DIALOGIC-DIALECTIC MATHEMATICAL ARGUMENTATION

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The importance of mathematical argumentation for learning is undeniable. Yet there seems to be an underlying complexity constituting a good mathematical argumentation, particularly in the context of classroom-based group work, where social goals as well as disciplinary ones are at play. This paper hopes to provide a better understanding on how we can study learners' behaviour during collective argumentation to inform the teaching and learning of argumentation in mathematics. In particular, it highlights how the potential and value of two seemingly distinct perspectives of understanding argumentation - dialogic and dialectic - can be used in concert for a more comprehensive understanding of mathematics argumentation.

INTRODUCTION

Mathematical proofs, as an essential aspect of the work that mathematicians typically engage in, help humans *understand*, *explore* and *think* about mathematics. Yet, more fundamentally, it can be argued that the underlying central processes they engage in are *mathematical argumentation* and *reasoning* (Schwarz et al., 2010). In the context of mathematics classrooms, it has been noted that learners are more likely to be engaging in argumentation (and reasoning) than in proving based on formal logic (Krummheuer, 1995). Mathematical argumentation has also gained attention in the twentieth century due to the reconceptualization of mathematics learning that places greater emphasis on understanding how mathematical knowledge is constructed and why it makes sense; and the shift in focus towards discursive activities in recognition of the benefits of argumentation for learning (Schwarz et al., 2010). However, despite the increasing acknowledgement of its importance to learning, mathematical argumentation does not necessarily or naturally happen in all classrooms, possibly due to its underlying complexity and ambiguity (e.g., Krummheuer, 1995; Schwarz et al., 2010). Without clarity in the specifics (of the various aspects) of mathematical argumentation, it will be challenging for researchers to analyse argumentations; for teachers to design and orchestrate meaningful argumentation in mathematics classrooms; and for mathematics learners to know how to argue and to learn through argumentation.

This paper will discuss how mathematical argumentation has been framed or analysed from two seemingly distinct perspectives, namely dialectic and dialogic. I will also discuss the potential in a coexistence of both perspectives in the mathematical argumentation process.

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MATHEMATICAL ARGUMENTATION - THE TWO PERSPECTIVES

Although there is no common definition for mathematical argumentation in the field of mathematics education, there seems to be agreement that its overarching focus resides in the rational reasoning and meaning-making process that mathematics learners engage in collectively. Many different aspects of mathematical argumentation have been studied. The more common ones include the cognitive, meta-cognitive and social-cultural aspects of argumentation (Krummheuer, 1995; Schwarz et al., 2010). However, according to Schwarz (2009), since "argumentation functions in two ways" (p. 104), there have been two strands of research on (mathematical) argumentation, which adopt either a dialectical approach or a dialogical one. The former places emphasis on the rationality behind argumentation. It focuses on how learners make connections in order to provide reasons to support or refute the multiple different ideas or claims proposed, before arriving at an agreement or consensus. As a result, it tends to focus on the cognitive aspect of argumentation. The latter, i.e. the dialogic perspective, places emphasis on the rationality that is situated within social-cultural rules and orientations that facilitate the progress and development of the collective argumentation process (Schwarz & Shahar, 2017). It thus tends to focus on the social-cultural aspects of argumentation. Notably, each perspective has been important for understanding the specifics of a good mathematical argumentation process.

THE DIALECTIC PERSPECTIVE

In mathematics education, it seems that understanding argumentation from the dialectic perspective has been the primary focus of research, due to its strong association with proofs. With respect to this perspective, research generally attend to how the rationality of a good mathematical argumentation can be established. In particular, to understand what makes a good mathematical argumentation process, the research literature has highlighted two key elements, namely, the structure of the argument; and the types of reasoning used in supporting the argumentation process.

Structured by Toulmin's Model of Argument

Toulmin (2003) proposed that arguments generally follow a structure where claims (C) are made based on data (D), the foundation for the claims. But claims need to be supported by warrants (W), usually "general, hypothetical statements, which can act as bridges" (p. 91) between the data and the claims. In other words, the warrants provide plausible reasons to explain the validity and soundness of the claims. Nevertheless, the warrants may only support the claims for some conditions and not others, as such the claims need to be specified further with qualifiers (Q). For the conditions where the claims may not be valid, they will then be refuted with exceptions or rebuttals (R). Lastly, the appropriateness and applicability of the warrants to the claims may need to be defended and elaborated with the necessary backings (B), which usually take the form of statements that are more absolute in nature. Thus, according to Toulmin's model, a logically valid argument should contain all the above six elements with each serving a different function. A cogent visual representation for Toulmin's model of argument is seen in Figure 1. Toulmin's model is helpful for us to understand how the rationality of learners' arguments can be established and is often used in research to analyse and understand individual mathematics learners' arguments.

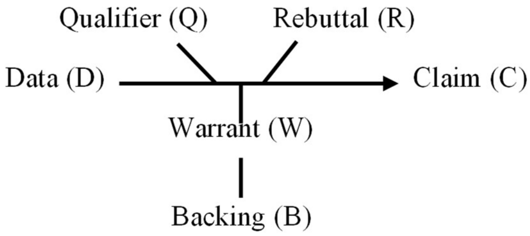

Figure 1: Toulmin's model of argument

Supported by Reasoning

Markedly, mathematics is often associated with deductive reasoning which preserves truths to establish valid conclusions from premises by following the rules of a well-specified logic (Meyer, 2010). However, non-deductive forms of reasoning have also been observed to be important and commonly used in mathematics. Above all, two types of non-deductive reasoning have been highlighted in research (Feeny & Heit, 2007). They are abductive reasoning, which seeks to provide the best explanations for the claims made from observations; and inductive reasoning, which infers generalizations from particular observations. While the order of the different forms of reasoning used may vary, a typical process will start with abductive reasoning to suggest a tentative explanatory hypothesis, followed using inductive reasoning to support or refute its plausibility. If the hypothesis is refuted, the search for another explanatory hypothesis will be required through abductive reasoning and this process will repeat until the best explanatory hypothesis, supported by inductive evidence, is formulated. After which, a proof can be pursued through deductive reasoning to establish the certainty of the conclusion (Meyer, 2010). As such, the type of reasoning that learners adopt in supporting their claims is likely to be dependent on the stage of argumentation and the context. The types of reasoning are also often used together with Toulmin's model to analyse individual learners' mathematical arguments.

THE DIALOGIC PERSPECTIVE

While research based on the dialectic perspective focuses on understanding the rationality of mathematical argumentation through the structure of their arguments, research taking a dialogic perspective aims to understand how the presence of multiple and differing perspectives enhances learning in the argumentation process. In particular, the dialogic perspective, which draws heavily on dialogism, emphasizes the importance of learners' interactions and their engagement with others' ideas and arguments where the focus is on how learners explore the relationship between diverse ideas without the need for a final synthesis or unification (Wegerif et al., 1999). Such a perspective seems to be in great contrast with the conventional expectations of mathematical argumentation, which draw on dialectical assumptions and focus on comparing or evaluating the validity and strength of the suggested arguments before converging and deciding unanimously on the best argument (Langer-Osuna & Avalos, 2015). This is probably also why research based on the dialogic perspective, which focuses on differences and learner interactions, seems to be less favoured as compared to those with a dialectic perspective.

Driven by Differences

Fundamentally, dialogism is based on differences (that are irreducible) rather than identity or unity, where knowledge is deemed to be formed through differences (Bakhtin, 1975/1981; Wegerif, 2008). In other words, object A is not known in and of itself, but in relation to object B (and other objects). It is often contrasted with the notion of dialecticism which emphasises the independent knowledge of A and B (through their respective properties) and then the logical synthesis that can arise by the overcoming of differences, towards uniformity and certainty. Dialogism and dialecticism might seem to be incompatible, in terms of their epistemological and ontological assumptions. Indeed, Bakhtin described dialecticism as being a dialogue in which different voices, perspectives, and emotions have been removed; and the "abstract concepts and judgements from living words and responses" extracted and presented as a single view (Bakhtin, 1986, p. 147, as cited in Wegerif, 2008).

In dialogism, meaning is not universal, but rather, a product of the different perspectives and contexts present. It cannot be constructed without the awareness of at least one other possible point of view, and this is how the dialogic approach differs from a monologic one. The consciousness of the multiple plausible ways of looking at something; the switch between different perspectives; and the context involved, is necessary for understanding to be developed (Wegerif, 2011). Meaning is also realised through dialogues which bring the differences or at least two perspectives together and situate them in a particular context. There is no need for agreement to be met to establish meaning. In fact, Bakhtin proposed that if common ground is achieved, the dialogue will be discontinued and there will be no further progress in meaning making as the presence of variation is the spark that opens up opportunities for perspectives to shift to allow for meaning to be made (Wegerif, 2011). Furthermore, the presence of differences does not necessarily imply that one perspective is correct or superior to their other. The various perspectives should be equally valued and accepted to reflect the multitude of meanings existing in the world such that "the world of meaning is essentially dialogic" (Wegerif, 2008, p. 349).

Augmented with the Other

Another key aspect of Bakhtin's dialogism is the mutual influence or dependency between the two (or more) participants present in the dialogue (Baktin, 1975/1981; Wegerif, 2008, 2011). It highlights their interrelatedness where any utterance is dependent on both parties - the speaker; and to whom it is directed at, the addressee.

The speaker must take into consideration who the addressee is before formulating the utterance as the message should be conveyed in a manner appropriate for the addressee, in terms of the language and style and keeping in mind how the addressee may react to it. Similarly, the addressee will also respond based on a projected view of the speaker. As such, they define each other mutually, where each utterance contains the voices of self and the other; and with each utterance, both the self and the other are being constructed and reconstructed. Eventually, the meaning and understanding constructed is not ascribed to any individual but an integration of the ideas put forth by both.

As a result of the mutual dependency, a metaphorical dialogic space (including position and time) that encompasses both the speaker and the addressee is generated, beyond a fixed and simple connection between them and their ideas (Wegerif, 2008, 2011). It is a shared and generative space where the two can relate to each other dynamically. Their positions in the space can change as the dialogue progresses (and where time changes too). They may take on each other's perspectives, or they may gradually or totally shift their initial positions as new insights emerge from their interaction.

EXPLICATING THE DIALECTIC AND DIALOGIC PERSPECTIVES

Although the dialectic and dialogic perspectives have been used extensively in their own respect within mathematics research, recent research evidence seems to be leaning towards a joint approach in analysing argumentation. Individually, each perspective may no longer be sufficient in providing a comprehensive understanding of mathematical argumentation and the conditions for it to happen, particularly in the case of pair-wise or group activity in the classroom. As such, I want to explore how the two perspectives possibly interact and complement each other during argumentation. I will use a short paradigmatic example of collective mathematical argumentation to illustrate some instances of the dialectic and dialogic perspectives. This example is situated in the context of an undergraduate geometry course where students were tasked to use a dynamic geometry program to form a conjecture and construct a proof for the following problem:

> Construct a circle with centre O and a fixed point Y in the interior of the circle. Let AB be any chord of the circle that passes through the point Y, when is the product of AY and YB a maximum?

Alex: Oh right. Hmm, will the maximum be when the chord is the diameter, so that the two parts have longer length?

Luke: Oh that's possible, since diameter is the longest possible chord, so we will be multiplying two biggest numbers! Let's drag the chord until it passes through the centre and see the length.

Alex: Yes, yes that's what I was thinking. See, the length of this part is 8.5, that is bigger than the lengths of both parts of that chord we started with that doesn't pass through the centre.

Nora: Hmm, wait, wait this doesn't sound quite right. Yes I agree that this part is definitely longer but the other part is shorter, which will cause the product

	to be smaller, isn't it? Your claim is only valid if both lengths are longer but it's not the case.
Luke:	Ah, I think Nora has a point there. Let's calculate the product, here, 8.5 is longer but the other part is shorter at 3.9, so the product is about, hmm, about 33? For the other chord, can you drag it back? The two lengths are 7.2 and 4.6 which is also 30 something, [pause] about 33.
Alex:	Huh, really? That's not what I was expecting.
Luke:	Ya, the products are so similar. Let's try another one and see what's the product. [Drags point *A*.] How about this, 4.1 and 8.1? [pause] The product is also 33!
Nora:	Does that mean that the product is a constant? There is no maximum?
Alex:	I don't quite believe this. Can we move that point *Y* and see what happens? What if the fixed point *Y* is nearer to the edge, the circumference of the circle instead of the centre? I am still trying to test my conjecture about the diameter.

From the dialectic perspective, it can be observed that the group of learners were engaged in abductive reasoning. Alex was the first to offer a conjecture, that the maximum product occurs when the chord *AB* is the diameter. His conjecture seems to be based on abductive reasoning as a warrant that the longer the chord, the longer the two segments that make up the chord was provided to back his claim. However, there was no data from the problem to support his claim, which is characteristic of abductive reasoning. His claim was accepted as a possibility by Luke who further elaborated on the claim that the diameter is the longest chord and provided a backing that the product should be larger when the two lengths are longer. Luke also initiated the use of the dynamic geometry program to find supporting data. Alex then expanded on his argument as he thought that he found suitable data (that the length of one part was longer than that for the initial chord) to support his claim. But this was quickly countered by Nora who highlighted that the length of the other part was actually shorter than the corresponding segment for the initial chord. As a result, the tentative claim by Alex was refuted as the data did not satisfy the assumption that the two lengths *AY* and *YB* will be the longest with a longest chord (i.e. the diameter) to start with. Using Toulmin's model, this sequence of argumentation, supported by abductive reasoning, can be represented graphically as shown in Figure 2. As the conjecture was proposed in the form of a question, it seemed to suggest the uncertainty of its validity. As such, a qualifier was necessary to illustrate the learners' degree of confidence.

From the dialogic perspective, it can be observed that differences in the learners' ideas were evident throughout the argumentation process and contributed much to the development and refinement of their arguments. For instance, if Nora had not challenged the validity of Alex's initial conjecture, the group may have embarked on

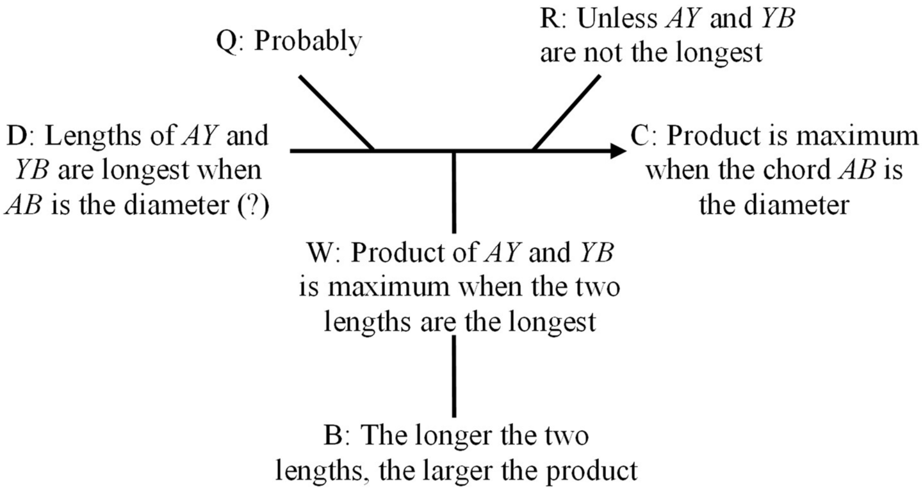

Figure 2: Alex's conjecture based on abductive argumentation

an attempt to prove an erroneous conjecture. It also led the group to discover another hypothesis to the problem which was rather different from the one before. The group shifted from a conjecture of having a possible maximum product to another stating a likely absence of a maximum as the product is a constant. This switch was made possible due to the presence of differing ideas which prompted a search for a better conjecture. Interestingly, despite the challenge from Nora and her supporting data from the dynamic geometry program, it was not convincing enough for Alex to switch his point of view on the problem. He appeared to be able to understand the alternative conjecture but was reluctant to shift his position and continued to make attempts to find evidence to support his own conjecture.

DISCUSSION AND CONCLUSION

Based on this short example, the dialectic and dialogic moments seem to occur almost successively. Rather than argue that the example must be read either in one way or the other, I will instead consider how each approach related to specific functions. While the learners had been observed to be able to articulate their ideas or claims with reasons or evidence, i.e. behaved dialectically, they were also able to see other perspectives and relate to each other's ideas and build on them, i.e. behaved dialogically. In particular, the two components appeared to complement each other at times. On the one hand, the more the learners questioned or challenged others' arguments, the more they had to be articulate in their reasoning and explanations in order to respond or counter-challenge the other. On the other hand, with an increase in the amount and depth of reasoning to present different ideas, the learners were more likely to be able to understand and engage with the differing views. Moreover, there appears to be moments when the dialogic aspect is more prominent as compared to the dialectic and vice versa, depending on the stage and function of the argumentation process. Specifically, the dialogic aspect seems to be more apparent when the two different conjectures were suggested and brought together to open the discussion while the dialectic may be more prevalent when the learners were trying to agree at a best conjecture and when there was more homogeneity in their ideas and arguments (e.g. between Alex and Luke at the beginning of the argumentation).

Hence, there is potential in using both the dialectic and dialogic perspectives in concert to better understand the complexity of mathematical argumentation in a collective setting. However, the potential and value of such an approach may not have been fully explicated through only one example. More authentic examples may need to be examined using this approach to increase its validity in understanding this phenomenon.

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