NUMBER STRUCTURE IN LEARNER WORKBOOKS

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This paper reports on the extent to which different representational modes in current learner workbooks conceptually signal the structuring of number (especially base-ten thinking) which research shows to be vital for learners to shift from counting to calculating strategies. Tasks contained in two learner workbooks currently used in Grade 1 classrooms across South Africa, i.e. DBE and Bala Wande, were contrasted in light of the conceptual signalling contained in the representations used. Analysis of these workbooks showed that the Bala Wande workbooks had more explicit conceptual signalling for working with number structure, which helps to address the wide-spread use of counting strategies that underpin poor learner attainment on the ground.

INTRODUCTION

Children usually start solving simple additive problems by counting in ones and also develop more sophisticated calculation strategies that are not based on counting, like near-doubles (e.g. 6 + 7 = double 6 + 1) or bridging through ten (e.g. 6 + 7 = 6 + 4 + 3 or 13 – 7 = 13 – 3 – 4). Developing calculation strategies builds on learners' facility with *structuring number*, which can be described as the skilful organisation of numbers using number relationships, number patterns and various combinations and partitions of numbers (Wright et al., 2006; 2009). Developing learners' facility with structuring number can be supported through the use of structured representations, that is, representations that can be 'read' as embodying a certain mathematical structure, like base-ten or doubles (Venkat, Askew, Watson & Mason, 2019). The importance of structuring number for enacting calculation strategies, and the access to structuring provided by structured representations, underlies this investigation into the nature of representations used in learner workbooks.

The enquiry reported on here is set in a context where an over-emphasis on counting in ones and an over-reliance on the use of concrete/unstructured materials hampers progression from counting to calculating strategies (Hoadley, 2012). Empirical research shows widespread use of unstructured representations of number (e.g. counters) in South African Foundation Phase classrooms (Grades 1 to 3, i.e. 6-9 year old) by teachers and learners (Ensor et al., 2009), the result of which is the confining of learners to counting-based strategies, which are inefficient and error-prone when working beyond the 1-10 number range.

NUMBER STRUCTURE AND BASE-TEN THINKING

A key form of number structure that children learn in the early grades is the base-ten structure arising from our use of the base-ten decimal number system and positional notation (Cobb & Wheatley, 1988). Activities that build learners' awareness of base-

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ten include learning the bonds of ten (e.g. 6 and 4, 8 and 2), adding and subtracting 10 to/from any number (35 + 10, 62 – 10), and flexibly splitting numbers into tens and ones (i.e. seeing 64 as 60 + 4 and as 30 + 30 + 4).

Base-ten thinking is a term related to structuring that stems from the body of work developed by proponents of RME. According to Wright and colleagues (2012), Freudenthal believed that "to make any progress in mathematics children must be inducted into base-ten thinking, developing a skilful habit of organizing numbers and calculations into 1s, 10s and 100s." (p.16). Freudenthal further argued that children need to skilfully structure numbers and calculations using base-ten thinking in order for them to get a handle on working with numbers larger than twenty.

A similar view about knowing the base-ten structure of number for working in higher number ranges is held by Anghileri (2006) who argues that learners who can add 10 to any number as a known fact is ready for 2-digit addition and subtraction. Learners who do not know that 32 add 10 is 42 as a known fact will need more practice with structured materials to establish these patterns of incrementing and decrementing by ten before attempting 2-digit additive problems (Anghileri, 2006).

It is widely accepted that children need to be facile in the use of the composite unit in base-ten representations of number and this facility is considered to be well within the reach of children in the first years of school, if not earlier (Perry & Docket, 2002). As noted above, such fluency is not attained by many learners in South Africa and this paper contributes to our understanding of why this might be so.

CONCEPTUAL SIGNALLING

In their evaluation of a workbook as a curriculum tool, Hoadley and Galant (2019) use *conceptual signalling* to refer to the extent to which the concepts/content/skills underpinning tasks or activities in the workbook are made explicit. Explicit conceptual signalling is communicated through explanatory notes, headings, sub-headings, text boxes or teacher notes. In the absence of explicit signalling, the likelihood of learners (and possibly teachers) becoming aware of the underlying mathematics is greatly reduced.

Drawing on the work of Hoadley and Galant (2019), we extend the notion of *conceptual signalling* to argue that various representations of a particular problem or concept may clearly signal some structure, with other representations not making the structure as 'transparent'. Structured representations can be effective at signalling number structure like doubles, base-five or base-ten. For example, the pairwise ten-frame in Figure 1 can signal the concepts of doubling and base-ten. Doubling, because eight can be seen as double 4, and base-ten, because eight can be seen as two away from ten – which can be linked to the number sentences '4 + 4 = __' and '8 + __ = 10', respectively.

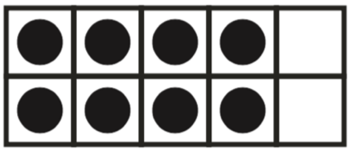


Figure 1.

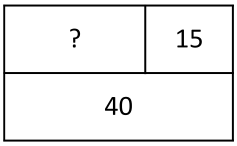
By extending Hoadley and Galant's sources of explicit conceptual signalling, we argue that representations in learner workbooks that exhibit mathematical structure (e.g. Part-part-whole (P-P-W) diagrams, images of 10-frames or bead strings) can also send explicit conceptual signals.

REPRESENTATIONS

Representations play an important role in teaching and learning mathematics (Heinze et al., 2009) and come in many forms such as drawings and inscriptions, pictures, symbols and concrete objects. Researchers agree that learning how to structure number can be supported by the use of structured representations, initially presented as concrete materials and gradually 'faded' until these representations merely need to be imagined by learners to support their mental calculation (Anghileri, 2006; Wright et al., 2012). Examples of structured representations usually found in Foundation Phase classrooms include: number tracks, number charts, number lines, part-part-whole (P-P-W) diagrams, arrays, triad diagrams, 5-frames and 10-frames.

Figures 2 and 3 below are two different representations of the same missing addend problem.

Figure 2. $\square + 15 = 40$

Figure 3. 

In Figure 3, the part-part-whole representation may enable a learner to see the structure in the problem as not being limited to finding a missing addend and become aware of the possible use of subtraction to find the solution. In the symbolic representation of the problem (Fig. 2) the possibility of using subtraction to find the missing addend is much less noticeable to young learners (who frequently add the given numbers). The P-P-W diagram (Fig. 3) thus conveys a stronger 'conceptual signal' about the relationship between addition and subtraction. Thus conceptual signals arising from structured representations can serve as an affordance to meaning making.

The caveat must be kept in mind that children do not simply 'see' a number concept or relationship embodied in a representation, but rather become aware of these concepts and relationships as they use these representations in their number work (Ellemor-Collins & Wright, 2009). One can say that learners become aware of number structure through the process of structuring number. For example, a learner who shows 32 on an abacus by counting in ones is not aware of the 10-ness embodied in the design of the abacus. But, when encouraged to show 32 by counting "ten, twenty, thirty, thirty-two" while moving over 3 lots of 10 beads and 2 beads on the abacus, the learner becomes more aware of the 10s structure of the abacus and can learn over time how to use this 10s structure for efficient enumeration. The structured representation and structuring actions on the abacus lead to understanding structure.

The importance of using certain modes of representation to encourage the structuring of number prompted using the notion of *conceptual signalling* to investigate to what extent representations used in workbook tasks might facilitate learners' use of base-ten thinking.

RESEARCH AIM AND QUESTIONS

In a context where learners' main access to mathematics structure is through workbooks, the aim of this study was to examine to what extent do workbooks conceptually signal base-ten thinking through the representations used in tasks. To achieve this aim we examined current Grade 1 mathematics workbooks used in South Africa, namely, the Department of Basic Education (DBE) Workbook and the Bala Wande (BW) Workbook (or Learner Activity Book). The selection of these workbooks was guided by the fact both workbooks are currently used in government schools: the DBE workbook nationally, the BW workbook in 3 of SA's 9 provinces. The specific research questions guiding this investigation were:

- How do two current workbooks used in Grade 1 conceptually signal base-ten thinking through the representations used?
- What are the implications of different types of conceptual signalling?

METHODOLOGY

Following Mason and Johnston-Wilder's (2006) distinction between task and activity, a 'task' in this report refers to what is presented in the pedagogical text as the focus of attention (these can be broken down into smaller parts) while the 'activity' describes what happens in the enactment of the task. The DBE and BW Grade 1 mathematics workbooks for 2021, covering all four school terms, were analysed and contrasted for this report; these workbooks were obtained in print and digital format.

The BW workbooks for Grade 1 are clearly divided into weeks and days. The 5th day of every week is for assessment and/or consolidation. All the activities planned for one lesson are seen as different parts of one task, therefore 1 day = 1 task. Most of the tasks in the BW workbooks consist of worksheets stretching over 2 pages. The BW workbook series for Grade 1 consists of 185 tasks in total, divided across Terms 1 to 4 as 45, 50, 50 and 40 tasks, respectively. The bilingual Sepedi-English version of the BW workbooks were used for consistency when referencing page numbers.

The DBE workbooks for Grade 1 are presented as two volumes: Book 1 for the first two terms and Book 2 for the last two terms. Book 1 and 2 each contain 64 discrete tasks: 32 tasks per term spread over eight weeks, i.e. four tasks per week (DBE, 2011 as cited in Fleisch, et al., 2011). Most tasks in the DBE workbooks consist of worksheets covering 2 pages (a few extend to 4 pages).

To start the process of analysis, each workbook series was carefully read to determine their overall structure. Using the task as the unit of analysis, a proforma was developed to capture the following data in tabular form: the total number of tasks per term, and the constitution of tasks: topic, intended activities and nature of representations used (i.e. structured or unstructured). This data was captured for DBE Workbook 1 (for Terms 1 and 2), DBE Workbook 2 (for Terms 3 and 4) and the Sepedi-English version of the four BW workbooks. Tasks in the BW workbooks that were used for assessment purposes were omitted because there are no comparable assessment tasks in DBE workbooks.

After this preliminary data capture, we re-looked at each task that used a structured representation and recorded additional information about the task: the number range used, the purpose of the structured representation/s (for illustration or for learners to act on/use to calculate an answer) and the number structure or number relationships signalled by the structured representation/s. We also made a note of tasks that used more than one structured representation for the same activity.

We were also interested in the frequency with which various structured representations were used in the workbooks. To this end we counted the number of times different structured representations were used across each term in both workbook series. If the same representation is used more than once in a task – e.g. BW Term 1 Week 5, Day 2, P-P-W diagrams are used in the whole class activity (p52), and the same representation is used again in the independent activity (p53) – this is counted as one instance of P-P-W used.

FINDINGS

The number of tasks in each workbook are shown per term, side-by-side in Table 1. Also recorded in Table 1 are the number of tasks that make use of an image of a structured representation and the number of tasks that use more than one structured representation for the same activity.

	Bala Wande				Total	DBE				Total
Term	1	2	3	4		1	2	3	4	
Number of tasks	45	50	50	40	185	32	32	32	32	128
Tasks with 1 struc. rep.	28	18	34	16	96	3	11	16	10	40
% of structured reps.	62	36	68	40	52%	9	34	50	31	31%
Tasks >1 structured rep.	13	0	5	0	18	0	4	2	3	9
% using multiple reps.	29	0	10	0	10%	0	13	6	9	7%

Table 1: Info on tasks in DBE and BW Workbooks.

From Table 1 it is evident that 96 of the 185 tasks in BW workbooks (+/- 52%) use an image of a structured representation while 40 of the 128 tasks in the DBE workbooks (+/- 31%) do so. About 10% of tasks in BW workbooks use more than one structured representation for one activity while about 7% tasks in DBE workbooks do so.

Table 2 shows the various structured representations present across both series of mathematics workbooks and the number of times these were used in tasks. Some tasks used more than one representation, thus the number of tasks that use an image of a structured representation (Table 1) does not match the number of instances a certain representation was used (Table 2). All nine structured representations used in the analysis are present in the BW series whilst five are present in the DBE workbooks. The 10-frame is the most frequently used structured representation in the BW series (50 instances) but not used at all in DBE workbooks. Other structured representations present in BW workbooks but absent from DBE workbooks are the array, P-P-W, triad diagram and 5-frame.

	BW				Total	DBE				Total
Terms	1	2	3	4		1	2	3	4	
Array	6	-	-	-	**6**	-	-	-	-	**-**
Hand or foot	2	-	-	-	**2**	-	4	7	1	**12**
Number chart	-	-	-	2	**2**	-	2	2	9	**13**
Number line	10	-	10	4	**24**	3	6	10	5	**24**
Number track	2	4	5	-	**11**	-	2	7	4	**13**
P-P-W diagram	8	13	6	10	**37**	-	-	-	-	**-**
Triad diagram	9	2	1	-	**12**	-	-	-	-	**-**
5-frame	5	-	-	-	**5**	-	-	-	-	**-**
10-frame	18	3	25	4	**50**	-	-	-	-	**-**
Total	60	22	47	20	**149**	3	14	26	19	**62**

Table 2: Structured representations used in workbooks.

Taken together, Tables 1 and 2 show that the BW series of Grade 1 mathematics workbooks contain a wider range (9 to 5, respectively) and a higher frequency (52% to 31%, respectively) of structured representational use across all tasks compared to the DBE workbooks.

DISCUSSION AND IMPLICATIONS

The use of structured representations of number can signal important number concepts or relationships that support learners' use of calculation strategies that are not based on counting. For many low attainers, who rely on inefficient and error-prone counting strategies, the use of structured representations can be the bridge to structuring number which in turn provides access to more sophisticated calculation strategies and working in higher number ranges. Using structured representations in an every-day resource like a workbook is one way of ensuring that learners have multiple opportunities to notice and use number structure.

In our investigation into how two current learner workbooks conceptually signal number structure, especially base-ten thinking, we found that the Bala Wande workbooks explicitly signal number structure by using structured representations more frequently than the DBE workbooks. This implies that learners who used the Bala Wande workbooks had greater access to representations that foreground number structure, and therefore had a greater chance of structuring number and using sophisticated calculation strategies, than learners who used the DBE workbooks.

Research shows that learners who are exposed to multiple representations of a concept, and who learn to seamlessly shift between representations, develop deeper conceptual understandings (Heinze, et al., 2009). Bala Wande workbooks provide stronger conceptual signals of representational flexibility than the DBE workbooks because they use a wider range of representations and use multiple representations for one task to a larger extent than their counterpart. The implication is that learners who used the former had a greater chance of shifting between multiple representations and developing deeper conceptual understandings than those who used the latter.

CONCLUSION

"Tasks lie at the centre of learning and teaching mathematics" (Askew, 2016, p1) thus their importance cannot be downplayed. In this report, tasks in Grade 1 learner workbooks were considered in light of the affordances provided through the conceptual signalling of number structure in the representations used. By focusing solely on the design of tasks, this report cannot claim that the presence or absence of a textual feature implies adequacy or inadequacy in the teaching/learning associated with such tasks. This report highlights the affordances and opportunities that are provided through workbook tasks that make use of a specific textual feature (i.e. structured representations), irrespective of implementation. This sends a strong message to workbook designers about being aware of the conceptual signalling in representations selected for tasks and ensuring that these signals align with their intentions for tasks. This is also a wake-up call to consumers of workbooks (teachers, parents, etc.) - closer attention must be paid to conceptual signals in representations used in tasks as this affects children's opportunities to learn.

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