STRATEGY USE IN NUMBER LINE TASKS OF STUDENTS WITH MATHEMATICAL DIFFICULTIES: AN EYE-TRACKING STUDY

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The number line is an important mathematical tool, especially in primary education. Previous research suggests that students with mathematical difficulties (MD) tend to have difficulties in empty number line tasks, but little is known about marked number lines. The aim of this study was to investigate if students with MD differ in their strategy use from students without MD in marked number line tasks. In our empirical study with fifth-grade students with and without MD (each n=20), we used eye tracking (ET), the recording of eye movements to gain insights into students' strategies. Based on ET video data, we inductively developed a category system of student strategies using qualitative content analysis. Our data analysis revealed significant differences: Students with MD used counting strategies more often—and less direct locating.

INTRODUCTION

Students with mathematical difficulties (MD) are characterized by difficulties in their learning of mathematics, essentially in basic arithmetic at primary school level (e.g., Moser Opitz et al., 2017; Scherer et al., 2016). Fundamental to learning mathematics is the development of the concept of numbers. This involves several aspects, including an ordinal understanding of numbers (Fuson, 1988). To develop this understanding, tools are often used where numbers are arranged linearly, such as the number line (Diezmann & Lowrie, 2007). On number lines, in addition to the ordinal arrangement of numbers, the relational interpretation of numbers to each other is important (Schulz & Wartha, 2021). Number line tasks are commonly used to investigate the learning of mathematics and the development of mathematical skills, and students' performance on number line tasks has been shown to correlate with their overall mathematical achievement (Schneider et al., 2018). Previous research has indicated that students with MD have difficulties in number line tasks, such as less accurate locating of numbers (Landerl et al., 2017). Further studies using eye tracking (ET) as research method to investigate students' work on empty number line tasks have indicated that ET is insightful for analyzing students' strategy use on these number lines and that students with MD show less flexible strategy use than students without MD (e.g., van Viersen et al., 2013; van't Noordende et al., 2016). In this paper, we investigate students' strategy use on a marked number line. We present students' strategies as well as differences in the use of strategies between students with and without MD.

NUMBER LINE

The number line is one of the most important tools in mathematics teaching and learning, especially at primary level (Diezmann & Lowrie, 2007). Different types of

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number lines can be distinguished: For example, the empty number line, where only the start and end points are labelled, and the marked number line with more hatch marks. There are many possibilities for the presentation of marked number lines: They can be fully or partially marked with hatch marks and labelled with numbers. Hatch marks (even without being labelled with numbers) can be visual reference points. Depending on the different ranges of numbers that marked number lines can represent, there are different markings and scales, so that distances on the number line must be interpreted differently (Schulz & Wartha, 2021). Number lines are used to develop and deepen an ordinal understanding of numbers (Diezmann & Lowrie, 2007). This involves understanding numbers as ranks or positions on the number line. In order to use the number line adequately, it is necessary to understand that all natural numbers have unique positions—even if these are not always visibly marked—and that all numbers are equidistant to each other (Schulz & Wartha, 2021). Furthermore, the relational interpretation of the given structuring features (markings and labelled numbers) on the number line is a way to make numbers accessible (Schulz & Wartha, 2021), that is, numbers have to be interpreted in relation to other numbers.

Difficulties with number line tasks at preschool age are predictive of later mathematical difficulties (e.g., Bull et al., 2021). School-age students' performance in number line tasks has been shown to correlate with general mathematical achievement (for a meta-analysis, see Schneider et al., 2018). Research on number lines is therefore important with respect to mathematical skills and thus for research on MD.

MATHEMATICAL DIFFICULTIES

Students with MD show difficulties in understanding basic arithmetic concepts (e.g., Moser Opitz et al., 2017; Scherer et al., 2016). MD can involve both a conceptual level—for example, in understanding the decimal system and place values—and a procedural level—for example, in the flexible use of calculation strategies (e.g., Moser Opitz et al., 2017; Scherer et al., 2016). MD are also associated with students having difficulties using adequate strategies when working on different mathematical tasks (for quantity recognition, e.g., Schindler et al., 2019). Difficulties, which occur in primary school, can become manifest over the course of the school years (Scherer et al., 2016) and are also observed on secondary school level (Moser Opitz et al., 2017).

Students with MD also tend to have difficulties in number line tasks: For example, they are often less accurate in locating numbers than students without MD (Landerl et al., 2017). Furthermore, previous research on students' strategies in empty number line tasks indicates that students with MD use strategies for locating numbers on the number line less adaptively than students without MD (van Viersen et al., 2013; van't Noordende et al., 2016).

These results, along with the preceding findings that performance in number line tasks correlates with general mathematical achievement, point to the importance of exploring in more detail how students with MD handle number line tasks.

EYE TRACKING

ET—the recording of eye movements (Holmqvist et al., 2011)—is becoming increasingly important in mathematics education research (Lilienthal & Schindler, 2019; Strohmaier et al., 2020). ET is used as a research method to study cognitive processes, and several studies have shown the potential of ET to provide insights into students' strategies in mathematical tasks in different domains. ET provides an opportunity to reveal differences in strategy use between students with and without MD: For example, in an ET study on quantity recognition by Schindler et al. (2019), students with MD tended to use counting strategies frequently over all tasks, whereas students without MD adapted their strategies more often to the tasks. There are also ET studies investigating number line tasks that have examined students' strategies and possible differences in strategy use between students with and without MD (e.g., van Viersen et al., 2013; van't Noordende et al., 2016). These studies have shown that ET is valuable for analyzing number line tasks, also for students with MD. ET appears to have added value in number line tasks: A study by Simon and Schindler (2020) has indicated that ET can provide more detailed insights into students' strategies than thinking aloud protocols, especially for students with MD. These findings were all to be found on empty number lines. To date, little is known about how students use the marked number line. A study by Simon et al. (2022) has indicated the potential that ET provides for gaining insights into strategies for marked number line tasks. However, to the best of our knowledge, there are no ET studies yet examining the strategy use of students with MD in marked number line tasks.

The aim of this study is to investigate if students with MD differ from students without MD in their use of strategies to locate numbers on the marked number line. We ask the following research question: *Do students with and without MD differ in their use of strategies in marked number line tasks?*

THIS STUDY

Participants. A total of 165 fifth graders from a German comprehensive school worked on the number line tasks. Before conducting the ET study, we administered a standardized arithmetic test, HRT (Haffner et al., 2005), to all students to diagnose MD. According to the HRT, students with a PR≤10 are considered to have MD. Students with a PR>25 are considered not to have MD. Students with a PR in between are "at risk" for MD (Haffner et al., 2005). Based on the results of the HRT, we selected 40 students: 20 students with MD (12 girls; mean age: 10.11 years, SD: 0.7 years) with the lowest scores in HRT (mean t-value: 29.5, SD: 1.8) and 20 students without MD (8 girls; mean age: 10.6 years, SD: 0.7 years) with the highest scores in HRT (mean t-value: 53.9, SD: 4.9). We did so to represent students at the lower and upper ends of the performance spectrums.

Tasks and procedure. Students in fifth-grade, in the transition phase from primary to secondary school, may still have difficulties with the number line (Rodriguez et al., 2001), which was also evident in our piloting of the tasks. Since we addressed students with MD, we decided to use tasks with a low difficulty level with number lines ranging from 0 to 100 (e.g., Department of Education, 2013). The study took place in individual sessions in a quiet room at school. We used two different number line tasks: In the "position-to-number-task" (PN) (Figure 1), we showed the students different positions (red cross) on the number line (numbers: 80, 40, 60; in that order) and asked them to name the corresponding numbers.

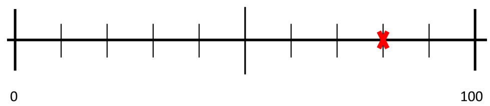

Figure 1: Position-to-number-task

In the "number-to-position-task" (NP), the number line did not have a red cross, but the students were asked to place presented symbolic numbers (70, 30, 90; in that order) on the number line. The numbers were displayed on the screen in the upper left corner before the number line appeared. Students were instructed to read the number aloud, to be sure that the number was perceived correctly. After that the number line appeared. Students were instructed to point at the place of the target number and to fixate this place with their eyes. Before each type of tasks, there was a practice task for the students. In between the tasks, the students were instructed to fixate a star displayed on the screen in the upper left corner, so that for all tasks and students the gazes started from the same place. We recorded students' oral responses with an audio-recorder. The students were not given feedback to their answers.

ET device. We used the Tobii Pro X3-120 eye tracker (120 Hz, binocular, infrared) to record the students' eye movements. This eye tracker was attached to a 24" full HD computer screen on which the tasks were displayed. Students were seated approximately 50 cm from the screen. The accuracy for our ET data was 0.8°.

Data analysis. For our data analysis, we used gaze-overlaid videos (eye gazes represented as a semi-transparent dot) provided by Tobii Pro Lab software. We analyzed the data inductively, following Mayring's (2014) qualitative content analysis. First, we *described* student's eye movements in the video. Then, we *paraphrased* the elements relevant to student strategies. Last, we *developed* the *categories*, that is, we inductively assigned categories with corresponding descriptions, and then *revised* the *categories*. From the content analysis, the following six categories of strategies emerged that the students used to locate numbers on the marked number line:

1. Direct locating: Students located numbers without looking at reference points (e.g., starting point or endpoint), that is, their gazes went immediately to the correct position.

2. Starting point use and counting: Students counted the given marks. They looked at the marks one by one—from the starting point to the target position or vice versa.

3. Midpoint use and direct locating: Students looked at the midpoint of the number line and located numbers directly from this reference point in one step. The midpoint strategy was used for numbers located to the left or right of the midpoint.

4. Midpoint use and counting: Students located numbers by looking at the midpoint of the number line and counting marks starting from that reference point.

5. Endpoint use and direct locating: Students looked at the endpoint of the number line and located numbers directly from this reference point, that is, they looked at the endpoint, and located the number from there in one step.

6. Endpoint use and counting: Students located numbers by looking at the endpoint of the number line and counting marks starting from the endpoint.

We made an a priori estimation of which strategies can be expected for each task (Table 1). For tasks with numbers larger than 50 (i.e., 60, 70, 80, 90), strategies with *endpoint use* are applicable. For tasks with numbers smaller than 50 (i.e., 30, 40), using the endpoint is theoretically possible, but unlikely being used here. For the numbers 40 and 60, when the strategy is *midpoint use*, there is no difference between *direct locating* and *counting*. The same applies to the number 90 and *endpoint use*. Therefore, these gazes are categorized as *midpoint* or *endpoint use* and *direct locating*.

		Number					
		30	40	60	70	80	90
Startpoint	Direct	x	x	x	x	x	x
	Counting	x	x	x	x	x	x
Midpoint	Direct	x	x	x	x	x	x
	Counting	x			x	x	x
Endpoint	Direct			x	x	x	x
	Counting			x	x	x	

Table 1: A priori estimation of the expected strategies
("x" indicates expected strategies for the respective number)

Based on the category system, one rater experienced in analyzing gaze-overlaid videos (first author of the paper) coded all data and another rater (second author) coded 20% of the ET videos independently. We calculated the interrater reliability using Cohen's kappa. The interrater agreement was 0.83, which is considered to be almost perfect.

Statistical analysis. To analyze differences in the use of strategies between students with and without MD, we carried out chi-square tests using SPSS 28. Effect sizes were calculated using *Cramér's V*. A total of 240 tasks were included in the analyses (60 tasks per group for PN; 60 tasks per group for NP). There was no data loss.

RESULTS

In the following, we answer the research question: *Do students with and without MD differ in their use of strategies in marked number line tasks?*

A chi-square test with the accumulated strategies for all tasks together (Figure 2) revealed significant differences, with small effect size, in the distribution of strategies between the students with and without MD: χ^2 (5) = 16.97, p = .005, V = .27.

In detail, cell tests for differences between the two groups revealed that students without MD used strategy *direct locating* (i.e., strategy 1) significantly more often (small effect size) than students with MD (χ^2 (1) = 14.65, $p < .001$, $V = .25$).

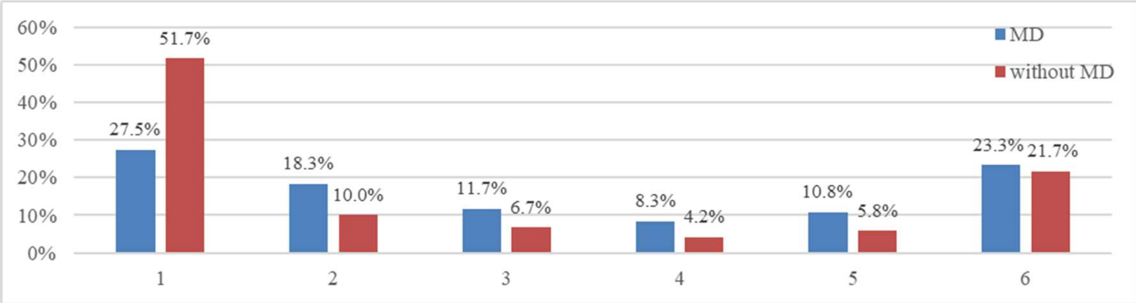

Figure 2: Distribution of student strategies over all tasks

A comparison of the summarized strategies that can be classified as *counting* (i.e., strategies 2, 4, 6) and the summarized strategies that can be labelled as *direct* (i.e., strategies 1, 3, 5) (Figure 3) showed that students with MD used *counting* strategies significantly more often, whereas they used direct strategies significantly less often than students without MD (χ^2 (1) = 4.92, $p = .027$, $V = .14$) (small effect size).

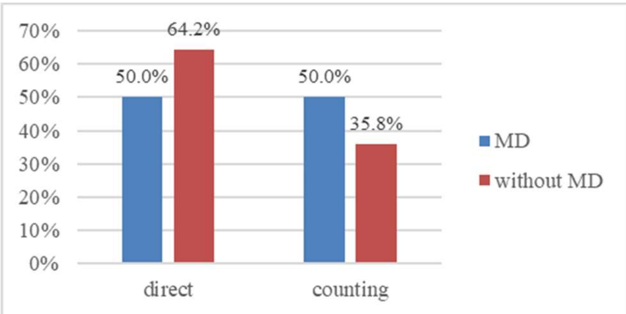

Figure 3: Distribution of students' counting and direct strategies over all tasks

DISCUSSION

The aim of this study was to investigate if students with MD differ from students without MD in their use of strategies to locate numbers on the marked number line. To pursue this aim, we qualitatively analyzed ET videos and developed a category system. We then investigated students' strategy use based on the qualitative ET data.

Our analyses indicate that strategy use differed significantly between students with and without MD. This relates to findings on different strategy use on the empty number line of students with MD compared to students without MD (e.g., van Viersen et al., 2013; van't Noordende et al., 2016). While students without MD used direct locating more often, students with MD counted more often. This is consistent with findings in previous studies on quantity recognition: Students with MD predominantly used counting strategies, whereas students without MD more often used structures and direct strategies (Schindler et al., 2019; Schindler et al., 2020).

Even though effect sizes are small, our findings indicate that students with MD differ in strategy use from students without MD. This is interesting given that the tasks were supposedly easy for the age of the students (e.g., Department of Education, 2013). This implies that locating numbers on a number line needs to be partially supported at the beginning of secondary school—even for this range of numbers. For students with MD, ways of accessing numbers on a number line should be fostered: For example, the relational interpretation of given structuring features should be addressed (Schulz & Wartha, 2021) as well as different reference points and their use for locating numbers.

As mentioned earlier, the number line is an important tool in mathematics education that can contribute to the development of the concept of numbers (Diezmann & Lowrie, 2007; Schulz & Wartha, 2021). Our study provides insight into student strategies when working on marked number line tasks. Our analyses of ET video data revealed students' strategies on the marked number line that had not been previously reported in this form. In the future, further research could examine if even more clear differences between students with and without MD are evident in more advanced tasks. A possible limitation of our study is the relatively small number of tasks. Further research should investigate if our results can be generalized to a larger set of tasks.

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