PROPOTIONAL REASONING OF LOWER GRADE STUDENTS THROUGH LEARNING TRAJECTORY AND LESSON STUDY

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Most research on proportional reasoning in mathematics education focuses on students in secondary schools or upper grade students in primary schools. Alternatively, other studies that examined preschool children or lower grade students in primary schools pointed out the problem of underdeveloped knowledge in the multiplicative structure of numbers (e.g., Resnick & Singer, 1993). This study approaches the question of developing mathematics lessons to foster the proportional reasoning of lower grade students by adopting a design research methodology (Cobb et al., 2017). We propose a learning trajectory of proportional reasoning for this group of students and use it as a tool for examining the processes of learning. The researchers, including primary school teachers, conduct lesson studies and investigate lessons for supporting the learning of students based on the learning trajectory. Theoretically, we develop, test, and modify conjectures about learning trajectory and the strategies for supporting learning. The learning trajectory is based on two types of ratios, namely, scalar ratios (SRs) and functional rates (FRs; Vergnaud, 1994). Initially, we set five stages for SRs and three stages for FRs. In this presentation, we describe two lesson studies conducted in 2021 in four Grade-3 classrooms (8–9 years old) in Japan. One of these studies intended to support the SRs' processes of unitizing and norming in the context of comparing and measuring a quantity. The researcher and teachers collaborated to design a series of lessons for two teaching units in a classroom and pursued learning among several students. The objective of the other lesson is to elicit the processes of SRs and FRs by designing tasks that require coordination between two quantities, which is not typical in lower grades in Japan. The researcher and teachers designed six lessons across one year. We describe the results in terms of the capabilities of the students and examine implications for the learning trajectory and strategies for realizing progress.

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