META-SCIENTIFIC REFLECTION OF UNDERGRADUATE STUDENTS: IS MATHEMATICS A NATURAL SCIENCE?

Patrick Fesser and Stefanie Rach

Otto-von-Guericke-University Magdeburg

Reflecting on the nature of mathematics is an important activity for undergraduate students. To analyse students' reflection, we address the questions how students categorize mathematics in the system of scientific disciplines and what arguments they use to support their decision, in particular. In an online-survey, we implemented two open-ended items to gather information about the meta-scientific reflection of 296 undergraduate students enrolled in a mathematics-related study program. By analysing students' answers, we identified nine subthemes that can be grouped in three themes: (1) the content, (2) the method, and (3) the purpose of mathematics. Most of the students concentrated on only one of the three themes. Based on these results, we discuss in which way prompts can support students' meta-scientific reflection.

INTRODUCTION

Citizens of modern societies are regularly confronted with problems that are somehow related to scientific findings: whether in their personal daily lives, or in their social responsibility. Those problems may be solved or caused by scientific findings and disciplines, respectively. One recent prominent example is an adequate dealing with information about the COVID-19 pandemic. A sufficient understanding, a reasonable dealing with scientific findings and a reflection of scientific ideas subsumed under the term "meta-scientific reflection", are key components of citizens' participation in their personal, social and public life. This reflection is mentioned e.g. in German school curricula under the term "scientific propaedeutics" ("Wissenschaftspropädeutik") which is an aim of upper secondary schools (KMK, 1972/2021).

So far, only few studies have examined whether students are able to reflect on a meta-scientific level or not (Oschatz et al., 2018). One possible reason for this absence of studies involving meta-scientific reflection is the lack of validated instruments for measuring meta-scientific reflection (Dettmers et al., 2010), especially on mathematics as a scientific discipline. Therefore, we developed an instrument to gain information about students' meta-scientific reflection on mathematics; in particular, to answer the questions how undergraduate students categorize mathematics in the wide system of scientific disciplines and what arguments they use to support their categorization.

THEORETICAL BACKGROUND

The reflection on scientific findings is important for the preparation of a university study program as well as being part of the modern scientific-orientated society. We call the concept *meta-scientific reflection* that covers reflection on an epistemological level:

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> Speaking about mathematics from an epistemological perspective, for example, about the characteristic distinctions between mathematics and other sciences, about the nature and the origin of mathematical knowledge, and so on (Neubrand, 2000, p. 256).

This quote emphasises the importance of reflecting on the differences between scientific disciplines and getting an orientation in the system of scientific disciplines. This perspective on meta-scientific reflection focuses on the development of skills that are helpful for understanding and evaluating scientific findings based on the used methods, conditions, and limits. To be able to reflect on scientific disciplines meta-scientifically, one has to have enough meta-scientific knowledge and an appropriate methodological awareness. Therefore, meta-scientific reflection is most difficult to achieve and maybe even not achievable by every high school student (Klafki, 1984).

The existing empirical research literature focuses strongly on meta-scientific reflection regarding natural sciences (e.g., Oschatz et al., 2018) or just scientific thinking and working in general (e.g., Dettmers et al., 2010). To the best of our knowledge, there are no validated instruments that measure meta-scientific reflection on mathematics. Literature addressing close-related concepts such as epistemological knowledge about mathematics can be found (e.g., Hoffmann & Even, 2021; Zazkis & Leikin, 2010). These projects investigated knowledge rather than reflection and mostly focus on teachers and not on high school graduates. Contrary to Rott and Leuders (2017), we are not interested in students' beliefs concerning mathematics, e.g., if mathematical knowledge is certain or not, but more general what mathematics look like. Our study on meta-scientific reflection of undergraduate students addresses this research gap.

Presenting the whole nature of mathematics, is not possible due to page restrictions. Thus, we use the following statement to illustrate the dual nature of mathematics:

> […] mathematics can be best understood as a framework for studying concrete real-world phenomena in terms of underlying abstract mathematical models. (Hansen, 2008, p. 1).

Hansen (2008) differentiates between an abstract and a concrete site of mathematics. Whereas abstract mathematics focuses on the analysis of formal structures and abstract ideas, concrete mathematics is related to the application of the abstract patterns and structures in real-life situations.

To further characterize scientific disciplines like mathematics, it is sensible to look at the various disciplines from different perspectives, namely in regard to (1) its contents (objects, structures and theories), (2) its methods and processes and (3) its purpose and goals etc. (Niss, 2014). These three aspects can be used to distinguish scientific disciplines from others.

The study in this contribution is a follow-up study of a study in which we investigated reasoning patterns that students used to argue whether mathematics is a natural science or not (Fesser & Rach, 2020). The findings indicated that more than half of the participating students think that mathematics is a natural science. Students' reasons for their decision referred to nine subthemes that can be grouped into three themes inspired by Niss (2014): (1) the content of mathematics, dealing with the question: What does mathematics contain? (2) the method of mathematical studies, which deals with the question: How are mathematical findings obtained? (3) the purpose of mathematics, dealing with the question: What are the goals of mathematics? (see Figure 1).

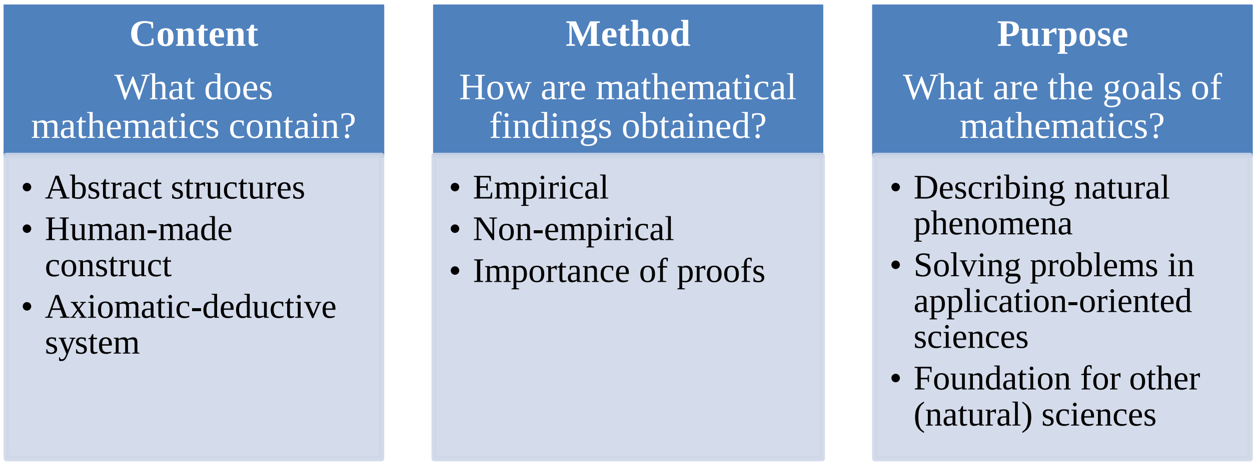

Figure 1: Framework for students' reflection on the nature of mathematics.

In the previous study (Fesser & Rach, 2020), we identified which themes students mentioned when reflecting on the question if mathematics is a natural science. Now, we analyse which themes and subthemes students use to agree or disagree to this statement to gain a deeper insight into students' meta-scientific reflection.

RESEARCH QUESTIONS

In the current study, we focus on undergraduate students who were enrolled in a Bachelor's Degree Program in which students have to participate in mathematics courses. Our aim is to investigate whether undergraduate students are able to reflect on the nature of mathematics. The research questions for this study are as follows:

- In which way do undergraduate students categorize mathematics as a scientific discipline?
- What arguments do undergraduate students use to support their categorization?

METHODS

Sample

The study was conducted at a middle-large university in Germany at the beginning of the winter term 2020/21. The target group of this study were first-year students who were enrolled in Bachelor's Degree Programs that are somehow related to mathematics (e.g., STEM subjects, economics, …). For this study, we focused on first-year students because we want to learn more about meta-scientific reflection of undergraduate students. The sample was collected via convenience sampling and participating was voluntary and anonymous. Out of 313 students who participated in the study, 296 participants (94.6%) answered the open-ended items. Those 296 participants (56.1% female, 82.4% younger than 22 years) were enrolled in the following study programs:

49.7% in economics-related, 21.3% in teacher training, 14.2% in technical-related, 7.4% in computer science, 4.1% in mathematics, and 3.3% in other study programs.

Data collection and data analysis

Due to the pandemic situation and other pragmatic reasons, we conducted the study via the online survey tool *SoSci Survey*. The main data source included answers to two open-ended items, which were implemented in a questionnaire. The two items to collect data about students' meta-scientific reflection were given in German and can be translated as follows:

> The term natural sciences includes e.g. sciences like biology, chemistry, and physics. Take position on the following statement "Mathematics is a natural science.".
>
> - What is in your opinion the strongest argument that (a) supports and (b) contradicts the given statement?
> - Compare both arguments and come to a decision whether mathematics is a natural science or not. Explain your answer.

At the beginning of this questionnaire, participants were instructed to think of mathematics as a scientific discipline. The participating students were asked to answer the questions within 2-4 whole sentences. In this contribution, we focus our analysis on the second item.

The qualitative data was gathered and then analysed, applying the summarizing qualitative content analysis (Mayring, 2015). In the first step (initial read-through), we read all the given answers and gained an overview about the whole data set. After that, we marked all sentences that dealt with a categorization of mathematics as a scientific discipline and possible arguments supporting ones positioning. Then we used the framework in Figure 1 and a peer-validated category system (Fesser & Rach, 2020) for the coding of the answers, enabling room for new categories if needed. Thus, the analysis was deductive and inductive, gathering also insight about adjacent themes.

FINDINGS

Students' answers on the categorization of mathematics as a scientific discipline and the related arguments to support the positions were associated with the three themes and the nine subthemes. A further analysis indicated that no more themes and subthemes emerged from the data. Firstly, we describe students' categorization of mathematics as a scientific discipline and secondly the used argumentation patterns. Thirdly, we give an insight about further findings and a new perspective mentioned by the participants.

Categorization of mathematics as a scientific discipline

The first research question deals with undergraduate students' categorization of mathematics as a scientific discipline: 137 participants categorized mathematics as a natural science (46.3%), 140 disagreed with the statement "Mathematics is a natural science" (47.3%) and 19 has not clearly positioned themselves (6.4%). Therefore, we have a rather equal distribution of students who agree and students who disagree with the statement that mathematics is a natural science (see Figure 2).

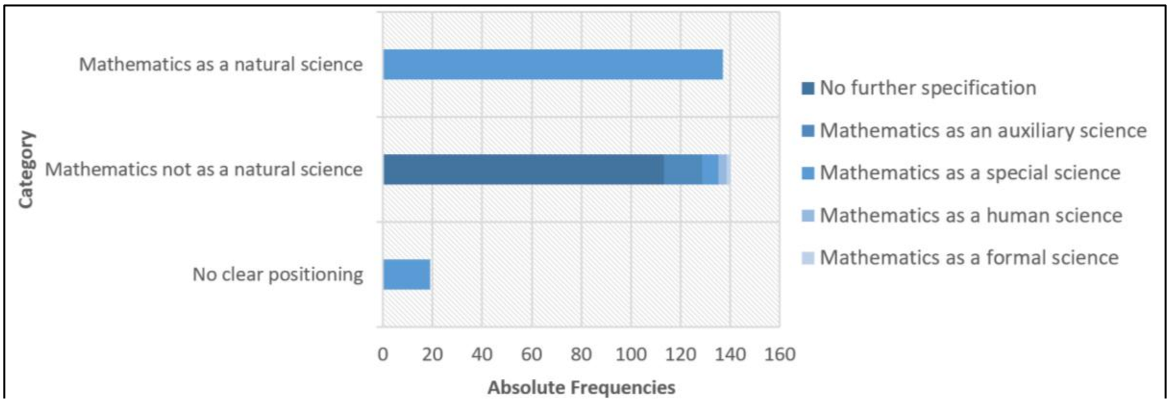

Figure 2: Students' categorization of mathematics as a scientific discipline.

Out of the participants who disagreed with the statement that mathematics is a natural science, 34 students took a further approach to specify mathematics as a scientific discipline. The students labelled mathematics mostly as an "auxiliary science" (55.9%), followed by "special science" (23.5%), "human science" (11.8%) and "formal science" (5.9%). One student who disagreed with the statement that mathematics is a natural science, also stated that mathematics is not a scientific discipline at all:

> As mathematics is a tool for explaining natural phenomena, I would say that mathematics is not a scientific discipline, rather than just a tool for natural sciences so that they [natural sciences] can keep working and researching (A170_2).

Argumentation patterns

Besides the categorization of mathematics itself, we also analysed the arguments that students used to support their categorization. 222 students gave a reason for their positioning. Out of those 222 participants, 195 students (87.8%) gave a sensible answer that we could further analyse. Whereas each of the three themes was addressed in the answers, the number of mentions varied substantially between and within the themes.

Content of mathematics

48 (24.6%) out of the total 195 students are associated with this theme, dealing with the questions: What does mathematics contain? Most of the students within this theme referred to abstract structures as the specific research objects of mathematics (56.3%), for example: "The research objects of mathematics are not natural phenomena, but the structure of formal objects" (A29_2). Fewer students referred to mathematics as a mental construct that is made by humans (29.2%); respectively mentioning that mathematics is characterized by its logical axiomatic-deductive structure (14.6%). Out of the 48 students that mentioned this theme, only one student supported the statement that mathematics is a natural science. It seems like referring to this theme relates to disagreeing with the statement that mathematics is a natural science.

Method of mathematics

The second theme could be emerged from 17 (8.7 %) students' answers, dealing with the question: How are mathematical findings obtained? Similar to the first theme only one out of the 17 students argued that mathematics is a natural science. Most of the students within this theme associated with mathematics being a non-empirical science (76.5%). These students often referred to differences between mathematics and the natural sciences, e.g., as mathematics is not an experience-based science, it does not generate findings based on experiments and observations. Further "it is not common to collect empirical data" (A85_2) for doing mathematics. Three students focused on the importance of proofs as the main criteria of evidence in mathematical research: Mathematics is characterized by "proving specific logical statements and theorems" (A91_2). Students use this subtheme to make clear that mathematical findings has to be proven prior to be used by other sciences.

Purpose of mathematics

Most of the students associated with the third theme (64.6%). Within this theme, we could find the following distribution to the subthemes: 62.2% mathematics as a foundation for the natural sciences, 26.0% mathematics as a tool for describing natural phenomena, and 11.8% mathematics for solving problems in scientific or daily life situations. Concerning the first subtheme, students reported that mathematics is a key component of the scientific working in the natural sciences. Mathematical methods are needed to predict developments and to represent natural phenomena via various diagrams. The first subtheme "mathematics as a foundation" was used to support (50.6%) and to oppose the statement that mathematics is a natural science (49.4%). On the one hand, students argued that mathematics is a foundation and therefore a part of the system of natural sciences and on the other hand, students argued that as mathematics is a foundation for natural sciences, it cannot be a natural science itself. That means that this subtheme is evenly distributed among the positions.

Further findings

Three students' answers could not be associated with any of the formulated themes. Those answers had in common that they are referring to their school experiences with mathematics, for example:

> I think mathematics is a natural science because that it is how I was taught in school. For me, that is like a fact. (A221_2).

Those argumentation patterns does not give any hint of a reflecting process, but are only based on prior experiences and knowledge (knowing that mathematics is a natural science) that was accumulated in school. Therefore, we does not expand our category system (see figure 1) with a new category or theme.

Apart from the findings above, we could also generate some interesting results concerning students' opinions and meta-scientific reflection on mathematics: Some students reported that it was their first time thinking about the place of mathematics in the wide system of sciences. One student even reported that reflecting about this question changed her/his former view on mathematics as a scientific discipline:

> For me, mathematics belonged to the natural sciences because that is how it was always portrayed to me. However, based on the arguments I found, I would say that mathematics is not a natural science but a scientific discipline that revolves around logic (A221_2).

We did not expect that students in higher secondary schools were taught that mathematics is a natural science. Likewise, we were surprised about students reporting that they had not reflected on the nature of mathematics before.

DISCUSSION

The conceptual framework based on Niss (2014) and the developed category system in a prior study (Fesser & Rach, 2020), was useful to examine how undergraduate students categorize mathematics as a scientific discipline and what arguments they use to support their decision. The analysis of students' answers showed that students mentioned all nine subthemes. For arguing that mathematics is a natural science, students referred to the purpose of mathematics: arguing against, they used the content and the method of mathematics. Thus, the purpose of mathematics as an abstract discipline seems not to be clear to students and therefore should be more explicated in mathematical classes to gain a more holistic understanding of mathematics. Besides referring to the representation of mathematics as a natural science in school, no additional themes were associated by the students.

As this study was implemented in a written survey, we were limited when analysing the given word material. Therefore, we were not able to ask students to explain deeply arguments they put forward, e.g., that mathematics was taught in high school. Future projects may collect data on meta-scientific reflection in interviews to get an insight about the quality of argumentations as to which Rott et al. (2014) provided results.

Our findings suggest that most of the students are able to categorize mathematics as a scientific discipline giving reasons. Even though most students gave reasons, the referred themes differ between students and many students ignore the abstract site of mathematics (Hansen, 2008). To support student in reflection, it seems not to be sufficient using reflection prompts only in surveys, but it has to be implemented in regular lessons (see Liebendörfer & Schukaljow, 2020). To gain a deeper insight into meta-scientific reflection, more research is needed to consider (1) whether the categorization is related to students' characteristics, (2) understand the differences between students' reasoning and (3) investigate whether mathematics is portrayed as a natural science in school or not.

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