THE PROCESS OF MODELLING-RELATED PROBLEM POSING – A CASE STUDY WITH PRESERVICE TEACHERS

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In real life, problems emerge from situations and often need to be posed before they can be solved. Despite the ongoing emphasis on the processes involved in solving modelling problems, little is known about the process of problem posing. To help fill this gap, the current study examined (1) what activities are involved in modelling-related problem posing and (2) the sequence in which they occur. For this purpose, we invited seven preservice teachers to pose a problem based on given real-world situations and analyzed their problem-posing activities. We identified the five most frequent activities that occurred in the sequence: understanding–exploring–generating–problem solving–evaluating. These results contribute to the uncovering of important activities and contribute to theories of modelling and problem posing.

INTRODUCTION

In mathematics classrooms, the ability to solve problems in the real world (i.e., mathematical modelling) is a key competency that needs to be learned to be able to function as a responsible citizen in society (Niss & Blum, 2020, p. 2). However, in the real world, problems often need to be identified and posed first before they can be solved. Therefore, posing problems in given real-world situations (i.e., modelling-related problem posing) is an important competency. In the past, a great deal of research has been conducted on mathematical modelling (Schukajlow et al., 2021). However, only a few studies have analyzed modelling-related problem posing. Posing one's own problems is a demanding process that has to be learned (Cai & Hwang, 2002). To improve the teaching and learning of problem posing, knowledge about the activities involved in the process is needed (Cai et al., 2015). Research on the activities involved in posing problems based on given real-world situations has largely been missing so far. To help fill this gap, we aimed to analyze the activities involved in modelling-related problem posing from a cognitive perspective.

THEORETICAL BACKGROUND

Problem Posing

Research in mathematics education has been focusing more on problem posing in recent years as it can be gainfully used for teaching and learning mathematics (Cai et al., 2015). Problem posing can be defined as the generation of new problems and the reformulation of given problems that can take place before, during, or after problem solving (Silver, 1994). Stoyanova (1997) differentiated between structured problem posing, which is based on an initial problem, and unstructured problem posing, which is less restricted and is based on a holistic description of a situation. The connection to

2022. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 355-362). PME.

reality of the given stimuli is another important characteristic of problem posing. Based on the classification of problems with and without a connection to reality (Blum & Niss, 1991), the stimuli given for problem posing can be intramathematical descriptions or real-world situations. The focus of the present study is on problem posing as the generation of new problems based on given real-world situations before solving them. In the following, we will refer to this type of problem posing as modelling-related problem posing. An exemplary real-world situation that can be used as a stimulus for modelling-related problem posing is presented in Figure 1.

Cable Car

For more than 90 years, the *Nebelhorn* cable car has taken numerous guests up into the heights. Now it can go into well-earned retirement. Beginning in the summer of 2021, a new cable car will transport enthusiastic outdoor fans to the Nebelhorn mountain. The aim of the project is to avoid long waiting times, provide seated transportation with an optimal view from every seat, and increase carrying capacity.

Technical data for the old cable car:

Model:	Large-cabin aerial tramway
Weight empty cabin:	1600 kg
Weight full cabin:	3900 kg
Height valley station:	1933 m
Height top station:	2214.2 m
Horizontal difference:	905.77 m
Speed:	8 m/s
Carrying capacity:	500 people/h
Power Unit:	120 PS

Figure 1: The real-world Cable Car situation

Problem-Posing Activities

Based on the given real-world situation, a variety of real-world problems can be posed (Galbraith et al., 2010; Hartmann et al., 2021). An exemplary problem that can be posed using the given real-world situation in Figure 1 is: What is the best way to reconstruct the cable car? To pose such a problem, creative thinking is necessary (Bonotto & Santo, 2015). Wallas (1926) used a four-phase model consisting of the phases *preparation (exploration)*, *incubation*, *illumination*, and *verification* to describe creative mathematical thinking process.

Some studies analyzed the activities that occur when a problem is posed (Baumanns & Rott, 2021; Christou et al., 2005; Pelczer & Gamboa, 2009). First, the situation has to be explored with respect to possible problems that can be posed in the given situation. This activity is called *editing, selecting* by Christou et al. (2005), *transformation* by Pelczer and Gamboa (2009), or *analysis, variation* by Baumanns and Rott (2021). Second, problems can be generated by formulating them. This activity is called *translating* (Christou et al., 2005), *formulation* (Pelczer & Gamboa, 2009), or *generation* (Baumanns & Rott, 2021). Third, the posed problems can be evaluated with respect to individual criteria (e.g., solvability or appropriateness) (Baumanns & Rott, 2021; Pelczer & Gamboa, 2009). Previous studies indicated that the sequence of posed problems was typically guided by the employed problem-solving strategies (Cai & Hwang, 2002). Therefore, thinking about a possible solution might already be part of problem posing. Moreover, some students might develop a possible solution plan while problem posing (Baumanns & Rott, 2021). Overall, it can be assumed that the problem-posing process consists of *exploring*, *generating*, and *evaluating* activities and might already involve *problem solving*. However, the studies revealed that the activities involved are by no means linear and that the process is instead characterized by jumping back and forth between the individual activities (Baumanns & Rott, 2021; Pelczer & Gamboa, 2009). Prior studies on problem posing used unstructured problem posing with intramathematical stimuli (i.e., graphs, tables, equations, and dressed-up stories) (e.g., Christou et al., 2005) or structured problem posing with intramathematical and dressed-up word problems (e.g., Baumanns & Rott, 2021; Pelczer & Gamboa, 2009). Regarding modelling-related problem posing, only a little is known about the activities that take place when posing problems based on given real-world situations. In problem posing based on real-world situations, students should understand and explore the situations, generate possible problems, and evaluate the problems regarding their solvability (Bonotto & Santo, 2015). However, these theoretical considerations have yet to be empirically evaluated.

RESEARCH QUESTIONS

The goal of the present study was to examine the modelling-related problem-posing process by investigating the activities involved in posing problems based on given real-world situations. For this purpose, we asked the following research questions:

1) What activities are involved when preservice teachers pose problems based on given descriptions of real-world situations, and how can these activities be described?
2) In which sequences do the problem-posing activities occur?

METHOD

Sample

Seven preservice mathematics teachers between the ages of 20 and 26 (M = 22.86, SD = 1.95) from a large COUNTRY/REGION university participated in our study (4 women). To select the sample, we used heterogeneity sampling regarding different mathematics performance levels, experience in problem posing and modelling, and participation in different university programs. Two of the participants studied in a middle-track secondary school teacher program and five of them in a higher track secondary school teacher program. All participants were experienced in solving modelling problems and six of them in posing problems. The study was approved by the ethics committee of the faculty.

Procedure and Instruments

To identify the cognitive processes and to gain deep insights into the processes of problem posing, we used a qualitative study that included thinking aloud and the stimulated recall method. The preservice teachers were instructed to first pose a problem based on the given real-world situations, and after posing each problem, to solve it. For both posing and solving, they were instructed to think aloud during all activities. All responses were videotaped. The videos included their voice, gestures, writing, and facial expressions. To initiate problem posing, we used three real-world situations as they are described in modelling problems and enriched them by adding further authentic information to allow a variety of problems to be posed. An example of a real-world situation is presented in **¡Error! No se encuentra el origen de la referencia.**.

Data Analysis

To analyze the recorded videos, we first transcribed the material from the problem-posing process and the subsequent stimulated recall with regard to content-bearing semantic elements and then analyzed them using Mayring's (2015) content analysis. The coding scheme is based on the theoretically assumed problem-posing activities (exploring, generating, evaluating, problem solving) described in the literature and was extended inductively on the basis of the given material by using subsumption.

Transcripts were coded by the first author. To test for interrater reliability (measured as Cohen's kappa), over 50% were coded by a well-trained second rater. Cohen's kappa was at least moderate ranging from $\kappa = .81$ to $\kappa = .95$ (Cohen, 1960). To gain an overall picture of the activities involved in modelling-related problem posing, we analyzed the data with respect to the realization of the individual activities, and then for the second research question (sequence of activities), we focused on the number and frequency of changes in activity.

RESEARCH FINDINGS

With regard to our first research question, which was aimed at describing the activities that take place when learners engage in modelling-related problem posing, the analysis revealed the involvement of the five activities understanding, exploring, generating, problem solving, and evaluating. **¡Error! No se encuentra el origen de la referencia.** gives an overview of the observed activities.

Category	Description
Understanding	Comprehending and understanding the given real-world situation and information based on the description of the situation.
Exploring	Discovering and gathering relevant information to develop possible problems and organizing the information.
Generating	Posing, formulating, and defining possible problems.
Evaluating	Evaluating possible problems on the basis of individual criteria (solvable, meaningful, complete, appropriate formulation and difficulty, suitable for a particular target group).
Problem Solving	Finding a solution plan to the self-generated question.

Figure 2: Activities involved in modelling-related problem posing

In the following, we focus on the realization of the individual activities:

Understanding involved building an understanding of the situation. Thereby, students read the given situation, summarized information, asked comprehension questions, and related the given information to their personal experiences. Exemplarily, Lisa questioned her understanding of the horizontal distance in the following excerpt.

> Um theoretically, I'm wondering right now if the horizontal distance really means that it's sort of between the valley station and the top station.

Exploring involved exploring the given situation for possible problems that could be posed. This included identifying relevant, irrelevant, and missing information, organizing the identified relevant information, and expanding the context with further information. In the following excerpt, Lisa identified relevant information (height of the mountain and valley station, horizontal difference) and linked them by making a drawing of the situation (see Figure 3).

> So, I'm sort of making a drawing for this because I know that I have here, let's say the (draws in a first point), the um top ah the valley station and the valley station here (draws in a second point).
>
> And I know that the height here at the valley station (labels one point) is 1933 m, and the top station (labels the other point) is 2214.2 m.

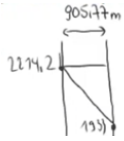

Figure 3: Lisa’s drawing

Generating was aimed at posing and writing down a problem. Thereby, possible problems were posed. From these, one question was then selected, formulated, and written down. In the following excerpt, Theo generated an idea for a possible problem based on the information he considered to be relevant.

> The goal of the project is to avoid long waiting times, seated transportation with an optimal view. Ok there you can perhaps consider how many people can realistically fit into such a cabin, so that each person sits at the window and has an optimal view and then consider whether you are exceeding the weight of a full cabin or not.

Evaluating included an assessment of the posed problems and referred to the assessment of appropriateness, solvability, and formulation. For example, Lea evaluated the appropriateness of her question as the following:

> So, you could somehow ask something about the weight in any case. But then the information is not relevant for whether we need a new one.

Problem Solving included solution plans for the self-generated problems. Thereby, mathematical operations or possible solution steps were identified. In the following excerpt, Max described a rather less detailed plan for solving the problem.

> You have to work through different steps bit by bit in order to solve it because I don't think you can come up with the solution directly in a calculation.

To find out more about the sequences in which the activities occurred (RQ 2), we analyzed the changes in activities (Table 1). All activities except understanding by evaluating followed each other at least once.

Followed by	Under-	Exploring	Generating	Evaluating	Problem

	standing				Solving
Understanding	-	*36 [73%]	5 [10%]	0 [0%]	2 [4%]
Exploring	18 [20%]	-	53 [58%]	7 [8%]	6 [7%]
Generating	1 [1%]	22 [19%]	-	59 [52%]	6 [5%]
Evaluating	2 [3%]	16 [20%]	38 [48%]	-	12 [15%]
Problem Solving	1 [3%]	6 [20%]	3 [10%]	12 [40%]	-

Table 1: Overview of the number of activity changes (Note: *Understanding was followed by exploring in 36 sequences, 73% of all understanding sequences.)

Regarding frequencies, understanding was predominantly followed by exploring and rather rarely by generating and problem solving. Exploring occurred frequently before generating but less frequently before exploring, evaluating, and problem solving. Generating was predominantly followed by evaluating, less frequently by exploring, and rather rarely by problem solving and understanding. Evaluating primarily occurred before generating, less frequently before exploring and problem solving, and rather rarely before understanding. Problem solving was followed most frequently by evaluating, less frequently by exploring, and rather rarely by generating and understanding. If we consider only the activities that follow one another most frequently, the idealized process model of a problem-posing route emerges (Figure 4). It presents a hypothesized process model for describing the idealized process of modelling-related problem posing. Importantly, the sequences of the activities while posing a specific problem by an individual (called individual problem-posing routes) are not linear and vary significantly (i.e., switching between different activities in the process model).

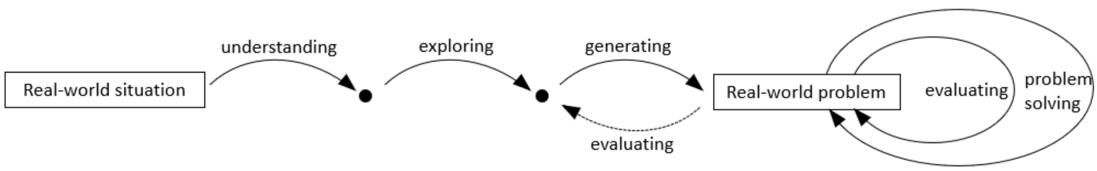

Figure 4: Hypothesized process model for modelling-related problem posing

DISCUSSION

Modelling-related problem posing included the activities understanding, exploring, generating, evaluating, and problem solving. These findings are partly in line with the activities found in prior studies on intramathematical problems and word problems (Baumanns & Rott, 2021; Christou et al., 2005; Pelczer & Gamboa, 2009). The activities of exploring, generating, and evaluating were observed in this and other studies. This finding indicates the commonalities between modelling-related problem posing and other problem-posing processes. In addition, the analyzed processes involved an activity in which possible solution steps are planned, similar to a study on structured problem posing based on a given word problem (Baumanns & Rott, 2021). This finding supports Cai and Hwang's (2002) assumption that problem posers are already thinking about a possible solution when posing a problem. However, modelling-related problem-posing activities differ in some ways from the activities found in prior studies. First, we were not able to identify the activities *transformation* and *variation* as described in studies on structured problem posing (Baumanns & Rott, 2021; Pelczer & Gamboa, 2009). A possible explanation is that stimuli had a different structure. As modelling-related problem posing is not based on a given initial problem, it is not necessary to transform the given problem. Second, we identified the activity understanding as being a part of problem posing. Understanding is an essential activity in the well-established models of the solution process of modelling problems, and it is important for problem posing as well (Niss & Blum, 2020, p. 17). However, prior studies on structured problem posing did not identify the activity understanding. A possible explanation could be that structured problem posing begins with the solution of the initial problem, and students already understand the initial problem before problem posing. In our study, two activities—exploring and evaluating—which were described in Wallas' (1926) model of creative mathematical thinking, were observed. Consequently, problem posing was revealed to be a creative process (Bonotto & Santo, 2015). However, we were not able to observe the activities incubation and illumination, probably because these processes are described as occurring subconsciously (Wallas, 1926), and hence, we were not able to capture them with our research method. Future research with methods such as eye-tracking or narrative interviews are needed to find out whether problem posing involves incubation and illumination.

Due to the design we chose, our study has some limitations. We used a qualitative research approach with a small sample to identify the process of modelling-related problem posing. The aim was to uncover problem-posing activities and develop an idealized hypothetical model of modelling-related problem posing. These findings must be verified in future studies. Additionally, limitations result from using specific real-world situations. Overall, our study contributes to research on problem posing from a cognitive modelling perspective. Our findings can be used to improve the teaching and learning of modelling-related problem posing by taking into account problem-posing activities and their ideal sequence.

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