FAIRNESS IN POLITICAL DISTRICTING: EXPLORING MATHEMATICAL REASONING

Sean Chorney

Simon Fraser University

For this paper, I explored the informal reasoning of undergraduate social science students in a mathematics class. They were looking into the mathematics of political districting, in particular gerrymandering. Using Sellars' notion of the space of reasons and analytic categories from the socioscientific issues literature, I examined the reasons students gave for the positions they took. I observed the way mathematics played a role in their reasoning and, how, when they addressed a social issue, their reasoning was more holistic. The analytic categories illuminated my data on how mathematics was integrated into the students' informal reasoning.

INTRODUCTION

Reasoning is an essential component of mathematics education, but in a recent article, Kollosche (2021) notes that there is a dearth of research on what reasoning takes place outside the formal kinds, such as deductive. He also states that there is no theoretical framing for reasoning in the mathematics education literature. In this study, I examine students' reasoning in a mathematics activity that engages with a social issue. I am interested in reasoning other than the objective, detached, formal kind. Pursuing this interest, I explored the question of fairness in political districting, a topic seen to be integrally related to mathematics (Staples & Evans, in press). I want to know how student use mathematics in their reasoning and how the mathematics interweaves with their personal values and the social context of the activity.

Research by Byers (2007) shows that connection to an authentic situation improves students' ability to reason. They are not just algorithmically completing a task, but grappling with ambiguity, values, and complexity. Addressing social concerns in mathematics education improves students' ability to conjecture and connect mathematical knowledge to social issues. This helps them becoming socially responsible citizens (Labaree, 1997). Applying mathematics to social issues moves education away from teaching of 'facts' *about* socio-mathematical issues, such as climate change or gerrymandering, to involving the reasoning about such issues.

In this paper, I use the theoretical framework of Wilfred Sellars' (1963) "space of reasons," which posits that knowledge is not accumulated through experience or rational thought but through justifying with reasons. That is, in the giving of reasons when grappling with an ill-structured situation a person becomes aware of their decisions and justifications (Chang & Chiu, 2008). In addition, I also draw on the informal reasoning categories of Sadler et al. (2007), as analytic tools. Sadler et al. are

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based in the socioscientific (SSI) field, which I posit aligns with Sellars' space of reasons. In this study I ask students to engage in a political districting activity. I ask what kinds of reasons are given and how mathematics fits in with those reasons.

THEORETICAL FRAMEWORK

Space of Reasons

The space of reasons (Sellars, 1963) is a philosophical notion that is a move away from mental representation toward increased awareness of the reasons one chooses to justify a claim. Giving reasons is a shift from problem-solving, in which the focus is based in trying to figure things out, to the contemplation of a situation in which contextual factors and emotional stance participate in the decision making of something that may not be able to be figured out. In this way, the space of reasons embraces broad reasoning, one not solely based in rational, objective thought. The purpose for using the space of reasons as a theoretical foundation is that it provides an opportunity to see students become aware of their own values and stance through the reasons they express. Mackrell & Pratt (2017) bring Sellar's space of reasons into mathematics education and suggest that: "human minds develop through an initiation into the space of reasons in that our thoughts and actions are increasingly guided by what there is reason to think about or do" (p. 427). I contend the sort of activity that I gave to the students in this study contributes to students reasoning because the context of a districting activity gives them an increased sense of what they can do.

Informal Reasoning

Sadler et al. (2007) who combine science with social issues, known in science education as SSI, gives currency to the space of reasons. Through their empirical work, they have come up with four kinds of reasoning in the context of working with a social issue: [1] complexity, [2] skepticism, [3] perspective taking, [4] inquiry. Briefly, [1] *complexity* is an unwillingness to commit to a simple solution because of an awareness of the multiple factors inherent in a situation. [2] *Skepticism* is an awareness of potential bias in a situation. [3] *Perspective taking* is looking at a situation from different positions and recognizing that in the social world different people have different priorities. [4] *Inquiry* is an exploration of a situation which may require further investigation.

The affinity between the space of reasons and the reasoning categories can be seen in the following passage:

> a key to interpreting a phenomenon as belonging within the space of reasons is whether the person holding the belief or desire or engaging in the action is aware that the belief, desire or action could be different and can ask the question whether their belief desire or action should be as it is (Mackrel & Pratt, 2017, p. 426).

The question of whether the students have an awareness of fairness in districting and whether their decisions "could be different" is the question that motivates this study. In particular, I ask what kinds of informal mathematical reasons are students giving in response to issues of fairness in a districting activity?

METHODS

In 2021, I taught an undergraduate mathematics education course at a university in western Canada. The course was intended for social science students who required a mathematics course to complete their degree. There were twelve 3-hour classes. There were 34 students registered and attending the class. The course was focused on quantitative reasoning and on topics such as the pigeonhole principle and proof by contradiction. The mathematics curriculum of the province in which the course was taught articulates two areas for mathematics teaching: content and competencies. Mathematical competencies are the ability to use mathematics rather than know the mathematics itself ("content"). According to the curriculum, students develop competencies through "reasoning." The curriculum recommends engaging in "inquiry", shifting "perspectives", and "reflecting" on activities (BCMoE, 2015). These correspond to three of Sadler et al.'s (2007) four categories.

Districting context

One of the topics I introduced this year was the redistricting of voting constituencies. One of the reasons I chose this topic (in addition to its connection to reasoning) was that there was a federal election in Canada occurring three weeks into our course. The winner of the election had fewer overall votes than the loser, not an uncommon result in a first-past-the-post electoral system. In the class we had the day after the election, I asked the students whether they thought the result of the election were fair. Almost all of them thought it was not.

Data

This topic of districting was taught from a mathematical perspective rather than a political one. The mathematics included comparing the general population and the district population and analysing how the proportions between these two depends on district boundaries, compactness metrics (such as the relationship between perimeter and area), and voter counts and its relationship with wasted votes. One mathematics formula introduced was the efficiency gap; it is a measure that compares the difference between each party's total votes minus wasted votes, divided by overall total votes. It was also essential to connect these mathematical ideas with the social issues of voting, taking into consideration concerns such as geographical obstacles, district contiguity, packing, and cracking.

To give a sense of the kinds of activities I presented to my students, I projected a grid like the one seen in Figure 1a and asked them to consider districting in different ways. Figures 1b and 1c show the extreme results of this activity.

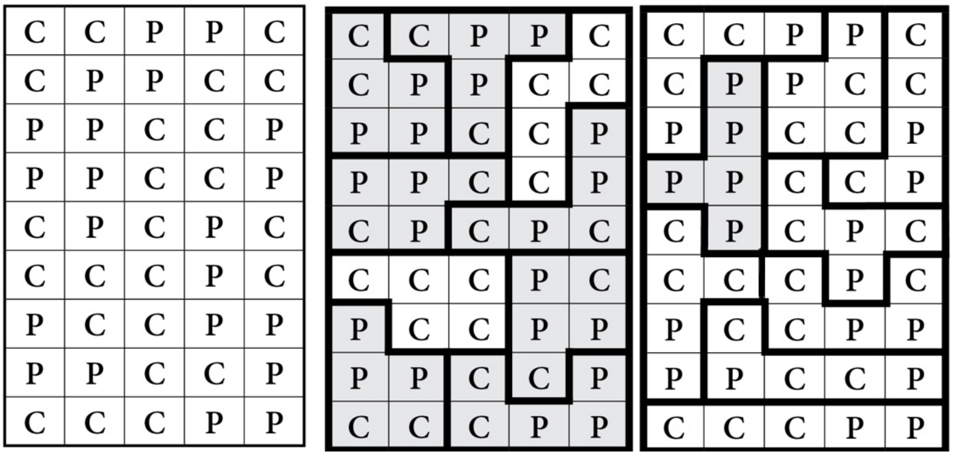

Figure 1: a) grid without any districts; b) districts favouring “P”; “P”s win 7-2; c) districts favouring “C”; “C”s win 8-1

Throughout the course, we investigated the redistricting prompts I gave through class activities, group discussions, and reflections. All data presented below comes from two take-home reflection assignments. The assignments were based on the notion of fairness, with the purpose of eliciting decisions and justifications. Due to the lack of definitive answers, mathematics, in this activity, can be considered a practice of giving reasons.

The data was rich and there was a large set of responses, but due to restrictions of space, I present responses from four students, two from one prompt and two from another. The examples presented below, however, are representative of the overall set of responses. All quotes below are verbatim. I present data and analysis together.

FINDINGS

In the following analysis, I report on two prompts.

Prompt 1

What is fair when districting a population?

Looking at the grids below, what is similar; what is different? Which set of divisions do you prefer and why?

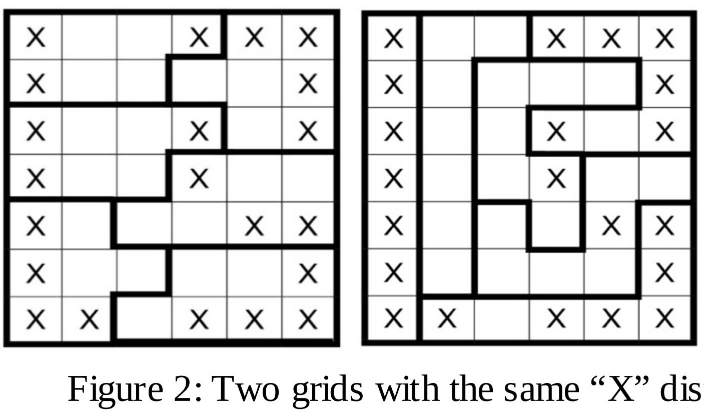

Figure 2: Two grids with the same “X” distribution but with different district boundaries

Inquiry

Xian wrote:

> The grid on the left is much more compact than the grid on the right; so it seems 'technically' better. However, both grids have the same (correct) outcome, showing that either type of division can be used 'fairly'. Having said that, I do think the type of divisions on the right can be used more effectively for gerrymandering ~ I tried to redistrict and gerrymander the left side while maintaining the compact districts and was unable to do so; but then tried the right and could not do so either ~ so really, is one better than the other? No in that regard.

Brown and Walter (1983) articulate a "what if" method as mathematical problem-posing. In Xian's response, we see an example of asking a "what if" question when she implicitly asks "what if" I try to manipulate the boundaries? Xian wondered whether the two grids had different potentials in terms of gerrymandering. Her justification introduced an inquiry of manipulating boundaries to see which grid was closer to the possibility of gerrymandering than the other. But since she found that neither could be easily gerrymandered, she reasoned that neither is better based on that metric. I had not taught anything related to whether grids are closer to or farther from possible gerrymandering. She reasoned with that idea on her own. There is also mathematical legitimacy here in the consideration of "close or not close" to gerrymandering in the mathematics field of "outlier analysis."

Skepticism

Barth wrote the following:

> Both sets of division share the same result of the X's and O's tying. The result of both sides are the same thus I PERSONALLY do not have a preference… But it is to be acknowledged that the diagram on the right can be seen as problematic as most districts are seemingly "packed", which can be an indicator of gerrymandering. Both districts also have the same amount of wasted votes (9) and have an efficiency gap of 0.

Barth's decision is strongly based in calculations; he calculates the winner, the number of wasted votes, and the efficiency gap. He positions these three calculations as central, and by emphasizing "PERSONALLY" he indicates that he feels uncomfortable with making a decision without more information. It is interesting to note that the calculations do not take precedence over his reasoning, in that he notices that there appears to be at least some potential for packing. But he cannot confirm or deny that there was. He notes it only as a possibility. In terms of skepticism, he is aware that there is not enough information and is therefore reluctant to express a preference.

Prompt 2

To simulate a random districting, draw a 5 by 5 district plan in an empty grid. Then flip a coin to propagate Xs and Os in the table as you move systematically through the grid. Is the result of this experiment appear fair for "X"s? If you can change your original district boundaries, what would you change to make it more fair?

Complexity

Helena wrote:

> The Xs occupy 13/25 cells and the Os occupy 12/25 cells. The Xs occupy 52% of the cells but 60% of the districts. I believe if we are basing this off majority/minority it is fair that the Xs won as they occupy the majority of the cells with over 50%. I believe this is fair as it represents an adequate picture of the reality within the cells....Let's say I created just simple linear districts going by columns, the Xs would still win with the same margin of 3/5. If we drew the districts by rows the Os would win, due to the packing of the Xs in the first row. I don't think I would make the change as it represents gerrymandering to allow the minority (O) to win by forcing packing of Xs in row number #1.

Helena’s outcome seems fair to her and she is content, but she also evaluates various configurations to see whether her original results are as fair as they could be. She checks what would happen if she uses columns as districts and then rows. In the case of columns, there is no change in outcome; but with rows, the outcome was that the Os would win overall since her cells in the top row were all Xs. She concluded that she would not choose boundaries based on rows because the count for the districts would not match the count for the population. Her comment on the “reality within the cells” addresses proportional comparison. She has a mathematical sense of fairness in that the proportion of the popular vote should align closely with the population of district outcomes. Proportion in pure mathematics is at its root a statement of equality: a/b=c/d, where c/d can be any magnitude bigger (or smaller) than a/b. Helena’s dissatisfaction was based on her acceptance of this proportional statement of equivalence.

Perspective Taking

In response to prompt 2, David wrote:

> Even though this experiment did randomly end up giving quite fair results, as there were not many wasted votes at all (4 wasted for X, and 6 wasted for O) even if the results have been more skewed, I don't think you can change them to be more “fair”. The districts were created completely randomly, and the grid was filled in that way too, which is essentially the fairest this can be. If you were to change it to try to reduce the amount of wasted votes for one side, it would just become unfair to the other side.

David is willing to accept any grid created randomly. He associates fairness with objectivity. David is relying on mathematics as objective, because it has no intent, it cannot be nefarious. He is using mathematics as a way to establish his stance. If there is human intervention in drawing and/or manipulating of boundaries, that is when problems can occur. He justifies this by recognizing that as soon as you reduce wasted votes for one party, you increase it for the other party. He noticed through a change of perspective that nothing would been gained if he were to interject. He would not solve anything, thus confirming his original stance.

DISCUSSION

This study is about how mathematics is used in student reasoning about social questions. I focused on the mathematical content and competencies that emerge in reasoning about fairness in a political districting activity. I wanted to see how students use mathematics when making decisions.

Returning to the notion of the space of reasons, it appears that students use both the social aspects of districting and mathematical justifications. Each student presented a unique approach to the reasoning in which they engaged. Xian engaged in a mathematical inquiry that paralleled Brown and Walters' "what if" method to explore two arbitrary grids in terms of their mathematical proximity to bias (gerrymandering). This inquiry not only aligns with Sadler et al.'s categories, but is also based on her personal sense of fairness. Barth feels uncomfortable expressing his own opinion and uses three calculations to ground his decision. But he remains skeptical about whether those calculations are satisfactory as there are still social variables that may be unaccounted for.

Helena grounds her stance in proportional equivalence and states her reasons as seeking the closest proportional equivalence between the population and the districts. She thinks that would be the fairest. She redraws district boundaries to determine whether there are better configurations. She, in fact, finds one that is worse. David relies on mathematical randomness as being fair and supports this by noting that in terms of wasted votes, he cannot improve the situation. Taking the perspective of one party and improving their situation will only negatively affect the other party.

In each case, the kind of mathematics is different. Xian, Helena, and David base their reasoning on mathematical competencies, and Barth bases his on mathematical content. It is also evident that the mathematical reasoning cannot be separated from the context or the student's values. This is confirmed by the various decisions made and how each student framed their stance, some confidently in mathematics, others without confidence in wanting to know more. The mathematics emerged from consideration of a social situation. For example, the proportionality between a population and its districts only exists because of the first-past-the-post system.

This study looks to assess how students use mathematics in the reasoning that informs their decisions. It is to understand that students can rearrange, count, calculate, identify, and verify to confirm a posed problem. That is, they develop reasons based on consequences of actions.

We could use more evidence-based classroom studies to advance our understanding of (1) how mathematics teachers can better prepare themselves to use social issues in teaching mathematics and (2) how students can broaden their views of mathematics and see how reasons develop in action.

This study cannot be generalised as only one class was studied, but the study is suggestive that engaging in a social issue in mathematics class may develop a more robust form of reasoning, one that interweaves the mathematics and the social.

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