THE ALGORITHMS TAKE IT ALL? STRATEGY USE BY GERMAN THIRD GRADERS BEFORE AND AFTER THE INTRODUCTION OF WRITTEN ALGORITHMS

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Solving addition and subtraction problems efficiently is an important goal of elementary school mathematics education. However, after the introduction of written algorithms, many students exclusively use these procedures to solve arithmetic problems, even if they are inefficient and error-prone. We explore the assumption that the dominance of written algorithms is due to the fact that students already previously had only used a very limited repertoire of strategies, which was then replaced by the written algorithms. We used data from a study of 222 German third graders. Sixty students received a brief training on computational strategies at the start of the school year and showed a broader strategy repertoire than their peers before the introduction of written algorithms. After learning the algorithms, the trained students still used a broader strategy repertoire (including short-cut strategies). We assume that students can succeed in flexibly using a broad strategy repertoire even after the introduction of the algorithms if they are supported in doing so from the beginning.

INTRODUCTION AND THEORETICAL BACKGROUND

One central goal of arithmetic education in the elementary school is the acquisition of computation skills. Meanwhile, arithmetic curricula in many countries also address number-based computational strategies (e.g., stepwise strategy, split strategy, compensation strategy, indirect addition), although the digit-based written algorithms continue to play an important role (Mullis et al., 2016). Skills in flexible use of strategies should help students to solve arithmetic problems efficiently with an appropriate strategy instead of using the same strategy for all problems. At the same time, learning different strategies is considered to promote conceptual understanding of numbers (e.g., Baroody, 2003; Verschaffel et al., 2007) and internalized computation strategies can be helpful to solve specific types of multi-digit arithmetic problems by purely mental calculation without paper-and-pencil computations.

In this report, we focus on strategies for multi-digit addition and subtraction problems. As mentioned before, these strategies can be categorized as digit-based (standard) written algorithms and number-based strategies. The latter can be further divided into universal number-based strategies, which are suitable for all addition and subtraction problems (stepwise: 462 + 299 via 462 + 200 = 662, 662 + 90 = 752, 752 + 9 = 761;

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split: 462 + 299 via 400 + 200= 600, 60 + 90 = 150, 2 + 9 = 11, 600 + 150 + 11 = 761), and short-cut strategies, which are very efficient for specific problem types (compensation strategy: 462 + 299 via 462 + 300 = 762, 762 - 1 = 761; simplifying strategy: 462 + 299 = 461 + 300 = 761; indirect addition: 702 - 697 via 697 + 5 = 702). Sometimes, students mix different strategies and if they have some routine they might also use short versions of the universal number-based strategies stepwise and split by combining sub-steps (e.g., 462 + 299 via 462 + 200 = 662, 662 + 99 = 761).

Although different computation strategies have already been implemented in curricula and textbooks in several countries for about 20 years, elementary school students show a low variation in applying different strategies and especially in applying short-cut strategies (e.g., Csíkos, 2016; Heinze et al., 2009; Hickendorff, 2020; Torbeyns & Verschaffel, 2016; Torbeyns et al., 2017). This indicates that acquiring skills in the flexible use of strategies is challenging for students. However, empirical research also suggests that these skills can be promoted through instruction (Hickendorff, 2020; Heinze et al., 2018; Nehmet et al., 2019; Sievert et al., 2019; Torbeyns et al., 2017).

Students' strategy use after the introduction of the written algorithms

Studies examining the development of students' strategy use in regular elementary school mathematics classes revealed that the use of number-based strategies decreased substantially after the written algorithms were introduced (e.g., Hickendorff, 2020; Nehmet et al., 2019; Selter, 2001; Torbeyns & Verschaffel, 2016; Torbeyns et al., 2017). Many students used the written algorithms almost exclusively to solve addition and subtraction problems, and there was little variation in the use of the strategies across the problems. Different possible explanations for this observation can be derived from empirical studies in the research literature. This research report takes a closer look at two of them which might apply to different groups of students.

A first possible explanation is that most students have used only a few strategies already before the introduction of the written algorithms. Empirical results suggest that there is a high proportion of students who initially use only one or two universal number-based strategies, like the stepwise and/or split strategy (e.g., Csíkos, 2016; Heinze et al., 2009; Torbeyns et al., 2017). Thus, there is also little flexible use of strategies before students learn the written algorithms. After the introduction of the written algorithms, the exclusively used universal number-based strategies are then replaced by the universal digit-based written algorithms. As a result, these students always use those universally applicable strategies that they learned last.

A second possible explanation is that students' skills in using strategies flexibly is not stable. Some students may have learned various number-based strategies (including short-cut strategies) in mathematics class before the introduction of the written algorithms. Then the written algorithms were explicitly introduced by the teacher and practiced intensively by the students for a longer period of time. Afterwards, on the one hand, students' knowledge and skills about the number-based strategies may have decreased again and, on the other hand, the algorithms may have gained a great importance in the students' perception. The findings of Nehmet et al. (2019) can be interpreted in this direction. They taught one group of students in the usual way, that is, the number-based strategies first and then the written algorithms. A second group of students learned all strategies interleaved. After the intervention the written algorithms were used significantly less and the short-cut strategies significantly more often in the second group than in the first group. Thus, if students spend long periods of time working exclusively on written algorithms, they may lose skills in other strategies.

PRESENT STUDY AND RESEARCH QUESTIONS

To examine the previously mentioned explanations, we use existing data from the intervention study of Heinze et al. (2018). This study monitored students of several school classes over the course of grade 3. A subsample of students was trained on number-based strategies and their flexible use at the start of the school year. In the second half of the school year, the written algorithms were introduced by the teachers in the regular mathematics class. Thus, the dataset covers two subsamples of third-graders. One subsample of students which participated only in the regular mathematics classroom and one subsample from the same classes which were briefly trained at the start of the school year. The latter showed better knowledge and skills of short-cut strategies and their flexible use than their peers before the introduction of the written algorithms.

Using data from this study, we explored the following research questions:

RQ1: What strategies do third-graders from a regular German mathematics classroom use before and after the introduction of the written algorithms?

RQ2: What strategies do third-graders use before and after the introduction of the written algorithms if they possess advanced knowledge and skills of short-cut strategies and their flexible use?

RQ3: What impact does more frequent use of short-cut strategies by students before and after the introduction of written algorithms have on the performance in addition and subtraction (in the sense of correct solutions)?

The third research question provides information on whether the two groups of students show a comparable arithmetic performance before the introduction of written algorithms. Further, we obtain information about whether the different use of strategies affects the solution rates.

METHODS

To investigate the research questions, we use data from Heinze et al. (2018) for a secondary analysis. In Heinze et al. (2018), 17 Grade 3 classes from Germany were considered. We selected those students who participated in all three tests we needed for our analysis. The sample comprised 222 third-graders (9-10 years old) from 15 classes, 162 of whom participated only in regular mathematics instruction, while 60 students received an additional training for the flexible use of computational strategies. The design on the study is presented in Figure 1.

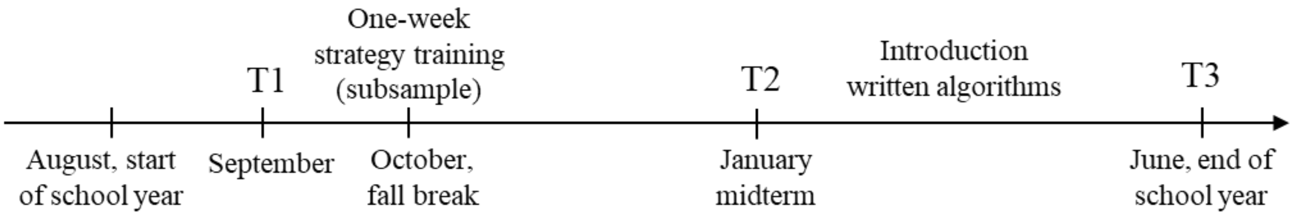

Figure 1: Design of the study, data collection at T1, T2 and T3 by test

According to the German Grade 3 curriculum, the number domain is extended up to 1000 and students learn addition and subtraction strategies for three-digit numbers. In the second half of grade 3, the standard written algorithms are introduced. The one-week training during the fall break was advertised in several schools. Students from each of the 15 classes participated voluntarily. They were taught five strategies (stepwise, split, compensation, simplifying, indirect addition). In the original study in Heinze et al. (2018), two instructional approaches were compared. Because their effectiveness did not differ, they are not distinguished in the current analysis here.

Data for strategy use was collected by trained university assistants in all 15 classes with a first test at the start of the school year (T1), a test 3 months after the training, but before the introduction of the written algorithms (T2), and a test at the end of the school year after students had learned the written algorithms (T3). Each test consisted of 8 multi-digit addition and subtraction tasks suggesting especially the short-cut strategies as efficient solutions. A core of 4 items was part of all tests (403-396, 1000-991, 398+441, 502+399). The item solutions were analyzed two times: firstly as correct or incorrect, and secondly by categorizing the applied strategies for the given task. For the latter, a bottom-up procedure to develop a category system with 21 strategy categories was applied (e.g., the ideal-typical strategies, as well as observed short versions and mixtures of these strategies). The assignment of a strategy to a category was judged independently by two persons with an acceptable inter-rater reliability ($\kappa > .70$). In case of different coding a consensual agreement was achieved after a discussion. In this report, we present a coarser category system in which the 21 categories have been combined into 5 categories (Table 1). We used χ^2-homogeneity tests to analyze the data for research questions 1 and 2, and a t-test as well as ANCOVAs for research question 3.

RESULTS

Table 1 presents the strategies the students used in the three tests. A comparison of columns No. 1 and 2 in Table 1 indicate that there was no significant difference in strategy use between the students of the training group and their peers at the start of the school year. The significant effects of the one-week training in October becomes apparent at T2 in January (columns No. 3 and 4): the trained group used much more short-cut strategies and less universal strategies than their peers.

Column No.	1	2	3	4	5	6
	T1 - start of school year		T2 - before introduction written algorithms (midterm)		T3 - after introduction written algorithms (end of school year)	
	Regular class	Regular class & training	Regular class	Regular class & training	Regular class	Regular class & training
Written algorithm	42 (3.4%) [a]	23 (5.0%)	168 (13.4%)	31 (6.5%)	583 (45.5%)	179 (37.8%)
Number-based universal strategies	692 (56.0%)	251 (54.7%)	558 (44.5%)	169 (35.5%)	280 (21.8%)	76 (16.1%)
Short version of number-based universal strategies	248 (20.1%)	79 (17.2%)	252 (20.1%)	57 (12.0%)	193 (15.1%)	29 (6.1%)
Number-based short-cut strategies	118 (9.6%)	57 (12.4%)	221 (17.6%)	211 (44.3%)	213 (16.6%)	185 (39.1%)
Not assignable	135 (10.9%)	49 (10.7%)	56 (4.5%)	8 (1.7%)	13 (1.0%)	4 (0.8%)
Total[b]	1235 (100%)	459 (100%)	1255 (100%)	476 (100%)	1282 (100%)	473 (100%)
χ^2	$\chi^2(4, N = 1694) = 6.47$		$\chi^2(4, N = 1731) = 139.41$		$\chi^2(4, N = 1755) = 109.28$	
p	.166		< .001		< .001	
Cramér's V[c]	.06		.28		.25	

[a] Percentages are column percentages, [b] Different total numbers due to a few missing solutions; theoretical maximum number of solutions was 1296 for regular class and 480 for regular class & training, [c] Interpretation of Cramér's V: weak association: < .20, moderate association: .20-.50 and strong association: > .50

Table 1: Number of applied strategy types for students in regular class and in regular class with additional training at start, midterm and end of school year

To analyze research question 1, we compared columns No. 3 and 5 which show the strategy use of the 162 untrained students before and after the introduction of the written algorithms. As expected, the use of the written algorithms drastically increased whereas the use of the number-based universal strategies decreased. The small amount of short-cut strategies remains stable (17.6% to 16.6%). For research question 2, we compared columns No. 4 and 6 and found a similar development for the trained students: strong increase of written algorithms, decrease of number-based universal strategies, and the amount of short-cut strategies remains more or less stable (44.3% to 39.1%). However, the difference to the untrained students is that the trained students used much more short-cut strategies before the introduction of written algorithms (44.3% to 17.6%) and the use of these strategies remains stable at T3 (39.1%).

For research question 3, we considered the test scores of the students (1 point for each correct solution). Table 2 presents the results for the different tests as well as the reliabilities. The t-test revealed no significant difference at T1, the start of the school year ($t(220) = 1.9$, $p = .066$, $d = 0.27$), despite the trained students ($M = 5.10$, $SD = 2.41$) showing higher scores than the untrained students ($M = 4.51$, $SD = 1.99$).

Accuracy strategy use (max 8 points)	T1 - start of school year *M* (*SD*)	T2 - before introduction written algorithms (midterm) *M* (*SD*)	T3 - after introduction written algorithms (end of school year) *M* (*SD*)
Regular class	4.51 (1.99)	4.98 (2.17)	5.44 (1.91)
Regular class & training	5.10 (2.41)	5.53 (2.18)	6.02 (1.81)
Total	4.67 (2.12)	5.13 (2.18)	5.60 (1.90)
Cronbach's α	.70	.74	.66

Table 2: Accuracy of applied strategies (mean values and standard deviations for correct solutions) for trained students and their peers at T1-T3

We ran two analyses of covariance with T1 as covariate and T2 as well as T3 as dependent variable. Neither at T2 ($F(1, 219) = 0.66$, $p = .417$, part. $\eta^2 = .003$), nor at T3 ($F(1, 219) = 1.84$, $p = .176$, part. $\eta^2 = .008$) significant effects occurred.

DISCUSSION

The results in Table 1 (columns No. 1 and 3) are consistent with previous findings that students without a specific support use only few strategies and, in particular, hardly use any short-cut strategies (e.g., Csíkos, 2016; Heinze et al., 2009; Hickendorff, 2020; Torbeyns & Verschaffel, 2016; Torbeyns et al., 2017). Table 1 (column No. 5) replicates findings that the written algorithms are dominant after their introduction (e.g., Hickendorff, 2020; Nehmet et al., 2019; Selter, 2001; Torbeyns & Verschaffel, 2016; Torbeyns et al., 2017). Regarding the previously mentioned two possible explanations for the dominance of the written algorithms, our findings support the first explanation. In the untrained group, mostly universal number-based strategies were used, which were then replaced by the written algorithms (Table 1, columns No. 3 and 5). The second possible explanation for the dominance of the written algorithms cannot be supported. The training group had used a high proportion of short-cut strategies before the introduction of the written algorithms (Table 1, column No. 4). This proportion remained essentially stable after the introduction of the written algorithms (Table 1, columns No. 4 and 6). Thus, it can be assumed that if students show skills to use short-cut strategies, this kind of strategy use will be maintained and short-cut strategies will not be replaced by written algorithms. Finally, we could show that the use of a variety of strategies (including short-cut strategies) is not at the expense of the correctness of the solutions.

Limitations

There are several limitations of the study. The analysis is based on tests consisting of only eight items, which in turn all suggested short-cut strategies. A longer test would be desirable, including items where short-cut strategies did not provide an efficient solution. Second, the items were the unit for analysis in Table 1; an analysis with the students as the unit will still be conducted. Third, there is no information about the mathematics instruction in the 15 classes. Given the weak results for the untrained students, we assume that there was not a strong emphasis on short-cut strategies. Fourth, the trained students participated voluntarily in the training during fall break. It might be the case that these students are more interested in mathematics. However, the data we presented above does not indicate that these students are only high-achieving students. Finally, there may be other possible explanations for why the written algorithms become dominant. For example, socio-mathematical norms perceived by the students could also play a role.

Educational practice and further research

Despite the limitations, suggestions for teaching practice can be derived from our study. For example, we found that promoting the use of different strategies (including short-cut strategies) before the introduction of the written algorithms leads to the retention and further use of these strategies after the learning of the written algorithms. We assume that the flexible use of different strategies can be further increased if it is addressed again after the introduction of the written algorithms. An appropriate range of tasks in textbooks could have impact on teacher action (Sievert et al., 2019). Such an approach and also approaches of interleaved learning of strategies (Nehmet et al., 2019) should be investigated in further studies.

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