NETWORKED MULTIPLE APPROACHES TO DEVELOPING FUNTIONAL THINKING IN ELEMENTARY MATHEMATICS TEXTBOOKS: A CASE STUDY IN CHINA

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By focusing on functional thinking (FT), the core component of algebraic thinking, this study aimed to explore the features of developing FT in textbooks from grades 1 to 6 by examining a popular reform-oriented mathematics textbook series in China. A framework of FT developed by Pittalis and colleagues (2020) was used to examine the FT related tasks in the textbook series. Based on a fine-grained coding analysis, it was found that multiple modes of FT are intended to be developed since the very beginning of elementary school. Multi-modes of FT have been developed and evolved simultaneously and progressively as grades increase, serving as an enhancement for arithmetic learning. Different types of FT tasks provide various opportunities for students to explore these multi-modes of FT while learning and consolidating arithmetic across grades.

INTRODUCTION

Researchers identified that the traditional sequence of "arithmetic-then-algebra" is the hurdle for students in learning algebra (Stigler, et al., 1999), and equation-entry toward learning algebra may limit students' learning of advanced mathematics (Thompson & Carlson, 2017). As one of four big ideas of algebraic thinking (Blanton & Kaput, 2011), functional thinking (FT) has been recommended as a better organizing concept for teaching and learning algebra than the concepts typically used (e.g., expressions and equations) (Stephen, et al., 2017). Although researchers have shown that elementary students are capable of engaging in generalizing and representing functional relationships (e.g., Blanton, 2008), the topic of FT has not been addressed purposefully and systemically in the early mathematics curriculum (Carraher, Schliemann & Schwartz, 2008), or neglected altogether (Stephen et al., 2017). Consequently, it remains largely unknown how FT can be systematically and effectively introduced and developed in elementary schools in tandem with numeration and arithmetic, as well as how to prepare students for formal algebraic learning.

Studies on Chinese mathematics textbooks have drawn international attention because Chinese 15-year-old students have consistently outperformed their counterparts in the Western countries on PISA (OECD, 2020). In the traditional Chinese mathematics textbooks, the function concept is not introduced formally, but function ideas are explored implicitly (Cai, Ng & Moyer, 2011). Since the implementation of the new curriculum in China in 2011 (MoE, 2011), which has adopted numerous research findings and innovative ideas from Western literature (Xu, 2013), the officially

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endorsed textbooks have been developed and refined over the years. Thus, a systematic examination of the most popular reform-oriented textbooks in the elementary school with a focus on FT could provide insight for international readers regarding curriculum and teaching materials related to developing early algebra thinking, and FT in particular.

ANALYTICAL FRAMEWORK

A three-dimensional framework was created for analyzing learning opportunities intended in the textbooks. It includes the modes of FT, types of function tasks, and grade levels (as shown in Fig.1). The function tasks are used as the smallest unit in the textbook analysis (Stylianides, 2009), and their types are categorized. Each FT task is determined whether it is intended to develop any mode of FT. Finally, we use the dimension of grade levels (lower grades (1-2), middle grades (3-4) and upper grades (5-6)) to analyze how the textbooks arrange different kinds of function tasks in sequence to develop different modes of FT.

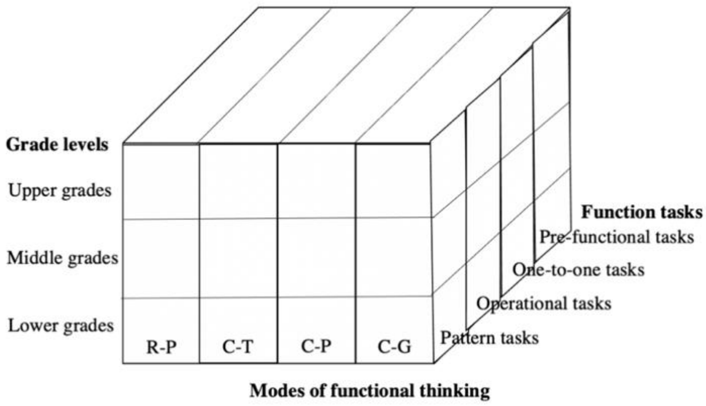

Figure 1: Analytical framework for the development of functional thinking

Modes of functional thinking

In this study, we adopted Pittalis et al.'s four modes of FT: recursive patterning (R-P), covariational thinking (C-T), correspondence relationships-particular (C-P), and correspondence relationships-general (C-G). Students exhibiting *R-P* focus only on one variable, finding variation within a sequence of values (Blanton & Kaput, 2011). *Covariation relationship* describes how two quantities co-vary simultaneously and students who hold this view keep that change as an explicit and dynamic part of a function's description (Blanton & Kaput, 2011), e.g., "if one can describe how x_1 changes to x_2, and how y_1 changes to y_2, he has described a functional relationship between x and y" (Confrey & Smith, 1991, p.57), which is *C-T*. Students exhibiting *C-P* means that they could notice the correspondence relations between corresponding pairs of values, e.g., one can complete a table involving two related quantities, while *C-G* means that students could identify and express the general relations between quantities or variables in word or symbols (Pittalis et al., 2020).

Function tasks

In this study if a task in the textbooks implicitly or explicitly reflects any mode of FT, it is called "a function task." Through coding and comparing the tasks in the textbooks, we combined similar types of tasks and grouped them into four categories: *pattern tasks, arithmetic operation set tasks*, *one-to-one corresponding tasks* and *pre-functional tasks*, which were described as below:

- *Pattern tasks* are problems which require one to seek patterns in numerical and geometric sequences;
- *Arithmetic operation set tasks* (Abbr. operational tasks) include function machine tasks and well-ordered arithmetic calculation items;
- *One-to-one corresponding tasks* (Abbr. one-to-one tasks) include comparing tasks in grade 1, and the corresponding tasks between numbers (or number pairs) and points on the number line and the Cartesian coordinate system.
- *Pre-functional tasks* are mainly the real-life problems which include two varying quantities, for example, using letters to represent varying quantities, and tabular problems which include proportional or linear relationships.

Different from Demosthenous and Stylianides (2018), we define the explicitness of a task regarding its relationship to modes of FT. If in answering one question in the task, students have to use one specific mode of FT, we consider this question to be "explicitly" developing this mode of FT; if students may or may not use one mode of FT, we call it "implicit."

The purpose of this study is to explore how FT evolves across grades in the selected reform-oriented mathematical textbook series in China (e.g., PEP textbooks). To achieve this goal this study seeks answers to the three research questions: how are function tasks arranged in the PEP textbooks to develop different modes of FT, how different modes of FT are embedded in the PEP textbooks and how do they evolve throughout the textbooks and if there are any particular routes for developing FT in the PEP textbooks.

METHODOLOGY

We selected Chinese PEP (People's Education Press) mathematics textbooks (Lu & Yang, 2012) for two reasons. The first is the reputation of the publisher, with this publisher being the only publisher to produce textbooks in China before 2001. The second is there are six sets of officially endorsed elementary mathematics textbooks in Mainland China, but the textbooks published by PEP are used by 63% of students there. All of the grade 1-6 student textbooks (12 volumes) and corresponding teacher guidebooks were selected.

In general, a content analysis was used to code and analyze the curriculum materials (Fan, 2013). There were two rounds of coding, first coding of function tasks, and second, coding of modes and explicit levels of FT. Four research assistants and the first author developed the coding system.

The first round of coding identified and categorized function tasks in the PEP textbooks. The third author read through the whole series of textbooks and identified function tasks based on the definitions of each of the four modes of FT (R-P, C-T, C-P and C-G). A research assistant read the teachers' guidebooks and marked all the points that declared that the design of the worked examples or practice problems intended to develop FT. Then a group meeting (including all the authors and the research assistants) was arranged to discuss the classification of the collection of function tasks into the four categories. Then all group members re-checked the functional tasks in the PEP textbooks, and the interrater reliability was checked using Cohen's kappa (usually kappa should be 0.7; Leech, Barrett & Morgan, 2008). The average kappa of 0.90 indicated good agreement between coders. Finally, we had a whole group discussion and all team members agreed with all the coding results.

Each identified function task included one or more questions, with the possibility of each question developing different modes of FT. So, there are two steps in coding the modes of FT reflected by each function task. Firstly, we counted the number of questions included in each task; and secondly, we determined the modes and explicit levels of FT developed by the question.

RESULTS

An overall distribution of tasks and relevant targeting modes of FT is shown in Fig.3. The figure reveals three salient features related to our research questions. (1) Multiple modes of FT are embedded in the math textbooks in all grades simultaneously;(2) Two routes could be identified for developing FT. These features are illustrated in the sections that follow.

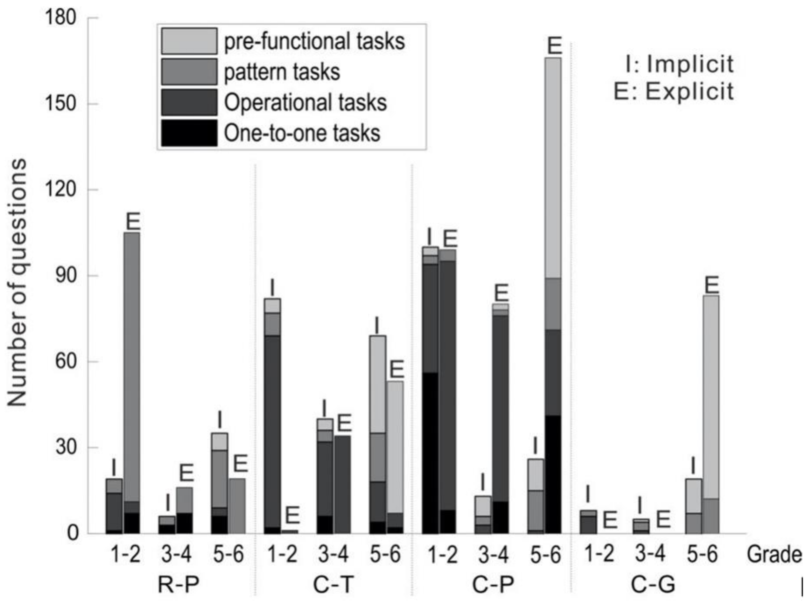

Figure 2: Distribution of function tasks that develop different modes of FT

Multiple modes of FT are developed simultaneously

Figure 2 illustrates that three modes of FT (R-P, C-T, and C-P) are developed simultaneously across three grade levels. In lower grades, two modes of FT (R-P and C-P) are developed explicitly through pattern tasks and operational tasks correspondingly. C-T is developed implicitly through operational tasks, while one-to-one tasks implicitly are used to develop C-P. In middle grades, C-T and C-P are emphasized simultaneously through the operational tasks. In upper grades, pre-functional tasks are used to develop C-T, C-P and C-G at the same time, and the pattern tasks, operational tasks and one-to-one tasks are still useful for developing C-P and C-T.

Grades	One-to-one tasks		Operational tasks		Pattern tasks		Pre-functional tasks		Total
Lower grades	30	20%	67	44%	55	36%	1	0%	153
Middle grades	9	16%	34	61%	6	11%	7	13%	56
Upper grades	26	24%	16	15%	22	20%	45	41%	109
Total	65	20%	117	37%	83	26%	54	17%	318

Table 1: Distribution of different types of function tasks across grades

Generally, there are 318 function tasks and 656 sub-questions in 12 textbooks, with the number of operational tasks being the most prevalent (117, 37%), and the number of pre-function tasks being the least (54,17%). The total number of function tasks in lower grades is the most (153), the number in upper grades is second (109), while the numbers in middle grades is the least (56).

Two routes are identified to develop functional thinking progressively

From Figure 2 and the above analysis, one can see two explicit and continuous routes in which FT is developed across grades. One explicit route is mainly using the pattern tasks, from R-P to C-G through C-P explicitly and C-T implicitly (Figure 3). The other is primarily utilizing the operational tasks and pre-functional tasks, from C-P to C-G, through C-T (Figure 4).

Route A is from R-P to C-G via C-P or C-T. The geometric patterns appear as repeated patterns in lower grades, simple growing patterns (additive or multiplicative) in middle grades, and complicated growing patterns (linear, non-linear) in upper grades. For the number pattern tasks, PEP textbooks combined number sense learning with the growing pattern. In grade 1 number patterns grow by 1, 3, 5, etc., which helps students understand the number sequence. In grade 2 the patterns might grow by 100s, 10s, or 1s, and in middle grades they might grow by 0.1s, 0.01s, 0.001s, etc., which is highly correlated with the knowledge of place value base 10.

In upper grades the PEP textbooks usually present number and geometric patterns together, and the figures help students to generalize the rules. In this way it is easier for students to match the figure and number (C-P), observe the change of both the figures and values (C-P, even C-T), and finally find out the rule (C-G). For some complicated patterns, the textbooks encourage students to observe the geometric figures and find the near-generalization term, which gives students opportunities to experience the non-linear relations which prepares them for the future leaning of functions.

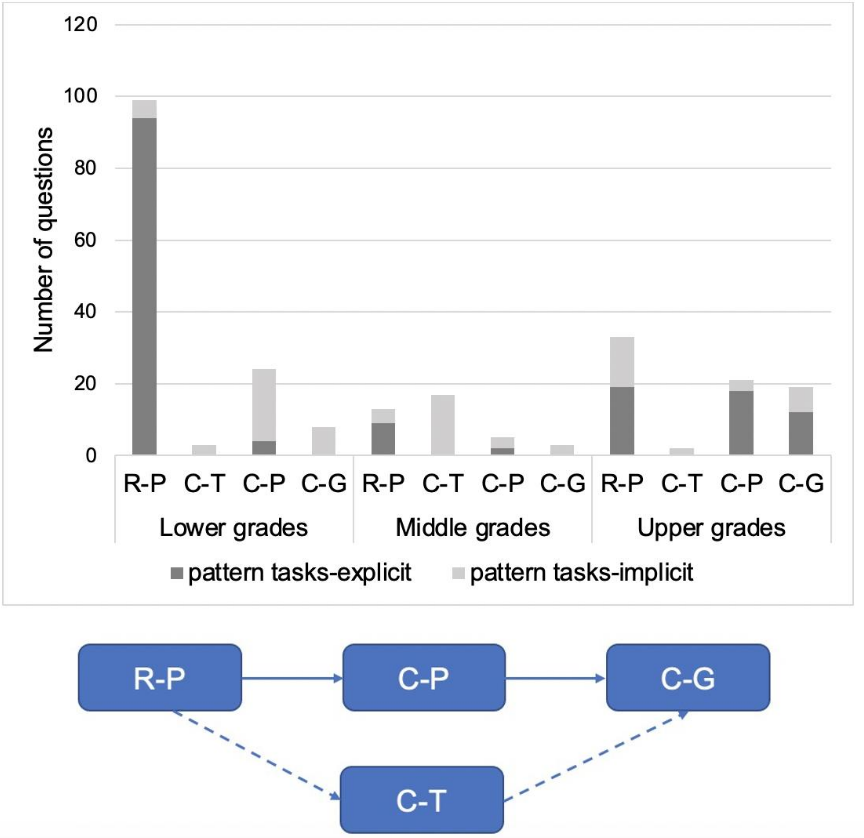

Figure 3: Route A develop FT from R-P to C-G

Route B is mainly from C-P to C-G through C-T. In the lower grades the textbooks usually used operational tasks and comparing tasks, explicitly developing C-P. Through utilizing the organizing table tasks, the textbooks begin to develop C-T implicitly. In middle grades, there are also function machine tasks which include fractions and decimals (explicit C-P), and some pre-functional table tasks, which frequently present the quantities in real-life situations, such as distance and time, price and cost, etc. These pre-functional tasks not only require students to fill in tables based on calculating (C-P), but also help students to experience the co-varying of two quantities. In upper grades, the pre-functional tasks usually involve direct or reverse proportional relationships, and they require students to generate the rules or judge the relationships (C-G) and also explicitly describe the co-variational relationship between the quantities. Many tasks develop C-P in upper grades, but some are by-products of C-G and C-T (e.g., find the corresponding values according to the rules they generate,), and some are preparation for future study (such as, non-linear functions).

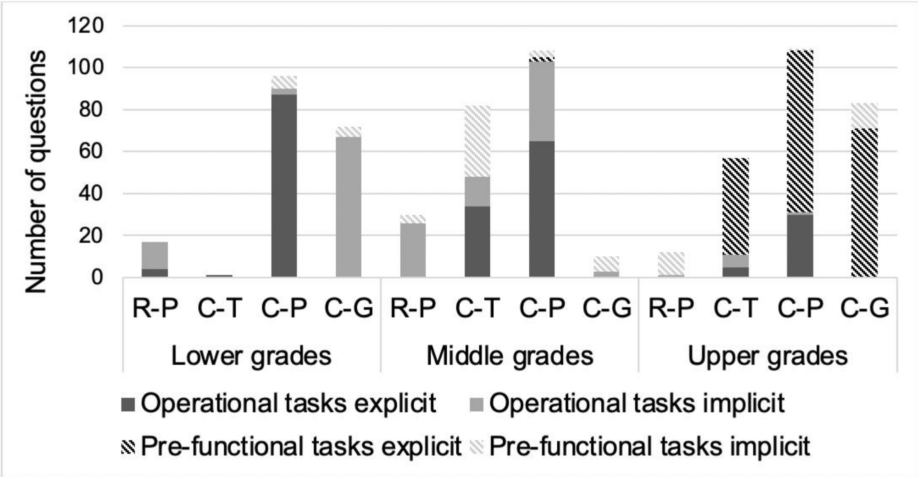

Figure 4: Route B develop FT from C-P to C-G & C-T

CONCLUDING REMARKS

This study provides several implications for developing FT in elementary mathematics. First development of FT should be embedded in learning arithmetic as an enhancement rather than an addition to the crowded existing content. Secondly, the 3D framework which integrates tasks, modes of FT and grade levels provides a useful analytical tool for examining textbooks regarding the development of functional thinking. Finally, we revealed two main pathways for the development of FT, which are aligned with the learning progression and pathways as described by other studies (Stephens, et al., 2017; Pitallis, et al., 2020). Thus, the ways of developing FT in elementary textbooks in China may provide insight for textbook development in other countries.

References

Blanton, M. L. (2008). *Algebra in elementary classrooms: Transforming thinking, transforming practice*. Portsmouth, NH: Heinemann.

Blanton, M. L., & Kaput, J. J. (2011). Functional thinking as a route into algebra in the elementary grades. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 5–23). Springer-Verlag.

Cai, J., Ng S. F., & Moyer, J. C. (2011). Developing students’ algebraic thinking in earlier grades: Lessons from China and Singapore. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 25–41). Berlin: Springer.

Carraher, D. W., Schliemann, A. D., & Schwartz, J. L. (2008). Early algebra is not the same as algebra early. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 235–272). New York, NY: Lawrence Erlbaum Associates.

Confrey, J., & Smith, E. (1991). A framework for functions: Prototypes, multiple representations, and transformations. In R. Underhill, & C. Brown (Eds.), P*roceedings of the thirteenth annual meeting of the north American chapter of the international group for the psychology of mathematics education* (pp. 57–63). Blacksburg, VA: Virginia Polytechnic Institute & State University.

Demosthenous, E., & Stylianides, A. (2018). Algebra-related tasks: Teachers' guidance in curriculum materials. *La matematica e la sua didattica, 26*(1), 7-27.

Fan, L. (2013). Textbook research as scientific research: Towards a common ground on issues and methods of research on mathematics textbooks. *ZDM the International Journal on Mathematics Education*, *45*(5), 765–777.

Leech, N. L., Barrett, K. C., & Morgan, G. A. (2008). *SPSS for intermediate statistics: Use and interpretation*. New York: L.

Lu, J. & Yang, G. (2012). *Compulsory Education Textbook: Mathematics*. People's Education Press.

MoE (2011). *Compulsory Education Mathematics Curriculum Standard.* Beijing: Beijing Normal University Publishing Group. [In Chinese]

Pittalis, M., Pitta-Pantazi, D., & Christou, C. (2020). Young Students' Functional Thinking Modes: The Relation Between Recursive Patterning, Covariational Thinking, and Correspondence Relations, *Journal for Research in Mathematics Education, 51*(5), 631-674.

Stephens, A. C., Fonger, N., Strachota, S., Isler, I., Blanton, M., Knuth, E., & Gardiner, A. (2017). A learning progression for elementary students' functional thinking. *Mathematical Thinking and Learning*, *19*(3), 143-166.

Stigler, J. W., Gonzales, P., Kawanaka, T., Knoll, S., & Serrano, A. (1999). *The TIMSS Videotape Classroom Study: methods and findings from an exploratory research project on eighth-grade mathematics instruction in Germany, Japan, and the United States* (Research and Development Report No. NCES 99-074). Washington, DC: U.S. Department of Education, National Center for Education Statistics.

Stylianides, G. J. (2009). Reasoning-and-proving in school mathematics textbooks. *Mathematical thinking and learning, 11*(4), 258-288.

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421–456). National Council of Teachers of Mathematics.

Xu, B. (2013). The development of school mathematics textbooks in China since 1950. *ZDM*, *45*(5), 725-736.