# MATHEMATICS TEACHER EDUCATORS' WORK TO FOSTER AN INQUIRY COMMUNITY

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*For several years, the study of mathematics teacher educators (MTEs) has been considered central in research. The present study is focused, in particular, on MTE expertise in generating documents for the work with teachers, during a professional development (PD) program. We aim to analyse how MTEs generate documents coherently with the goal they have set for the PD program itself. In the case study presented in this paper, MTEs' goal is building an inquiry community with the teachers. The results illustrate what kind of resources are involved in MTEs' documentational genesis and how MTEs' choices in their documentation work are connected with their goal for the PD program.*

## INTRODUCTION AND THEORETICAL FRAMEWORK

Working Group 3 (WG3) at PME44 conference was on the theme "Conceptualizing the expertise of the mathematics teacher educators" (Helliwell, & Chorney, 2021), in continuity with the WG on the same topic at PME 43 (Helliwell, & Chorney, 2019), testifying the centrality of mathematics teacher educators' (MTEs) role in current research. Two of the aims of WG3 at PME44 were: 1) "to formulate approaches and research questions around MTE expertise"; 2) "to explore and develop potential methodologies that support these approaches and research questions." (Helliwell, & Chorney, 2021, p. 97).

The study presented in this report is framed on these aims, which give space for new fields of investigation. The authors' research group has been involved for several years in the study of teacher professional development, introducing theoretical models for the analysis of teachers' and MTEs' practices as part of a community evolution process (e.g. Robutti, 2020; Robutti et al., 2021). The report is focused on MTE expertise in generating documents for teacher professional development (PD), analysed with the Documentational Approach to Didactics (DAD: Gueudet & Trouche, 2009; 2010; 2012). Specifically, we study MTE expertise in generating documents consistent with the goal they set for the PD program, through a case study involving a group of researchers in mathematics education. The researchers (including the authors) have the role of MTEs and have the task of designing and implementing a teacher PD program. We identify the relationships between their documentation work (Gueudet & Trouche, 2009; 2010; 2012) and their goal of building an inquiry community (Jaworski, 2006; 2008) with the teachers participating in the PD program.



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## Inquiry communities

Jaworski (2006, 2008) introduces the term "inquiry community", referring to didacticians (researchers, who are also teacher educators) and teachers working together, exploring and developing mathematics teaching-learning in classrooms. Jaworski's vision of inquiry communities is based on the concept of co-learning inquiry, that means people learning together through inquiry. Inquiry, in this context, is intended both at class level and at teachers and didacticians level, while exploring how to use inquiry-based tasks with their students (Jaworski, 2006). The idea of inquiry community is built on Wenger's (1998) idea of communities of practice, whose members experience engagement, imagination and alignment of shared practices. In this approach, alignment means engaging in forms of practice and ways of being, in order to conform to expectations and to the "normal desirable state". The difference with respect to communities of practice is that, in an inquiry community, the "normal desirable state" is continuously challenged, with a questioning attitude (Jaworski, 2008), called critical alignment (Jaworski, 2006). This means bringing a critical attitude to alignment - questioning, exploring and seeking alternatives – that renders possible to develop and change the normal state.

In our study, MTEs' goal is to build an inquiry community with teachers, by promoting a questioning attitude and by prompting critical alignment, resulting from a process of co-learning inquiry. To achieve this goal, MTEs engage teachers in an inquiry cycle (plan, act and observe, reflect and analyse, feedback), which led to a continuous process of reconceptualization and redesign (Jaworski, 2008) of teaching materials for their students, based on inquiry-based tasks. Besides contributing to the design of teaching materials, MTEs also have to design materials to be used with teachers during the meetings of the PD program. We analyze this latter design work with the theoretical lenses of DAD, because this framework allows to highlight exactly the aspects that interest us, related in particular to the resources on which MTEs relies.

## Documentational Approach to Didactics (DAD)

DAD framework (Gueudet and Trouche, 2009; 2010; 2012) focuses on teachers using different kinds of resources to prepare their lessons and to support students' learning. It is framed on the instrumental approach (Rabardel, 2002), which distinguishes between artifact (only object) and instrument (involving also the subject). DAD introduces a parallel distinction between resources and documents: a document consists of a set of resources and subject's utilization schemes for a particular class of situations. Documents may be material, or psychological entities, like instruments in the sense of Rabardel. Documentational genesis is the process by which documents are generated (Gueudet & Trouche, 2009), starting from resources and introducing utilization schemes, namely classes of situations (in which resources are used), rules of action (stable elements in the way the resources are used) and operational invariants (which are part of the set of beliefs and knowledge of the teacher).

Usually, DAD is used to investigate teachers' work and growth via understanding changes in their documentation work. However, there are also other possible studies: Kieran et al. (2013) apply DAD framework to researchers' documentational genesis: the documents generated by researchers are directly designed for the students and not for the work with teachers; Psycharis & Kalogeria (2018) analyse the documentation work of trainee teacher educators, who were themselves teachers (and not researchers). Their learning was expected to be developed through their engagement in designing resources for teachers during a PD program.

Aiming to broaden the horizons of these previous studies, and remaining situated in continuity with previous PME reports, we analyse the documentation work of MTEs, who are researchers in mathematics education, when they design documents for teachers' PD. Our research question, therefore, is:

*How do MTEs generate documents to foster the building of an inquiry community with teachers?*

Answering this question will allow us to have a deeper insight in MTEs' expertise in performing their documentation work consistently with the goal of the PD program.

## METHODOLOGY

### The context

The PD program is part of the Turin University project Scuole Secondarie Potenziate in Matematica (SSPM, https://frida.unito.it/wn_pages/tmContenuto.php/456_matematica-teorie-e-applicazioni/45/), which is part of the national project Liceo Matematico (https://www.liceomatematico.it/torino/). Through an agreement with the Mathematics Dept. of the University, the schools involved in the project provide additional mathematics hours to the students, taught by their mathematics teachers, who attend a PD program (30 hours per year), held by mathematics education researchers.

We examine here the community of lower secondary school (grades 6-8) mathematics teachers, created in 2017: they are 17 teachers, who have attended the program from the start. The MTEs are academics and two of them coincide with the authors. The data collected refer to the fourth year of teachers' attendance, when the program was held online, due to Covid-19 pandemic restrictions. The program for the teachers consists in: ten 2-hour meetings of PD – one per month - and additional work online through a platform in, and of 33 hours of classroom implementations, in charge of teachers. The meetings had a fixed structure: a first moment in a common session, in which the MTEs presented the activities to the teachers, followed by a moment in which the teachers worked in groups on the activities, and, in the end, a collective discussion orchestrated by the MTEs.

**Data collection**

All the analysed data are retrievable on the web platform (Moodle) used for the asynchronous interactions between teachers and MTEs, during the PD program.

In particular, we collected:

1. The ten "activity sheets" given to teachers during the meetings. They include ideas for mathematics tasks for students, whose design must be implemented and reflection questions for the teachers.
2. All the slides projected by the MTEs during the meetings, to introduce the activities.
3. The transcripts of the video-recordings of all the interactions, included collective discussions, occurred when teachers and MTEs were altogether in the main session of the on-line meeting (in the separated sub-sessions it was not possible to record, due to technical limitations).
4. Teachers' protocols, provided in response to MTEs' requests. They include the design of tasks for students, teachers' answers to reflection questions and reports on classroom experimentations.
5. Teachers' answers to a written questionnaire about their beliefs and practices, administered during the first meeting.
6. Transcripts of teachers' semi-structured interviews, conducted by an educator (one of the authors), remotely via a web platform.

The written questionnaire, mentioned in point 5., consisted of 25 questions: 23 open questions, a multiple choice and a Likert Scale.

**Data analysis**

As in DAD is analysed the reflective investigation of teachers' documentation work, in this study we base our analysis on the reflections of the MTEs, scrutinizing their documentation work. Since DAD points out the importance of an active involvement of teachers, because they have access to their documentation work and they can make visible some hidden resources (Gueudet & Trouche, 2012), here too we rely on the MTEs' reflective attitude towards their own documentational genesis.

The first two types of data enlisted above (points 1. and 2.) can be considered the "material part" of documents generated by the MTEs, for the work during the PD program. In addition, we analysed them also as resources for the generation of new documents, by MTEs, to be used in the subsequent meetings of the PD program. For all the documents, we identified the utilization scheme, composed by the class of situations, the rules of action and the operational invariants. Besides that, we identified the main resources on which the MTEs relied for their documentational genesis, proceeding with a backward analysis and having an overall look at the development of the entire year of the PD program.

The data enlisted in (points 3.; 4.; 5. and 6.) were analysed following the principles of qualitative thematic analysis (Braun & Clarke, 2006), with an inductive approach. Our aim, related to our research question, was to identify the different kinds of resources, embedded in these data, that were involved in the MTEs' documentation work and to study how they contributed to their documentational genesis.

In the end, we traced a connection between MTEs' documentation work and their goal of building an inquiry community with teachers, explaining the motivations at the basis of their choices in the process of documentational genesis. Particularly, we identified MTEs' attempts to promote critical alignment (Jaworski, 2006) in the teachers, through a process of co-learning inquiry, involving a critical attitude.

At every stage, the authors worked, at first, individually, especially focusing on the part of the documentational genesis in which they had been more involved. In a second moment, they met together to share and discuss the results of their analysis.

## RESULTS AND DISCUSSION

We will present an example of a document generated by the MTEs', identifying the main resources on which it is based and its utilization scheme. Other examples can be presented during the conference. We will trace, in this document, evidence of connections with the MTEs' goal of building an inquiry community with teachers.

### Document for the 4th meeting

In this example, we will show a MTEs' document, generated for the 4th meeting of the PD program. On that occasion, the MTEs presented some slides to the teachers, to prompt a reflection moment and a collective discussion. During the previous meetings, the teachers had been asked to propose task designs for their students and to report on the implementations of their classroom activities, based on what they had designed. The 4th meeting started with a slideshow, presented by one of the authors, which had the aim of triggering a collective discussion on the teachers' task design proposals and reports of classroom activities. We consider the slides as the "material part" of the document we are describing, associated with the utilization scheme that we will illustrate below.

The first slide shows the distribution of the answers to the Likert Scale question of the preliminary questionnaire (point 5.), administered to teachers during the first meeting. In this question, teachers were asked to express with a score from 1 to 6 how much they feel that certain tasks are central to the role of the mathematics teacher. The image that emerges from the answers is that of teachers who have to promote student centred activities, in which students are engaged in creative processes. The items with the highest scores are, in fact, those most in line with what is required of a teacher in laboratory, inquiry-based activities: "To create situations in which students have to make decisions and choices" (5,5 points), "To promote freedom of thought and creativity" (5,47 points) and "To promote students' awareness and critical sense" (5,44 points).

The second slide, presented during the 4th meeting, contains four questions for teachers, based on the answers showed in the first slide:

*"How and to what extent have awareness and critical sense been promoted with this activity?"; "How and to what extent have freedom of thought and curiosity been promoted?"; "How and to what extent did students have to make decisions and choices?"; "Could the previous aspects have been further promoted? How?"*

The MTEs' aim was to prompt a collective reflection, making a comparison between the reports, made by teachers, about the implementation of the activities they had designed for their students during the previous meetings and their answers to the Likert Scale question of the preliminary questionnaire. There was, in fact, an evident (at least for MTEs) inconsistency between what teachers had declared in the questionnaire and their task designs and the content of their classroom activities reports. These last, in fact, never mentioned opportunities for students to make choices, to exercise their freedom of thought, creativity, or their awareness and critical sense.

The utilization scheme of the document, whose material part is constituted by the slideshow described above, is the following:

*Class of situations:* collective discussion and reflection with teachers on their task design and classroom reports.

*Rules of action*: 1) Show a slide with the teachers' answers to a question of the preliminary questionnaire, about the teacher's role. 2) Ask teachers to reflect on the consistency between their answers to the questionnaire, the task design they proposed and the reports of the activities they carried out in their classrooms.

*Operational invariants*: 1) Teachers should be confronted with possible inconsistencies between their answers in the preliminary questionnaire and their practices. 2) The collective discussion should address possible issues, which prevented teachers from enacting practices coherent with what they had declared in the questionnaire.

**Resources**

**Resource 1. Answers to the Likert Scale question of the preliminary questionnaire** (point 5.), provided by the teachers participating in the PD program. These answers were used by the MTEs, during the 4th meeting of the PD program, as a stimulus for the collective discussion, because they appeared in contrast with the reports of the classroom activities, made by the same teachers during the meetings of the PD program.

**Resource 2 – Reports of classroom activities** (point 4.). In many reports about the implementation of the activities, designed by teachers in the first meetings, it emerged that teachers guided their students a lot, to try to lead them to find the solution of the proposed tasks. For example, in the reports related to the task design and the implementation in the classroom of an activity, intended for a VI grade class, there was

no evidence of peer discussion, sharing and comparison of conjectures or wrong discoveries. A teacher reported a justification of her task design, saying:

> Lucia: In grade VI, […] the questions must be made explicit as clearly as possible and, above all, they must be progressive. In grade VII you can also skip a question and make sure that, in order to answer another question, the students must have already answered the underlying one. Instead, in grade VI no, in my opinion they must be guided step-by-step to the solution […].

Based on the reports obtained during the first three meetings, the MTEs felt the need to deeper investigate the teachers' reluctance to engage both low and high-achieving students in activities, which promote higher order thinking, creativity, critical sense, freedom of thought and awareness. So, they generated the document for the fourth meeting, presented above, with this aim.

**Other resources**. There are, of course, other resources, in addition to those detailed above, that contributed to the MTEs' documentational genesis. Among them, we can list*:* transcripts of the collective discussions among teachers and MTEs, notes of the meetings among MTEs on the design of the PD program, literature in the field of mathematics education, national and international meetings with scholars who work in the field of teacher professional development.

## Building an inquiry community

The documentational genesis we described in the previous section is connected with the MTEs' goal of building an inquiry community among MTEs and teachers. This connection is testified by their effort to prompt collective discussions and reflections among teachers and MTEs, to promote a questioning attitude. The MTEs designed documents aimed to highlight possible issues, which can hinder the implementation of the inquiry-based approach by the teachers in their classrooms. These documents are thought to foster a co-learning inquiry process, in which teachers and MTEs try to address the emerging issues and to implement an inquiry cycle. The teachers, in fact, are requested to reflect on their task design and on their classroom implementations, in order to make improvements and redesign their teaching materials.

With this study, we obtained an insight in the expertise of MTEs, who are also researchers, interested in generating documents based on teachers' feedback and on the interactions during the PD program. Such an insight could also have an impact on MTEs' own professional development, which could be object of further research.

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