RELEARNING: A UNIFIED CONCEPTUALIZATION ACROSS COGNITIVE PSYCHOLOGY AND MATHEMATICS EDUCATION

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We propose a unifying conceptualization of "relearning", a construct that has a long history in the field of cognitive psychology and has recently been reconceptualized in the mathematics education with respect to teacher training. We argue that existing accounts of relearning are versions of the same phenomenon subjected to different motivations for relearning and intended relearning outcomes. Utilizing the existing theoretical rigor behind existing conceptualizations of relearning, we demonstrate the utility of the unified conceptualization in using findings from one section to suggest new avenues for others, and in addressing issues posed by a lack of theoretical framing in the studies of remedial mathematics education and repeated mathematics courses.

In this report we argue for the utility of a conceptualization of "relearning" in mathematics education, or the experience of learning about mathematical content one has tried to learn about before. Global pushes for widespread access to higher education combined with the hierarchical structure of mathematics has resulted in an increased number of relearning experiences for college mathematics students. In the United States, this can be seen in increasing enrollment in remedial mathematics courses (Chen, 2016) in particular. While such remedial courses are less common outside of the United States, they have begun to gain popularity more globally (Rienties et al., 2008; Brants & Struyven, 2009). Measures concerning their effectiveness remain limited due to the lack of research describing student experiences with teaching and learning in such courses (Grubb, 2001; Cox & Dougherty, 2019). Despite calls for research that investigates the relationship between students and mathematical content in remedial courses (Sitomer et al., 2012; Mesa, Wladis & Watkins, 2014), there exists no theoretical perspective useful for structuring investigations into the phenomenon of relearning in this context, or in college contexts more broadly. We argue that such a perspective may be built by combining and expanding on two similar lines of inquiry: the study of memory in cognitive psychology, and relearning in content courses for future mathematics teachers.

EXISTING CONCEPTUALIZATIONS OF RELEARNING

In cognitive psychology, the term *relearning* is attributed to psychologist Hermann Ebbinghaus' studies of memory and retention. In 1885, Ebbinghaus documented the number of verbal rehearsals necessary for him to memorize strings of randomly-ordered nonsensical syllables as the lengths of the strings varied. He then recorded the number of rehearsals necessary to recite the strings of syllables again from memory after varying intervals of time. Ebbinghaus labeled his experience of

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trying to memorize the same strings of syllables through verbal rehearsal a second time as “relearning”. Of particular significance was his ‘savings in relearning’ result (Nelson, 1985; Murre & Dros, 2015), or the observed inverse logarithmic relationship between the amount of time elapsed from the first learning trial to the relearning trial and the number of rehearsals required in the relearning trial for the individual to reproduce the material perfectly from memory. Ebbinghaus hypothesized that this change in time was proportional to the amount of the syllable string stored in one’s memory. Thus, by studying the amount of time “saved” in each relearning trial, one could estimate the rate at which content held in memory was forgotten.

Such an estimation is undoubtedly valuable in educational contexts, and has led to the adoption of the technique of successive relearning (spaced relearning that anticipates Ebbinghaus’ retention curve; comparable to other types of techniques to promote retention such as self-explanation, spaced practice, or mnemonics) in psychological studies of college students (Dunlosky & Rawson, 2015). However, as noted by more contemporary critiques (Bahrick, 1979), interventions in authentic educational contexts centered around Ebbinghaus’ conceptualization of relearning remain limited in applicability. Namely, because the intended learning outcome under this conceptualization is a successful reproduction of content from memory in as little time as possible, application to contexts involving more complex systems of knowledge such as the structure of a language (Hansen, Umeda, & McKinney, 2002), or mathematics (Rawson, Dunlosky & Janes, 2020) is more limited.

To this discussion, we add that while relearning motivated only by *retention* may have limited applicability in mathematics, relearning motivated by insufficient understanding of mathematical content previously learned is very common. In fact, this traditional tie to memory us on memorization may explain why an entirely separate theory of relearning has recently been developed by Zazkis (2011) in the field of content courses for preservice mathematics teachers. While relearning as a term was used to describe the learning experience of preservice mathematics teachers colloquially prior to Zazkis (2011) (e.g. Nicol, 2006), her work marked the first acknowledgement of relearning as a phenomenon of theoretical significance in undergraduate mathematics education. Zazkis argued that “contemporary” understandings of how people learn mathematics such as constructivism or situated cognition were insufficient in this context, "since prior cognitive structures have been constructed in the learner's mind some time ago, the reconstruction and reorganization processes involved [in relearning] are more challenging for the learner as well as for the instructor" (p. 13). Zazkis’ notion of relearning may be distinguished from relearning as it is conceptualized in studies of memory in cognitive psychology by two features: the intended learning outcome, and the motivation behind relearning.

Under Zazkis’ conceptualization, the intended outcome of relearning in teacher education is “restructuring knowledge,” or revisiting previously-held knowledge in order to reorganize it in a particular way seen as better-suited for the purposes of teaching. This reconstruction is motivated by an insufficient understanding of mathematical content from K-12 experiences, either due to "prior misleading learning" that resulted in misconceptions on the part of the student, or a K-12 experience in which the content was presented with limited depth (Zazkis & Rouleau, 2018). In this way, relearning in cognitive psychology and teacher education have very different proficiency criteria within their motivations. While relearning in cognitive psychology requires that the content be "learned" (memorized) successfully when first introduced in order for relearning to occur in the second encounter, relearning under Zazkis' conceptualization in teacher education *requires* a previously-insufficient content understanding to take place. Furthermore, while agree that Zazkis' definition of relearning is more useful for researchers of preservice teacher mathematics education because it expands learning beyond the notion of retention, we see potential for a more expanded conceptualization. Namely, while Zazkis' conceptualization is useful for describing the *intended* outcome of content courses for future teachers, it has limited utility in describing the phenomenon as it actually occurs for preservice teachers. Student experiences with relearning have been noted to be fraught with resistance from preservice teachers (e.g. Nicol, 2006; Barlow et al., 2018) given that they have seen the material before and may be more comfortable with their previous understandings. Thus, outcomes other than restructuring are not only possible in such courses, but a common point of concern for teacher educators. We contend that a theory meant to describe student experiences learning about content seen before in this context would benefit from the inclusion of such outcomes.

Despite their surface differences, we argue that the inherent phenomenon being described as 'relearning' across the aforementioned fields is inherently the same. Their ostensible dissimilarity comes from the fact that they both describe different types of relearning subject to restrictions that are relevant to the foci of their respective fields. However, by viewing them as separate instantiations of the same general phenomenon, we contend both fields would increase the likelihood of theoretical advancements for mathematics educators. Divorcing the term *relearning* from the norms of a particular context allows for the focus to shift from answering the question: 'what outcome *should* students get as a result of this experience?' to 'what outcomes *are* occurring and how do the circumstances of this particular context determine which outcomes are possible?' Furthermore, considering these areas of research to be contributing to the same overarching field of study means that researchers have access to a wider range of perspectives with which to consider issues of interest.

PROPOSAL OF UNIFIED CONCEPTUALIZATION

At the most basic level, we contend that relearning requires three things: some (mathematical) content, a "time 1" (T1) representing a past occurrence in which one has tried to learn about that content, and a "time 2" (T2) representing the most recent time one has tried to learn about that same content again. Although the name re*learning* appears to suggest some degree of mastery of content at T1, we make no such assumption in our treatment of this construct. That is, T1 learning need not cross any threshold or meet any criteria for relearning to be said to occur at T2. This is not to say that different levels of proficiency do not matter, but instead that a particular level of proficiency at T1 is not *required* for the phenomenon to take place. Furthermore, while the content at T1 and T2 need not be identical, it does need to cross a particular threshold of similarity such that the content learning goals at T2 are essentially the same as those at T1. For some studies of memory in cognitive psychology this criterion is more clearly filled as the materials to be memorized are completely identical at T1 and T2. In the field of mathematics teacher education, the issue of determining content similarity is more complex because mathematics content courses for future teachers often have additional learning goals related to pedagogy that would not be considered in the K-12 context. However, the focus of the mathematical content remains the same.

If these components are considered sufficient to defined a relearning experience, then several other common college mathematics experiences would fall under this category such as retaken college mathematics courses and remedial math courses. In the United States, remedial math courses are either semester-long courses or corequisite sections of courses in college whose content mirrors that of algebra courses offered in the middle and high school settings. This similarity to content learned at a time T1 is often noted as a point of concern for semester-long prerequisite remedial math courses (Stigler, Givvin, and Thompson, 2010) which are sometimes referred to as “high school all over again,” (Ngo, 2020). By placing additional restrictions on the basic components in terms of *motivation* and *intended learning outcome,* we can recognize and compare sub-types of relearning as they are currently conceptualized (Table 1).

Context	Motivation for Relearning	Intended Learning Outcome
Cognitive Psychology	Reduce likelihood of forgetting previously-memorized content.	Content is successfully reproduced from memory in as little time as possible.
Mathematics Teacher Education (Zazkis, 2011)	Mathematical knowledge previously demonstrated to be insufficient for teaching	Restructuring: address misconceptions from T1 and widen “domain of applicability” of content.
Traditional Remedial Mathematics Education	Mathematical knowledge previously demonstrated to be insufficient for subsequent course in mathematics.	Acquire ideal understanding from K-12 experiences; impact on understanding brought to T2 undefined.
Corequisite Remedial	Mathematical knowledge previously demonstrated to	Acquire understanding of only K-12 content related

Mathematics Education	be insufficient for current math/statistics course.	to credit-bearing course; impact on understanding brought to T2 undefined.

Figure 1: Constraints Among Relearning Contexts

By *motivation* we mean the main rationale that justifies the beginning of the relearning experience for the individual. Importantly, this question is asked of the *relearning context* rather than of the individual. For instance, an individual required to participate in a psych study of memory for course credit and an individual required to take a math content course for future teachers might both list 'academic requirement' as their motivation for beginning the relearning experience. The motivation behind the inclusion of relearning in the two scenarios, however, is very different. Historically, contexts involving relearning have used both proficiency-based motivations and memory-based motivations.

By *intended learning outcome* we mean the intended impact on the understanding of material gained at T1 by the end of a relearning experience. This is *not* a grade or an indication of passing/failing. For a scenario in which one is learning for the first time, we ask what content was learned. This may, more or less, be determined by examining a student's answers to a well-designed exam. The same is not true for a relearning scenario. In asking about intended learning outcome, we mean to answer the question: what was the intended additional value of learning about the material this time around? The answer to this question requires one to reference the understanding of content that was developed at T1 as well as to define the impact of the relearning experience on that understanding. Unlike Zazkis' theorizing of relearning within teacher education, relearning has yet to be theoretically investigated within remedial mathematics courses. While there is a general sense that students should reach a level of competency with material that was desired at T1, there is no consensus as to what the impact should be on the understanding of content that the students begins with at T2. However, as we will discuss in the implications, there is existing literature on student understanding in these courses that may serve as starting points for such a theorization.

Comparison to Alternative Conceptualizations

Due the hierarchical structure of mathematics, one could argue that you would be hard-pressed to find any college mathematics course that didn't include learning about at least *some* content that a student had seen before. Thus, one might argue that instances of relearning are really simply special cases of students building on prior knowledge. Recall that in order for a scenario to be labelled as relearning, the content learning goals at T2 are essentially the same as the content learning goals at T1. This would exclude cases, for instance, in which calculus instructors reference common algebraic errors when teaching students how to find critical values of functions whose derivatives involve fractions. The content learning goals are focused on the novel

Calculus concepts of derivatives and local maxima and minima, not the algebra that might be involved in solving a problem related to these concepts.

While it would be possible to view relearning scenarios through the theoretical lens of prior knowledge, we contend that this would be less advantageous for understanding student experiences. Consider the comparison between the above examples from calculus with the educational scenarios described in Cox (2015). In her analysis of instructional activities across six remedial mathematics courses, Cox describes different strategies for teaching students about fraction representations. For instance, one strategy involved positioning the idea of fraction division within a larger domain of part-whole relationships between numbers by asking students to produce problems whose solutions would be represented by various fractions rather than prioritizing simplification of an expression like 3/.25. In evaluating the effectiveness of the strategies, one could conceptualize the phenomenon taking place in this classroom as students building on prior knowledge to produce a new type of understanding of what previously may have been only a mathematical “rule”. However, considering this to be a task of helping students *relearn* algebra allows one to shift the focus from the content covered to the relationship a student would build with their already-established understanding of that content in the current context. The primary area of focus would not be that another representation of 3/.25 was learned, but rather how it was learned by students relative to their previous learning experiences. We would argue that the relearning lens is more useful in this context because it attends to the defining features of the classrooms Cox observed (i.e. the situation of learning about the same content again), whereas the use of prior knowledge would work equally-well for analyzing an instructional strategy for learning about algebra for the first time.

Relearning may also be distinguished from McGowen and Tall’s notion of a *met-before* (McGowen & Tall, 2010). A met-before is defined as “a mental structure that we have now as a result of experiences we have met-before,” (p. 171). McGowen and Tall use met-befores to construct mental models of students’ understanding of content by considering how students employ mental structures formed by previous experiences with mathematical content to learn new things. The notion of a met-before is not incompatible with the notion of relearning, but the two terms represent different types of entities. Met-befores are mental structures containing previously seen content, whereas re-learning is an experience that takes place when a student is learning about the same content at a different timepoint. However, met-befores may be a useful concept when examining how a relearning context restricts the kinds of learning outcomes that are possible for students given that they are capable of being both supportive and unsupportive according to the context in which they are encountered.

IMPLICATIONS FOR RELEARNING FIELDS

We have proposed a unified conceptualization of relearning in mathematics education along with the constructs of *motivation* and *intended learning outcomes* that have traditionally been used to define relearning within various sub-disciplines. In doing so, we hope to broaden opportunities within each sub-discipline in two ways. First, in the realm of cognitive psychology and teacher education, we encourage researchers to move beyond the learning outcomes that are intended or desirable within their particular context in order to explore the realm of *possible* learning outcomes that students may encounter. For instance, while the restructuring outcome addressed by Zazkis earlier is the intended learning outcome of a content course for future teachers, it will not always be achieved depending on student engagement with the material. In under-theorized sub-disciplines such as remedial mathematics, looking to the learning outcomes that are possible in other sub-disciplines may serve as a starting point by which to begin to examine student experiences. It may be the case that Zazkis' notion of reconstruction would fit the intended learning outcome for some types of remedial courses, whereas student-generated descriptors of remedial mathematics courses as "refreshers" (Cox & Dougherty, 2019) of their memory, may point to connections to cognitive psychology's treatment of relearning instead. It may also be the case that multiple learning outcomes could exist simultaneously for one individual such that he or she may be reconstructing their understanding of some mathematical topics while achieving different outcomes for others. Determining the range of outcomes that exist in a relearning experience and comparing it to the desired or range of desirable outcomes would be one of the first ways in which one could begin to determine which contextual elements are or are not supporting students in meeting course expectations.

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