STUDYING MATHEMATICS TEACHERS' DESIGN OF TASKS INSPIRED BY AUTHENTIC PRACTICES

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This paper reports a case study research aiming to explore the potential of authentic workplace situations in mathematics teaching in upper secondary classes. For this purpose, seven teachers participated in a group aiming to connect the teaching and learning of mathematics with the marine navy context (ship navigation). We use the notion of the mathematical working space to compare the tasks designed by two of the teachers for their lessons inspired by authentic ship navigation practices. The results indicate substantial differences of the two designed working spaces in terms of their semiotic, instrumental and discursive dimensions.

INTRODUCTION

In recent years there has been consensus among researchers about the shift in mathematics teaching for the 21st century to promote making real-world connections (Gravemeijer et al., 2017). Therefore, a discussion has emerged among researchers about the potentiality of using authentic workplace tasks in mathematics classrooms by suggesting the idea of using authentic practices as a source of inspiration for designing educational materials (Dierdorp et al., 2011). Many researchers find the above idea very promising since authentic workplace practices are rich and meaningful and offer students' chances for inquiry activities; engage students with challenging problem-solving practices and support students' development of mathematical reasoning skills (e.g., Dierdorp et al., 2011; Wake, 2015). However, workplace research informs about the complexity of identifying mathematics in professional practice. Researchers in this field argue that school and workplace mathematics are different practices with different goals, types of tools, and genres of mathematical language and community rules while workplace mathematics is black-boxed in professionals' routine tasks (Williams & Wake, 2007). Hence, to reach a modus vivendi between authenticity and classroom mathematics teaching seems to be a challenge for teachers who should provide students' proper familiarization with workplace tools and discourse and at the same time engage students in a rich mathematical activity (Nicol, 2002). Research has highlighted the need for more research on how a workplace context orients a working space putting under investigation how teachers might introduce this working space in their teaching as a mathematical working space (Kuzniak et al., 2016). The aim of the reported study is to contribute in this direction by exploring the potential of naval navigation as a context for mathematics learning in secondary schools. Our focus is on the choices made and

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the challenges faced by secondary teachers when they are engaged in designing tasks for their mathematics classrooms inspired by authentic ship navigation practices.

THEORETICAL FRAMEWORK

Mathematical working spaces (MWS) offers a framework relevant for the study of mathematical work in an educational context. Under this perspective, mathematical work is understood as an intellectual work of production, the development of which is oriented and supported by mathematics.

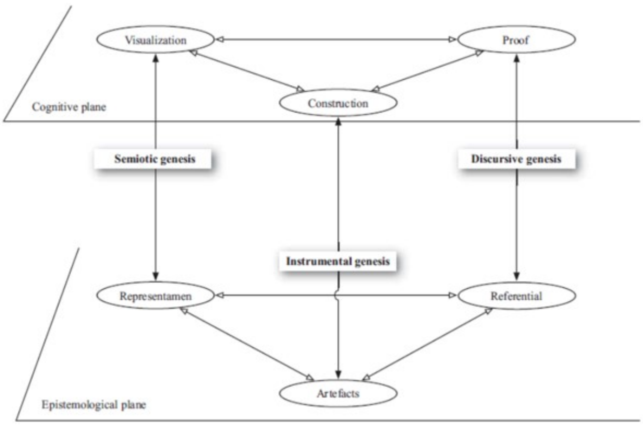

Figure 1: The MWS diagram

According to Kuzniak et al. (2016), an MWS consists of two planes, the epistemological and the cognitive (Figure 1). The epistemological plane is related to the mathematical content and the tasks that will take place, while the cognitive is related to the students' way of thinking and the reasoning followed during their work on a task. The two levels are each made up of three components. More specifically, epistemological plane consists of: a set of distinct objects (representamen); a set of objects such as drawing instruments or software (artefacts); and a theoretical referential framework consisting of definitions, properties and theorems. The cognitive level is composed of the following three components: visualisation, which corresponds to the creation, manipulation and interpretation of the symbols of the specific representation of each field of work; the construction, that refers to the form of reasoning that depends on the tools used and the relevant techniques; and the proof that emerges through the creation of mathematical arguments and validation. Under the lens of MWS, the mathematical work of an individual is evolving through intertwined generative developments (i.e., geneses) between the epistemological plane and the cognitive plane defining three dimensions: semiotic, instrumental and discursive (Figure 1). The semiotic dimension concerns the use of algebraic symbols, geometric representations, graphs, diagrams, etc. That is, it creates connections between verbal, conceptual, functional expressions and geometric constructions with symbolic expressions. The instrumental dimension refers to treating objects as tools and using them for the necessary mathematical and non-mathematical constructions and is used to explain how artefacts are transformed into learning tools through the interaction of teachers and students. Finally, the discursive dimension refers to processes of justification and proof and concerns the production of mathematical meanings. In this paper, we use MWS as a tool to analyze tasks designed by teachers for engaging their students in ship navigation activities in the classroom. In terms of the MWS theory, we analyze how the *suitable MWS* is shaped by teachers' design choices. Our focus is on how teachers conceive authentic practices and adapt them for their lessons and how the dimensions of mathematical work defined by MWS are considered in their designs.

METHODOLOGY

A group was set up for the study consisting of seven mathematics teachers working in different schools, two researchers, and one researcher/teacher who conducted the research (Vroutsis et al., 2018). The main goal of the group was to inform the teachers about the context of the workplace, and to design and implement authentic tasks inspired by naval navigation in their classrooms. The group was supportive in providing feedback on task implementation. The researcher/teacher had experience of ship navigation; he studied official navigation textbooks and collaborated with a professional captain in order to get more familiar with the workplace. There were four group meetings over three months. The researcher/teacher acquainted the group with the context of the workplace, the naval chart, the captain's authentic tools, and the original measurements used for fixing the ship's position through relevant professionals' videos.

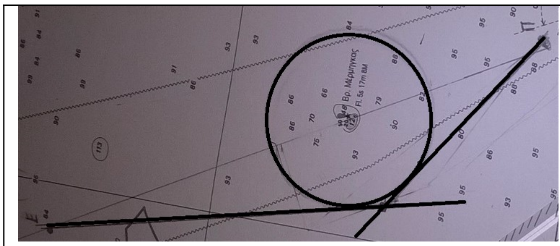

Figure 2: Avoid Obstacle

The teachers were introduced to the following authentic tools and measurements: nautical chart, nautical divider (its legs end in sharp edges and are longer than the legs of a common divider); parallel rulers (two connected rulers moving in parallel lines); compass rose (protractor integrated to the nautical chart); bearing (the clockwise angle between the direction of an object and that of true north); range (the distance between two objects). In addition, the teachers had the opportunity to experiment with the above elements of the workplace through small tasks given to them (e.g., "Plot and determine the course from the port of Syros to the port of Naxos through the use of bearing"). Finally, the researcher/teacher introduced the group to the authentic practices of the captain provoking discussion about the mathematical content that is black boxed in these practices. Later the teachers designed and implemented tasks inspired by the aforementioned professional's authentic practices. In this paper, we focus on two teachers' (A and B) designs. Both teachers had a master's degree in mathematics education while Teacher A had experience in the workplace as he had served in the Navy. The tasks were implemented in two general secondary education schools (grade 9 classes, teacher B; grade 10 classes, teacher A). We analyse the tasks of the two teachers inspired by the authentic practice "Avoid Obstacle". Professional captains apply this practise to avoid an obstacle in ship's course. They consider one imaginary circle around the obstacle (safety distance). The new route consists of two tangent lines in the circle, one from the starting point and one from the destination (Figure 2).

The collected data consisted of: transcriptions of recording of the group meetings; teachers' personal notes; teachers' resources and materials (lesson plans, worksheets); semi-structured interviews of the two teachers after the implementation. The analysis was performed in two phases. Initially, each subtask in the worksheet presenting the main task was analysed in the light of the MWS through the triplet of dimensions (semiotic, instrumental, discursive) and their elements (e.g., visualisation, construction). In the next phase, teachers' design choices that emerged from the analysis of worksheets were cross-analysed with the teachers' interviews so as to synthesize design choices and underlying reasons.

RESULTS

Both teachers noted the importance of familiarizing students with the workplace. Teacher A said, *"I had no particular problem communicating the tasks to the students as in the first two hours of the implementation the students had acquired the skills to handle the new elements brought from the workplace."* Thus, reaching the task "Avoid Obstacle" the students had been acquainted to the basic elements of the workplace, needed to get involved in the main tasks, through small activities. Table 1 lists the task of teacher A. With bold writing in the left column we have located the quotes of the task that we focus on based on the three dimensions of the MWS. In the left column, we quote the corresponding dimension of the suitable MWS in which we include it.

We want to go from Kavia Bay (**southwest in Kea island**) to Mavriana Bay (**southwest in Kithnos island**). For **safety reasons** throughout the course, we should not approach Cape Tamelos **less than two nautical miles.**	*Semiotic* (workplace terminology and restriction, visualizing the distance between two landscapes on the nautical chart').
Q1: **Locate** the forbidden **area on the map**.	*Instrumental* (plotting the desired area as a circle - *construction*). *Discursive* (reasoning with use of geometric locus).
Q2: **Map the route** that you consider to be the shortest possible length.	*Instrumental* (plotting tangents - *construction*). *Discursive* (reasoning on the shortest length).

Q3: **Record the courses zl_1, zl_2** that you used.	*Semiotic* (interpreting symbolic authentic notation, using bearings to determine zl - *visualisation*). *Instrumental* (using authentic artefacts, parallel ruler and compass rose). *Discursive* (reasoning on ship's course though).
Q4: Find **the total length** of the route.	*Semiotic* (interpreting symbolic authentic notation, visualizing the segment as distance of two landscapes). *Instrumental* (using authentic artefacts, nautical divider). *Discursive* (reasoning on ship's course length).
Q5: **Record the coordinates** of the point S at which **a change of course** takes place	*Semiotic* (interpreting symbolic authentic notation, geographical coordinates - *visualisation*). *Instrumental* (using authentic artefacts, position fixing on the nautical chart).
Q6: The captain of the ship decided to turn when he has **the proper bearing** of Cape Tamelos. **Can you figure** out what **this bearing**? **How many degrees** will the ship turn at point S?	*Semiotic* (interpreting symbolic authentic notation, ship turn, angle - *visualisation*). *Instrumental* (using authentic artefacts, parallel ruler and compass rose; parallel line displacement). *Discursive* (calculating ship's turn through subtraction of bearings).
Q7: We consider an alternative route in which we start with the path zl_1 that you calculated before, **continue one nautical mile** after the point of change of course of the previous route and then turn. **Calculate the new path zl_2** that we must now follow to move at the place of destination.	*Instrumental* (using authentic artefacts, nautical divider and parallel ruler). *Discursive* (calculating ship's course through the use of bearing zl).
Q8: Can you **use your knowledge of Geometry to show that** the new path is necessarily longer than the original?	*Discursive* (reasoning through mathematical proof).
Q9: Confirm the previous one by **recording the length** of the new route.	*Instrumental* (using authentic artefacts, measuring on the map with nautical divider). *Discursive* (justifying on Q8).

Table 1: Teacher's A suitable MWS

We present in the same way the analysis for teacher's B task in the Table 2. It is obvious that teacher's B task is shorter. Also, the quotes in parentheses in the left column are explanations that the teacher himself had added to help the students.

You are **in the role of "cadet"** who helps the captain set the course and steer your boat properly. You have in your hands a text of the sailor, which probably describes the path to a forgotten chest. The text says: “From where we left the key of the chest to go to Cape Cyclops (Serifos Island) you have to travel **at least 12.27 n.m.,** while from Agios Dimitrios (Kithnos Island) you will travel **at least 11.4 n.m.”** Don’t waste time**. Where are you heading?**	*Semiotic* (visualizing a situation of treasure hunting, using authentic data and marine measurement units). *Instrumental* (using authentic artefacts, position fixing on the nautical chart; making specific measurements with nautical divider). *Discursive* (using authentic measurements to identify position fixing through two ranges).
Q1: What will be **your course? (Zλ =….)**	*Semiotic* (interpreting symbolic authentic notation, using bearings to determine zl - visualisation). *Instrumental* (using authentic artefacts, parallel ruler and compass rose, parallel line displacement). *Discursive* (calculating with authentic measures the course bearing zl).
Q2: **Plot the route** from the Baths of Kythnos to the chest position, **avoiding the dangerous waters** and **determine it with zl** = … (the "angle" of the course according to the compass rose of the area).	*Semiotic* (interpreting symbolic authentic notation range, circle radius). *Instrumental* (plotting tangents, use of parallel ruler and compass rose, parallel line displacement - *construction*). *Discursive* (developing mathematical model to overcome a problematic situation).

Table 2: Teacher’s B suitable MWS

By recording similarities and differences between the tasks, at the semiotic level we note that both teachers built a narration in order to engage the students into the tasks more effectively. Teacher A presented the tasks through a story about a ship's voyage and the dangers it faces *“I wanted to give students a complete story so they would make sense of their involvement.”* Teacher B reported on a treasure hunt, through which he introduced the tasks, giving students the role of a professional *“I gave them a realistic scenario to challenge them and get them into the role of the professional.”*

On the other hand, although both give students two tasks, teacher A breaks them into individual small sub tasks while teacher B does not follow the same approach. Explaining his choice, teacher A talks about his anxiety for the students to complete the task and that is why he chose to "guide" them in this way.

> Teacher A: I owe it to my anxiety to complete the task. That way I could guide the students, when needed. On the other hand, I had the option to skip questions that turned out to be insignificant and save time.

Another issue in which the two teachers present differences is their view on the authenticity of the tasks they gave to the students. Both teachers used workplace terminology and jargon. In addition, the measurements given to the students were realistic, and the tasks required the students to handle authentic tools and to interpret the professional's measurements. However, teacher A speaks clearly of a dominant authentic framework while teacher B speaks of realistic rather than authentic tasks.

> Teacher A: The framework is highly authentic (nautical chart, authentic tools and practices), even workplace restrictions affect students' mathematical activity. The context is dominant; overall, the application had clearly authentic character.

> Teacher B: I have doubts about the authenticity, because I do not know the workplace context very well. A professional may have recognized situations, in the tasks that were either incompatible with reality or "ideal". On the other hand, the scenario is realistic, as is the data given to the students and the context itself puts the students in the role of the professional.

At the instrumental level both teachers used authentic tools in their tasks, with which the students plotted ship courses, bearings and distances on the nautical chart. The above constructions had also mathematical meaning for the students, for example, they treated the distance as a radius of a circle and the course of the ship as tangent to a circle. However, teacher A seems to seek, sometimes explicitly, to connect the authentic elements with the mathematical concepts hidden in them (e.g., "How many degrees will the ship turn"; "Calculate the new path zl_2").

The above discrimination is clearly visible in the discursive level of the MWS targeted through the tasks, which is ultimately the element that differentiates the approaches of the two teachers. Teacher A emphasized the importance of students' engagement with school mathematics in the new context and prioritised validation within mathematics. This is also manifested in his words (interview).

> Teacher A: I seek students to apply in a new context different from school mathematics the geometric properties hidden in the authentic practical ... Yes, it was my intention to ask for geometric proof and mathematical validation.

On the other hand, teacher B described clearly in his interview his choice to engage students in mathematical exploration within the authentic context.

> Teacher B: I seek for students to build strategies to try them, possibly reject them or adapt them ... It is more in the direction of solving a problem. Although I do not deny the role of math teacher, I would like students to explore their solutions through the new context of the workplace and less with the use of school mathematics.

CONCLUSIONS

We analysed two experienced teachers' tasks inspired from an authentic ship navigation practise so as to address the design of their suitable MWS. Comparing the two suitable MWS designed by the teachers, the analysis indicated similarities and differences. As regards the similarities, both MWS were based on tasks that: involved explicitly elements of the ship navigation practice; were based on stories related to the authentic situation; were implemented on the nautical chart; and the values of the measurement data given to students were realistic. However, the analysis brought to the fore distinct differences. Teacher B chose to introduce the authentic context through a game-like task (treasure hunting), similar to the "imaginative" tasks reported by Nicol and Crespo (2005), without reference to any obvious mathematical content or aim. The task offered space for students to work with the original measurements and tools, explore, and develop strategies to discover the solution. In the initial questions, teacher A used an authentic story to engage students in the situation providing them the role of a captain who faces an authentic problem (danger). In the next parts of the worksheet the authentic context fades and the tasks become quite guided asking mainly for calculations and mathematical validation. A comparative look at the dimensions of the MWS framework indicates that both suitable MWS are characterised by rich instrumental and semiotic dimensions in terms of mathematics and workplace signs (e.g., terminology) and instruments (e.g., artefacts, measurements). Teachers' choices in the discursive dimension determine the balance between authenticity and school mathematics in the designed tasks and reveal differences. While Teacher B provides space for students to explore the problem and validate it either through mathematics or workplace, Teacher A guides students towards a solution targeting validation within mathematics. Further analysis including classroom data is expected to allow us getting a deeper understanding of how the suitable MWS is transformed in actual teaching and its potential for exploring further what makes authentic tasks meaningful for students.

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