# MAKING VISIBLE A TEACHER'S PEDAGOGICAL REASONING: AN ASPECT OF PEDAGOGICAL DOCUMENTATION

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*Much of a teacher's practice and professional learning remains unseen despite recent calls to incorporate practice-based and inquiry-based approaches to improve mathematics instruction. Although the idea of pedagogical reasoning and action can provide a way to unpack these unseen aspects of practice, it remains to be seen how a teacher's actions and thinking can be made visible. In this paper, we present a case of how a teacher's pedagogical reasoning is made visible through pedagogical documentation, which suggests the possibility of using documentation to unpack these unseen aspects of a teacher's practices.*

## INTRODUCTION

Preparing teachers to learn from teaching is a powerful way of thinking about professional learning. Hiebert et al. (2007) proposed that teachers should learn to specify the learning goals, collect evidence of learning from classroom observations, think about the effectiveness of their instructional approaches, and improve their instruction based on the evidence collected. In other words, teachers should have opportunities to examine their understanding of content, curriculum materials, learning and instruction (Sherin, 2002). Despite recent developments in adopting practice-based and inquiry-based approaches to improve mathematics instruction, much of the complexity surrounding teacher learning and the different elements of a teacher's practice remains unseen. Shulman's (1987) proposed model of pedagogical reasoning and action can be seen as "a starting point for unpacking the unseen aspects of practice" (Loughran et al., 2016, p. 388). Yet, whether a teacher has gained new comprehension (Shulman, 1987) from reflection, and how this new learning has taken place still resides in a black box. This paper presents how a teacher's pedagogical reasoning can be made visible using pedagogical documentation.

## THEORETICAL CONSIDERATIONS

Teaching "begins with an act of reason" and "continues with a process of reasoning" to culminate in a series of pedagogical decisions (Shulman, 1987, p. 13). In other words, teachers need to learn how to apply their knowledge for teaching to provide justifications for their instructional decisions. Doing this involves taking one's understanding about content and "making it ready for effective instruction" (Shulman, 1987, p. 14), through a cycle of activities involving comprehension, transformation, instruction, evaluation, and reflection, leading to new comprehension. According to Shulman (1987), comprehension refers to how teachers can understand what they teach and relate these ideas to other ideas within and beyond the subject in different ways. A



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teacher then transforms his or her knowledge into "forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (Shulman, 1987, p. 15). Transforming this knowledge involves preparation, representation, instructional selections, adaptations of these representations and tailoring the representations to specific students' profiles. Although comprehension and transformation can occur at any time during teaching, Shulman (1987, p. 18) sees these two processes as "prospective", occurring before instruction, an "enactive" performance in the classrooms. Shulman (1987) then highlights evaluation as the process of assessing students' understanding to provide feedback about the teacher's instruction. However, it is when a teacher reflects on the instructional experiences that learning from teaching can occur. This new learning in the form of better understanding about teaching and learning will then be part of a teacher's new comprehension, which becomes the starting point for planning future lessons.

While Shulman's model may provide a lens to examine a teacher's instruction, much of a teacher's pedagogical reasoning remains invisible. How can we document a teacher's thinking about instruction to make it more visible? For this, we turn to the idea of pedagogical documentation (Dahlberg & Asen, 1994; Lee-Hammond & Bjervås, 2021), which is widely practised in early childhood education settings. The practice of pedagogical documentation involves teachers in collecting written notes, audio and video recordings, photographs, or students' learning artifacts for describing what and how students learn, which then serve as a basis for reflection and making instructional decisions (Lee-Hammond & Bjervås, 2021). In this way, the documentation is both a product and a process, and has been demonstrated to support teachers in professional learning. However, pedagogical documentation is scarce in mathematics education contexts, and we wonder if this practice could be incorporated as part of a mathematics teacher's everyday activities to enhance professional learning. In this paper, we present a case of how a teacher's pedagogical reasoning is made visible through pedagogical documentation, unpacking the unseen aspects of teaching and learning. The key question framing this paper is: What aspects of a teacher's pedagogical reasoning and action are captured in her pedagogical documentation?

## METHODS

The data presented in this paper were collected as part of a larger project, aimed at developing a proof of concept for a sustainable professional learning model for mathematics teachers. Drawing on current theoretical perspectives of teacher noticing (Dindyal et al., 2021; Fernandez & Choy, 2019), we conceptualized professional learning sessions where teachers have opportunities to work and co-learn with us in a community of inquiry (Jaworski, 2006). At the time of this present study, face-to-face sessions with teachers were not feasible due to prevailing Covid-19 restrictions. Hence, we conducted two online professional learning (PL) sessions for six elementary school teachers: In the first session, we elicit teachers' ideas about ratio and challenges associated with teaching ratio; in the second session, we shared ideas about

proportional reasoning and discussed the teaching of ratio for Grade 5 students (age 11). After the PL sessions, we followed up with two of the teachers, who volunteered their lessons for the entire unit on ratio for us to observe. In this paper, we uncover the pedagogical reasoning of one of these experienced teachers, Kathy (pseudonym), as she planned, taught, and reflected on a series of four lessons on ratio.

Data were generated from the voice and video recordings of the lessons, Kathy's lesson plans and instructional materials, and an interview with Kathy at the end of the study. In addition, we leveraged on the idea of pedagogical documentation to capture teachers' thinking about content and their pedagogical reasoning as they reflected on the planning and teaching of the lessons. More specifically, we used Padlet (https://padlet.com/), a digital notice board, as a platform for Kathy to curate her pedagogical documentation. We did not impose any number for the reflections—instead, we asked her to post her reflections, photos, videos, or documents related to any incident that she had found interesting on Padlet—and we left all instructional decisions to Kathy. Our role was to observe what she had learned from our sessions, her considerations for the selection of tasks and the instructional decisions made during her lessons. Findings were developed through identifying and analyzing critical incidents (Goodell, 2006), which are "everyday" events "encountered by a teacher in his or her practice that makes the teacher question the decisions that were made and provides an entry to improving teaching" (p. 224), during her planning and teaching. We analyzed these critical incidents by a "thematic approach" (Bryman, 2016, p. 578) to highlight aspects of concepts related to ratio, students' confusion about ratio, and instructional decisions before we tried to relate these incidents to Shulman's (1987) model of pedagogical reasoning and action.

## FINDINGS

For this paper, we present one of these critical incidents, which centred about Kathy's reflections on her selection, modification, and implementation of a colour mixture task (see Figure 3). We begin by highlighting aspects of Kathy's comprehension of the ratio concept and making explicit her thinking about the colour mixture task before and after the task was implemented from her pedagogical documentation.

### Kathy's comprehension of the ratio concept

In the first PL session, we asked teachers to share their understanding about ratio and anticipate the possible confusion that their students might have. Referring to Figure 1, we observed that Kathy was aware of some important ideas about ratio. She understood ratio "as a way of comparing 2 or more quantities", without specifying whether the quantities are of the same kind (Lamon, 2012). Kathy also highlighted that working with ratio "involves proportional reasoning" (Tourniaire & Pulos, 1985) and ratios are connected "to other topics like fraction and decimal". Moreover, she was cognizant of students' tendency to "use the additive idea" instead of multiplicative thinking when working with ratios (Clark & Kamii, 1996). Students' inability to apply multiplicative thinking strategies to solve missing-value problems was also

highlighted by Kathy with a specific example of "8 : 12 = ? : 15". Lastly, Kathy also surfaced the issue that students might not understand ratio as an ordered comparison.

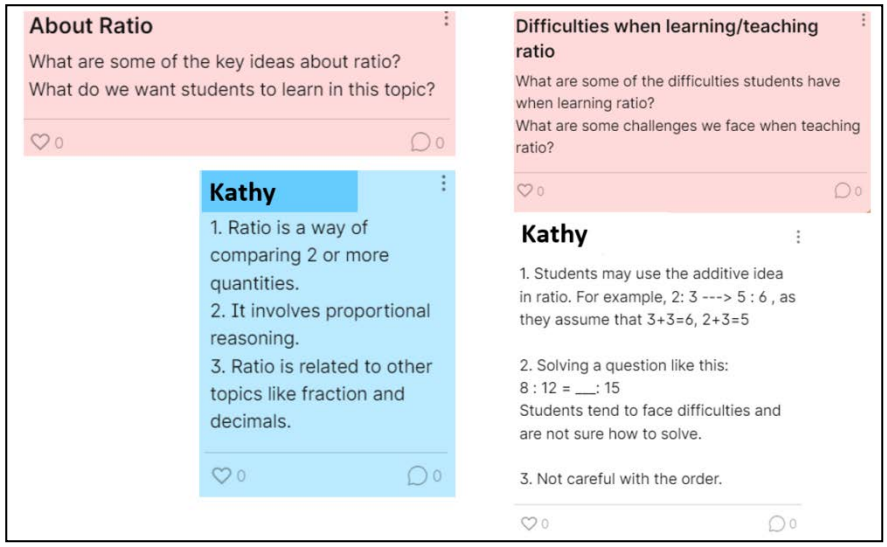



Figure 1: Snapshots of Kathy's responses on padlet.

In the second PL session, we shared other nuanced notions of ratio, emphasizing ideas such as absolute comparisons, relative comparisons, part-part comparisons and part-whole comparisons, as well as making a distinction between ratio, proportion, rate, and proportional reasoning (Yeo, 2019). We then invited Kathy to post her thoughts and reflections whenever something interesting came to her mind during the planning and implementation of her lessons.

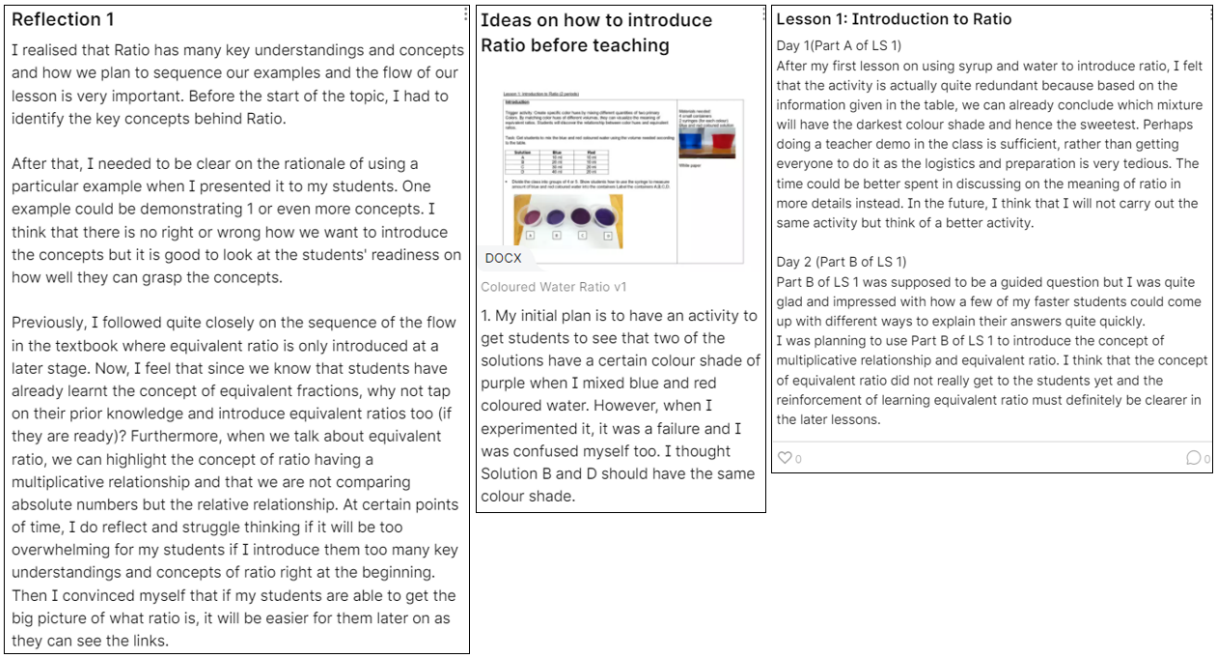



Figure 2: Snapshots of Kathy's pedagogical documentation.

Figure 2 shows a snapshot of her pedagogical documentation at the end of the unit. As seen from her reflections, Kathy became more aware that "ratio has many key

understandings and concepts". She was able to highlight how equivalent ratios are premised on "multiplicative relationship" and the difference between "absolute" and "relative" comparisons. Interestingly, Kathy agonized over the ideas and the sequence in which they should be introduced. She was also thinking about the profile of her students and considered the possibility of introducing the various inter-related ideas about ratio instead of presenting them in the sequence as presented in the textbooks. In particular, she entertained the idea that equivalent ratios could be presented to her students earlier even through the concept was introduced much later in the textbook. Her enriched comprehension of the concepts also contributed to the changes in her choice of the initial ratio task.

**Kathy's versions of the colour mixture task**

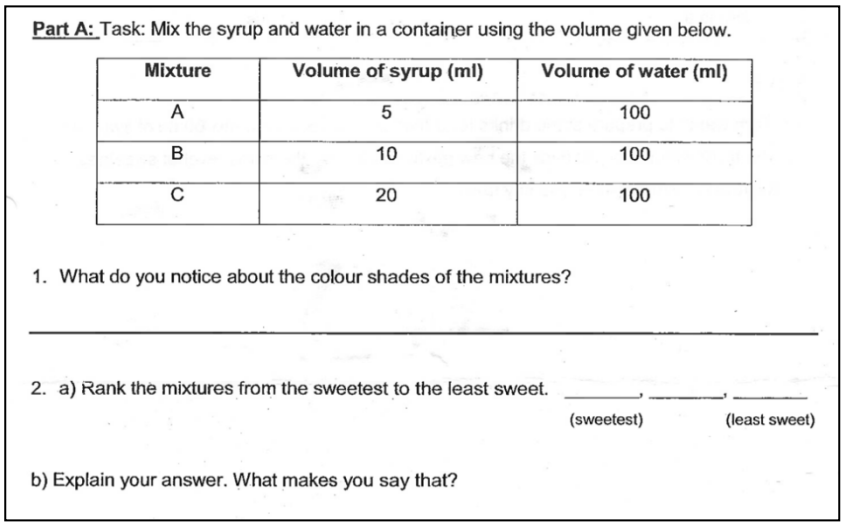
In her original lesson plan, she wanted to introduce the concept of ratio through an activity involving students making a dough using different number of cups of flour and water. After the two sessions we had conducted, Kathy began to think about the use of a colour mixture task (Figure 3), where students had opportunities to think about how the different amounts of blue and red dye contributed to the colours of four different solutions. The mixture problem, and its variations, has been used in other studies to develop students' proportional reasoning (Lamon, 2012; Tourniaire & Pulos, 1985). In Kathy's version, she used measurement units instead of non-standard units like cups.

Task: Get students to mix the blue and red coloured water using the volume needed according to the table.

| Solution | Blue | Red |
|---|---|---|
| A | 10 ml | 10 ml |
| B | 20 ml | 10 ml |
| C | 30 ml | 20 ml |
| D | 40 ml | 20 ml |

Figure 3: Excerpt from Kathy's colour mixture task.

However, as we can see from Kathy's reflection ("Ideas on how to introduce ratio before teaching"), she tried the activity but did not get the expected outcomes ("B and D should be of the same shade"). On one hand, the use of same units may help students to see ratio as comparison of quantities of the same kind and the fact that ratio has no unit (Yeo, 2019). On the other hand, the use of "quantities expressed in the same unit may be more confusing" (Tourniaire & Pulos, 1985, p. 184) as in the case of Kathy. Another possible point of confusion is that the volumes of blue and red dyes are different, and each mixture had a different volume, which may lead to a discussion on rate rather than ratio. This may be difficult for students who are formally learning ratio for the first time. It is possible that Kathy might have taken that into consideration by keeping constant one of the volumes in the second version of the task as shown in Figure 4.



**Part A:** Task: Mix the syrup and water in a container using the volume given below.

| Mixture | Volume of syrup (ml) | Volume of water (ml) |
|---|---|---|
| A | 5 | 100 |
| B | 10 | 100 |
| C | 20 | 100 |

1. What do you notice about the colour shades of the mixtures?

2. a) Rank the mixtures from the sweetest to the least sweet. _____, _____, _____
(sweetest) (least sweet)

b) Explain your answer. What makes you say that?

Figure 4: Kathy's second version of the colour mixture task.

Referring to Figure 4, we see that Kathy had changed the context of the task from comparing colour of a mixture to that of comparing taste (albeit through the colour shades) of the mixture. Kathy had intended her students to mix the syrup with water in class as evidenced in the lesson plan. Although Kathy had "addressed" one issue by keeping the volume of water at 100 ml for all three mixtures, it created another issue of students being able to solve the problem without mixing the syrup and water. Furthermore, the numbers made the solution obvious, which then reduced the demands of the task. While one may argue that Kathy could have caught the problem before the task implementation, it is noteworthy that she noticed the issue after the lesson. To be clear, the lesson went on well and the students were engaged with the task. But as Kathy had noted in her reflection on Lesson 1 (see Figure 2), she realised that the task was "redundant", and she could have done "a teacher demo" and spent the time "discussing the meaning of ratio" in greater depth. Here, we see Kathy's reflection of her instruction and assessment during the lesson leading to her new comprehension of how ratio could be approached differently. This new comprehension reinforced the importance of thinking about the first examples as highlighted in her "Reflection 1" (see Figure 2), which could potentially lead to her thinking about a "better activity".

## DISCUSSION

In this paper, we gave an account of what Kathy understood about ratio and how she reflected on her selection, modification, and implementation of a mixture task by making her pedagogical reasoning visible using pedagogical documentation. For example, Kathy's reflections about the content provide a window into her understanding of ratio, highlighting the aspects of her mathematical knowledge for teaching ratio. More importantly, we could "see" how Kathy transformed her understanding into the design of the task and how she eventually reflected on her instruction to modify her thinking about the lesson design for future lessons. Thus, Kathy had gone beyond documenting her practice—before, during, and after

lessons—and used her documentation as a basis for reflection to make instructional decisions (Lee-Hammond & Bjervås, 2021).

The power of pedagogical documentation to make visible a teacher's pedagogical reasoning has important implications. For researchers, the idea of pedagogical documentation can be repurposed to focus on teacher learning and be extended to include teacher artifacts before and after lessons. The use of such documentation helps to pinpoint the areas for intervention and support as mathematics educators work with teachers to improve their instruction. For teachers, documenting their practices provide opportunities for them to scrutinise and negotiate among three aspects of their teacher knowledge: understanding of mathematics, curriculum materials, and knowledge of how students learn (Sherin, 2002), a pre-requisite for teachers to learn from their own teaching. Moreover, a teacher's pedagogical reasoning and action can also be made visible to other teachers as part of their professional learning activities. Discussions around teachers' pedagogical documentation can then form the basis of pedagogical shifts in one's daily teaching activities not just for a teacher, but for the whole community.

But it is challenging and time-consuming for teachers to document their practices in ways that enhance their pedagogical reasoning and action. As Kathy had said during the final interview:

> I don't really like [documentation] because it takes some time. But it's good, and, you know, you got asked to write all this stuff. It really forces us to think, you know, what are the things that is in our mind? And then we can refer to that [documentation] later, even after a long period of time.

While the benefits of pedagogical documentation may justify the efforts needed to document one's practices, such tensions about effort and benefit should not be ignored if we want to move towards the idea of learning from one's teaching. What else can teachers do to document their practices? How can teachers be supported to document their practices? These are the important questions for future research.

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