A DESIGNATED PROFESSIONAL DEVELOPMENT PROGRAM FOR PROMOTING MATHEMATICAL MODELLING COMPETENCY AMONG LEADING TEACHERS

Hadas Handelman, Zehavit Kohen

Israel Institute of Technology, Technion

Mathematical modelling is an important component of STEM education in the 21st century. This study examines how a designated professional development program impacts teachers' perceptions of mathematical modelling instruction and their mathematical modelling competency. The perceptions were assessed by a pre-post questionnaire and their modelling competency was measured by their solutions to modelling tasks, over three different timepoints. The results show a positive change in teachers' perceptions of modelling instruction and a positive trend in their competency to apply certain stages of the mathematical modelling cycle. In the study, methodological and practical contributions are discussed with respect to promoting and assessing mathematical modelling competence among mathematical teachers.

INTRODUCTION

Mathematics is considered as the foundation of all the STEM fields, yet studies indicate that there is a gap between the relevance of mathematics as taught in classes, compared to the applicability of mathematics in real-life, particularly in STEM-related fields (Blum, 2015; Kaiser, 2017; Kohen & Orenstein, 2021; Verschaffel et al., 2020). The use of mathematical modelling (MM) provides a method for demonstrating students the applicability of mathematics, as it reflects a transition from a real situation to a mathematical model. MM is a cyclic process that begins and ends with real-world situations unrelated to mathematics, in which a translation is made from the real-world context into mathematical terms toward a mathematical solution to the real-world situation (Blum & Leiß, 2007; Kaiser, 2017; Perrenet et al., 2012). Yet, students face a variety of challenges when it comes to MM, as they are faced with questions that arise from the reality, which they must apply mathematical knowledge to (Ferri, 2017). MM instruction, particularly when it involves a STEM-related context is a significant challenge for mathematics teachers as well. As teachers, they are required to deal with the difficulties of their students who are unfamiliar with modelling as part of formal math lessons (Verschaffel et al., 2020), as well as deal with the same difficulties themselves in applying modelling skills in solving modelling problems with different contexts (Kramarski & Kohen, 2017). Also, since this sort of instruction is not often addressed in formal math classes, it is important that teachers have positive perceptions towards MM instruction (Kohen, Orenstein, & Nitzan, 2019). It is therefore imperative that teachers be trained to have these skills, both as learners and as teachers, through a

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supportive professional development (PD) environment (Darling-Hammond et al., 2017).

THEORETICAL FRAMEWORK – MATHEMATICAL MODELLING

Blum and Leiß (2007) presented an MM cycle which consists of seven stages that reflect the transition from reality and mathematics (see Figure 1). This model describes the actions that a solver must perform in order to solve a real-world problem using mathematical methods. The modelling cycle suggested by Blum and Kaiser's has four main steps. The first two steps involve idealizing of a real-life situation and making it into a realistic model. Mathematization that is the third step is the transition from reality to the mathematical world upon choosing a mathematical model to solve the real-world model, and involves investigating of the mathematical model through the use of mathematical algorithms, routines, and procedures. The last three steps involve ensuring that the results of the model are comparable with reality by interpreting them. The MM cycle closely resembles PISA's mathematical literacy cycle. PISA defines mathematical literacy as the ability to think mathematically in order to solve problems in a variety of real-world contexts (OECD, 2018) (see Figure 2). In terms of the PISA conceptual framework, mathematical literacy includes three main stages that fit to the MM cycle: Formulate, Employ, and Interpret. These matches to PISA's cycle are visualized in figure 1 as follows: formulate related stages are marked with orange frame, employ related stages are shown in blue frame, and whereas interpret related stages are shown in green frame.

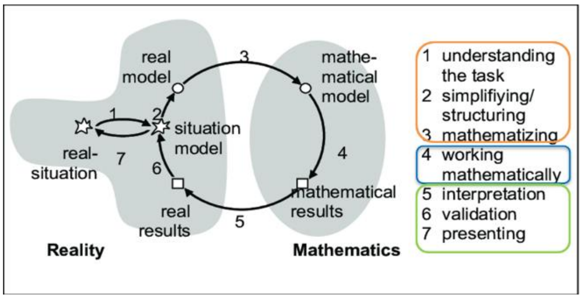


Figure 1. The modelling cycle (Blum & Leiβ, 2007)

Figure 2. PISA's Mathematical literacy model (OECD,2018)

The modelling tasks applied in this study reflect the narrowed modelling cycle. However, as these tasks are suited for formal school mathematics (Kohen & Orenstein, 2021), the idealizing stage that takes place within the reality is explicitly provided in the task, thus reflecting a more constrained type of a modelling problem. With that, these modelling tasks have much similarity with the PISA framework, which led us to use the PISA conceptual framework to represent the MM stages.

This research is based on MM tasks with a real-world context, that is retrieved from technology and engineering authentic applications, and an example of that is the 'Iron Dome' task. 'Iron Dome' is an advanced defence radar system that can detect the trajectory of rockets and can calculate the expected impact zone. As soon as the rocket enters the free fall stage, the Iron Dome system uses the information it receives to trace its trajectory, by using mathematical calculations that are based on a quadratic equation. In accordance with the MM processes, formulating is realized in this task with the use of a main question that reflects the transition from reality to the mathematical world: 'How can the rocket trajectory be predicted?' Then, the employing stage involves applying mathematical procedures and graphic representations, to derive the mathematical solution, that is based on the identification of three points in the rocket trajectory and calculating a quadradic equation. In the interpretation stage, based on the mathematical solution and the predicted landing area, students can estimate whether or not the Iron Dome will intercept the rocket.

The purpose of this study is to explore the impact of a designated PD program on the advancement of teachers' MM competencies, as well as teachers' perceptions toward modelling-based instruction. The research question is: What are the changes (if any) in teachers': a) perceptions towards MM instruction, and b) MM competencies?

METHODOLOGY

The context of the study - a PD program for modelling-based instruction.

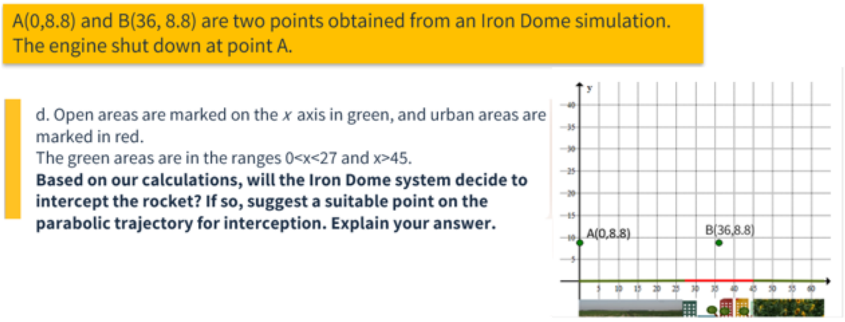
The current study was conducted as part of a designated 60-hour PD program for leading mathematics teachers, which is held for the goal of training teachers to apply modelling-based instruction. During the PD program meetings, the coaches introduced the modelling framework, and its correspondence to the PISA's framework, introduced modelling tasks with real-world technology or engineering context, and discussed pedagogical content to support the adaptation of modelling-based instruction.

Participants were about 40 math leading teachers who took part in the PD program. Some of the teachers are math coordinators, instructors, or hold key positions in the Ministry of Education in Israel. The teachers have varied teaching experience, with most of them having more than seven years of experience in the education system.

Research Tools and analysis.

The study uses two main tools. The first tool was a pre-post self-reported questionnaire for measuring teachers' perceptions toward MM instruction, on a six-level Likert scale (1, not true at all, and up to 6 - almost always true). The questionnaire aimed to assess teachers' perceptions toward the application of the various modelling processes, i.e., formulate, employ, and interpret in their instruction during math lessons. The questionnaire was distributed to participants at the beginning and the end of the PD program. The second tool was a solution to a modelling problem, which aimed to assess the teachers' MM competency. The teachers were asked to explicitly write all the phases of their solution. Below is an example of a MM problem, that was retrieved from the 'Iron Dome' task (see Figure 3).

The teachers were asked to solve three modelling tasks throughout the PD program in three different time points. The Iron Dome task described above was applied at the beginning of the program as a starting point. Four months later, the teachers solved a problem retrieved from the 'Autonomous Car' modelling task, which involves using ultrasonic sensor technology, that is based on sound speed, and is related to a motion problem where the distance equals time multiplied by speed. Towards the end of the program, the teachers were asked to solve a problem retrieved from the GPS modelling task which deals with how satellite signals are received and analysed in order to determine a GPS receiver's location, which solution is based on a motion problem followed by a Pythagorean theorem.



A(0,8.8) and B(36, 8.8) are two points obtained from an Iron Dome simulation. The engine shut down at point A.

d. Open areas are marked on the *x* axis in green, and urban areas are marked in red.
The green areas are in the ranges 0<x<27 and x>45.
Based on our calculations, will the Iron Dome system decide to intercept the rocket? If so, suggest a suitable point on the parabolic trajectory for interception. Explain your answer.

Figure 3. An example for a MM problem, retrieved from the 'Iron dome' task

DATA ANALYSIS

Quantitative data retrieved from the questionnaires was analysed using dependent T-test to determine changes over time. Further, for assessing the teachers' MM competency, we analysed their solutions to the modelling tasks, and graded them in a 3-level process as described below. We then conducted One-Way ANOVA with repeated measures to evaluate the change over time in teachers' modelling competency, as measured referring to the three investigated modelling tasks. We demonstrate the analysis process, based on the modelling problem that was retrieved from the iron dome task, and is presented in figure 4.

Phase 1 – Assessing the level of the various modelling components of the task.

This phase was conducted prior to the tasks being responded to by the participants and was designed to objectively evaluate the modelling competencies the tasks require. Based on a valid rubric (Kohen & Gerrah-Badran, in press) for assessing an authentic MM task, each task was evaluated. This rubric allows to determine the MM competency that are summoned in modelling tasks, referring separately to each of the modelling processes, which is given a grade on a scale of 1 (low level of modelling competency) to 3 (high level of modelling competency).

For the 'Iron Dome' task, the coding was as following. Formulating was assigned to level 2 (medium) since the problem requires working efficiently with two representations (graphic and algebraic), while taking assumptions, such as “falling in an open area” means no interception, so there must be a cut point with axis X. However, there is no requirement to create a new representation, but to work with a familiar one, so the level is medium and not high. Employing was assigned to level 3 (high) since the problem requires planning strategies for a solution while reasoning the mathematical solution such as choosing the cut point on axis X and then finding the quadratic equation. Finally, interpreting was assigned to level 2 (medium) since the problem requires reasoning to provide justification and adaptation to a representation of a situation in the real world, such as the selection of the appropriate point for interception. As there is no requirement to explain the process of drawing conclusions, the level of this stage is medium.

Phase 2 – Using indicators for recognizing modelling components in teachers' solutions.

This phase is based on evaluating the teachers’ responses to the various tasks. An indicator (0/1) was given for each component of the modelling process that was recognized in teachers' solutions, based on an indicator that was developed for each task specifically. The indicator development included a validation process performed by math-education experts. It should be noted that the formulating component of the modelling process is a component without which the employing cannot be reached. Therefore, even if the formulating process is not expressed in the written answer, but the employing process was carried out correctly, it was assumed that the teacher went through the formulating process while thinking about the solution. The following example demonstrates the solution of Michael (pseudo) to the iron dome task, and the indicators that were given for this solution, referring to each of the modelling components: “After finding the equation of the function (by placing it in the vertex representation 18,25) it is possible to select a point whose X-rate is for example 15 that will need interception as it enters a built-up area (24.55, 15)”.

In this solution, Michael goes directly to the mathematical procedures and explain what procedures should be performed to solve the mathematical aspect of the question. Thus, an indicator of 1 was given to the employing component. In this case, it can be assumed that the formulating process was conducted within his mind, thus this component was also marked with indicator ‘1’. Then, there is a reference in his solution of returning to the real-world context of the task, but it seems as if Michael did not fully understand the question (the determination weather the Iron Dome system will or won't intercept the rocket, based on its expected fall location). Thus, an indicator of ‘0’ was given to the interpreting component as he reached a mathematical solution but failed to draw conclusions out of it.

Phase 3- Determining a grade for the teacher's MM competency.

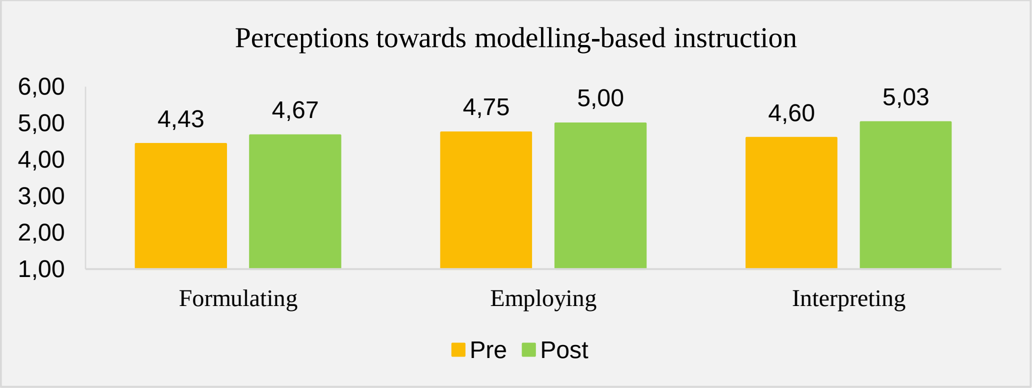
In this phase, a merge of the two previous phases was conducted to determine the modelling competency of the teachers that was reflected in each of the investigated tasks. For each component of the modelling process, we multiplied the objective grade that was given to each modelling component on phase1 by the indicator that was given to teachers’ solutions on phase 2. Then we summed up the results and divided it by the sum of grades from phase 1 for the purpose of normalizing the score, so the grades ranged between 0 to 1. Table 1 presents the assessment of Michael’s MM competency, as was determined based on the problem retrieved from the Iron Dome task.

Modelling stage	Phase 1	Phase 2	Phase 3	Grade
Formulating	2	1	$2 \cdot 1 = 2$	$\frac{2+3+0}{7} = 0.71$
Employing	3	1	$3 \cdot 1 = 3$	
Interpreting	2	0	$3 \cdot 0 = 0$	

Table 1. The assessmnet of Michael's MM competency

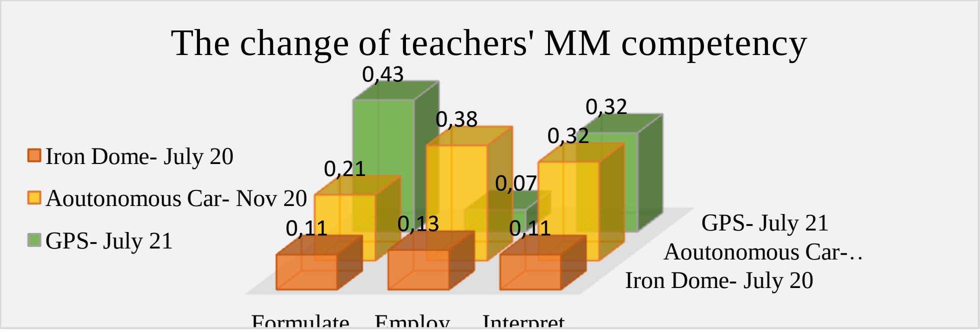
FINDINGS

Findings revealed no significant difference in teachers’ perceptions toward modelling-based instruction, with respect to all modelling components, $-1.91<t<-1.05$, $p>.05$. Yet, post-hoc analysis according to Cohen's d effect size indicated that the teachers demonstrated more positive perceptions toward modelling instruction that is based on the formulate ($d = 0.36$) and employ ($d = 0.42$) modelling processes, and particularly with respect to modelling instruction that is based on the application of the interpret process ($d = 0.67$). As Graph 1 below demonstrates, there is a positive trend in teachers' perceptions before and after participating in the program.



Graph 1. Teacheres’ Perceptions towards modelling-based instruction, before and after participating in a PD program

Graph 2 below presents the change over time in the teachers’ modelling competency. Findings revealed a significant multivariate effect for the three latent variables as a group in relation to three times of measures, indicating higher modelling competency toward solving the third modelling task, $F(3,40) = 10.83$; $p<.0001$, $\eta^2=.619$. Simple main effect tests with Bonferroni adjustment indicated that teachers’ competency of formulate and interpret during solving the third task was significantly higher that the competency they demonstrated during solving the first task, and the second task (with respect to merely the formulate competency). For the employ competency, they demonstrated an improvement of this competency during the second task, which decreased during solving the third task.



Graph 2. The change in teachers' MM competency

CONCLUSION AND CONTRIBUTIONS OF THE STUDY

The study findings indicate an improvement in both teachers' modelling competency and their perceptions toward instruction-based modelling. Based on previous studies conducted on the field with the aim of improving the MM capabilities of students, Niss (2001) concluded that applications and modelling capabilities can be learned. Teachers play an essential role in promoting modelling competency among their students (Doerr & English, 2003). However, for learning to occur, teachers must devote time and effort to implementing modelling tasks. Thus, their positive perceptions towards modelling-based instruction, as well as their own modelling competencies are significant in promoting MM among their students.

The most significant improvement in teachers' modelling competency was detected in the formulating and interpreting stages of the process. These two stages represent the main difference between a standard mathematical word problem and a MM one, as they reflect the transition from the real world to the mathematical one (Ferri, 2017; Kaiser, 2017; Perrenet et al., 2012). This finding reinforces the importance of supporting teachers' modelling competency as leaners, through PD programs. However, it remains to be seen whether the employing stage is directly impacted by the PD program or if it is primarily influenced by teachers' previous knowledge that is required to solve the MM task during the employing stage.

An effective PD program allows teachers to progress professionally and changes the way they apply new or improved methods of instruction (Darling-Hammond et al., 2017). The practical contribution of this study is reflected in the presented designated PD program that was found to be effective in enhancing the participating teachers' MM competencies and their perceptions toward modelling-based instruction. In terms of the study's methodological contribution, we produced a tool for measuring teachers' modelling competency, which can be also valuable as a practical tool for teachers and other researchers.

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