EXPLORING THE AFFORDANCES OF A WORKED EXAMPLE OFFLOADED FROM A TEXTBOOK

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In designing a set of instructional materials to use in his classroom, a teacher heavily offloaded items (e.g., worked examples, practice questions, exercises) from school-based materials and textbooks. At a cursory level, one may easily dismiss this as a thoughtless lifting of curricular materials. But upon careful analysis – as is detailed in this paper – a different picture emerges. In this paper, we describe and analyse how this teacher adapted one of many worked examples, beyond its typical use, during instruction to develop students' conceptual understanding of proportionality. We argue that he noticed and harnessed multiple affordances in a single item that most teachers may overlook, without the need to modify the example, and propose a notion of "affordance space" as a lens to view teachers' design of instructional materials.

INTRODUCTION

Emerging research on Singapore mathematics teachers as designers of instructional tasks and materials has illustrated the innovative ways that teachers can adapt and improvise tasks, representations, and sequencing to achieve various instructional goals (e.g., Cheng et al., 2021; Leong et al., 2019). However, there are teachers who choose to heavily rely on tasks and procedures from curricular materials for instruction, otherwise known as *offloading* (Brown, 2009). While using an item directly from a textbook may appear to be inherently less complex and involve less "design thinking", Brown (2009) noted that offloading should not be mistaken for being inferior to adapting or improvising, nor does it necessarily imply teachers who offload are negligent or less competent. In a study conducted by Amador (2016), four teachers with 1 to 17 years of teaching experience engaged in offloading, as well as adapting and improvising; two of whom initially offloaded and shifted to adapting during a lesson. Furthermore, as Choy and Dindyal (2021) demonstrated, despite offloading "typical" tasks from past-examination papers, a teacher, Alice, was able to implement them in unexpected and productive ways to develop students' conceptual understanding. They proposed this was due to the teacher's ability to "effectively *notice* and *harness the affordances* of these materials in mathematically productive ways" (p. 196). We build on Gibson's (1986) idea and refer to the set of possibilities for how a task may be used as the affordances of a task. In addition, we follow Choy and Dindyal (2021) in seeing that the affordances of a task are always there, "independent of teachers' ability to perceive them" and "do not vary as teachers' instructional goals or needs change" (p. 198).

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In comparison to the abundance of research on the affordances of “challenging” and “rich” tasks, there is an underrepresentation of research on the affordances of “typical” and “routine” items. Hence, the aim of our study is to examine the affordances of a worked example that was offloaded from a textbook by a secondary mathematics teacher, Peter (pseudonym), but somehow implemented in a non-typical and non-routine way. We hypothesize that Peter engaged in a nuanced form of offloading based on noticing and harnessing *multiple* task affordances—which may not always be immediately obvious—simultaneously. We propose the notion of an *affordance space* to describe the cognitive space in which teachers work with tasks whose dimension is dependent on the number of affordances they perceive. The more affordances a teacher perceives in a task, the great number of ways they can use the task beyond its “typical” procedural use. Further details of the affordance space will be discussed later. Our research questions are: What affordances does a teacher perceive in a typical worked example that influenced their decision to offload? And how do these affordances influence their implementation of the worked example?

METHODS

The data reported is drawn from a larger study on secondary mathematics teachers’ design of instructional materials (IMs). Four teachers from two local secondary schools in Singapore engaged in 3 to 6 design cycles involving individual design of IM drafts, one-on-one semi-structured interviews after each draft, and subsequent professional learning community (PLC) discussions with their colleagues. The topics of their IMs were Ratio and Rate, with an underlying emphasis on proportionality. Then, the teachers implemented their IMs and one-on-one semi-structured interviews were conducted after every lesson. The teacher discussed in this paper is Peter (pseudonym). At the time of the study, Peter had over 10 years of mathematics teaching experience, predominantly at upper secondary (Year 11-12), and it was his first-year teaching Year 9 mathematics. He implemented his IMs over four lessons, each lasting 40-70 mins. All interviews, PLC discussions, and lessons were recorded and transcribed. Peter’s IM drafts and the curriculum materials he used—a set of school-based worksheets and a textbook—were collected.

To analyse the data, we adopted two grain sizes of analysis. Firstly, at the *item-level* we examined the individual items (e.g., worked examples, practice questions, investigation tasks) within Peter’s worksheets to determine: (i) instances of offloading, adapting, or improvising; and (ii) potential task affordances that influenced Peter’s offloading, adapting, or improvising. Then, at the *set-level* we examined Peter’s tasks as a collective, to determine overarching instructional goals. This dual item-level and set-level analysis was conducted initially on Peter’s worksheets and his design interviews, then the implementation and post-lesson interviews were used to triangulate the affordances and goals.

FINDINGS

In this section, we begin by summarising Peter's selection of items for his IMs before we present a vignette of how Peter had used one of the worked examples to develop students' understanding of proportionality. We then highlight two of the affordances inferred from Peter's use of the example. Table 1 summarises the offloads, adaptations, and new items we determined in our first round of item-level analysis. Out of a total 35 items, 27 items were offloaded, suggesting that Peter heavily relied on the school-based worksheet and textbook. Due to length constraints, we will focus our discussion on one item that was offloaded from the textbook in his Ratio worksheet to explore the affordances Peter noticed and harnessed to adapt the task during instruction.

	School-based worksheet		Textbook		Peter's new items	Total items
	Offloaded	*Adapted*	*Offloaded*	*Adapted*		
Ratio	14	2	6	1	1	24
Rates	6	2	1	2	0	11
Total	20	4	7	3	1	35

Table 1: Summary of items in Peter's instructional materials

The worked example Peter offloaded resembled those *typical problems* (Choy & Dindyal, 2021) found in any textbook or examination paper about ratios (Figure 1). It shows how to find a ratio between two quantities that have different units, followed by two short questions for students to ponder. In general, worked examples are used to demonstrate a solution method for students to imitate. Hence, most teachers would typically read these with students, possibly bringing key steps to students' attention, before applying the same method to a similar problem. This is how one would expect Peter to use the worked example, especially given that he directly offloaded it from the textbook into his worksheet and followed it with a similar question ("Andrew and Sueda took 90 seconds and $\frac{21}{3}$ minutes respectively to answer an IQ question. Find the ratio of Andrew's time to Sueda's time.").

Yet, this was not how Peter implemented the item, nor was it his intention to use the worked example as a demonstration for the subsequent question. Instead, Peter used the task to engage the class in a discussion about a fundamental concept of proportionality over a 10-minute episode. He briefly went over the working in four short sentences, and then quickly moved to focus on question (a):

Peter: You're supposed to find the ratio of Bobby's time to Aravin's time. Do take note if you are comparing using the same units. In this case, Aravin's time converted into minutes, that should give you three over two minutes. Then we actually can compare the ratio. Now, question! What if instead of converting Aravin's time from seconds to minutes, what if we compare in seconds?

Students: Times 60!

Peter: Before we even calculate, do you think the ratio would be the same?

Students: No... (some students begin to write)

Peter: Wait, ah! Don't calculate first. Wait, wait, wait! What happens if we compare them in seconds? Who says it will be different? Raise your hand.

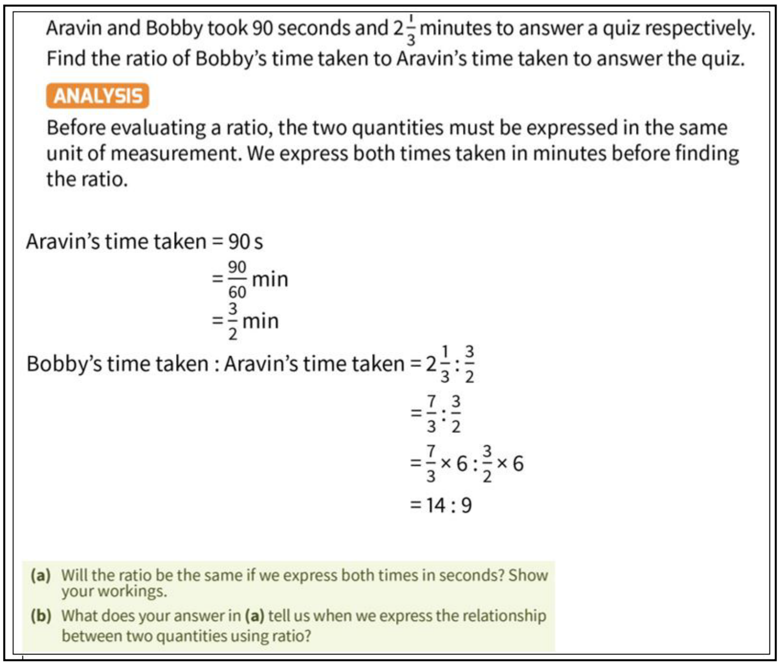
The students looked around the classroom. Those who had initially raised their hands lowered them slowly, and those who still believed it would be different sheepishly kept their hands up as low as possible. Peter asked them again:

Peter: Who thinks the ratio will be different? It's okay. I remember seeing three or four hands, then becomes two hands now? I was pretty sure I heard more than one voice. Who says it will be the same? Raise your hand!

Some students began writing on their worksheets while others continued to look around the classroom. Out of a class of 38 students, eight students raised their hands. It was evident that there was uncertainty amongst the class and clearly the worked example was not useful in resolving this. Peter had fostered curiosity amongst the students, creating the need for the class to investigate this before moving on to the next task. Peter orchestrated a whole-class discussion in which he asked the students to suggest the actual working of the solutions to the same question in a different unit. As he followed their instructions, he drew arrows on the side of each step (Figure 2) and said, "Whatever you do to one side, you do to the other side". When the students shouted out the solution without stating their reasoning, Peter asked them, "How do you know?" Eventually the class arrived at the solution 14 : 9 and numerous students yelled, "They are the same!" One student exclaimed, "They are *equivalent*!".

Affordance 1: Developing conceptual understanding about proportionality

As an experienced teacher, Peter was likely aware that worked examples are commonly used for demonstrating the steps to solving a problem. However, it was not used to ensure students understood the necessity of converting quantities to the same units, nor was it about how to simplify ratios. Instead, Peter's requests for students to think about whether the ratios would be the same or different "before we even calculate" illustrated that his intention was more focused on developing students' conceptual understanding about ratio. In an interview about one of his worksheet drafts, he mentioned that it was "a thought I'd like to plant in their heads". He paid little attention to the solving procedure and utilised the worked example as a foundation for exploring with students the preservation of proportionality (i.e., the ratio will be the same, regardless of the units). In the post-lesson interview, Peter revealed that he deliberately spent more time on the worked example because he didn't "want them to think proportionality questions always [involved] systematically comparing the process. I also want them to think in context as well." Furthermore, this use of worked examples to develop students' conceptual understanding about proportionality was not exclusive to this item. It was observed in another worked example on comparing the rate of fuel consumption of a car using two different units (Figure 3). Evidently, Peter saw the affordance of using worked examples to go beyond demonstration of procedures.



Aravin and Bobby took 90 seconds and $2\frac{1}{3}$ minutes to answer a quiz respectively. Find the ratio of Bobby's time taken to Aravin's time taken to answer the quiz.

ANALYSIS

Before evaluating a ratio, the two quantities must be expressed in the same unit of measurement. We express both times taken in minutes before finding the ratio.

Aravin's time taken = 90 s

$= \frac{90}{60}$ min

$= \frac{3}{2}$ min

Bobby's time taken : Aravin's time taken $= 2\frac{1}{3} : \frac{3}{2}$

$= \frac{7}{3} : \frac{3}{2}$

$= \frac{7}{3} \times 6 : \frac{3}{2} \times 6$

$= 14 : 9$

(a) Will the ratio be the same if we express both times taken in seconds? Show your workings.

(b) What does your answer in **(a)** tell us when we express the relationship between two quantities using ratio?

Figure 1: Worked example offloaded from the textbook onto Peter's worksheet

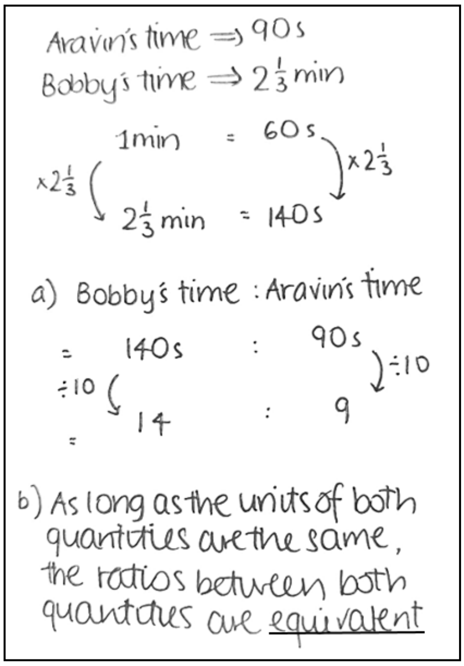


Figure 2: Peter's written working on the whiteboard (rewritten for readability)

Affordance 2: Representations that make proportionality more visible

If Peter had written the students' working on the board in a similar manner to the worked example, he would have still been able to show that the ratios were the same. Yet, he chose to adapt from the worked example and adopt the use of a new representation, the arrows (Figure 2). With the worked example projected onto one side of the whiteboard and Peter's writing on the other, a comparison of the two would show that the underlying proportionality in simplifying ratios is more visible when using the arrows. On top of serving as a reminder to students that simplifying ratios requires treatment to both quantities, it illustrates why proportionality is preserved because of the equal treatment to both quantities. Hence, an affordance of offloading this worked example directly from the textbook was also to be able to demonstrate in contrast to another representation of proportionality that would aid students in making sense of the solving procedure.

There was no clear evidence in the worked example about how Peter came to using arrows. However, when we analysed the implementation of other worked examples, which did not have arrows present on the worksheet, we found Peter had also used this arrow method as an alternative representation (Figure 4). Furthermore, he asked students "I know in the example there isn't an arrow, but can you please write in the arrow in the example just for you to see, so you can follow" on another worked example. Our analysis of the 35 items on the worksheets at the set-level identified only three instances of some form of arrows; one was an improvisation, and two others were adaptations. However, when we zoomed out to examine the 35 items implemented during the lesson, we noticed he had adapted them all by consistently using arrows as a representation of proportionality. This consistent and well-rehearsed use of arrows suggests that although he offloaded most of his worksheet items from the school-based worksheet and textbook, he intended to adapt the implementation all along.

A car travelled 450 km on 40l of petrol. Find the rate of petrol consumption in
(i) km per l, **(ii)** l per 100 km.

Solution

(i) Rate of petrol consumption of the car $= \frac{450\text{ km}}{40l}$
$= 11.25$ km/l

(ii) Rate of petrol consumption of the car $= \frac{40l}{450\text{ km}}$
$= \frac{40}{450} \times 100l$ per 100 km
$= 8\frac{8}{9}l$ per 100 km

Qn: Is the rate of petrol consumption of the car the same in (i) & (ii)?

Figure 3: Another offloaded worked example

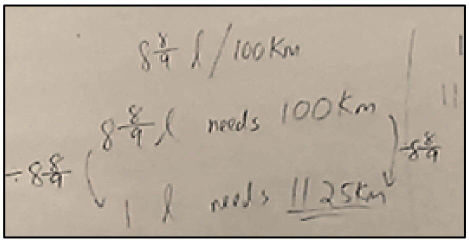


Figure 4: Another instance of Peter's use of arrows

In this 10-minute episode, Peter's implementation of the worked example was noteworthy for two reasons. Firstly, he demonstrated how typical worked examples need not be used for imitating solving procedures but could instead be a catalyst for whole class discussions on fundamental components of a concept. Secondly, although he essentially ignored the procedural elements of the worked example, he was still able to target procedures related to proportionality through his use of arrows to make the reasoning process more visible to students. This dual achievement of both conceptual and procedural developments is an example of how multiple affordances can be noticed and harnessed within a typical item.

DISCUSSION AND CONCLUDING REMARKS

The research questions of our study were: (i) What affordances does a teacher perceive in a typical worked example that influence their decision to offload? And (ii) how do these affordances influence their implementation of the worked example? Instead of using the worked example in the usual manner to demonstrate a solving procedure, Peter perceived a key affordance as being able to facilitate an investigation about the preservation of proportionality when forming ratios involving a unit conversion. Furthermore, he utilised and demonstrated to students how adopting a different representation—the arrows—when simplifying ratios could be useful in making the underlying proportionality in ratio problems more pronounced and easier to follow.

Building on the work of Choy and Dindyal (2021), we propose the notion of an affordance space. On the basis that the potential of a task is dependent on the teacher's ability to notice and harness its affordances, teachers who see a task's sole affordance to facilitate procedural development can be said to be working in a one-dimensional affordance space and therefore less likely to use the task in adaptive or productive ways. However, teachers who notice multiple affordances of a task work in an affordance space of higher dimension and can take the task in various directions beyond procedural development. As Alice in Choy and Dindyal's (2021) study and Peter in this paper demonstrate, research on the affordances of typical task can make clearer the work of teachers, while also demonstrating the complexity of teachers' work in the interesting ways they may use such tasks. Unfortunately, amongst this sea of innovative teachers adopting challenging tasks and adapting and improvising others, teachers like Peter and Alice are easily missed or disregarded.

Lastly, Brown's (2009) definition of offloading does not seem to fully capture the phenomena we observed with Peter. If we adopt the notion that offloading is fundamentally judged on the instructional outcomes, then Peter cannot be said to be offloading at all. But what does that mean for his design of instructional materials which clearly demonstrate the offload of the task from one resource to his worksheet? Furthermore, Amador (2016) noted that Brown's (2009) description of teachers' interactions with curriculum resources implied a static interaction. However, in her study, as well as ours, she documented two teachers who shifted from offloading in lesson design to adapting during instruction. While their shift was triggered by unexpected incidents that meant students would be unable to achieve the instructional goals, interestingly, in the case of Peter, his shift was not triggered during the lesson. His adapting was evidently planned due to the casual and well-rehearsed way he skipped through the solution method to focus on the preservation of proportionality, as well as his consistent use of arrows throughout his implementation.

This brings to question the need to redefine offloading, or at least elaborate and extend on it to encapsulate such instances. In our analysis of Peter's implementation, we wondered if there was possibly no such thing as completely offloading because every teacher brings with them their own unique knowledge and contexts. On a broad-grained scale we might see teachers simply carrying out the task as described in the textbook—or as Brown (2009) gave the example of teachers reading from the curriculum materials—but when we zoom in to the teaching episode, we can likely capture teachers asking additional questions or even very nuanced moments where the teacher provides some alternative scaffolding that was not prescribed in the textbook.

As our findings pertain to a single teacher, and are hence not generalizable, future research should aim to study the various affordances that teachers notice and attempt to *simultaneously* harness in typical tasks to develop the concept of affordance space. In particular, instances where there appears to be a disconnect (or shift) between how teachers interact with tasks during lesson design and implementation would be worthwhile pursuing. To do so, a similar item-level and set-level analysis approach used in this study would help to identify and examine shifts in teachers' interactions at different grain-sizes.

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