HOW DO FIRST-GRADE STUDENTS RECOGNIZE PATTERNS? AN EYE-TRACKING STUDY

Lukas Baumanns[1], Demetra Pitta-Pantazi[2], Eleni Demosthenous[2], Constantinos Christou[2], Achim J. Lilienthal[3], Maike Schindler[1]

[1]University of Cologne, [2]University of Cyprus, [3]Örebro University

Recognizing patterns is an important skill in early mathematics learning. Yet only few studies have investigated how first-grade students recognize patterns. These studies mainly analyzed students' expressions and drawings in individual interviews. The study presented in this paper used eye tracking in order to explore the processes of 22 first-grade students while they were trying to recognize repeating patterns. In our study, we used numerical and color pattern tasks with three different repeating patterns (i.e., repeating unit is AB, ABC, or AABB). For each repeating pattern task, students were asked to say the following object of the given pattern. For these patterns, we identified four different processes in recognizing repeating patterns. In addition, we report differences in the observed processes between the patterns used in the tasks.

INTRODUCTION

Mathematics can be described as the science of patterns (Steen, 1988). In early mathematical learning, patterns play a decisive role in the development of algebraic thinking (Carraher & Schliemann, 2007; Clemens & Samara, 2007). Being aware of patterns provides primary students with a mindset that is useful in the later study of algebra (Schoenfeld, 2007). In addition, pattern recognition is a central content of mathematics education in primary school (NCTM, 2000), which makes it a significant topic for mathematics education.

Yet, pattern recognition poses challenges to many students. For example, Clarke et al. (2006) found that for repeating patterns (e.g., ABABAB) first graders are successful in recognizing and extending them in only 31% of the tasks. This calls for support of young children in their ability to recognize patterns. For being able to foster children's pattern-recognition ability, it is crucial to investigate and understand their pattern-recognition processes (Lüken, 2018; Papic et al., 2011). Empirical studies mostly explore pattern-recognition processes based on children's expressions and drawings. The present pilot study in the context of the Erasmus+ project DIDUNAS investigates students' pattern-recognition processes using eye tracking, the recording of eye movements. Eye tracking has shown to be useful for investigating processes of children on early primary level before (Schindler et al., 2020; Sprenger & Benz, 2020). Our study uses eye movement video analysis to explore pattern-recognition processes.

In this pilot study, the aim is to investigate (1) what processes first-graders use in recognizing patterns and (2) whether there are differences in students' use of pattern-recognition processes between different kinds of patterns.

2022. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 59-66). PME.

THEORETICAL BACKGROUND

Pattern recognition in early mathematics learning

One goal of early mathematical learning is the development of algebraic thinking (NCTM, 2000). Algebraic thinking in early grades includes noticing structures (Kieran, 2004). Fostering students in their ability to notice structures and recognize patterns can therefore contribute to their development of algebraic thinking. (Carraher & Schliemann, 2007). In the first grades of primary school, repeating patterns can be used to foster students' ability to recognize patterns (Clemens & Samara, 2007). Repeating patterns refer to patterns that have one unit (e.g., AB) that repeats multiple times (ABABAB). Patterns differ from each other if the repeating unit is of different length or the elements of a repeating unit are arranged differently. For example, pattern ABABAB is different from pattern ABCABCABC, or pattern AABCAABC is different from ABACABAC.

For pattern recognition, it is of particular importance to recognize the recurring unit of a pattern (Papic et al., 2011). The repeating unit can be represented, for example, symbolically (e.g., AB), numerically (e.g., 2 5), or by colors (e.g., ● ●). The present study uses numerical and color patterns. Common pattern tasks for preschoolers and first graders consist of identifying and extending given repeating patterns (Rittle-Johnson et al., 2015). For example, students are given a color pattern (e.g., ● ● ● ● ● ● ●) and asked to say what color the next dot should be. In this paper, student approaches to extend repeating patterns, which are anticipated to entail identification of the repeating unit, are referred to as pattern-recognition processes.

Research on pattern recognition at the beginning of primary school

Some studies investigate the abilities of preschoolers or first graders in the context of repeating or growing patterns. Clarke et al. (2006) found that 76% of the students at the beginning of first grade can copy a repeating pattern, but only 31% can extend it. Rittle-Johnson et al. (2015) found similar results in a study with 64 four-year-old preschoolers. Further studies have used children's expressions and drawings in individual interviews to investigate processes of preschoolers and first graders in recognizing patterns (Lüken, 2018, Papic et al., 2011). For example, Lüken (2018) found that three- to five-year-old children use a process of *comparison* to compare the beginning of the repeating pattern with the part that has to be extended. Lüken also found that the repeating unit was identified and used by the students. Papic et al. (2011) identified similar pattern-recognition processes.

In this study, we were interested in pattern-recognition processes of first-grade students. In contrast to previous studies on this topic, we use eye tracking videos (not student utterances or drawings) to explore student pattern-recognition processes, since eye tracking has proven itself valuable to identify student processes in mathematics (e.g., Schindler et al., 2019, 2020). We ask the following research questions.

1. What processes do first-grade students use in recognizing repeating patterns?
2. Are there differences in students' use of pattern-recognition processes between different kinds of repeating patterns?

METHODS

Participants, procedure, and tasks

The study was conducted with 22 first-grade students (age: M = 6.80 years; SD = 0.24 years) from a primary school in Cyprus. Fourteen (~63.6%) of the students had Greek as their mother tongue, the others Arabic (n=7, ~31.8%) and Bulgarian (n=1, ~4.5%).

In addition to the eye-tracking study (see below), we conducted the standardized mathematics test ZAREKI-K for assessing students' mathematical performance level at the transition from kindergarten to primary school (von Aster et al., 2009). We used the adapted version by Walter (2020). The test indicates that twelve of the 22 students (~54.5%) are not at risk for developing math difficulties, but ten are at risk. Thus, the sample has a good spectrum in terms of performance levels.

In the main part of the study, the students worked individually on eight pattern tasks on a computer screen (see Figure 1). Each task consisted of at least three repetitions of a unit followed by a white blob. The students were asked to name the number or color of the object that was hidden behind the white blob (e.g., 1, yellow). Before the first numerical and the first color pattern task, the students worked on a sample task, to ensure that the students understood the task correctly. The following three repeating units were used in the pattern tasks: (1) AB (four tasks), (2) ABC (two tasks), and (3) AABB (two tasks). The students answered by saying aloud the number or color they thought was behind the white blob. The students did not receive feedback and incorrect answers were not corrected. Four tasks had numerical repeating units in form of digits (e.g., 4 1), four in the form of colored dots (e.g., ● ●).

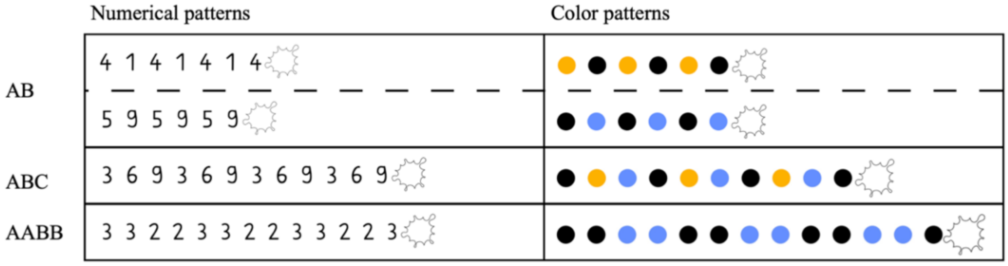

Figure 1: Numerical and color repeating pattern tasks used in the study.

Eye tracking

Students' eye movements were recorded with the screen-based eye tracker Tobii Pro X3-120 (infrared, binocular), with a sampling rate of 120 Hz. The tasks were presented on a 24" monitor. The students' heads were about 60–65 cm away from the monitor. The eye-tracking data showed an average accuracy of 1.37°, which corresponded to an error of about 1.44–1.55 cm on the screen at a head distance of 60–65 cm. The centers of the digits and dots in our tasks were on average 4.16 cm apart from one another on the display on the monitor (3.2–5.5 cm), which means that the eye-tracking accuracy was sufficient to reliably determine what element the students looked at.

Analysis of eye-tracking data

Raw gaze-overlaid videos provided by Tobii Pro Lab software were used to analyze students' pattern recognition processes. In addition, notes were taken during the data collection describing student actions (e.g., when students pointed to the monitor). Although pattern-recognition processes have already been elaborated in research, an inductive approach was chosen for this study. We performed a qualitative content analysis through a data-driven inductive category development (Mayring, 2000), similar to Schindler et al. (2019, 2020), in four stages: *Stage one*: A randomly selected half of the gaze-overlaid videos were viewed and for each video, the gazes were described. *Stage two*: Similar descriptions of the gaze-overlaid videos were subsumed into one category, while categories in this study refer to pattern-recognition processes. A first general description of the respective pattern-recognition process was formulated. *Stage three*: The second half of the gaze-overlaid videos were viewed and coded using the pattern-recognition processes elaborated in stage two. During this coding, existing descriptions of pattern-recognition processes were revised and specified, and new processes were added when existing processes did not seem suitable for describing the gaze-overlaid videos. *Stage four*: Finally, with the complete category system, the first half of the gaze-overlaid videos were re-coded to check the fit of the revised process descriptions and to assess the emergence.

All gaze-overlaid videos were coded by the first author. 22.7% of the videos were analyzed independently by the last author. The interrater agreement was calculated using Cohen's Kappa (Cohen, 1960). With $\kappa = 0.87$, the inter-rater agreement is almost perfect (Landis & Koch, 1977).

Statistical analysis

To determine differences between the pattern-recognition processes and the different patterns, a two-tailed Fisher–Freeman–Halton exact test for $r \times c$ contingency tables was performed (Freeman & Halton, 1951) using SPSS 28. This test is an extension of chi-square test and is especially suited for small sample sizes for which the chi-square approximation does not hold (Fagerland et al. 2017). For this analysis, we have grouped the different patterns according to their structure (see Figure 1). For example, the pattern tasks with repeating unit 4 1 and ● ● in Figure 1 fall into group AB.

RESULTS

Pattern-recognizing processes

In the following, we describe the pattern-recognition processes found through the analysis of gaze-overlaid videos of the first-grade students. We use gaze plots to visualize the processes for this paper, even though the analysis was based on the videos. Figure 2 shows idealized gazeplots of these processes as illustrations. These idealized gazeplots are not actual gazeplots of children, but idealized illustrations for the identified processes.

(1) Identifying one repeating unit of the pattern

The gazes go to one repeating unit (sometimes multiple times)—mostly the repeating unit before the white blob. The gazes partially also touch one dot/number before the repeating unit.

(2) Identifying one repeating unit and validating/applying it

(a) Identifying and validation: The gazes go to the repeating unit before the blob (sometimes multiple times) and then go to another repeating unit in the pattern (sometimes multiple times). Afterwards, the pattern is extended by continuing the repeating unit before the blob.

(b) Identifying and application: The gazes go over a repeating unit in the beginning or the middle of the pattern (sometimes multiple times). Afterwards, the pattern is extended by continuing the repeating unit before the blob.

(3) Looking at each element

The gazes go to each dot/number of the pattern individually usually from left to right (sometimes multiple times). Up to three dots/numbers are skipped from the beginning.

(4) Unsystematic jumping over the pattern

The gazes jump fast over the pattern. Often the blob is not looked at. There is no systematic process recognizable.

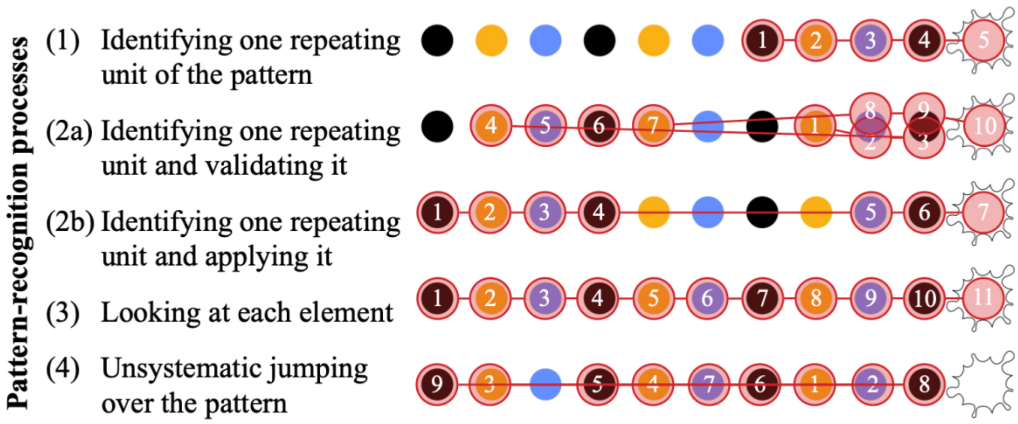

Figure 2: Idealized gazeplots of the pattern-recognition processes with numbers indicating the order in which the students looked at each dot.

Differences in pattern-recognition processes between different patterns

To determine differences between the identified pattern-recognition processes (see Figure 2) and the different kinds of patterns (see Figure 1), Fisher–Freeman–Halton exact test for $r \times c$ contingency tables was performed (see Table 1). The test revealed that the pattern-recognition processes used by the students differed significantly between the three kinds of patterns (p = 0.031). Cramér's V = .20 indicates a moderate relationship between the kind of pattern and pattern-recognition processes.

		Pattern-recognition process				
		(1)	**(2)**	**(3)**	**(4)**	Total
Repeating unit	**AB**	35	24	23	0	82
	ABC	12	22	5	1	40
	AABB	15	16	47	2	40
		63	61	35	3	162

Table 1: Observed number of pattern-recognition processes for the different patterns.

Figure 3 shows the distribution of the pattern-recognition processes over the three kinds of pattern tasks based on the absolute values in Table 1. Figure 3 illustrates that for patterns of kind AB, students used process (1) identifying one repeating unit, the most with 42.68%. Process (2) identifying one repeating unit and validating it, and (3) looking at each element, were equally distributed in the pattern tasks with the repeating unit AB. Process (4) unsystematic jumping over the pattern, in contrast, did not occur. In the pattern tasks with repeating unit ABC, process (2) was identified most often with 55%. The other processes occurred less often. Processes (3) and (4) together were identified only half as often as process (1). For patterns of kind AABB, processes (1) and (2) were identified with almost equal frequency, 37.5% and 40%, respectively. Processes (3) and (4) were identified slightly more often than in patterns of kind ABC. Category (4) appeared exclusively together with wrong answers.

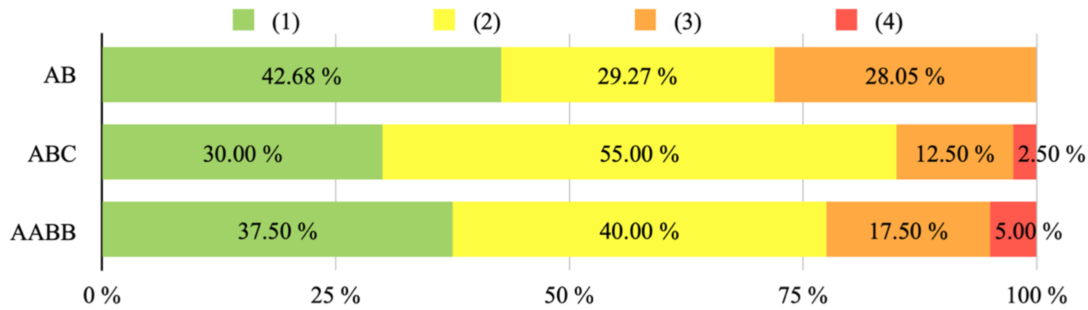

Figure 3: Distribution of pattern-recognition processes over the kinds of patterns.

DISCUSSION

This pilot study aimed to investigate what processes first-grade students use in recognizing repeating patterns and whether there are differences in students' use of pattern-recognition processes between different kinds of repeating patterns. The study was conducted with 22 first-grade students and eight repeating pattern tasks in which students were to name the number or color of the next object of a given pattern. The results of our study show that the first-grade students used four different pattern-recognition processes in numerical and color repeating pattern tasks: (1)

Identifying one repeating unit of the pattern, (2) identifying one repeating unit and validating/applying it, (3) looking at each element, and (4) unsystematic jumping over the pattern. Our results connect to some of the pattern-recognition processes identified in previous studies (Lüken, 2018; Papic et al., 2011) and extend them. Furthermore, we found that these processes were used differently between different kinds of patterns (see Figure 3).

These results should be interpreted considering the following limitations: With 22 students, a relatively small sample was available. It cannot be discounted that with a larger sample, additional pattern-recognition processes could be identified. Also, in future studies, more than eight pattern tasks should and will be conducted. In particular, other patterns than those used in this pilot study (i.e., AB, ABC, AABB) need to be investigated (e.g., AAB, AABC, ABAC).

The results of this study hinted at the value of using eye tracking to explore students' pattern-recognition processes. With regard to the overall purpose of supporting children in pattern recognition processes, the study has shown that eye tracking can inform about student strategies and that these insights can help to support students adaptively and individually in developing further their pattern-recognition processes. In line with this, one aim of the DIDUNAS project is to develop teacher materials that serve to support students, for example, in pattern recognition. The results of this pilot study provide initial insights for the development of such teacher materials.

Acknowledgment

This project has received funding by the Erasmus+ grant program of the European Union under grant agreement No 2020-1-DE03-KA201-077597.

References

Carraher, D. W., & Schliemann, A. D. (2007). Early algebra and algebraic reasoning. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 669–705). NCTM.

Clarke, B., Clarke, D., & Cheeseman, J. (2006). The mathematical knowledge and understanding young children bring to school. *Mathematics Education Research Journal, 18*(1), 78–102.

Clements, D. H., & Samara, J. (2007). Early childhood mathematics learning. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 461–555). NCTM.

Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychological Measurement, 20*(1), 37–46.

Fagerland, M. W., Lydersen, S., & Laake, P. (2017). *Statistical Analysis of Contingency Tables*. CRC.

Freeman, G. H., & Halton, J. H. (1951). Note on an exact treatment of contingency, goodness of fit and other problems of significance. *Biometrika, 38*, 141–149.

Kieran, C. (2004). Algebraic thinking in the early grades: What is it? *The Mathematics Educator*, *8*(1), 139–151.

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics*, *33*(1), 159–174.

Lüken, M. M., Peter-Kopp, A., & Kollhoff, S. (2014). Influence of early repeating patterning ability on school mathematics learning. In P. Liljedahl, S. Oesterle, C. Nicol, & D. Allan (Eds.), *Proc. of the Joint Meeting of PME 38 and PME-NA 36*, Vol. 4 (pp. 137–144). PME.

Lüken, M. M. (2018). Repeating pattern competencies in three- to five-year old kindergartners: A closer look at strategies. In I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank, & C. Benz (Eds.), Contemporary Research and Perspectives on Early Childhood Mathematics Education (pp. 35–53). Springer.

Mayring, P. (2000). Qualitative content analysis. *Forum: Qualitative Social Research, 1*(2), Art. 20.

NCTM (2000). *Principles and standards for school mathematics*. NCTM, Reston.

Papic, M. M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education*, *42*(3), 237–269.

Rittle-Johnson, B., Fyfe, E. R., Loehr, A. M., & Miller, M. R. (2015). Beyond numeracy in preschool: Adding patterns to the equation. *Early Childhood Research Quarterly*, *31*, 101–112.

Schindler, M., Bader, E., Lilienthal, A.J., Schindler, F., & Schabmann, A. (2019). Quantity recognition in structured whole number representations of students with mathematical difficulties: An eye-tracking study. *Learning Disabilities: A Contemporary Journal*, *17*(1), 5–28.

Schindler, M., Schovenberg, V., & Schabmann, A. (2020). Enumeration processes of children with mathematical difficulties: An explorative eye-tracking study on subitizing, groupitizing, counting, and pattern recognition. *Learning Disabilities: A Contemporary Journal*, *18*(2), 193–211.

Schoenfeld, A. H. (2008). Early algebra and algebraic reasoning. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the Early Grades* (pp. 479–510). Routledge.

Sprenger, P. and Benz, C. (2020). Children's perception of structures when determining cardinality of sets—results of an eye-tracking study with 5-year-old children. *ZDM – Mathematics Education*, *52*, 753–765.

Steen, L. A. (1988). The Science of Patterns. *Science*, *240*(4852), 611–616.

Walter, J. (2020). Ein Screening-Verfahren zur Prognose von Rechenschwierigkeiten in der Grundschule. *Zeitschrift für Heilpädagogik*, *71*, 238–253.

Zippert, E. L. (2020). Finding patterns in objects and numbers: Repeating patterning in pre-K predicts kindergarten mathematics knowledge. *Journal of Experimental Child Psychology*, *200*, 104965.