TEACHERS' PD UPTAKE: HOW VISUAL REPRESENTATIONS IMPACTED MATHEMATICS TEACHING

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This study provides an in-depth examination of two teachers who participated in a mathematics professional development project that focused on linguistically responsive teaching and what the teachers took up and used in their classrooms 3-4 years after their participation in the project. Survey, interview and classroom video data were analysed in order to explore the ways in which the teachers' learning from the PD endured over time. Results indicate that the teachers remembered, continued to use and hone their use of visual representations as a strategy to provide access to English learners. These strategies and implementation use were aligned to the goals and intention of the PD. Furthermore, they extended and transferred this knowledge to other content areas and to remote teaching settings.

INTRODUCTION

High-quality professional development (PD) in mathematics education is considered the key to improvement of students' mathematics learning. One commonly accepted trajectory of teaching and learning suggests that gains in teacher knowledge can lead to changes in instruction, which in turn has a positive impact on student learning (e.g., Yoon et al., 2007). Current trends related to research on mathematics PD have started to show more evidence of change, mostly incremental changes in teachers' knowledge and instruction, and somewhat less in terms of the more distal, student achievement (Jacobs, et al., 2019). We posit that teacher learning and implementation of new ideas and strategies takes time. This paper examines the instructional changes related to the goals and intentions of a US nationally funded PD that focused on linguistically responsive teaching (LRT) 3-4 years after participating in the project. Specifically, the research question that guided this work is: What specified and intentional instructional LRT practices did the teachers take up and use 3-4 years post PD experience?

THEORETICAL FRAMEWORK

Mathematical Knowledge for Teaching – Visual Representations (MKT-VR)

Ball and Bass (2000) were the first to coin and define *mathematical knowledge for teaching* (MKT), the requisite knowledge to effectively teach mathematics to K-12 students. This knowledge is complex and includes both content and pedagogical knowledge and provides the field with baseline knowledge to focus on in teacher preparation programs and professional development experiences for teachers (Jacob et al.,2017). DePiper and Driscoll (2018) were inspired to think about the need to further theorize the MKT constructs and created the MKT-VR theoretical framework. They define visual representations (VRs) as graphic creations such as diagrams or drawings

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that illustrate quantities, quantitative relationships or geometric relationships. Using models or representations is an important component of doing mathematics and especially important for English learners, a cornerstone for LRT (DePiper et al., 2021). VRs support students to make sense of problems by identifying quantities and the relationships between quantities in order to use VRs to reason with and ultimately justify mathematical solutions (Ng & Lee, 2009). However, this knowledge is complex and requires challenging skills including a strong grasp of the content, anticipation of students' thinking, and selecting the most appropriate VRs for particular purposes.

OVERVIEW OF TADD PROJECT

This paper highlights a project that is part of a large three-year impact study, *Taking a Deep Dive* (TaDD), that collects qualitative data from three large U.S. National Science Foundation PD projects in order understand what teachers take up and use and the factors that influence uptake 3-4 years post PD experience.

Visual Access to Mathematics (VAM)

The VAM PD, the focus PD of this paper, is a "60-hour blended, face to face and online course to build teachers' knowledge of and self-efficacy about LRT strategies to strengthen English Learners (Els) problem solving and discourse in middle grades" (De Piper et al., 2021 p. 491). The goals and intentions of VAM were to cultivate in teachers the fluent use of representations, anticipation of students' strategies, the ability to interpret and construct various mathematical solutions, and to reason with and across representations. Teachers learned how to strategically select and align VRs with their instructional goals, anticipate student thinking and misconceptions, and then implement lessons using these strategies in their classrooms. Once implemented they would share experiences and student work, and collaboratively and independently reflect on the teaching cycle in the VAM PD online workshops.

In particular, VAM focused on two VRs, the double number line (DNL) and tape diagrams. Both VRs are effective tools that have the potential to foster students' understanding of proportional reasoning and reinforce students' conceptual understanding of rational numbers (DePiper & Driscoll, 2018). The DNL is a representation that uses a pair of parallel lines to represent equivalent ratios. Tape diagrams, also referred to as bar diagrams, are rectangular representations that illustrate number relationships. Both diagrams represent quantities and the relationships between quantities, allow students that think more additively to "see" multiplicative relationships and the relationships between quantities with the representation. VAM focused on problem solving with rational number tasks that were easily represented on a DNL or tape diagram and actually many different representations. Subsequently these VRs were used as a communication tool to show and explain students' mathematical thinking in a very concrete and conceptual manner. We selected VAM as a case because the PD illustrates evidence of high uptake and at the same time provides evidence that teachers continued to hone their craft, modify and expand their use of VRs using different mathematical problems, domains and contexts.

Participants and Settings

This paper focuses on two teachers, Kimberly and Rachel (pseudonyms), 3-4 years post participation in the VAM PD to address the research question: *What specified and intentional instructional LRT practices did teachers take up and use 3-4 years post the PD experience?* We purposefully selected two teachers that showed high levels of uptake in the surveys we initially administered (n = 66) across all three projects. We wanted to dig deeper to better understand the factors that influenced their high levels of uptake. They were also chosen because they used and modified strategies and VRs learned in the PD while teaching remotely during the COVID 19 pandemic. Although the intent of this project was not to study remote teaching and learning, the pandemic changed the nature of the classroom data we collected and allowed us to explore the ways in which participating teachers used and modified representations in the online setting. Both teachers taught middle school mathematics in the northeast US and both used online platforms to teach synchronous mathematics lessons to their students in the data collected in this study.

Data Collection and Analysis

Multiple data sources were collected in order to comprehensively understand of the participants' uptake and enable triangulation of findings (Cresswell & Poth, 2018). These data sources included survey data, interviews, and videotapes of classroom instruction. The survey included questions that asked participants to reflect on their PD experience and characterize their past and/or current use of the PD content, pedagogy and materials. The survey included both Likert scale questions, in which participants responded to statements on a scale of 1-10, as well as follow up questions that allowed the participants to explain and provide more details about their numeric responses.

Following the completion of the survey, 17 case study teachers from across the three projects were asked to videotape their classroom once a month and identify clips in which they believed that they were using content, pedagogy and/or resources from the PD they participated in. Kimberly and Rachel were interviewed twice during spring and fall 2021 school year by the TaDD project research team for approximately one hour each. The first part of these interviews included questions aimed to understand the teachers' experiences with the PD, what they remembered related to the goals and intentions of the PD and what strategies, content and resources they used from the PD in the past and continue to use currently in their classrooms. The second part of these interviews followed a *think aloud* protocol, where teachers watched video clips they selected and described their perceived uptake and implementation of content, pedagogy, and resources from the PD. These interviews were recorded on Zoom and transcribed.

At least two of the research team members reviewed and took detailed notes on the survey, interview and video data several times to create a profile for each teacher. These profiles were analysed and coded for segments that related to participants' use of representations using the MKT-VR theoretical framework. Sample codes included

use of DNL, use of tape diagrams, alignment of DNL/tape diagrams with teaching goals, and use of VRs to represent quantitative relationships. The members of the research team then met to discuss what they noticed, and identified salient themes and patterns that emerged. The three themes that emerged were related to (a) knowledge and uptake of new VRs, (b) use of VRs in remote settings and (c) application of VRs to other content areas. Findings from all the varied sources were validated through a triangulation process. For example, data from the individual teacher's surveys, interviews and classroom videos were matched for convergence and divergence. Following this, narratives were written for each of the three themes and reviewed for consistency and alignment across the data sources.

FINDINGS

Knowledge and uptake of new visual representations

Survey Data. Kimberly and Rachel both perceived high levels of uptake in terms of content, pedagogy and resources in their initial survey responses. They mentioned that they learned new VRs in VAM, in particular the DNL and rectangular tape diagrams. Rachel mentioned that she saw a DNL prior to VAM and wrote:

> I never really understood the purpose of them until I saw how many different ways they can be used to represent a situation and solve problems.

Kimberly reported that she enjoyed the DNL activities and shared the applet with her students. From both teachers indicated on their surveys that they implemented the VRs and perceived them as relevant and helpful teaching strategies.

Interview Data. Prior to participating in the PD, neither participant was familiar with using these VRs to solve problems and had not used them in their classrooms. In her interview, Kimberly said:

> I remember when the Common Core Standards came out and we were like, ‘What’s a tape diagram? I don't know what a double number line is. How can I figure out how to use these tools in my classroom?

Similarly, Rachel mentioned in her interview:

> I didn’t know a lot of the representations that they were teaching us [in VAM]. I had been teaching middle school math for 8 years and I had never used a DNL. I was solving these problems using proportions or equations and I never knew this thing existed.

Prior to the PD, Rachel mentioned that she used equations to solve proportions, as she did not know about the VRs options. This aligns with the research indicating that when teachers are unfamiliar with how to use these tools they typically rely on algorithmic thinking to solve these types of ratio and proportional reasoning problems (Orrill & Brown, 2012).

After the PD, Kimberly and Rachel began to incorporate these representations in their teaching and continue to use them. Interview data reflected their uptake of the tape diagram and DNL in their classrooms. For example, Rachel noted in her interview:

> I started to take the DNL and completely change the way I teach ratio and proportion and percent and I started to use the tape diagrams and the DNL for everything. I still am using the materials from VAM for those units.

Video Data. The classroom video data aligns with the perceptions they shared with us during the interview and on their surveys about their use of the representations in their classrooms. Video clips include examples of them modelling the creation of DNLs and tape diagrams, asking students to examine different tape diagrams and DNLs and having students create their own tape diagrams and DNLs. When talking about one of her video clips, Rachel discussed her use of the DNL in a unit rate lesson and how she would never have thought to use it before VAM. She mentioned that her goal in the lesson was to help students make connections between what was going on in the context of the problem and the visual representation of it.

The two teachers also strategically chose VRs and used them to promote quantitative reasoning. They encouraged their students to use the diagrams to reason about the relationships among quantities in the problems. The teachers also articulated why they chose to use certain representations. In discussing one of her classroom clips in which she displayed tape diagrams to represent different percent amounts, Kimberly noted:

> In VAM, we used a lot of tape diagrams which I don't think I had used as much beforehand. So that was a newer model to me. I think a lot of teachers want to go back to the pie, you know the fraction pie, but I think the tape diagram leads us to the DNL which is so useful with percent so I definitely took that away-- starting with that tape diagram talking about percent and fractions and leading them to the DNL later on.

Both teachers demonstrate this strategic use of representations throughout their video clips and explained how their choice of representations in their lessons align to their instructional goals in the interview data.

Using visual representations in a remote setting

Video Data. Kimberly and Rachel adapted their use of VRs to their remote settings. In some instances, they had students draw VRs on their papers and hold them up to the camera. In other situations, they used online platforms such as PearDeck to display teacher created representations as well as to allow students to create and display representations to the class. One feature of these online tools that they noted was to the ability to give immediate feedback to the students about their representations. The teachers saw the students' diagrams in real time and gave feedback on the labelling or accuracy of the representation. The video of the two teachers showed their use of these tools to strategically select students' representations to share with the class, which allowed students to communicate their mathematical thinking.

Interview Data. Both participants noted in the discussion of their video clips is that one of the ideas that resonated with them from the VAM PD was the use of VRs as communication tools. Kimberly mentioned:

> I think one thing I took away was getting kids to talk about their models or look at each other's models because I don't think we always did that before. We had them draw their models and then that was it. But I think one thing I really like to do is have kids draw a model, and then have somebody else look at it and try to see what connections that student made. That was one thing we definitely did during the PD that I continue to do.

In one of Kimberly's classroom videos, she demonstrated how she used representations as a communication tool by having students sometimes hold up the VRs they drew and at other times, displaying one of the VRs on the slides they had created from the online PearDeck slides. Then, another student explained the diagram that was displayed. Similarly, in one of Rachel's videos, she selected and displayed two tape diagrams students created during the lesson. She then asked the class to compare them and discuss what they noticed and what they would change about the representations. In the interview, both teachers noted how the online tools made it easy to select and display students' representations.

The two teachers also noted some challenges that occurred in the remote setting. One was related to creating classroom records of work. Kimberly mentioned:

> I usually annotate a representation as a student is explaining in the classroom and leave that up and that's harder to do in the electronic world as they sort of disappear as we go slide to slide.

They also discussed the challenges of group work in an online setting and the ways in which they need to adapt tasks and use technology to address these challenges.

Application of representations to other content areas

Interview Data. Although the VAM PD focused on proportional reasoning, interview and video data from both participants indicated that they applied what they learned about VRs to other content areas. Kimberly noted:

> I like to have visual representations in everything we do. The VAM PD focused on ratio/proportion but I have taken it though the whole curriculum. They didn't give us models for other topics but when I see them I know what they should look like. You leave with this idea of this is what a good model looks like.

Kimberly added that she now uses mobiles to teach algebra because they allow students to develop a visual understanding of the quantities in an algebraic equation.

Video Data. Similarly, a video clip from Rachel demonstrated how she uses tape diagrams in her algebra unit. She noted:

> I have never taught tape diagrams with expressions and equations until this year. But it was a really interesting way to think about not just the distributive property (to show them how you can chunk things up differently and still get the same value) but in terms of solving equations thinking of what you can get rid of immediately and what you have left.

Years after the PD, she incorporates what she learned about VRs in new ways and applies these ideas to new content areas. While the VAM PD did not focus

representations in these content areas, the teachers credited the PD with their extension of representations throughout their other units.

DISCUSSION AND IMPLICATIONS

This in-depth examination of teachers' classroom practices 3-4 years after participating in the VAM PD showed that their uptake of VRs continued in ways that connected to the goals and intentions of the VAM PD. The two teachers explain and demonstrate the ways in which they select and use specific representations, in particular tape diagrams and DNLs, in their classrooms. Classroom video data supports their survey and interview data and illustrates the ways in which these teachers use VRs to foster an understanding of important mathematical ideas related to ratio and proportional reasoning. The teachers also use representations to assist student' communication and explanation of their mathematical thinking. This knowledge and use of representations aligns to the goals and intentions of the PD developers.

These findings provide insight into the ways in which teachers continue to take up ideas that they learned in PD, years after their participation and the ways in which they adapt and apply them to novel contexts. In the case of these two teachers studied, knowledge related to VRs endured over time and is evidenced in their practice. Furthermore, while the initial PD did not focus on remote settings or content areas outside of proportional reasoning, these participants were able to transfer and extend their knowledge and use of VRs to these contexts. They also use online tools to provide immediate feedback on the accuracy and labelling of students' representations as well as to strategically select students' representations to share with the class. They incorporate the online tools to allow students to communicate their thinking related to the representations they create and to allow other students to comment and unpack the diagrams that their classmates create. This adaptation to remote learning aligns with the initial goals of the PD of having teachers strategically select VRs and use them as communication tools.

From our case studies presented here, it appears that learning strategies focused on LRT were transformative. Specifically, VRs added to teachers' pedagogical toolboxes in ways that cut across mathematical domains, types of teaching (online to face to face) and uses for different learners. We hypothesize that this type of flexibility that allows teachers to hone their craft over the four years and show high levels of uptake. Additionally, this PD was optional and teachers that elected to participate were teaching in schools with high numbers of Els so the strategies that they picked up were important for their context in order to reach and be successful with students learning English. These predictors of uptake need further examination to identify the other possible predictors of high levels of impact. All classroom video included in the analysis were of remote instruction and we are still unclear how this translates to in-person classroom practice. Our next steps are to continue this work as teachers transition back to in-person teaching. We also plan to conduct cross-case analyses in order to understand the similarities and differences that may exist across different PD

projects. While our paper reports on a small-scale case study, it contributes to how we conceive and theorize about teacher learning and the importance of recognizing that learning happens over time. Over the past two decades, the studies on teacher PD that show incremental change are possibly the seeds which call for further exploration, as a pre- post randomized control study might not provide enough time for planning, implementation and reflection to support teacher learning.

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