

TEACHING INTEGRAL CALCULUS USING RECOGNITION AND HEURISTIC SEARCH

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Successfully solving integrals involves recognition of which techniques are useful and, when recognition is incomplete, heuristics on how to effectively explore and reduce the remaining problem space (Gobet, 2015). Based on work by Kop et al. (2015) we developed an expertise framework that distinguishes multiple levels of recognition and corresponding heuristics. This framework provides direction on what students need to learn and how this can be taught.

The research question was: how can a series of lessons based on the framework contribute to the development of recognition and heuristic search in calculating integrals for students in upper secondary education?

In the series of lessons, the techniques of integration were taught using a whole task approach with reflective questions. The framework was extended step-by-step, incorporating newly learned techniques and heuristics.

Data included written exams, thinking-aloud protocols when calculating integrals, and card-sorting tasks in which students, at the end of each lesson, had to categorize functions according to the different techniques of integration.

The results of the card-sorting tasks showed that students were able to make increasingly precise distinctions between the categories. These results were validated by the thinking aloud protocols, which showed that students used similar approaches. These thinking aloud protocols also showed that students expanded their repertoire of heuristics during the series of lessons. The written exams showed that most students could perform the techniques properly after the lesson series.

Step-by-step explicit teaching of recognition and heuristics seems to be a promising direction to support students' learning of integral calculus.

References

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