# USING ONLINE DISCUSSION FORUMS FOR THE PROFESSIONAL DEVELOPMENT OF MATHEMATICS TEACHERS

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*We present a study of a model for professional development of mathematics teachers, based on their participation in a collaborative problem solving in online discussion forums, in two roles. At the first stage of the study, 47 high-school mathematics teachers participated in the forums as students. At the second stage, they mediated forums as mentors. The first stage of the study showed gradual development of group synergy among the teachers-as-students. The second stage showed that the experience of group synergy gained by the teachers at the first stage has supported the development of their mathematical fluency in teaching.*

## INTRODUCTION

There is a broad consensus in the mathematics education community that mathematical reasoning in problem solving, critical thinking, and the ability to work collaboratively are the key components of students' learning (OECD, 2019). This approach to students' learning implies that teachers should develop knowledge and skills of mathematical communication with students in real time, including the ability to listen, interpret and respond to the student's reasoning, and conduct effective mathematical discussion in the learning process. The proficiency in these skills is referred to as *mathematical fluency in teaching* (MFT) (Ball et al., 2008). In addition, MFT assumes the teacher ability to evaluate alternative solutions, understand students' unfinished ideas, and identify sources of their mistakes.

Studying the forms of teacher professional development (PD) that can contribute to the development of MFT is one of the priorities in the field of research on teaching mathematics (Hoover et al., 2016). Several studies have demonstrated the potential of PD models, in which teachers act as learners while tasting and developing the skills they would like to develop in students (e.g. Kramarski & Kohen, 2017). The current study makes one step further and examines a PD model based on teachers' participation in collaborative problem solving in online discussion forums while assuming two roles. At the first stage, the teachers participate in the forums as students. At this stage we target the growth of *group synergy* (Clark, et al., 2014; Stahl, 2021), which is referred to as continuous interaction among problem solvers who monitor and develop each other 's problem-solving ideas. At the second stage, the same participants assume the role of leaders of problem-solving forums (PSF henceforth). This study aims at testing



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the following hypothesis: the development of group synergy among teachers in the process of their participation in collaborative problem solving in PSF contributes to the development of their MFT.

## THEORETICAL FRAMEWORK

In the last decades, many studies explored knowledge and skills that mathematics teachers need to develop (Chapman, 2015). Mathematical fluency in teaching (MFT) has been identified as one of the most important teaching skills (Ball, et al., 2008; Hoover, et al., 2016). It is broadly agreed that for the development of the MFT, it is necessary for the teachers to deepen their mathematical knowledge, in order to be in position to quickly navigate among approaches to understanding and solving mathematical problems that students may have. One of the methods of deepening mathematical knowledge is systematic engagement in solving challenging mathematical problems (Polya, 1945). Additionally, experiencing problem solving by teachers is necessary in order to strengthen their pedagogical skills for better understanding how students think (Chapman, 2015).

A number of PD models developed for deepening mathematical and pedagogical knowledge of teachers is described in the professional literature. For example, Koellner et al. (2007) described a PD model consisting of the following cycle: the teachers first solve mathematical problems, then analyse videotaped problem solving by school students who are given the same problems, and then discuss how they would use the problems in their classrooms.  Koellner et al. (2007) showed that this model has undeniable potential for strengthening the link between the mathematical knowledge for teaching and teaching practice. However, the study did not attend to the exchange of mathematical ideas among the teachers in the problem-solving process, as well as to the enactment of the accumulated knowledge with students in real time.

A number of studies have demonstrated the potential of PD models, in which teachers act as learners, testing and developing skills that they would like to develop in learners (e.g., Kramarski & Kohen, 2017). The present study continues both of these directions: the development of mathematical knowledge of in-service teachers through problem solving and the testing of new teaching methods by teachers, on themselves as students. This article discusses the model of the PD of teachers in the process of their participation in the joint solution of mathematical problems in small groups in the role of students, with the subsequent transfer of the accumulated experience to teaching. According to many researchers, synchronous online forums are a conducive environment for successful group interaction due to more precise wording of arguments and a greater willingness of participants to express alternative views and critical ideas (e.g., Asterhan & Eisenmann, 2009; Stahl, 2021). For this reason, PSF were chosen as the environment in which two-stages discussions of mathematical problems took place in our study.

The question of the necessary conditions for productive collaborative work on tasks is broadly studied. In particular, Stahl (2021) studied interactions aimed at involving learners in "research participation" (Stahl, 2021, p. 493). In addition, the importance of interactions, in which learners attempt to understand each other thinking – so-called "other-monitoring" (Goos et all, 2002) has been pointed out. Over time, these types of interactions can lead to the emergence of group synergy. Interaction is considered a group synergy if it is a series of interrelated messages from different participants, in which they either continue and develop each other's ideas, or test the ideas expressed, based on theoretical knowledge and logical conclusions drawn from them. The result of such interaction is progress in understanding the problem and its solution, expressed in new ideas on the way to solving the problem or in the recognition of the fallacy of the proposed idea (Clark et al., 2014). Such cooperation presupposes the ability to delve into the mathematical ideas of colleagues in real time, quick reaction and the desire to reach mutual understanding about the ways of solving problems, that is, those qualities that determine MFT in communication with students and are the key to improving the mathematical education of teachers (Hoover, et al., 2016).

This study answers the following questions: (1) How does group synergy develop in interactions among teachers during their continued involvement in PSF as problem solvers? (2) How is teachers' own experience of group synergy reflected in the MFT of when the teachers interact with students as PSF mentors?

## METHODOLOGY

### Participants and research progress

The study was conducted as part of a PD program for mathematics teachers at the Faculty of Education in Science and Technology, Technion, Israel. The study involved 47 high school teachers with an experience of 5 to 20 years. At the first stage of the study, as part of the course "Foundations of Geometry. Plane Transformations", each teacher participated as a student in a group of 3-5 in six PSF meetings, mentored by the first author of this article. Each meeting was devoted to collaborative discussion and solving one challenging geometry problem. The second stage took place in the course "Methods of teaching mathematics", when each of the participants acted as a mentor (teacher) at two PSFs. The learners in these forums were students studying for B.Sc. in mathematics education. They also solved complex geometric problems. At this stage, the teachers were tasked with organizing and leading a discussion at the PSF. This article analyses the activities of one of the groups, consisting of 5 teachers. The group consisted of the same participants in all six PSF of the first stage. Then, the experience of one teacher from that group is tracked in his capacity of a PSF mentor. This group is quite representative of the other groups, as the data obtained for this group reflect similar learning processes.

## PSF

The technological platform for the PSF in this study was the social network WhatsApp. A WhatsApp group was opened for each group of teachers in which meetings took place. The duration of each meeting was about one and a half hours. Each online meeting approximately consisted of 180 messages with an average frequency of 5 messages per minute. Most of the messages were text messages. Participants also posted photographs of drawings and, in some cases, resorted to short voice messages.

## Data and data analysis

In the course of the study, 96 PSF protocols were obtained and analysed. Of these, 72 forum protocols in which teachers acted as students (12 groups with a permanent membership) and 24 forum protocols in which teachers acted as mentors. When analysing the protocols, the unit of interaction was a message (post) sent by one of the participants. In order to answer the first question of the study, the protocols of the forums in which teachers acted as students were analysed. To assess the dynamics of group synergy, we have defined the concept of "synergetic chain", which is understood as a block of interrelated posts of various participants concerning the discussion of one mathematical issue. An example of a synergistic chain is the following episode of the forum during the discussion of a geometric problem:

| | | |
|---|---|---|
| 33 | A.: | I think BE = EC |
| 34 | B.: | This is true since they are chords from equal inscribed angles |
| 35 | A.: | And also, triangle EHC is isosceles |
| 36 | C.: | Yes, because in it the height coincides with the median |
| 37 | B.: | Means BHCE kite. How have I not seen this before? This will help us a lot. |

This episode refers to group synergy, as it contains several interrelated messages containing an element of monitoring (34, 36), the development of each other's ideas by the participants (37) and the progress of the group in understanding the task, since B. expresses his conclusions aloud, referring to the whole group (37). Each synergistic chain has its own length (the number of messages included in it). The length of the chain reflects the duration of the interaction between the participants. We use the average length of all synergy chains included in this forum as one of the characteristics of group synergy in it. In our study, this characteristic was named Syn1. So, in the given example, the length of the synergistic chain is 5. If four threads are found on the forum, containing respectively 5, 2, 7, 4 messages, then the Syn1 characteristic will receive the value Syn1 = 4.5, which shows the group's ability to long-term interaction. An additional characteristic Syn2 characterizes the share of group synergy among other interactions and is calculated as the ratio of the total number of messages included in a particular synergy chain and the number of messages in a given forum. So, if a forum containing 187 posts, contains 4 synergistic chains, 5, 2, 7 and 4 posts long, the Syn2 characteristic is calculated as follows: Syn2 = (5 + 2 + 7 + 4) /187=0.096 (9.6%).

To answer the second question of the study, a qualitative analysis of the content of the messages that the teacher published in the PSF, in which he was a mentor, was carried out. The situation in which the group was at the moment of the mentor's intervention was characterized. Examples of characteristics attended to are as follows: lack of activity in the discussion, the development of a wrong idea, or presence of a right idea that escapes the attention of the students. Then we inductively deduced from the above analysis which qualities of the MFT the teacher showed in his intervention. Finally, the forums in which the teacher-mentor acted as a student were characterized in order to identify situations that could be the prototypes of this intervention. Examples follow.

## FINDINGS

Below are graphs showing the change in the indicators of group synergy in the selected group in the process of its participation in six PSFs as learners.

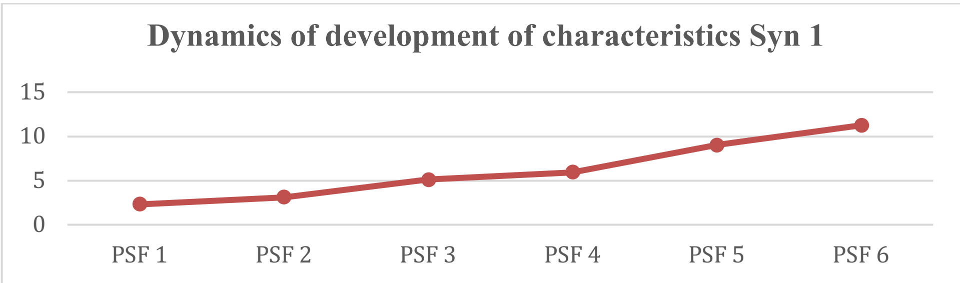



Figure 1. The development of group synergy (Syn1) in six forums

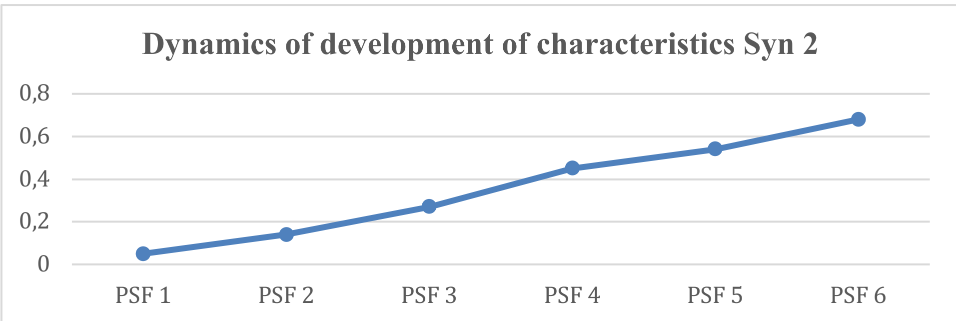



Figure 2. The development of group synergy (Syn2) in six forums

The graphs show an increase in the indicators of group synergy in this group, both in terms of the share of group synergy among the interactions of forum participants, and in terms of increasing the length of synergistic chains. In the last forum, group synergy becomes the main type of interaction, where 70% of messages are in synergy chains, that is, they are part of a brainstorming session. A similar pattern was observed in the other groups participating in the study.

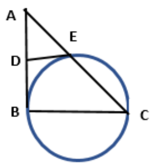
The analysis of the content of messages included in various synergistic chains led to the identification of different types of synergies. For example, the above episode demonstrates the complementarity of participants' mathematical ideas that propelled the group forward in solving the problem. As a result, together with other members of the group, everyone achieves more than he could achieve himself. Another type of group synergy refers to the case when one of the participants explains his idea, and other members of the group monitor it. Often, as a result of such a discussion, it turns out that the idea requires development or turns out to be incorrect. An example is the following snippet of the discussion:

| | | |
|---|---|---|
| 62 | A.: | I have proved the similarity of triangles in two corners. |
| 63 | B.: | What angles are equal? |
| 64 | A.: | There are two inscribed, resting on equal arcs |
| 65 | C.: | That's right, they are equal |
| 66 | A.: | More right angles. One inscribed at the diameter, and the second at the tangent point |
| 67 | B.: | Wait a minute, but, after all, we do not know whether the radius comes to the point of tangency. |
| 68 | C.: | It is definitely not a radius; it cannot go through the centre. |
| 69 | A.: | But then the angle is not right either. I think I was wrong. |

Group synergy also arises when a group makes a collective effort to explain ideas it finds to a straggler or misunderstood comrade. Often during such an explanation, shorter paths are found or details are clarified. The final stage of the work is characterized by a group via reflective discussion of the problem.

The results of the analysis of messages, which supported the work of the PSF by teacher A. from the described group in the role of a mentor, illustrate the answer to the second question of the research. The following task was proposed for discussion in the forum:

*A circle of radius R is given (see drawing). BC is the diameter of the circle, AB is the tangent to the circle at point B, D is the midpoint of the segment AB. The ACB angle is β. It is required to express the ratio of the areas of triangles ADE and ABC using R and β.*




The following exchange of ideas took place between the students:

| | | |
|---|---|---|
| 21 | M.: | DE is the middle line of the triangle. |
| 22 | N.: | The figure shows that DE is equal to BD by the two-tangent theorem |
| 23 | K.: | Then β can be found. It is equal to 45 °. |

All messages were received within one minute. The teacher was required to understand and evaluate the statements made in real time. That is, to show MFT skills. He should have noticed that N.'s statement (22) is true but requires proof. And the assertion M.

(21) is true only in the case $\beta = 45°$ and cannot be the basis for solving the problem in general. K.'s assertion (23) was based on trust in previous allegations, which could later lead the group in the wrong direction. After assessing the situation, the teacher had to make a decision about the usefulness and form of the intervention. He decided to intervene and sent a message: I don't fully understand why DE = DB? The success of the question from the point of view of organizing a mathematical discussion was proved by the subsequent reasoning of the students, during which they proved that DE is a tangent, but not necessarily a middle line. Between this episode and the episode described earlier, when the joint observation of A.'s statement (62) in the role of a student about the similarity of triangles led to an understanding of the fallacy of reasoning. It can be assumed that this experience was used by A. to stimulate discussion and monitoring of ideas while working as a mentor. Working in a group in the role of students, A. and his colleagues did not know whether the statement he proposed was true, and only a joint analysis led them to understand. In the role of a teacher, A. did not point out to the students that the ideas were correct or erroneous. Instead, he asked a specific question (similar to the way colleagues asked him why the angles he named were equal). Thus, with the help of a specific question, A. created a situation that entailed discussion and progress in understanding. One of the components of the MFT is the ability to conduct a mathematical discussion. In particular, it is necessary to involve students in the conversation, to push them to participate in the discussion. A. supported the discussion, using his own experience of participation in the PSF. For example, when there was a long pause at the beginning of the forum, A. stimulated the activity of the participants with the message: "Throw in ideas. The more ideas there are in the discussion, the more chances that some of them will lead to a solution". A similar proposal was addressed to each other by members of group A. when they participated in the FOP as students. A.'s experience of participating in PSF as a student was also reflected in the fact that he supported and guided the discussion, using encouraging and guiding comments, which the instructor in his group encouraged the discussion. For example: "This is a great idea. You should discuss it "or" This is a good idea, but worth discussing if it is always correct. "

## CONCLUDING REMARKS

Based on our findings, we concluded that PSFs are a conducive environment not only for collaborative learning, as shown in previous studies (Stahl, 2021), but also for the PD of teachers. Various forms of group synergy have been found to grow and develop with the continued participation of teachers in PSF as learners, demonstrating improvements in listening, critically analysing and developing others' ideas in real time. Thus, teachers develop MFT skills, which are a necessary component of successful teaching of mathematics in the modern world (Ball et al., 2008; Chapman, 2015). The experience of mathematical communication acquired in the forums was used in the work of teachers as mentors of the forums, where MFT manifested itself in the ability to delve into students' ideas in real time, interpret them, and quickly choose the reaction that was most useful for learning. This study responds to a request for the need to study models of mathematics teacher PD that, on the one hand, will be relevant for teachers in terms of their work, and on the other hand, will correspond to the goals set for the mathematical education of teachers (Hoover, et al., 2016). The methodological contribution of this study is the quantitative method presented in this study for assessing group synergy in the joint solution of mathematical problems, which adds to methods of qualitative analysis developed in the past studies (Goos et al., 2002; Clark et al., 2014; Stahl, 2021).

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