

INTERNATIONAL PERSPECTIVES ON PROOF AND PROVING: RECENT RESULTS AND FUTURE DIRECTIONS

David A Reid¹, Yusuke Shinno², Taro Fujita³

¹University of Agder, Norway; ²Hiroshima University, Japan;

³University of Exeter, UK

This working group is a continuation of the working group on international perspectives on proof and proving at PME 43 (Reid et al., 2019a, 2019b). The group was not able to meet in 2020 and 2021 due to the pandemic. The aim is to foster research on proof and proving from an international perspective by bringing together research on proof and proving and international comparison. The long-term goal is to present the results of a number of comparative international researches on proof and proving in a PME Research Forum.

The past two decades have seen a strong increase in research into proof and proving in mathematics education. Much of this has been conducted in single national and cultural contexts, although there have been, and continue to be, a few comparisons that have compared proof in a few contexts. For example, since the studies referenced by Reid et al. (2019a), there has been a comparison of word use in curricula and standards in Norway, the USA and Germany (Reid, 2022), a proposed framework from a cultural perspective (Miyakawa & Shinno, 2021), a study of proof-related reasoning in upper secondary school textbooks in Sweden and Finland (Bergwall, 2021), and a study of Estonian and Finnish students' views about proof (Viholainen et al., 2018). This slowly growing research base on proof and proving from an international perspective is much needed as it remains unclear whether existing research results from single national and cultural contexts are transferable, or, indeed, if the assumptions on which the studies are based are valid elsewhere. Notwithstanding the small amount of existing comparative research on different aspects of proof and proving, comparatively little information exists about the role of proof and proving in educational contexts from an international perspective. Additional international comparisons involving a wider range of countries could shed light on the *teaching and learning of proof and proving* in areas such as curriculum (including textbooks and other teaching and learning materials); student learning and achievements; teaching (including teaching practices, teachers' knowledge, and teacher education or professional development of teachers); and assessment.

At PME 43 subgroups were formed. These were selected based on the interests of those present, and include some overlap. Three groups formed around issues specific to education levels, and three others around topics across education levels. The topics chosen were:

- Pre-Primary and Primary Argumentation and Proof

- Secondary Level Argumentation and Proof
- University Level Proof Teaching and Learning
- How are Argumentation and Proof conceptualised internationally?
- Proof in the Primary & Secondary school Curriculum
- Visualisation and Proving

Each group identified research questions and continued sharing information over the year following. As part of the PME 45 Working group sessions there will be brief reports from some of these subgroups. Some groups will not be present to report on their progress but the organisers will be able to summarise their activities. All groups are very much welcoming new participants.

STRUCTURE OF THE SESSIONS

Involvement of participants from different countries is essential to the functioning of the group. Over the two sessions the following activities are planned:

- Introduction to the working group, its aims, goal, and history.
- Reports from participants in the subgroups formed at PME 43, and the questions they have considered and steps they have taken.
- Proposals of new themes based on the interests of new participants in the working group.
- Discussion in subgroups, including possible theoretical and methodological approaches.
- Whole group discussion of ways to expand collaborations among researchers.

References

- Bergwall, A. (2021) Proof-related reasoning in upper secondary school: characteristics of Swedish and Finnish textbooks, *International Journal of Mathematical Education in Science and Technology*, 52(5), 731-751.
- Reid, D., Jones, K., & Even, R. (2019a). International perspectives on proof and proving. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proc. of the 43rd Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 181–182). PME.
- Reid, D., Jones, K., & Even, R. (2019b). Working Group 4 report: International perspectives on proof and proving, *PME Newsletter*, 2019 Issue 2, 7–8.
- Reid, D.A. (2022) ‘Reasoning’ in national curricula and standards. Paper presented at CERME-12 to TWG-1.
- Viholainen, A., Lepik, M., Hemmi, K., Asikainen, M., & Hirvonen, P. (2018). Research study about Estonian and Finnish mathematics students’ views about proof. In E. Norén, H. Palmér, H., & A. Cooke (Eds.), *Nordic Research in Mathematics Education, Papers of NORMA 17*, (pp. 119–127). SMDF.
- Miyakawa, T., & Shinno, Y. (2021). Characterizing proof and proving in the classroom from a cultural perspective. In Inprasitha, M., Changsri, N., & Boonsena, N. (Eds.). (2021). *Proc. 44th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 242–250). PME.