FOCUSING ON NUMERICAL ORDER IN PRESCHOOL PREDICTS MATHEMATICAL ACHIEVEMENT SIX YEARS LATER

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The development of numerical ordering of number symbols, unlike numerical ordering of other stimuli, such as sets of everyday items, has recently gained growing research interest. Here, we report a nine-year follow-up study with 36 three-year-old children. We investigated how children's focusing on numerical order develops alongside number sequence production and cardinality recognition skills. Results showed large individual and developmental differences in children's focusing on numerical order from the ages of 3 to 6 years. Preschool focusing on numerical order and spontaneous focusing on numerosity predicted curriculum-based math achievement at 12 years of age.

INTRODUCTION

Children learn many important mathematical skills at preschool age. At around 3 years of age, they already practice reciting number word sequence and start to recognize cardinal values for small sets of items by subitizing (Fuson, 1988). Next, at roughly 3.5 years of age, children begin to learn to recognize the numbers of items by counting (Wynn, 1990). Counting skills develop gradually, and there are many subskills involved (Fuson, 1988; Sarnecka et al., 2015). It has been well established that these early mathematical skills predict later academic achievement (Duncan et al., 2007), and a great number of studies have described how early mathematical skills develop (Clements & Sarama, 2007).

Children's tendencies to spontaneously focus their attention on mathematical aspects have drawn increasing research interest (Verschaffel et al., 2020), following research on spontaneous focusing on numerosity (SFON) (Hannula & Lehtinen, 2005). Numerous studies show developmentally significant and domain-specific individual differences in children's tendency to focus on exact numerosity in situations that are not explicitly mathematical, even if they have the mathematical skills to do so (for a review, see McMullen et al., 2019). Children with a higher SFON tendency have been shown to have an advantage in the development of early and later mathematical skills in elementary school (for a review, see Verschaffel et al., 2020).

Studies of numerical order development have found that children who are better at deciding whether three numbers are in numerical order or not have better arithmetic skills later in life (Malone et al., 2021; Attout & Majerus, 2018; Lyons et al., 2014). Children's ability to process numerical order is a unique predictor of later arithmetic skills, and the predictive force seems to increase from Grade 1 to Grade 6, exceeding

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cardinal skills as a predictor of arithmetic skills (Lyons et al., 2014). However, studies regarding numerical order have mainly focused on the order of number symbols; only a few studies have used non-symbolic stimuli (Spaepen et al., 2018), and even rarer are studies using play-based methods with sets of toys.

Numerical order is not just some artificial order related to number symbols. It is a consequence of the corresponding cardinal values, which have the order of each cardinal value being exactly one more than the previous one (Spaepen et al., 2018). Recent studies have shown that even if children acquire counting and cardinal skills, they still lack the generalized knowledge of how counting up the number sequence is related to cardinal value increasing by one (Spaepen et al., 2018; Cheung et al., 2017), which is the mechanism of how all natural numbers are constructed. Understanding how cardinal and ordinal aspects of numbers are integrated during early development is a question that has yet to be answered.

THE PRESENT STUDY

A developmentally important aspect of numeracy has been neglected in previous studies: the recognition of the numerical order of nonsymbolic items. Here, we report longitudinal data on a novel task that was developed for measuring children's focusing on numerical order in the context of a three-year longitudinal study on early numeracy (Hannula & Lehtinen, 2005) with a follow-up study that took place nine years after the first testing. The data sought to emphasize that the use of numerical order in action first requires focusing on the exact numerosity of items in sub-sets, followed by recognizing the number of items in the sub-sets, only after which can the focusing on and recognition of numerical order of sub-sets take place. Focusing on numerical order may thus require well-integrated cardinal and ordinal aspects of numbers (Anderson & Cordes, 2013), and it may thus appear only after children have learned to fluently recognize the cardinality of a set by counting. The research questions are as follows:

1) How does focusing on numerical order develop from the age of 3 to 6 years?
2) How do preschool mathematical skills, such as spontaneous focusing on numerosity, focusing on numerical order, number sequence production, and subitizing-based enumeration, predict math achievement at 12 years of age?

METHOD

Participants

Thirty-six Finnish children (18 girls and 18 boys) with no developmental delays from Finnish-speaking families in daycare participated in this study. The mean age of the children was 3.0 years (*SD* = 1.5 months) at the start of the first data collection.

Procedure and tasks

Preschool data collection took place at the ages of 3, 4, 5, and 6 years. Children were tested for their focusing on numerical order, number sequence production, and cardinality recognition skills at every time point. In addition, the children were tested for their subitizing-based enumeration skills at the age of 5 and their SFON tendency at the age of 6 years. A follow-up was conducted at the age of 12 years, where the children were tested for their curriculum-based math achievement.

Children's *focusing on numerical order* was assessed with a novel, previously unreported caterpillar ordering task. In this video-recorded task, the child was shown similar boxes. Each box contained a caterpillar with a unique number of legs and a picture of a matching number of socks (see Figure 1). At the ages of 3 and 4 the child was shown five boxes (1–5 legs) and seven boxes (1–7 legs) at the ages of 5 and 6. First, the experimenter helped the child notice that each caterpillar had its own box of socks with as many socks as the caterpillar needed (Figure 1a.). The boxes were placed on the table and the caterpillars next to their own boxes. Then, the experimenter took the caterpillars away from their boxes and said, "Let's organize these boxes of socks like this. Every box has its own place." With the socks being visible, the experimenter organized the boxes in a vertical row in an increasing numerical order (Figure 1b.), and the child was asked to remember where each box was. The child was left to notice themselves that the boxes were in numerical order. Then, the experimenter closed the boxes, handed the caterpillars in front of the child, and asked the child to show each caterpillar where its own box of socks was (Figure 1c.). The highest number of caterpillars in the correct order was recorded and regarded as the score in the task.

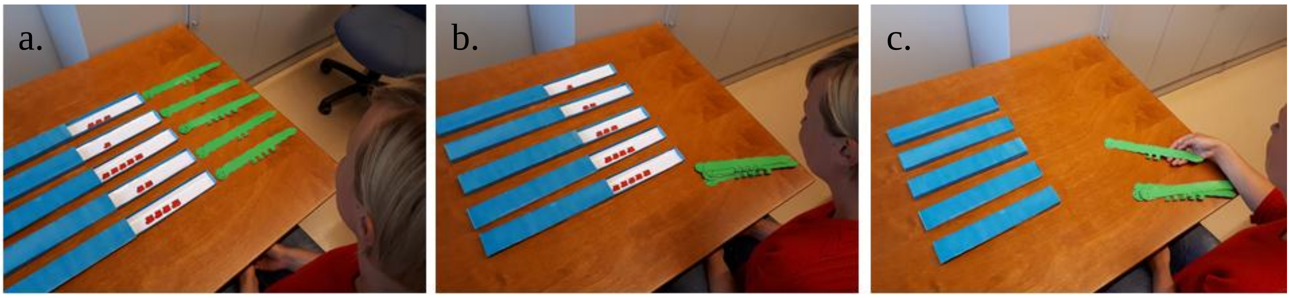

Figure 1: Caterpillar ordering task used to measure children's focusing on numerical order.

Children's *SFON tendency* was measured at the age of 6 years with a sum score of Imitation, Model, and Finding tasks (for details, see Hannula & Lehtinen, 2001; 2005) in which it was assessed how frequently the child noticed and used exact number of items spontaneously, i.e. without any guidance or explicit task instructions. *Cardinality recognition skills* were assessed at the ages of 3 and 4 with the caterpillar task ("Bring the caterpillar as many socks as it needs", max 10 [see Hannula & Lehtinen, 2001; 2005]) and at the ages of 5 and 6 years with an object counting task ("Count aloud how many 'turtles' there are on the table", max 23 [see Hannula & Lehtinen, 2005]). In the *number sequence production* task, children were asked to count as far as they could, or until fifty, where they were stopped (for details, see Hannula & Lehtinen, 2005). Children's *subitizing-based enumeration skills* were measured with a computer-based test, in which the child was asked to identify which of the four groups with different numbers of dwarves had stolen the groups of objects that had earlier been flashed on the laptop for 120 ms (see Hannula et al., 2007). Curriculum-based *math achievement* was measured using the Finnish standardized RMAT test (Räsänen, 2004), which includes 56 items of multi-digit arithmetic and simple algebra.

RESULTS

The results in Table 1 show large individual differences and how children's focusing on numerical order developed from 3 to 6 years, where most participants did not really understand the task at 3 years old to almost all of them mastering it at 6 years old in the caterpillar ordering task.

	Highest number of correct order produced							
Age	0	1	2	3	4	5	6*	7*
3 years	87	10	3	0	0	0		
4 years	44	18	13	3	0	23		
5 years*	10	18	8	5	5	0	0	54
6 years*	0	2	8	10	0	0	0	80

Note. * The total number of boxes used was five at the ages of 3 and 4, and seven at the ages of 5 and 6.

Table 1: The highest number up to which the boxes were numerically correctly ordered in percentages of the sample (N = 36).

Figure 2 demonstrates the differing ranges for children in number sequence production, cardinality recognition (or object counting), and focusing on numerical order. Importantly, this is only indicative of children's number ranges in real life due to the task maximums differing in various measures. However, a closer look at the individual children's developmental data indicates that the developmental order of the skills followed the same pattern. First, children learned to recite a list of number words, then to recognize small numbers of items and develop object counting skills for the first three or four numbers; only then did they start to notice the numerical order of items. The differences in the numerical ranges of the children's skills mirrored the developmental order of the skills as well.

Next, we explored the associations between preschool mathematical skills (number sequence production, subitizing-based enumeration, and focusing on numerical order at the age of 5, and SFON tendency at the age of 6) and math achievement at 12 years old. We found significant correlations within preschool mathematical abilities, and also between all preschool mathematical abilities except for subitizing-based enumeration and general mathematical abilities at 12 years (Table 2).

We further investigated whether preschool mathematical skills predict mathematical skills at the age of 12 by means of multiple linear regression analysis. We included number sequence production, subitizing-based enumeration, focusing on numerical order at 5 years, and SFON tendency at 6 years as predictors in the model. The results presented in Table 3 indicate that there was a collective significant effect between preschool mathematical skills and general math achievement by the age of 12. A closer look at the predictors indicates that only focusing on numerical order at 5 years and SFON tendency at 6 years were significant predictors of general mathematical abilities, unlike number sequence production at 5 years or subitizing-based enumeration at 5 years old.

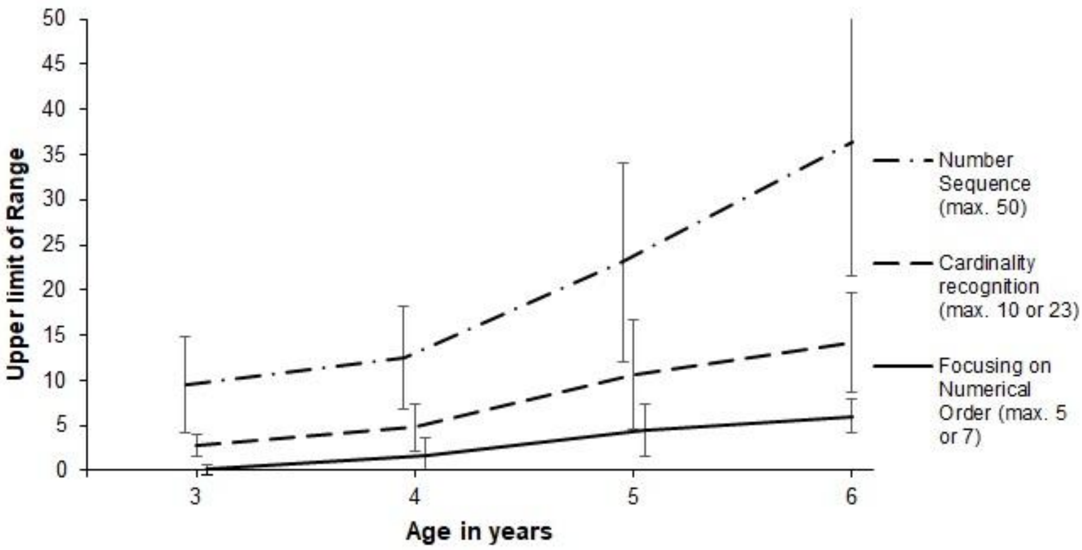

Figure 2: Children's accurate number ranges in number sequence production, cardinality recognition/object counting, and focusing on numerical order.

Variable	*M*	*SD*	(2)	(3)	(4)	(5)
(1) Number sequence production at 5 years	23.42	6.53	.47**	.53***	.46**	.39*
(2) Subitizing-based enumeration at 5 years	2.64	1.38	--	.39*	.40**	.09
(3) Focusing on numerical order at 5 years	4.61	2.84		--	.51**	.54***
(4) SFON tendency at 6 years	2.58	1.76			--	.54***
(5) Math achievement at 12 years	36.03	6.53				--

Note. * $p < .05$, ** $p < .01$, *** $p < .001$.

Table 2: Descriptive statistics and correlations between preschool mathematical skills and general math achievement at the age of 12 years ($N = 36$).

Variable	β (*SE*)	95% *CI*
Number sequence production at 5 years	.14 (.08)	[-.12 – .28]
Subitizing based enumeration at 5 years	-.27 (.75)	[-2.81 – .23]
Focusing on numerical order at 5 years	.36* (.39)	[.04 – 1.64]
SFON tendency at 6 years	.40* (.61)	[.23 – 2.71]
Total R^2 = .44***		

Note. * $p < .05$, ** $p < .01$, *** $p < .001$ ($N = 36$)

Table 3: Regression analyses predicting math achievement at 12 years.

CONCLUSION AND DISCUSSION

The current study investigated the three-year development of children's focusing on numerical order and its predictive effect on general mathematical abilities. We found evidence that children have individual differences in focusing on numerical order as well as an increase in focusing on numerical order from 3 to 6 years of age. Our finding seems to be in line with an earlier study that showed that cardinal recognition skills develop before the ability to order sets numerically (Spaepen et al., 2018). In fact, Cheung et al. (2017) have suggested that only after acquiring cardinal skills can children place sets of objects in correspondence with the number sequence, which is also reflected in our data.

Next, we found that focusing on numerical order and SFON tendency were significant predictors of children's math achievement, even after controlling for number sequence production and subitizing-based enumeration skills at preschool age. This indicates that focusing on numerical order might be an important aspect of early mathematical skills. Interestingly, number sequence production, which many studies have reported to be a strong early predictor of mathematical skills (Koponen et al. 2016), did not significantly predict later mathematical skills when the two numerical focusing tendencies were included in the model.

These results may indicate a similar reciprocal development between early mathematical skills and focusing on numerical order, as was reported in earlier studies of SFON (Hannula & Lehtinen, 2005). Initial number skills enable the spontaneous use of these skills in various situations, in this case noticing and making use of numerical order, which subsequently leads to enhanced mathematical skills. In this study, the caterpillar ordering task had hints toward the numerical nature of the task, so the task did not yet measure children's *spontaneous* focusing on numerical order. In addition, our sample was small, and the numerical order focusing task had only one test item. Thus, our results need to be treated as suggestive. Future studies with larger samples and more tasks should investigate whether focusing on numerical order could be another member of the "spontaneous mathematical focusing tendencies" (McMullen et al., 2019), called Spontaneous Focusing On Numerical Order (SFONO).

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