

COMPUTATIONAL THINKING IN DENMARK FROM AN ANTHROPOLOGICAL THEORY OF THE DIDACTIC PERSPECTIVE

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In this paper, we study the external didactical transposition of programming and computational thinking (PCT) into the Danish compulsory school mathematics curriculum. Taking an anthropological theory of the didactic approach, we focus on the nature of mathematical and PCT knowledge being transposed into the new curriculum as prescribed praxeologies. Two main critiques arise. First, PCT knowledge being transposed into mathematics is broad and has no immediate relation to definitions from the literature. Second, the new competence and subject-matter areas are barely juxtaposed to those from mathematics. As a consequence, the responsibility of developing meaningful integrations in concrete teaching—the internal didactic transposition—lies entirely on teachers and material developers.

INTRODUCTION AND BACKGROUND

In 2006, Jeannette Wing revived the concept of computational thinking (CT) initially introduced by Papert (1980), and argued that it is a “fundamental skill for everyone, not just for computer scientists” (Wing, 2006, p. 33). Wing’s (2006) paper sparked a new wave of interest, which has led to research in programming and computational thinking (PCT) teaching and learning and revisions of national curricula to include this (Bocconi et al., 2016). In many countries, these revisions have consisted of adding PCT elements to the mathematics curriculum. In the literature, there is consensus that mathematics and CT share a focus on logical structures and modelling (e.g., Gadanidis et al. 2017), and that there are potential educational synergies in combining the two areas (Benton et al., 2017). Still, there are not yet conclusive findings in terms of how we establish such synergetic relations between mathematics and PCT. In part, this knowledge gap is due to the fact that PCT is still an ambiguous term, defined and practiced in a diversity of ways (Shute et al., 2017). While PCT is closely related to the scientific domain of computer science, it sometimes also draws on other fields, such as sociology and philosophy (see, e.g., Helenius & Misfeldt, 2021). In turn, the specific PCT content that is being implemented at a curricular level varies substantially. In this paper, we tackle the nature of PCT and mathematical knowledge in the interplay between scholarly notions and knowledge aimed to be taught, focusing on the particular case of Denmark. While several countries (e.g. Sweden, England and Norway) have already implemented PCT in their respective compulsory school

national curricula, the Danish Ministry of Education (UVM) took a more cautious approach. It consisted of trying out a new subject called Technology Comprehension (TC) at 46 schools, and then evaluating its outcomes under two implementation strategies (UVM, 2021). These ideas were to inform a future decision for mainstream implementation. Whereas the first strategy regarded TC as a subject in its own right, the second strategy inserted TC competence areas and learning goals in existing subjects, including Danish (language 1), mathematics, arts, physics/chemistry, science, craft and design, and social science. Both approaches were supported by a newly drafted curriculum, including mathematics (UVM, 2019). In order to address the interrelation between out-of-school types of knowledge and those depicted in the Danish curriculum, we activate appropriate concepts from the didactic transposition (Brousseau, 2006) and the anthropological theory of the didactic (ATD). Below, we elaborate on the theoretical underpinnings and describe the empirical foundation for our analysis to formulate our research questions.

THEORETICAL ELEMENTS FROM ATD

ATD has been developed as an epistemological perspective to the teaching and learning of mathematics (Chevallard & Sensevy, 2014). It focuses on the nature of mathematical knowledge regarded as a human activity, and how it is disseminated and taught (Bosch & Gascón, 2006). According to Chevallard and Sensevy (2014), knowledge to be taught can be modelled as a *praxeology*, which consists of two main building blocks: *praxis* and *logos*. The *praxis* block includes the type of *tasks*, i.e., the concrete challenges to be confronted by students, and specific *techniques*, which describe how the tasks should be handled. The *logos* block consists of technology and theory. *Technology* covers the discursive knowledge that builds the language needed to talk about the tasks and techniques. *Theory* is defined as the logical and conceptual frames that explain and justify the technological components, and relate these to other areas with their respective theories. Although ATD focuses on knowledge as being experienced in human activity, we are investigating knowledge as depicted in the curriculum, which we might label as *prescribed praxeologies*. The second element of interest is the transition between these domains of knowledge and the knowledge that ought to be taught in a school setting. In the language of ATD, this process is called *external didactical transposition* (Bosch & Gascón, 2006). In contrast, the *internal didactical transposition* takes place from the knowledge meant to be taught (e.g. curriculum) to that factually being taught (i.e. teaching practices). Typically, ATD regards the point of departure for the external didactical transposition to be scholarly mathematical knowledge and the role of mathematical knowledge and skills in society and everyday life. Analyses commonly regard praxeologies as situated within a discipline's subfield, such as 'algebra' within mathematics. However, as Helenius and Misfeldt (2021) point out, the integration of PCT into mathematics requires the mobilization of knowledge from several domains. This diversity calls for specific

attention to the level of coordination between the elements of the didactical transposition coming from the different sources. Such coordination can happen on the level of external and on the level of internal didactical transposition (Schmidt & Winsløw, 2021).

RESEARCH QUESTIONS

We first aim to characterize the diversity and structure of the fields of knowledge—mathematical or otherwise—involved in this new Danish TC curriculum, leading to the first research question:

- What domains of knowledge make up the new PCT-related praxeologies in the Danish mathematics curriculum?

In the above-described openness to the domains of knowledge, we formulate our second research question:

- To what extent is the interplay between knowledge from different domains (mathematics versus TC) contained in the external didactical transposition, and which elements are left to the internal didactical transposition?

DATA: THE DANISH TC CURRICULUM

The Danish exploratory project to implement TC at the 46 schools began with developing a curriculum for TC as a subject in its own right. This curriculum included four competence areas, namely: digital empowerment; digital design and design processes; computational thinking; and technological agency. Further, each competence area was defined by 3–5 subject matter areas presented as pairs of skillset and knowledge. For example, Table 2 displays those included in mathematics.

As our primary source of data, we draw on the official mathematics goals overview with integrated TC published by UVM. This document includes a general declaration and description of competence goals for mathematics in Denmark's compulsory K-9 education. These descriptions are organized into four competence areas: (1) mathematical competencies (see Niss & Højgaard, 2019); and subject-matter areas, (2) numbers and algebra, (3) geometry and measure, and (4) probability and statistics. This experimental version of the curriculum then adds TC as a fifth competence area, including elements of PCT (see Table 1).

Although the exploratory program planned to experiment with two implementation strategies (as an independent subject and integrated into others), it was decided that both should address the same curriculum components. Hence, in order to integrate TC in existing subjects, the individual competence areas of the curriculum for TC, as a subject in its own right, were to be distributed among the subjects in which TC should be integrated. In the case of mathematics, six TC components were integrated into mathematics: digital design and design processes; modelling; programming; data, algorithms and structures; user studies and redesign; computer systems (see Table 1).

Mathematical competencies	Description of six competencies: Problem treatment; Modelling; Reasoning and thinking; Representation and symbol treatment; Communication; Aids and tools
Algebra and numbers	Description of five areas: Numbers; Calculation strategies; Equations; Formulas and algebraic expressions; Functions
Geometry and measurements	Description of three areas: Geometric properties and relationships; Geometric sketching; Placement and movements; Measurement
Statistics and probability	Description of two areas: Statistics; Probability
Technology comprehension	Description of six areas of comprehension: Digital design and design processes; Modelling; Programming; Data, algorithms, and structures; User studies and redesign; Computer systems

Table 1: TC added to the description of the mathematics curriculum for Danish K-9. The objective of TC in relation to the mathematics curriculum reads: “The student can act with judgment concerning the use of digital technologies in working with open problems from the surrounding world” (UVM, 2019, p. 4, our translation from Danish). Table 2 illustrates this for the six components of TC.

TC components	Skillset	Knowledge
Digital design and design processes	The student can design digital artefacts through an iterative design process that will benefit the individual, the community and society	The student has knowledge about complex problem solving and iterative design processes
Modelling	The student can construct and act on digital models of the real world and assess the range of the model	The student has knowledge about how models of the real world can be used to describe and treat this
Programming	The student can modify and construct programs for solving a given task	The student has knowledge about methods for stepwise development of programs
Data, algorithms, and structures	The student can recognize and utilize patterns in structuring of data and algorithms with a departure point in specific problems	The student has knowledge about patterns in structuring data and algorithms

User studies and redesign	The student can plan and carry out investigations of users' perspectives and applications of digital artefacts	The student has knowledge about users' perspective and application of digital artefacts
Computer systems	The student can assess different computer systems' possibilities and limitations	The student has knowledge about how number systems, encryption mechanisms, and network protocols affect the basic construction and mode of operation of computers and networks

Table 2: The six TC components related to the mathematics curriculum for Danish K-9 (our translation from Danish).

ANALYSIS

While the new TC competence and subject matter areas are added to the curriculum, they are not explicitly related to existing mathematical competencies and subject matter areas. We may model this situation as a didactical transposition following, for instance, the example of Schmidt and Winsløw (2021). We see that the external didactical transpositions from TC and mathematics are only superficially coordinated by juxtaposing the curricular learning objectives and providing examples of teaching materials. This situation resembles the model in Figure 1.

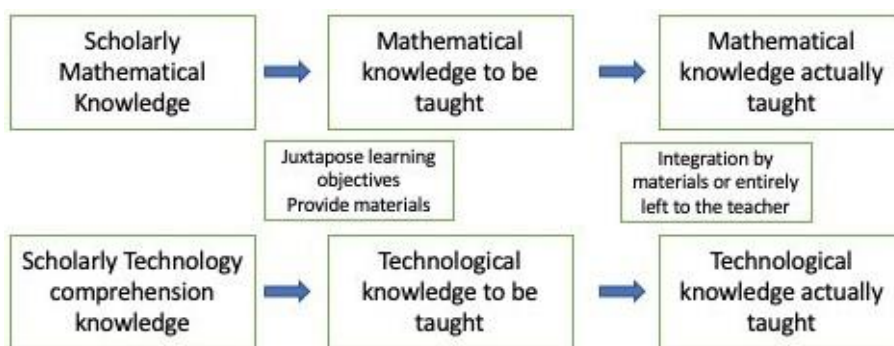


Figure 1: Superposed didactical transposition in the Danish exploratory integration of TC and mathematics. The external didactical transposition is weakly coordinated

The inclusion results in a curriculum consisting of juxtaposed components from TC and mathematics, thereby placing the responsibility of developing meaningful integrations in concrete teaching on the shoulders of teachers and, in part, on curriculum material developers. Given the existing level of PCT skills among the participating Danish mathematics teachers, it seems a quite unrealistic and unreasonable requirement (UVM, 2021). This problem has been acknowledged and addressed in the Danish exploratory TC project by developing rather targeted teaching sequences available to the public in tekforsøget.dk/forlob. In a sense, this combination

still leaves the teacher with most of the formal responsibility to interpret the interplay between mathematics and TC. Moreover, it does so without providing the teachers with any real influence on the matter. The specific content from TC that is suggested integrated into mathematics is undoubtedly relevant to the subject of mathematics. Still, if we look at Table 2, it is not entirely clear how this relation can be articulated and promoted in actual student activities. All six objectives clearly can be related to mathematics but in a myriad of different ways.

Digital design and design processes as well as *user studies and redesign* are described completely generic with no specific relations to mathematical praxeologies. For *modelling* and *programming* the case is almost similar, even though these objectives are more closely related to PCT, and thus perhaps easier to interpret in a mathematical direction. The two components, *data, algorithms, and structures* and *computer systems*, do have some relations to mathematics in the short description provided. Data, algorithms, and structures contain elements of algorithms, data and patterns that are obviously relevant to a number of mathematical activities and insights. In relation to computer systems there are references to number systems and encryption that could support the development of relevant mathematical praxeology.

DISCUSSION

Above, we have seen how the external didactical transposition prescribes the integration of PCT and mathematics in the Danish exploratory subject TC. It seems clear that TC is the result of transpositions of several scientific disciplines and knowledge domains, not only computer science. We also notice that the external didactical transposition does not form or prescribe new mathematical praxeologies as such. Rather, independent TC praxeologies are suggested to the mathematical curriculum by juxtaposition, leaving the teachers without guidance.

Evidently, this situation is not ideal, but could it have been avoided? Let us take a closer look at how this problem has been handled in other countries that have tried to integrate PCT into mathematics education. We can see that the UK has chosen not to formally integrate PCT and mathematics, but instead has an independent Computing subject. On the other hand, Sweden has made a more specific and detailed integration of PCT into algebra, among other mathematics subject areas (Bråting & Kilhamn, 2021; Helenius & Misfeldt, 2021). In England, the computing subject is formed by transposing a university-level computer science topic. The Danish TC curriculum is formed from a much broader range of topics, including computer science, sociology (e.g., democracy and surveillance), and philosophy (e.g., ethics). The didactic transposition in the two countries, however surprisingly, share the feature that none of them prescribes specific praxeologies that explicitly include both mathematics and PCT. For England this is not surprise, since the computing subject shares no structural overlap with the mathematics curriculum. However, it is remarkable that the didactical

transposition of PCT and TC into the Danish curriculum merely consists of juxtaposing new praxeologies from TC to the existing mathematics curriculum. The task of finding meaningful ways of connecting mathematics and PCT is, in both cases, a matter to be handled in the internal didactical transposition, i.e., as mentioned above, it is left to curriculum material producers and teachers. Nevertheless, the difference is that the Danish exploratory topic requires this integration to occur, whereas the English subject does not. The Danish case is an example of a required yet unsupported, superposed didactical transposition, whereas the English case is neither supported nor required. Both approaches are rather different from how computer programming is embedded in the Swedish mathematics curriculum, where, for example, algorithms have been embedded in problem-solving and algebra (see Helenius & Misfeldt, 2021).

Our first research question aimed at understanding what domains of knowledge make up the new PCT praxeologies in the Danish mathematics curriculum. This inquiry is actually not easy to answer with an outset in the core curriculum document. The juxtaposed TC components open a wide range of possibilities yet give little clear direction for practitioners. Nevertheless, the areas *data*, *algorithms*, and *structures*, *programming*, and *modelling* are all parts of what Wing (2006), Shute et al. (2017), and Weintrop et al. (2016) refer to as CT. Hence, these areas are unquestionably in play. The lack of clarity regarding the first research question, in a sense, answers the second one. Here we asked to what extent the interplay between knowledge from different domains (mathematics versus TC) are contained in the external didactical transposition and which elements are left to the internal didactical transposition. The answer seems to be that the majority of the work is left to the practitioners in the internal didactical transposition.

We thus see this as a case of a superposed didactical transposition supporting transdisciplinary education by integrating knowledge from different domains into coherent educational scenarios. This makes it difficult, although necessary, to be specific in the external transposition.

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