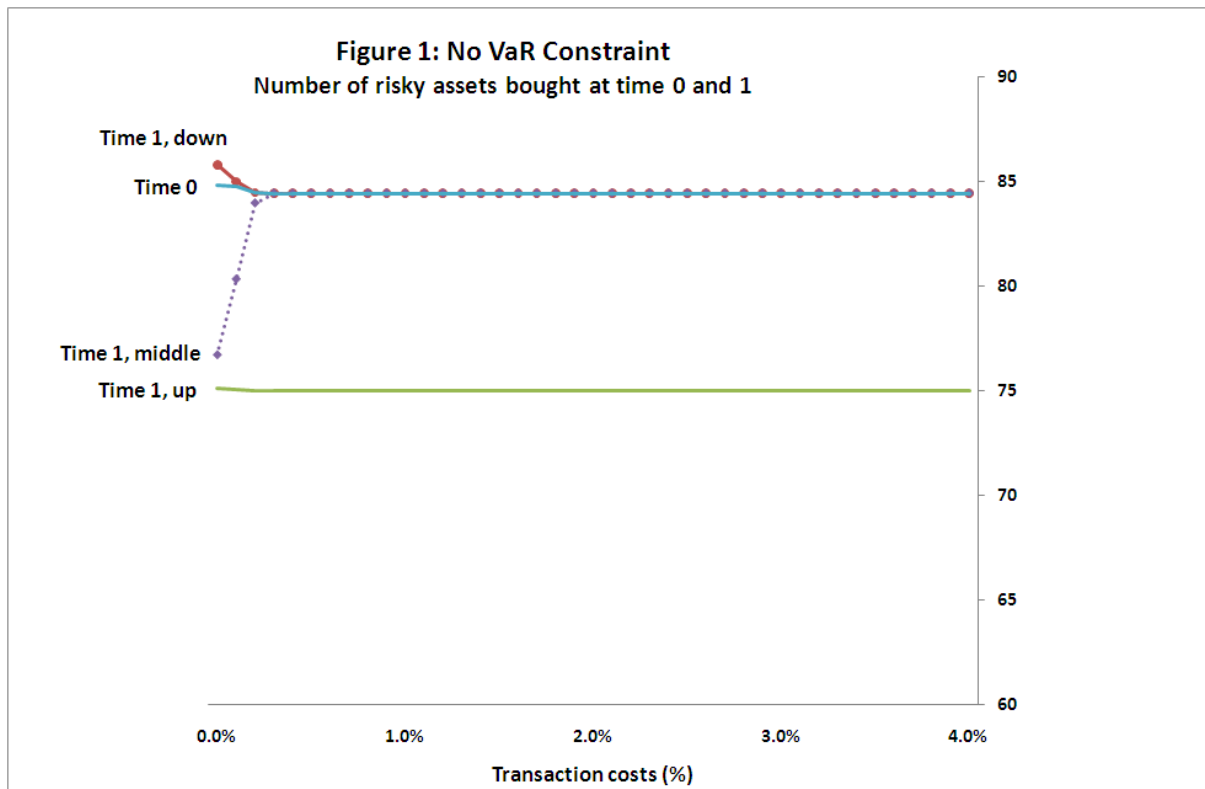
This equation also applies to price changes from time 1 to 2. Volatility is also stochastic. At time 0,  $\sigma_0$  is equal to 15%, and after 1 period it is 10% in the up and middle state, and 20% in the down state. Finally, transaction costs are proportional to the price and only occur after one period, so not at time 0 or 2. We consider two settings. One with constant transaction costs of  $L\%$  in all three states of the world, and one with transaction costs of  $(0\%, L\%, 2L\%)$  in the up, middle and down states. In the latter case liquidity is correlated with volatility and returns.

The investor maximizes CRRA utility of terminal wealth after 2 periods over the amount invested in the risky asset at time 0 and at time 1 (in each of the three states of world), under Value-at-Risk constraints. The VaR constraint is such that at each point in time (time 0 and 1), the investors' wealth should be at least 4 times the 99% VaR. The VaR is calculated using a normal distribution for the return over the next period (using the drift of  $\mu\sigma_t^2$  and conditional volatility  $\sigma_t$ ). As mentioned above, we see the trinomial tree as approximation of this normal distribution. These VaR constraints should be strictly satisfied in all states of the world. Note that any trading costs are subtracted from the wealth used for the VaR constraint, and these trading costs are endogenous as they depend on the chosen portfolio weights. This VaR constraint gets more binding as the volatility increases and when losses are incurred on the risky asset (leading to lower wealth).

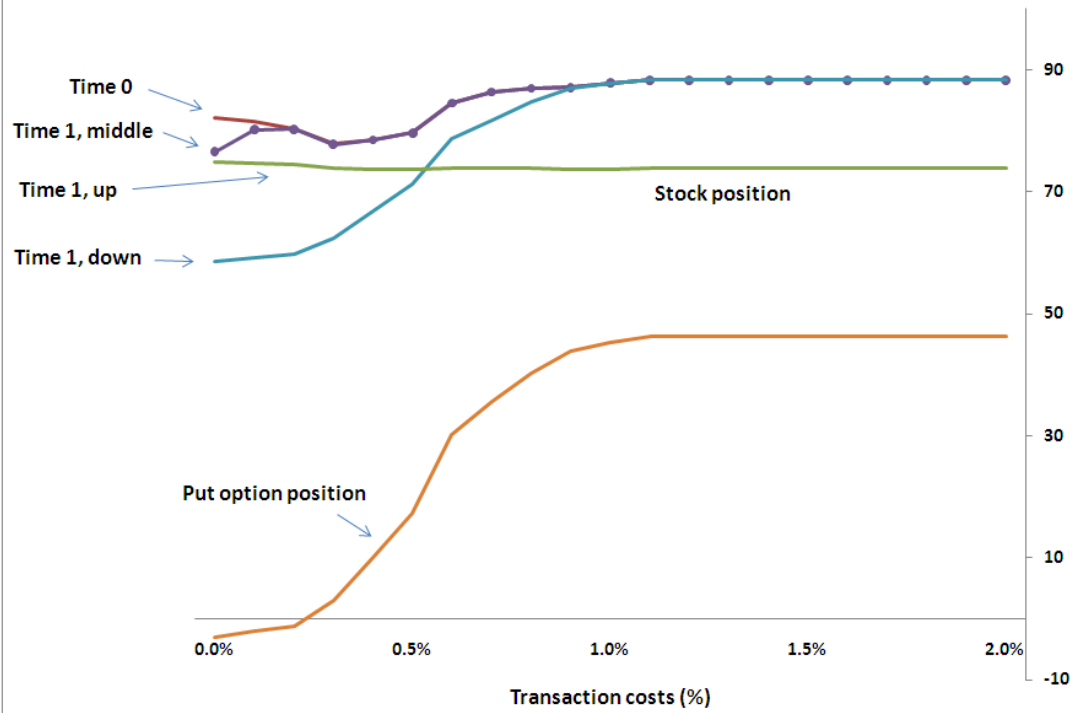
Some final numerical settings: we set  $\mu\sigma^2=0.05$ ,  $r_f=0.02$ ,  $W_0=100$ , and a CRRA coefficient equal to three.



Figures



**Figure 3: Position in put option to hedge liquidity risk**



**Figure 4: Average monthly return US stock portfolios: 1962-2008**

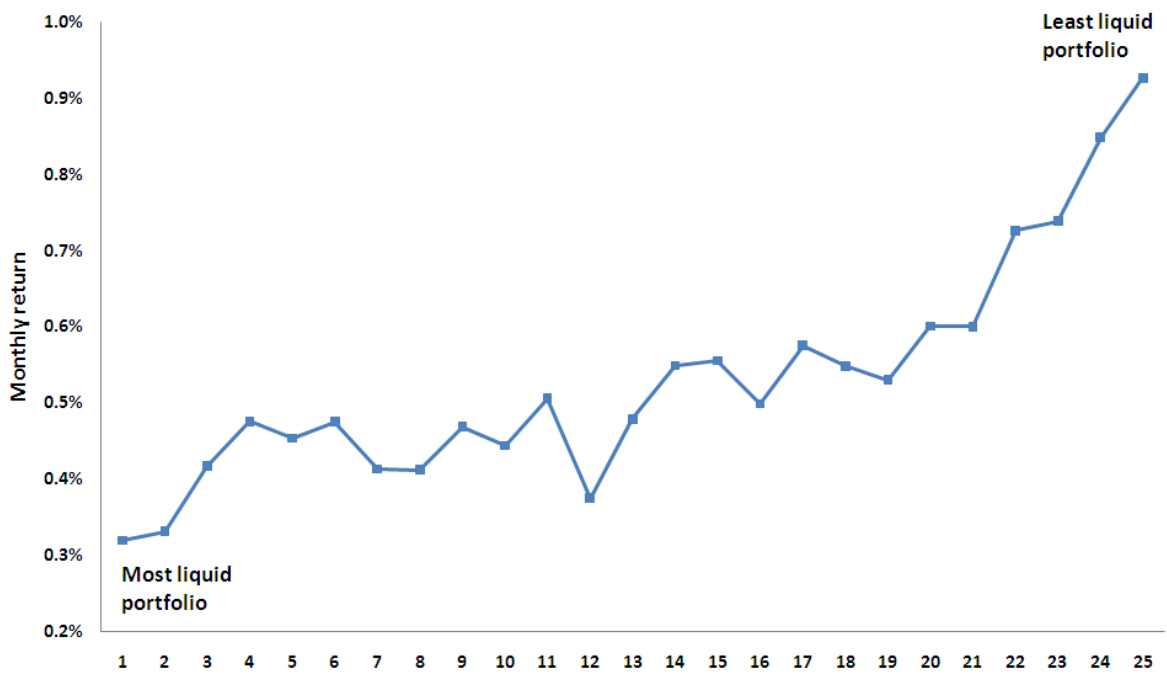


Figure 5: Monthly turnover US stock market: 2007-2008

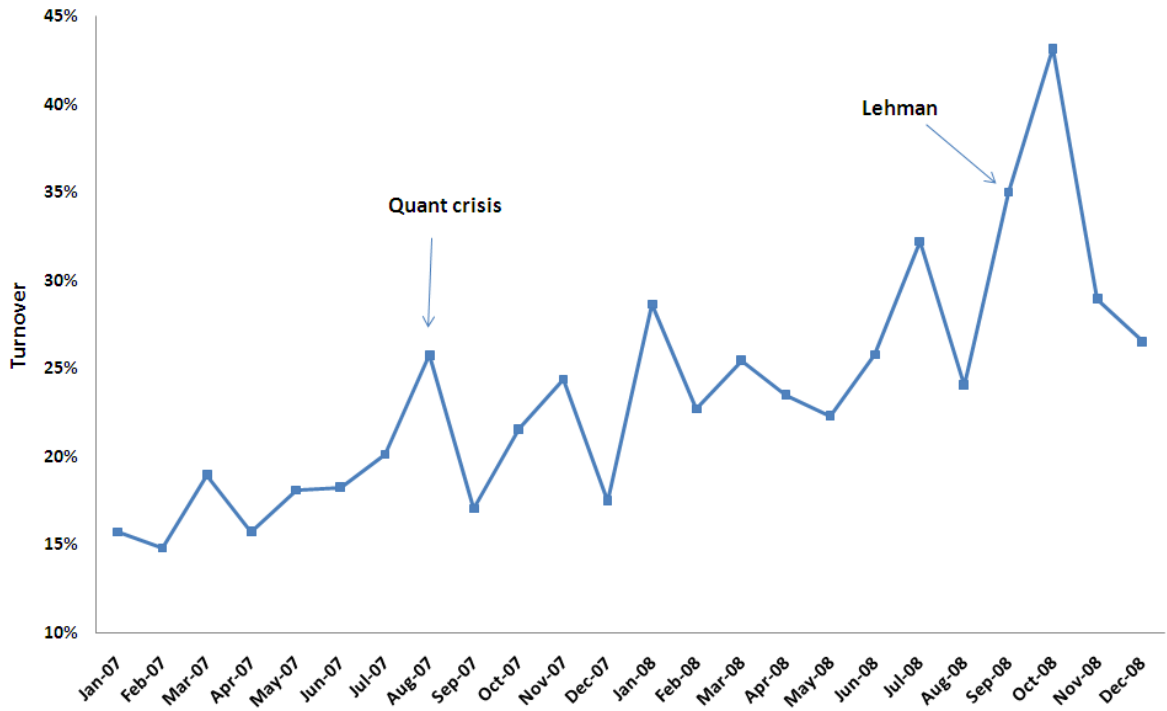


Figure 6: Expected CDS returns

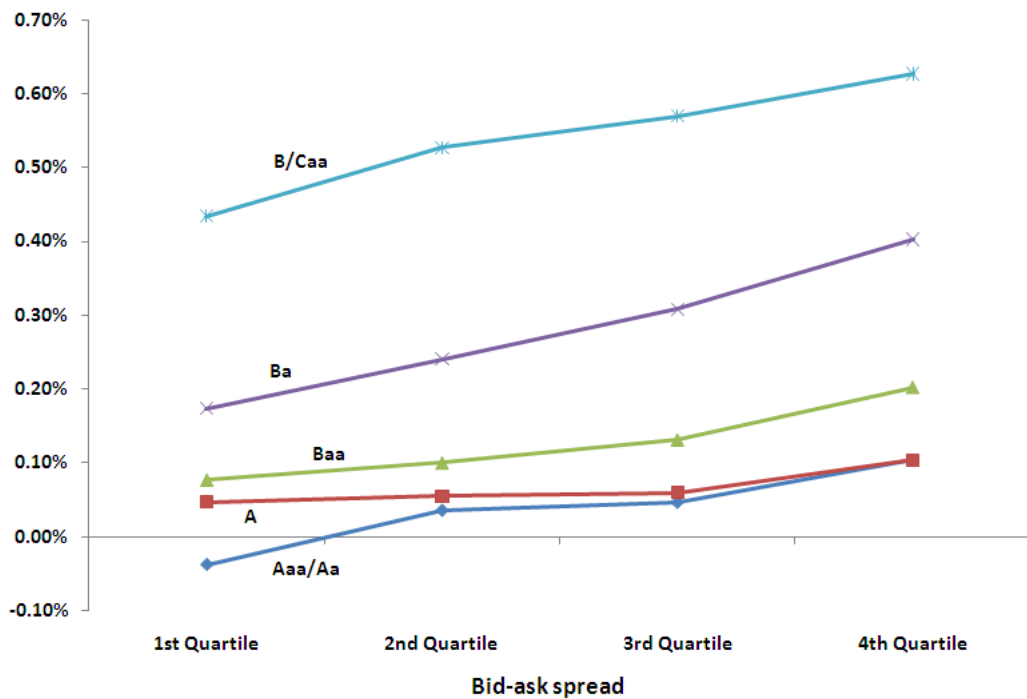


Figure 7: Effect (%) of option and stock illiquidity on option price

