COMPARING THE USAGE OF PROOF-RELATED WORDS IN GERMAN AND JAPANESE MATHEMATICS TEXTBOOKS

Ryoto Hakamata[1], Fiene Bredow[2], Christine Knipping[2], Takeshi Miyakawa[3], and Yusuke Shinno[4]

[1]Kochi University, [2]University of Bremen, [3]Waseda University, [4]Hiroshima University

How can we better understand the linguistic specificities of proof and proving to be taught in different countries? Looking at proof in curricular documents (e.g., textbooks, national curricula) in a given country, one may notice that what is called a proof and what is taught as proof differs, within and across these documents. International comparisons are one way to investigate the diversity related to proof. In this presentation, we aim to analyse and compare the usage of proof-related words in German and Japanese mathematics textbooks. Although the educational systems of the two countries are different, 'proof' is introduced in middle schools in both countries.

We chose five series of textbooks for each country and explored some common chapters. As a result, *argumentieren* (argue), *begründen* (justify), *beweisen* (prove), *erklären* (explain) and *zeigen* (show) were identified as proof-related words in the German textbooks, as well as 説明する *setsumei-suru* (explain), 証明する *shōmei-suru* (prove) and 示す *shimesu* (show) in the Japanese textbooks. Comparing different 'functions' (roles of proofs) of these terms (Miyakawa & Shinno, 2021) revealed that different functions can be attributed to different words in the textbooks. For example, both *beweisen* and *shōmei-suru* emphasize the function of verification, providing the logical value of a statement (Duval, 1991). Other words (*erklären*, *begründen*, and *setsumei-suru*) are associated with the function of explanation, gaining an understanding of the reason why a statement is true. In addition, both *erklären* and *setsumei-suru*, in their ordinary usage, point to the function of communication or illumination, which are used for expressing one's thinking in a social context. Regarding the formulation of 'structures' (organisation of reasoning) (Miyakawa & Shinno, 2021), *beweisen* indicates students should demonstrate a proof by a deductive chain of reasons, while *begründen* often allows no specific form of reasoning. The usage of *beweisen* is almost identical to that of *shōmei-suru*. But what is asked by *setsumei-suru* in the proving context is different and varies before or after *shōmei* is introduced. Similarly, the meaning of *erklären* varies in the context of proving.

References

Duval, R. (1991). Structure du raisonnement déductif et apprentissage de la démonstration. *Educational Studies in Mathematics*, *22*(3), 233-261.

Miyakawa, T., & Shinno, Y. (2021). Characterizing proof and proving in the classroom from cultural perspective? In M. Inprasitha et al. (Eds.), *Proceedings of PME44* (Vol. 1, pp. 242-250). PME.