CONCEPTUAL AND PROCEDURAL MATHEMATICAL KNOWLEDGE OF BEGINNING MATHEMATICS MAJORS AND PRESERVICE TEACHERS

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In light of known challenges in the transition from school to university in mathematics, we investigate differences in the (mathematical) prerequisites of mathematics majors and preservice mathematics teachers. Results show that although there are no significant differences in high school grade point average, mathematical prerequisites of mathematics majors are significantly better than those of preservice mathematics teachers. Differences are higher in conceptual than in procedural knowledge with medium effect sizes between mathematics majors and preservice higher secondary teachers and (very) large effect sizes between mathematics majors and preservice lower secondary or primary school teachers. These results are discussed regarding transition challenges and the fit of prerequisites and chosen study program.

MATHEMATICS PRESERVICE TEACHERS AND MAJORS COMPARED

In Germany teacher training at university is organized in separate degree programmes, but not always in separate lectures. While preservice higher secondary teachers attend advanced mathematics courses together with mathematics majors, primary and lower secondary teachers usually attend specific mathematics courses with more basic mathematical content (Gildehaus et al., 2021). The credits to be taken in mathematics are accordingly less in lower-secondary and primary programs.

In joint courses with mathematics majors, preservice teachers perform slightly lower on exams (Göller et al., 2022) and report less satisfaction with their studies than mathematics majors (Kosiol et al., 2019). Mathematics (preservice) teachers in general, tend to question the relevance of advanced mathematical content for their teaching profession (Gildehaus & Liebendörfer, 2021; Zazkis & Leikin, 2010).

However, it is unclear whether these differences arise from acculturation at the university or are rooted in different prerequisites already at the beginning of the studies: For example, differences in dissatisfaction can be partly explained by different interest profiles at study entrance (Kosiol et al., 2019). Preservice teachers are in mean more interested in school mathematics (especially in using calculation techniques) and less interested in university mathematics (e.g., proof and formal representations) than mathematics majors (Ufer et al., 2017).

In addition to such affective variables, cognitive variables are relevant factors for academic success and related study satisfaction. The high school grade point average (HSGPA) has empirically proven to be one of the best indicators for predicting study

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success across different study programs (e.g., Richardson et al., 2012; Schneider & Preckel, 2017; Westrick et al., 2021). Mathematical knowledge assessed in entrance tests is found to be an even better predictor of later academic performance in mathematics courses (Eichler & Gradwohl, 2021; Greefrath et al., 2017; Halverscheid & Pustelnik, 2013; Rach & Ufer, 2020).

In terms of such cognitive prerequisites, the differences between mathematics preservice teachers and majors are less evident: Blömeke (2009) found no differences in high school grade point average (HSGPA) between mathematics preservice teachers and mathematics majors, however, mathematics majors performed better in a mathematics test at study entrance than mathematics preservice teachers (Pustelnik & Halverscheid, 2016). To elaborate on these findings, we report on a study following the idea that differences in students' mathematical interests (Ufer et al., 2017) might be mirrored in different types of mathematical knowledge.

CONCEPTUAL AND PROCEDURAL MATHEMATICAL KNOWLEDGE

In mathematics tests for beginning university mathematics students, mathematical knowledge is usually conceptualized and surveyed as a unidimensional construct. To investigate whether different types of mathematical knowledge, corresponding to the different mathematical interests of students (Ufer et al., 2017), can be empirically distinguished, we build on the subdivision of mathematical knowledge into *conceptual knowledge*, which is thought as a network of relationships connecting different pieces of information, and *procedural knowledge*, which comprises knowledge about algorithms or a series of steps for completing mathematics tasks (Hiebert, 1986). Although conceptual mathematical knowledge seems to be theoretically (Gray & Tall, 1994; Gueudet & Thomas, 2020) as well as empirically (Hailikari et al., 2007; Rach & Ufer, 2020) more important for later academic success in mathematics at university, many of the mathematics tests used at study entrance rather measure procedural knowledge, such as basic arithmetic skills (Heinze et al., 2019). We do not know of any study that explicitly examines differences between mathematics preservice teachers and majors in terms of conceptual and procedural mathematical knowledge. We thus explore the following research questions:

RQ 1: Can conceptual and procedural knowledge of mathematics students at study entrance be empirically distinguished?

RQ 2: How do mathematics students of teacher and non-teacher study programs differ in their (mathematical) prerequisites at the beginning of their studies?

METHODS

To answer these questions, we refer to data from a medium-sized German University with 310 participants in a pre-university mathematics course in September 2021, about one month before the start of their studies. Participants can be subdivided into three groups (with regard of their different study programs):

- *Group 1* (majors) consists of 15 mathematics and 70 computer science majors.
- *Group 2* (higher secondary teachers) consists of 55 preservice higher secondary mathematics teachers enrolled in a study program with joint mathematics courses with mathematics majors (Group 1).
- *Group 3* (primary & lower-secondary teachers) consists of 170 preservice primary and lower-secondary school mathematics teachers enrolled in a study program without joint mathematics courses with mathematics majors (Group 1).

The participants self-reported their high school grade point average as well as their last math grade from school (1 = best, 6 = poorest) and worked for 60 minutes on an online mathematics test with 21 tasks (12 (complex-)multiple-choice items, 9 with open numerical input) of which 11 were classified as conceptual items and 10 as procedural items. Conceptual items comprised tasks that required connecting different pieces of information such as changing between different representations (e.g., relating terms and graphs, modelling) or using given information for argumentation (see Figure 1).

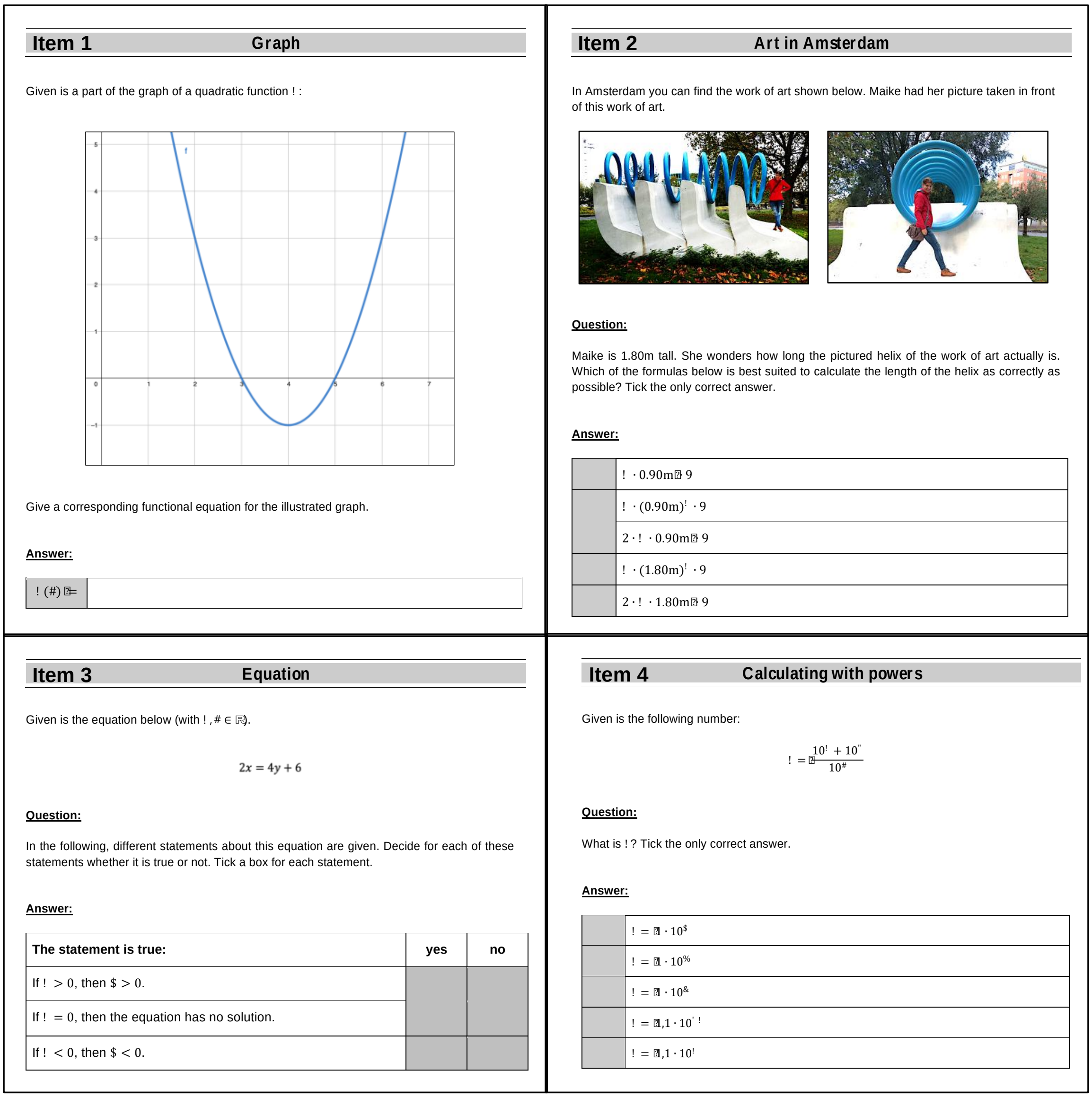

Figure 1: Examples of test items. Items 1,2,3 conceptual items, Item 4 procedural item. Items translated from German by the authors (cf. Besser et al., 2020)

Procedural items comprised tasks that required algorithms or a series of (calculation) steps to be completed. Examples of procedural items are e.g. calculating the derivative of $f(x) = (3 - x^2)^6$ or simplifying the expression $\frac{x^{k-n}}{y^{2n}} : \frac{x^{2k-n}}{y^{n-1}}$ for $x, y \neq 0$ and collecting the variables (Hochmuth et al., 2019) as well as Item 4 of Figure 1.

Participants' answers were coded dichotomously (0 = not correct, 1 = correct; missing answers were coded as not correct) and analyzed using the R-package "mirt" regarding a unidimensional 2-parameter logistic IRT model as well as a two-factor 2PL IRT model distinguishing conceptual and procedural items. Person scores were extracted and further analyzed using analyses of variance in SPSS.

RESULTS

Addressing RQ 1, both considered models show acceptable to good model fit statistics (cf. Table 1). Noteworthy, the two-factor model that distinguishes conceptual and procedural knowledge fits the data significantly better.

	RMSEA	TLI	CFI	AIC	BIC	$\chi^2(5)$	p
Unidimensional	0.044	0.941	0.947	7667.14	7824.07		
Two Factors	0.029	0.976	0.979	7621.56	7797.18	55.576	<.001

Table 1: Fit statistics of the unidimensional and the two-factor (conceptual-procedural) IRT model

The latent factor correlation of conceptual and procedural knowledge is r = .70, indicating that they measure different (yet correlated) constructs. Table 2 shows the bivariate correlations below the diagonal and reliability measures on the diagonal. Also, the bivariate correlation of conceptual and procedural knowledge indicates different (yet correlated) constructs with r = .51 for the IRT person scores.

	1	2	3	4	5
1. HSGPA	-				
2. Math Grade	.65*	-			
3. Unidimensional Model (IRT Unidim)	-.23*	-.28*	.83		
4. Conceptual Knowledge (IRT Concept)	-.21*	-.30*	.87*	.73	
5. Procedural Knowledge (IRT Proced)	-.18*	-.20*	.86*	.51*	.74

Table 2: Bivariate correlations of school grades and IRT person scores below the diagonal and empirical reliability measures on the diagonal. *p < .01

Regarding RQ 2 we first give some descriptive statistics in Table 3. In the mean, students solved approximately half of the tasks. Means of Group 1 (mathematics majors) are the highest for all measured variables, followed by Group 2 (preservice higher secondary teachers), while means of Group 3 are the lowest.

	Full Sample				Group 1		Group 2		Group 3	
	Min	Max	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
HSGPA	1.00	3.60	2.28	0.53	2.21	0.62	2.22	0.56	2.34	0.47
Math Grade	1.00	5.00	2.32	0.91	2.04	1.00	2.18	0.84	2.51	0.85
Sum Test	1.00	21.00	11.46	4.62	14.82	4.32	12.75	4.07	9.37	3.73
Sum Concept	0.00	11.00	6.25	2.83	8.28	2.52	7.05	2.50	4.98	2.36
Sum Proced	0.00	10.00	5.21	2.42	6.54	2.27	5.69	2.20	4.39	2.23
IRT Unidim	-2.23	2.21	0.00	0.91	0.68	0.90	0.25	0.81	-0.42	0.70
IRT Concept	-1.89	1.64	0.00	0.85	0.62	0.76	0.25	0.74	-0.39	0.70
IRT Proced	-1.97	1.95	-0.01	0.86	0.46	0.84	0.16	0.81	-0.30	0.77

Table 3: Descriptive statistics. For HSGPA (high school grade point average) and Math Grade (1 = best and 6 = poorest). Sum Test = Sum of correctly solved items, Sum Concept = Sum of correctly solved conceptual items, Sum Proced = Sum of correctly solved procedural items. Group 1 = majors, Group 2 = higher secondary teachers, Group 3 = primary & lower-secondary teachers

The results of the ANOVA (Table 4) show that the means of the three groups do not differ significantly regarding high school grade point average (HSGPA). Mean differences in the last mathematics grade are only between Group 1 (math majors) and Group 3 (preservice primary & lower-secondary school teachers) significant with medium effect size. Mean differences in the math test are higher, especially for conceptual knowledge and the total test, with medium effect sizes between Group 1 and Group 2 (preservice higher secondary teachers) and (very) large between all other groups. Differences in procedural knowledge are somewhat smaller, with Group 3 again performing significantly lower than the other two, with medium to large effect sizes.

	F	p	η^2	d_{G1-G2}	d_{G1-G3}	d_{G2-G3}
HSGPA	2.11	.123	.014	-0.01	-0.24	-0.25
Math Grade	8.76	<.001	.054	-0.16	-0.53*	-0.39
IRT Unidim	59.84	<.001	.280	0.49*	1.42*	0.92*
IRT Concept	59.04	<.001	.278	0.50*	1.45*	0.89*
IRT Proced	27.04	<.001	.150	0.35	0.95*	0.59*

Table 4: Results of the ANOVA. d_{Gi-Gj}: Effect size (Cohen's *d*) of the mean difference between Group *i* and Group *j*. *Differences are significant (post hoc tests with Bonferroni correction, $p < 0.05$)

DISCUSSION

The results show that conceptual and procedural knowledge of mathematics students at study entrance can be empirically distinguished and measured (RQ 1). Furthermore, they show that although differences between mathematics preservice teachers and majors are not significant for HSGPA and rather small regarding the last mathematics grade from school, they differ significantly with regard to the math test scores, with medium to very large effect sizes (RQ 2). Overall, these results suggest that students chose (have chosen) a study program that fits their mathematical abilities, as reflected in the mathematics test but not (barely) in their school grades: Preservice higher secondary mathematics teachers (Group 2) who attend joint mathematics lectures with mathematics majors (Group 1) are almost at the same level with their performance while preservice primary and lower secondary school teachers (Group 3) who attend mathematics lectures on a less advanced level start their study on a significantly lower mathematical knowledge base. Nevertheless, the mathematical prerequisites of the preservice higher secondary mathematics teachers (Group 2) are lower than those of the mathematics majors (Group 1) which might contribute to explanations of preservice teacher' dissatisfaction with university mathematics contents (Gildehaus & Liebendörfer, 2021) as well as their slightly lower performance in mathematics exams compared to mathematics majors (Göller et al., 2022).

Noteworthy, the differences in mathematics performance are higher in conceptual than in procedural mathematical knowledge. On the one hand, this is in line with preservice teachers interests who are in mean more interested in school mathematics (especially in using calculation techniques) and less interested in university mathematics (e.g., proof and formal representations) than mathematics majors (Ufer et al., 2017). On the other hand, this suggests that university pre- and bridging courses should focus (even) more on building conceptual knowledge in order to compensate for inequalities and to prevent frustrations or other difficulties accompanying the transition from school to university in mathematics (Göller & Gildehaus, 2021).

When interpreting the results, the following limitations, to name but a few, should be taken into account: 21 Items of a one-hour test cannot capture overarching constructs such as mathematical, conceptual, or procedural knowledge in their entirety, which means that the results are of course influenced by the operationalization of the mathematics test used for this study. Since participation in the pre-course, in which the test was taken, was voluntary and test-taking was anonymous, and thus performance on the test had no consequences for the participants (apart from feedback for themselves), selection effects are possible, which are likely to be influenced in particular by the interest of the participants. In addition, the relatively small sample should be considered, which consists of students from only one university.

Accordingly, further research is desirable to better understand the (mathematical) prerequisites of university students and, based on this, to advance the teaching and learning of mathematics at the university.

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