

# THE DIALECTICAL PROCESS OF PROBABILISTIC THINKING

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We present a qualitative analysis of the probabilistic thinking process of six high school senior students in a Statistics course. The students were selected from a sample of size (n=270) as their answers were clear, detailed and representative of different thinking processes regarding predictions and justifications in three learning activities related to probability and their answers to five questionnaires.

The objective of the research was to find out what justifications are considered to predict or explain a result, or to make decisions when trying to win a game. The justifications were classified as intuitive (IJ), formal (FJ), or combined (CJ).

In the first activity, students threw a pair of dice 25 times, predicting the sum in each throw. Students answered a questionnaire (Q1A1) about what they thought would happen before the activity and answered another questionnaire (Q2A1) after the activity focused on their initial approaches to generalization. In Q1A1 only one student was classified as FJ, everyone else was classified as IJ and their predictions were very varied. On the other hand, in Q2A1 the student who was classified as FJ changed to CJ and everyone else maintained the IJ classification, although some of them observed that their previous predictions of Q1A1 had no sense. In the second activity, a simulator was used for throwing two dice 25, 100, and 1000 times. All students deduced that 7 was the most probable number based on frequency. All students' justifications in a questionnaire (Q2A2) were classified as FJ. The third activity was a game: a turtle race. A set of turtles were numbered from two to twelve. Two dice were thrown determining the movement of the turtle that coincide with the sum of the points of the dice. In the questionnaire (Q1A3) students had to choose one winning turtle justifying their choice. Most of them selected a number different than 7 but close. Four students' answers were classified as CJ and the rest of them as FJ. After the game in the questionnaire Q2A3 four of them were classified as FJ and the rest as IJ.

The results show a dialectical nature in the probabilistic thinking process, intuitive justifications were present again and again and live together with the formal ones. We concur with Batanero (2020) on the necessity to consider student's informal ideas stressing that probability should be viewed as a mathematical model.

## References

Batanero C. (2020) Probability teaching and learning. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 491-496). Springer.