SINGLE UNIT COUNTING – AN IMPEDIMENT FOR ARITHMETIC LEARNING

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In this paper we direct attention to the single unit counting strategy that is observed to be limiting students' opportunities to develop their arithmetic skills. We describe what impediments single unit counting may entail when encountering novel subtraction tasks, and how these impediments can be explained. From a sample of 121 interviews of students aged 7-8 we have chosen nine who were using single unit counting as their dominating arithmetic strategy. An analysis based on variation theory reveals that the impediments are related to the students not experiencing numbers as composed units and thereby lack in discerning number relations necessary to handle multi-digit subtraction. Educational implications are discussed grounded in the theoretically driven findings.

INTRODUCTION

In the large body of research on young children's arithmetic development, many scholars have described strategies for arithmetic problem solving (e.g., Fuson, 1992; Baroody & Purpura, 2017 for overviews), ending up in recommendations for how to teach arithmetic strategies (e.g., Baroody, 1987; Torbeyns et al., 2004). What seems to be lacking is however a critical view on the observed strategies, whether the observed trajectory in fact reflects a powerful path to proficient arithmetic problem solving skills, which likely have implications for teaching practices. In this paper we aim to raise attention to particularly one of the basic strategies observed in early arithmetic development and in teaching – single unit counting – and raise some issues on what implications students' single unit counting strategies may have in a long-term perspective. The specific research questions are: 1) what impediments may single unit counting as the dominating arithmetic strategy entail when encountering novel subtraction tasks, and 2) how can these impediments be explained? To answer these questions, we analysed task-based interviews with school beginners (age 7-8 years in Swedish schools).

RESEARCH ON EARLY ARITHMETIC STRATEGIES

There are many studies describing the trajectory of arithmetic skills development, but in this paper, we direct specific attention to single unit counting. Counting strategies are ways to keep track of counted units, either by raising one finger at a time (representing the single units counted) or by making markers, one for each counted unit. These are commonly observed among young children (Laski et al., 2014) and

2022. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.). *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 363-370). PME.

some researchers find this to be a normal step in the trajectory of learning arithmetic (Fuson, 1988). Furthermore, counting by ones may also entail double counting that is keeping track of the sequence of words that become the entities to be counted, a strategy which Steffe (2004) interprets as the student having constructed a scheme of the number sequence which in turn bridges to strategic arithmetic reasoning and is thereby considered a higher level of functioning.

Single unit counting strategies (such as “counting all” and “counting on”) may solve simple arithmetic tasks but do not *per se* support students’ recognising a part-whole structure of an arithmetic task, which becomes necessary to solve more advanced tasks, for example multi-digit subtraction. The shortcomings of single unit counting as the dominating strategy among young students were shown in Neuman’s (1987) studies and later in studies by Ellemor-Collins and Wright (2009), in which students who rely on single unit counting strategies did not develop efficient arithmetic skills. Research influenced by cognitive science conclude that students are ”forced” to develop more advanced strategies when the number range exceeds 10 and concrete units like fingers can no longer support their keeping track (Carpenter & Moser, 1982). However, there seems to be a need for educational interventions for some students to learn to discern the relation between and within numbers and thus make use of more efficient strategies than single unit counting. What is it then that these students lack in order to develop and broaden their repertoire of arithmetic strategies? This becomes a critical issue in educational research and practice, because studies from teaching interventions show that learnt single unit counting strategies are not easily abandoned by students (Cheng, 2012).

When arithmetic strategies’ limitations are discussed in the literature, this is mainly in relation to students having mathematics difficulties (e.g., Ostad, 1998; Geary et al., 2004). Not surprisingly, these studies show that students who rely on counting strategies have difficulties solving novel problems because they lack in conceptual understanding of arithmetic. From the research literature, we see that students are observed making use of single unit counting strategies but often abandon these for more powerful strategies. Nevertheless, research also shows that this is not true for all students and normally developing students may prefer the cumbersome strategies even when encountering larger number ranges (Ellemor-Collins & Wright, 2009). It surprises us that not more attention is directed towards the single unit counting strategy and the limitation it may entail for students’ developing arithmetic skills.

METHODS

This is a study of young students solving arithmetic tasks, as part of a larger project focusing on early arithmetic teaching and learning. Teachers and students from five elementary schools participate in the project. The classes were selected due to their teacher’s interest in participating in a practice-based research project where their teaching was target for development and study. To follow any learning progress among the students in these classes, each student was asked to participate in task-based interviews at three occasions. The students' legal guardians were asked for written consent, which included the option of participating in video-recorded interview or only audio-recorded. This ended up in 121 students participating in the interviews at three occasions.

Procedure

The interview guide consists of arithmetic tasks given orally or on paper. To increase verbal reasoning and reflections among the students, no manipulatives were given to the students. The interviews were conducted individually at the students' own schools, by researchers trained in interviewing young students. The interview occasions were done at the beginning of Grade 1 (Interview I), at the end of Grade 1 (Interview II) and at the end of Grade 2 (Interview III).

For the purpose of this particular analysis, we selected subtraction tasks from the interviews that were the same in all three interviews: **10–6=_, 15–7=_, 24–_=15**, 14–_=6, a new subtraction task given in Interview II: **32–25=_**and new ones in Interview III: **57–38=_,** 83–7=_, 204–193=_, 204–12=_ (**bold** = oral context based tasks, e.g., "you have ten candies and eat six of them, how many are left?", normal text = written tasks with only numerals).

Analysis

All students participating in the interviews were coded for strategy use, either *Counting* or *Structuring.* If a student was coded as structuring, it meant the student reasoned her way to an answer to a particular task by experiencing numbers in the task as parts and whole, dealing with larger units than one (e.g., task 15–7=_ "I take five from the seven, that makes ten and then two more and I have eight left). For the purpose of our research interest, we selected those students who at the *first interview only made use of a single unit counting strategy* when attempting to solve the three subtraction tasks. We chose subtraction as target tasks, since these are more likely to induce counting-strategies if a student does not have a repertoire of number facts or know how to apply the complement principle (thus experiences numbers' part-whole relations and being able to use structuring strategies). This ended up in a sample of 39 students. These were followed through the second and third interview, resulting in three groups: students who abandoned the single unit counting strategy and approached subtraction tasks in the later interviews by structuring numbers (N=26), students who expressed a mix of structuring and single unit counting strategies (N=4) and students who remained using single unit counting as the main strategy in all three interviews (N=9). The nine students who remained using single unit counting through all three interviews are chosen for further analysis in this paper.

To answer RQ1, we did a qualitative analysis of the nine students who remained using single unit counting throughout all three interviews, to find out how they encountered novel tasks. To answer RQ2, principles from variation theory (Marton, 2015) were used as analytical tools. The theory states that powerful strategies stems from powerful ways of experiencing, which presupposes the discernment of critical aspects of what is learned (the object of learning). From a variation theory perspective, learning difficulties, e.g. to solve a subtraction task like 83–7= is explained in terms of not (yet) having discerned certain aspects of the task, the numbers involved and relations between and within them. This way of analysing students' responses to arithmetic tasks ends up in categories that reveal qualitative different ways of experiencing numbers in a task. The answer to our research questions is thereby shown in such categories, where those aspects of numbers that a student discerns constitute his or her way of knowing and thus what strategies he or she is able to execute in completing the arithmetic task. The categories found among the nine single counting strategy users in our sample are presented in the following.

RESULTS

In the first and second interview we observe the students solving the subtraction tasks by counting down in ones and using fingers to keep track of counted numbers, or if the numbers exceeded the student's fingers, using other objects (e.g. sheets of paper). The strategy counting single units was thereby considered strong in the selected group of students for our further analysis. When analysing the students' ways of solving novel tasks in the larger number range in interview III, certain problems emerged that direct attention to aspects that seem to become critical for these students to discern in order to develop arithmetic skills that allow them to try to solve novel tasks in a larger number range. All of the nine students were primarily counting down in ones, but encountered difficulties when the subtrahend (to be counted down) was larger than they managed to keep track with their fingers. Thus, they had to make use of some other strategy to complete the task, usually operating with numbers in similar positions (similar to a written algorithm line-up). The strategies these students apply in their completing the tasks may bring an answer to the task, sometimes even correct ones, but as a recurring strategy we here aim to interpret how such encounters may become an impediment in the students' development of arithmetic skills.

Cannot create a composite unit of single entities

Our observations show that the students rely on counting single units as a primary strategy. In novel tasks where the number range is larger, they operate the task as a "counting down" act on the number sequence.

Task: 204–12=_

Jonas: 203, 204 (folds down one finger for each said counting word) Wait. 203, 202, 201 (stops) 200 (stops) 199 (stops) eeh, 198, 196, 194, 193 (hesitates) 192, 191 (still folding one finger for each counting word).

This observation of the student Jonas is typical. The students operate on the number sequence, but need to keep track with their fingers. What stands out is that each number (counting word) appears as a single unit and particularly bridging hundreds (or tens) does not indicate a benchmark to them. When experiencing numbers as single units in this way, ten or hundred do not mean a composed set of "ones" and thus becomes one number just like any other number in the long line of numbers in a sequence. This way of experiencing numbers makes the counting sequence an important asset to apply the single unit counting act on, which we in the observation above can see becomes an obstacle when counting "backwards" while having to keep track of the number of counted (spoken) counting words. When the number of counted single units (the subtrahend) is large, this entails a severe challenge, because of the difficulties to keep track of counted units.

Task: 57 – 38 =_

George: (unfolds one finger at a time on his right hand, then on his left hand and on the right hand again) Thirty. It's thirty.

Interviewer: Did you count up or down?

George: Up, no down, down, down from 57. To 25, I think.

The student George encounters a problem when it becomes necessary to keep track of single units and does not experience any benchmark in the counting sequence that could indicate larger units to relate to. The same student George responds to the task 204–193=_ by saying: "Wait, this one is impossible. It's too difficult". His response indicates that the strategy he executed in earlier subtraction tasks would not be helpful in solving the subtraction task with such a large subtrahend (that is, counting down 193 single units). He does not either try to solve the task by any other strategy.

Number relations – What to add and what to subtract

Students who realize they cannot execute the "counting down in single units" strategy when encountering the subtraction tasks may turn to another strategy based on an algorithmic-like approach. This means, the students are operating on the numbers based on their position, reminding of written calculations. However, when executing this strategy mentally, our observations reveal that these students do not necessarily experience multi-digit numbers as composed units, but rather operate on the numbers as if they were single units. We can see expressions of this way of experiencing numbers when students complete the task 83–7=_ by first operating on the three and the seven, then realizing the eight should also be part of the operation, for example as one of the students, Vera, starting with "three plus seven", then continuing saying "eighty… eight-hundred-ten, no, eighty, eight-hundred-one". This way of reasoning indicates difficulties in experiencing how numbers relate to each other and particularly how ones and tens, as well as hundreds relate. Below is another example of a similar way of experiencing numbers that frequently appear in our sample when encountering larger number ranges.

Task: 204–193=_

Jenny: (unfolds index finger, folds it again) It's one hundred ninety one. Because you take the four minus three, and then the zero minus nine makes nine and then two minus one, that's one.

The student Jenny also seems to experience numbers as single units that are to be treated as individual entities rather than composed units of tens or hundreds. The student Vera seems though to experience some sense of value difference between numbers, since she claims the result of her operation cannot become "more" as in eight hundred one. Nevertheless, the relation of ones and tens are not discerned by her. The same way of experiencing numbers is observed in how the student Jenny attempts to solve 204–193=_ by subtracting each number as "taking the smaller from the larger" and disregarding any meaning of the positions that the numerals are presented – the numbers are not related to one another as would be necessary to experience the idea of the base ten system and positioning of numbers. This way of experiencing numbers induces that, what is part and what is whole are not discerned. The students seem to attend to some kind of algorithmic-like strategy but they do not experience numbers' relations within the task, such as how digits in a multi-digit number represent tens or hundreds.

DISCUSSION

The conclusion we draw from the analysis above, is that single counting units becomes an impediment for these students when encountering multi-digit subtraction tasks, which confirms Ellemor-Collins and Wright's (2009) as well as Neuman's (1987) observations. We add to these observations that these students' ways of completing subtraction tasks may be explained by their way of experiencing numbers and the meaning assigned to numbers and their relations. When students are experiencing numbers as single entities rather than composed units, they are not discerning the relations between parts and whole within numbers and thereby not relations between numbers either. That is, ten is not seen as a benchmark either. Tens and hundreds are merely experienced as single numbers in a long line of numbers and do not represent composed units, which is why ten is not taken as a benchmark to help structure their problem solving.

To recognize and make use of number structure builds on the student experiencing numbers as composite sets that can be decomposed, and that there are numerical relations between and within numbers. For example, in subtraction the subtrahend can be decomposed into two parts in order to bridge the nearest ten (e.g., 83–7=_ , 7 is decomposed and 80 is a benchmark, 83–3=80, 80–4=76). Number relations do not appear when counting single units, for instance when keeping track of counted units on the fingers or by making markers, because number relations and experiencing units larger than one are not needed to solve the task. To prevent un-developable strategies among students and support conceptually founded knowledge, some researchers advocate that a structural approach to arithmetic problem solving, which primarily directs attention towards relationships between numbers in a task (Venkat et al., 2019) and making use of part-whole relations rather than single unit counting strategies, should be emphasised already in the early years (Brissiaud, 1992; Davydov, 1982; Neuman, 1987; Polotskaia & Savard, 2018). In following reports, we will do analyses of the teaching conducted between the interviews, to find possible keys for how teaching may influence arithmetic development that apparently is necessary for the students in our sample.

ACKNOWLEDGEMENT

This work was supported by the Swedish Institute for Educational Research [Grant number 2018-00038].

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