FOSTERING A CONCEPT OF FUNCTION WITH COMBINED EXPERIMENTS IN DISTANCE AND IN-CLASS LEARNING

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Hands-on experiments and simulations foster functional thinking (FT) in different ways. Both benefits can be combined effectively, when a focus is set on the difficult aspect of covariation through a qualitative approach. Self-directed learning in such settings produces significantly higher gains in FT that rather numeric consideration of experiments (Digel and Roth, 2021). Both settings were implemented as in-classroom (N=219) and distance learning environments (N=113) respectively, within the given constraints due to COVID-19. The results for distance learning Hammerstein et al. (2021) report in their meta-study are inconsistent, but with clear negative tendency. In the study reported here both learning modes show comparable results and the overall differences between covariational and numeric setting persist in both modes as well.

DISTANCE LEARING DURING THE PANDEMIC SITUATION

The restrictions due to COVID-19 set a focus on digital teaching strategies and revealed deficits in the school system concerning this topic. Despite intensive efforts of many schools, the average increase in learning during the first lockdown in Germany was comparable to that during the summer holidays, i.e. without school operations (Hammerstein et al., 2021). With regard to mathematics, the picture in Germany is not entirely consistent at the first glance. In their study on digital learning environments (bettermarks), Spitzer and Musslick (2021) found performance increases in the cohort with lessons under corona conditions comparable to the previous year's cohort. In an annual school performance study in Baden-Württemberg, there were significant learning deficits compared to previous years, especially in operational, mathematical skills, while performance in arithmetic (calculation-related) skills was at the level of previous years (Schult et al., 2021). The authors interpret these findings as indicators for an arithmetic focus in mathematics distance learning. Considering that the digital learning environments of bettermarks also strongly emphasise arithmetic, lower gains for operational and conceptual learning can be assumed in this case as well.

Distance Learning of weak learners

Related to pre-COVID performance, distance learning also increased the differences in learning achievement. In particular, low achievers showed lower gains compared to previous years (Schult et al., 2021), which could also be due to the significantly shorter learning time for this group. In addition, learners with low SES background had significantly poorer conditions for distance learning (Hofer et al., 2022). Our study does not replicate these negative findings. The learning environments for FT based on

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experiments with hands-on and digital material showed comparable learning gains in both grammar and comprehensive schools during distance and in-class learning.

DEVELOPING A CONCEPT OF FUNCTION

The concept of functions is a major concept and at the same time a major hurdle in mathematics at school. Hence a considerable amount of research has been dedicated to the teaching and learning of functions. For the learning environments used in this study we try to bring together several branches of evidence to a coherent approach to the concept of functions. Breidenbach et al. (1992) used the Action-Process-Object-Scheme (APOS) theory for a developmental perspective on students' conceptualization of functions. The action concept on the lowest level is limited to the assignment of single output values to an input. With the more generalized process concept students consider a functional relationship over a continuum, enabling the reflection on output variation corresponding to input variation. Finally, functions conceptualized as objects can be transformed and operated on. Students with an elaborate concept of functions are supposed to be able to use the action, process or object conception depending on the mathematical situation (Dubinsky & Wilson, 2013).

Aspects of functional thinking

The developmental stages of APOS are in line with key elements of a function concept, that are described as aspects of functional thinking (FT) by Vollrath (1989) as follows: the correspondence of an element of the definition set to exactly one element of the set of values; the covariation of the dependent variable when the independent variable is varied and the final aspect, in which the function is considered as an object. Although with the APOS perspective one might deduce a teaching sequence with an initial focus on correspondence, then covariation and finally object, current research advocates for a major role of covariation. Thompson and Carlson (2017) argue that the correspondence aspect alone does not evoke an intellectual need for the new concept function and difficulties with functional relationships are mainly rooted in lacking ability and opportunity to reason covariationally. Johnson (2015) points out that correspondence induces a static view on a functional relationship, while a dynamic perspective is a prerequisite for covariation and a process concept. These arguments lead to the call for a qualitative approach to functional relationships in school.

Experimenting fosters functional thinking

Learning environments with experimentation activities have proven to be beneficial for functional thinking (Lichti & Roth, 2018). One possible explanation could be the proximity of functional thinking to scientific experiments as illustrated by Doorman et al. (2012): with a given variable as starting point, a dependent variable is generated in an experiment. Relating the output to the input clearly addresses the correspondence aspect and the action concept. Following manipulations of the input and concurrent observation of the output make the covariation of both variables tangible and enables a process view. Another benefit of student experiment is the inherit constructivist learning approach that leads to higher learning gains in combination with digital technologies (Drijvers, 2019). Lichti and Roth (2018) implement the scientific experimentation process – preparation (generate hypotheses), experimentation (test the hypotheses) and analysis (conclusions) – in a comparative intervention study to foster functional thinking of sixth graders with either hands-on material or simulations and report learning gains for both approaches (ibid.), but a closer look reveals disparities that can be explained with the instrumental genesis.

Hands-on experiments and simulations in the light of instrumental genesis

The instrumental approach (Rabardel, 2002) and its distinction between artefact and instrument can be useful when interpreting these results: while the artefact is the object used as a tool, the instrument consists of the artefact and a corresponding utilization scheme that must be developed. This developmental process - the so-called instrumental genesis - depends on the subject, the artefact and the task in which the instrument is used. Hence, different artefacts lead to different schemes. Artefacts that are more suitable for the intended mathematical practice of a task appear to be more productive for the instrumental genesis and facilitate the learning process (Drijvers, 2019). When using simulations, schemes that develop are dynamic and concerned with variation as well as transition and hence support the covariation aspect (Lichti, 2019). Measurement procedures of the hands-on material induce static schemes for values and conditions, fostering the correspondence aspect (ibd.). While hands-on material stimulates basic modelling schemes, relating the situation to mathematical description, a simulation already contains a model of the situation. When used as multi-representational systems, the simulation illustrates connections between model and mathematical representations (e.g. graph and table) that evoke schemes for these representations and their transfer. The study presented here attempts to make use of both beneficial influences on the instrumental genesis through an appropriate combination of hands-on material and simulations in experimental activities to foster functional thinking.

Fostering the conceptual development

To foster FT we combine hands-on experiments and simulations with the premise of a productive instrumental genesis as follows: hands-on material at the beginning initiates modelling schemes. Subsequently, simulations facilitate the representational transfer (table – graph; situation/animation – graph), enable dynamic exploration of the relationship and systematic variation, thus fostering an understanding of covariation. Finally, measurements with hands-on material convey the correspondence aspect. The two different settings developed for this study are outlined as a scientific experimentation process with the three phases hypotheses, experimentation, analyses.

The numeric setting follows the APOS steps sequentially and gives the measurement procedure a dominant role in the experimentation phase. This sets a focus on the correspondence aspect. In the analyses phase the learners access the covariation aspect with simulations, that connect an accordingly designed animation with the dynamic representations of the relationship in a table and a graph.The second, covariational setting consequently fosters a dynamic view on the relationship and the related variables. It is implemented with two shorter scientific experimentation processes. After initial hypotheses with hands-on material, the experimentation phase with simulations immediately sets the focus on (co-)variation through. The analyses phase complements the animation with a dynamic representation of the relationship in a graph. Only after this phase, measurement data is generated with hands-on material and fed into the simulation to test the results on the relationships drawn so far.

Both settings use a story of two friends preparing to build a treehouse and contain identical contexts, hands-on material and simulations. The tasks of each setting are similar, but adapted to the numerical and covariational focus respectively. Both settings can be accessed in digital classrooms (www.geogebra.org/classroom numerical Setting: HQX7 UZRQ and covariational Setting: D3XM DDSB).

STUDY DESIGN

A comparative intervention study (pre-post design) is implemented both in distance and in-classroom learning mode with seventh and eighth graders at grammar and comprehensive schools. It contrasts the covariational and numerical settings and includes an additional control group with the simulation only implementation of Lichti and Roth (see above). The intervention is designed for six lessons (split into three sessions). It is preceded and followed by a short test on functional thinking (FT-short, online version: www.geogebra.org/m/undht8rb, Rasch-scalable, 27 items, see Digel & Roth, 2020), to compare the learning outcomes in both settings. Students work in teams of two pairs. A pilot study (ibid.) verified the comparability of the covariational and numerical setting in terms of processing time and difficulty. In this paper we present results with a focus on school form and learning mode:

RQ 1: Which setting is most beneficial for FT in the different school forms?

Hypothesis Grammar > Comprehensive: Large studies on student assessment regularly show lower competence levels in comprehensive schools to grammar schools (OECD, 2019), a gap that is getting wider (Guill et al., 2017). Regarding the different settings, the focus on the difficult covariation aspect in the covariational setting could overburden lower competence levels and thus increase the competence gap. Dubinsky and Wilson (2013) in contrast foster low achievers on all APOS levels of the concept of function successfully.

RQ 2: Does the learning mode (in-class/distance) have an impact on the learning gains in the compared settings?

Hypothesis In-Class > Distance: All three settings focus on conceptual competences, while arithmetic competences are secondary. According to the discussion in the first section in this paper, lower learning gains can be expected in distance learning, especially in comprehensive schools.

METHOD

Data analysis was conducted according to Item Response Theory. The dichotomous one-dimensional Rasch model and a virtual persons approach were used to estimate item difficulties for FT-short. The person ability was then estimated with fixed item difficulties. We applied mixed ANOVAs (between factors: setting, school form, learning mode; within factor: time) after controlling data for normal distribution and homogeneity of variance. Pairwise t-tests were used to investigate differences of the settings. A statistical power analysis (3 groups, 2 measurements, power .9, α =.05) for a medium effect (η_p^2 = .06) in a mixed ANOVA gave a desired sample size of 204.

RESULTS

Here we present quantitative results of the main study (N = 332, 121 female, 187 male, age M = 13.0, SD = 4.8). The distribution of the sample over the settings and constraints is shown in table 1. The estimation of the Rasch-model, used to determine the person abilities for the total sample, showed good reliabilities in the pre- and post-test: $EAP\text{-}Rel_{pre}$ = .86 and $EAP\text{-}Rel_{post}$ = .80 as well as $WLE\text{-}Rel_{pre}$ = .85 and $WLE\text{-}Rel_{post}$ =.80.

Table 1: Sample sizes and effect sizes Cohens d (pre/post) of subgroups

	Covariational Setting		Numerical Setting		Control Group		Total
	N	d	N	d	N	d	N
Total	114	.51***	125	.25***	93	.27***	332
Comprehensive/ Grammar	39 75	.63*** .48***	52 73	.32*** .27***	66 26	.34*** .28***	157 175
Distance In-Class	36 78	.48*** .56***	39 86	.33** .28***	38 55	.36** .30**	113 219

Comparisons of the settings under constraints

Regarding the **school form** (see Figure 1 left) the mixed ANOVA showed a significant main effect for time (F(1, 326) = 197.34, p <.001, η_p^2 =.38) and a significant effect of school form (F(1, 326) = 87.82, p <.001, η_p^2 =.21). Above, there are two significant interaction effects: between time and setting (F(2, 326) = 5.92, p <.005, η_p^2 =.018) and between time and school form (F(2, 326) = 9.57, p <.005, η_p^2 =.029). The grammar school students outperformed the comprehensive school students in the pretest significantly (t(174) = 8.09, p <.001, d = .61), but for both school forms students' ability increased significantly with a small to medium effect (grammar: t(425) = 7.08, p <.001, d = .34; comprehensive: t(216) = 5.84, p <.001, d = .40). In both school forms students in the covariational settings showed the highest learning gains (see Table 1).

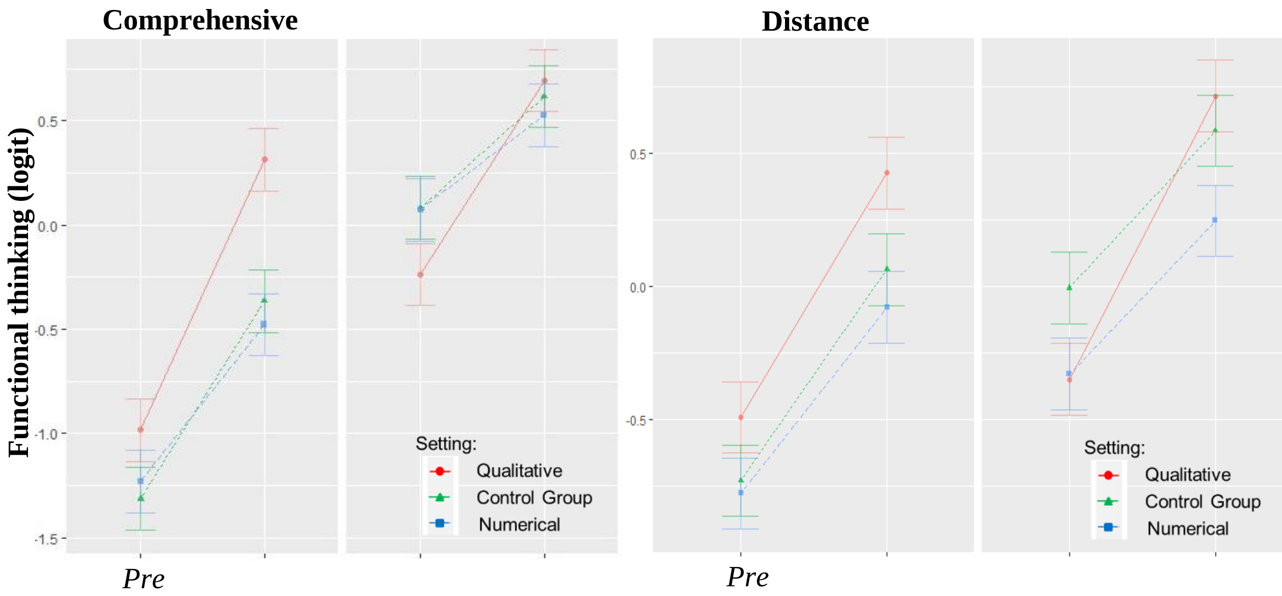

Figure 1: Increase in FT pre/post by setting, school form (left) / learning mode (right)

The mixed ANOVA for **learning mode** (see Figure 1 right) resulted in one significant main effect for time ($F(1, 326) = 170.88$, $p <.001$, $\eta_p^2 =.34$), one for learning mode ($F(1, 326) = 10.85$, $p < .001$, $\eta_p^2 = .03$) and a significant interaction effect of time and setting ($F(2, 326) = 3.63$, $p < .005$, $\eta_p^2 = .02$). In both learning modes the covariational setting shows the highest learning gains (see table 1). The students with in-class learning showed slightly higher results in the pretest ($t(153) = 2.19$, $p < .05$, $d = .18$). The overall learning gains in distance learning ($t(150) = 3.48$, $p < .001$, $d = .28$) are comparable to those in-class ($t(149) = 4.57$, $p < .001$, $d = .38$).

DISCUSSION

First of all, the results are not generalizable without reservation, since they depend on the concrete settings developed in the study. Another restriction is the disbalance of subgroups, caused by altering pandemic restrictions in participating schools. Nonetheless, results of the total sample show that both settings foster FT, while the covariational setting is significantly more beneficial for FT than the numerical setting, but the learning effects in the latter do not differ significantly from those in the control group (see Digel & Roth, 2021). Two characteristics of the covariational setting seem most influential: first, the early focus on the dynamics of the observed variables provides opportunities to reason variationally and to develop a dynamic view on functions. Second, replacing early measurement with investigation and observation of the relationship initiates practice in covariational reasoning.

The significant advantages of the covariational settings also appear in both school forms (RQ1). Significant difference in pretest between grammar and comprehensive schools in FTshort ($d = 0.61$) are as expected, but these disparate competence levels are not reinforced by the intervention, in contrary, learning gains in the comprehensive school sample outperform those of the grammar school sample. FT seems to be accessible in the three settings to learners on all competence levels and the covariational focus is also beneficial to lower levels of FT and not restricted to high achievers, which replicates Dubinsky and Wilson (2013).

The results regarding RQ2 are limited through a possibly lower level of engagement and focus in the pre- and post-tests in distance learning. Nevertheless, we can conclude from the results that all three learning environments promote functional thinking in distance and in-class to a comparable extent. This is contrary to previous studies on the effectiveness of distance learning, especially in the case of conceptual skills, such as FT here. There are three different explanations for this: On the one hand, motivational influences may have favoured the learning process in distance, since hands-on experiments set in everyday contexts and group work with individual coaches stand out positively. Secondly, inquiry-based learning with open tasks contrasts distance learning which is rather dominated by arithmetic and initiates intensive interaction with the concept as well as higher cognitive activation. A continuous interaction with partners/teams enables co-construction processes and mathematical communication about ideas, hypotheses, approaches and thus intensifies interaction with the content.

To sum up, the covariational approach to functions with experiments (1) attains higher learning gains across competence levels, (2) successfully transfers in-class activities to distance learning with comparable learning gains, (3) makes the covariational aspect accessible for high and low achievers and (4) benefits from the combination of hands-on material and simulations. In classroom practice (distance or in-class), an approach to functions designed accordingly has the potential to enhance learning gains.

REFERENCES

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. *Educational Studies in Mathematics, 23,* 247–285.

Digel, S., & Roth, J. (2020). A qualitative-experimental approach to functional thinking with a focus on covariation. In A. Donevska-Todorova, E. Faggiano, J. Trgalova, Z. Lavicza, R. Weinhandl, A. Clark-Wilson & H.-G. Weigand (Eds.), *Proceedings of the 10th ERME Topic Conference on Mathematics Education in the Digital Age (MEDA) 2020* (pp. 167-174). JKU. hal-02932218

Digel, S., & Roth, J. (2021). Do qualitative experiments on functional relationships foster covariational thinking? In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 218–226). PME.

Doorman, M., Drijvers, P., Gravemeijer, K., Boon, P., & Reed, H. (2012). Tool use and the development of the function concept: From repeated calculations to functional thinking. *International Journal of Science and Mathematics Education*, *10*(6), 1243–1267.

Drijvers, P. (2019). Embodied instrumentation: Combining different views on using digital technology in mathematics education. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 8–28). Utrecht University.

Dubinsky, E., & Wilson, R.T. (2013). High school students’ understanding of the function concept. *The Journal of Mathematical Behavior, 32*(1), 83–101.

Guill, K., Lüdtke, O. & Köller, O. (2017). Academic tracking is related to gains in students’ intelligence over four years: Evidence from a propensity score matching study. *Learning and Instruction, 47,* 43–52.

Hammerstein, S., König, C., Dreisörner, T. & Frey, A. (2021). Effects of COVID-19-Related School Closures on Student Achievement-A Systematic Review. *Frontiers in Psychology, 12*:746289.

Johnson, Heather L. (2015). Together yet separate: Students’ associating amounts of change in quantities involved in rate of change. *Educational Studies in Mathematics, 89*(1), 89–110.

Lichti, M. (2019). *Funktionales Denken fördern: Experimentieren mit gegenständlichen Materialien oder Computer-Simulationen* [Fostering functional thinking: Experimenting with real materials or computer-based simulations]. Springer Spektrum.

Lichti, M., & Roth, J. (2018): How to Foster Functional Thinking in Learning Environments: Using Computer-Based Simulations or Real Materials. *Journal for STEM Education Research 1*(1-2)*,* 148–172.

OECD (2019). *PISA 2018 Results (Volume I): What Students Know and Can Do*, PISA, OECD Publishing.

Rabardel, P. (2002). *People and technology: A cognitive approach to contemporary instruments.* Université Paris. hal-01020705

Schult, J., Mahler, N., Fauth, B., & Lindner, M. A. (2021, March 11). *Did Students Learn Less During the COVID-19 Pandemic? Reading and Mathematics Competencies Before and After the First Pandemic Wave*. PsyAr-Xiv. https://doi.org/10.31234/osf.io/pqtgf

Spitzer, M. W. H., & Musslick, S. (2021). Academic performance of K-12 students in an online-learning environment for mathematics increased during the shutdown of schools in wake of the COVID-19 pandemic. *PLOS ONE, 16*(8)

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421–456). National Council of Teachers of Mathematics.

Vollrath, H.-J. (1989). Funktionales Denken [Functional thinking]. *Journal für Mathematikdidaktik*, *10*(1), 3–37.