

MATHEMATICS TEACHERS' INTERACTIONS WITH RESOURCES THROUGH A LANGUAGE LENS

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INTRODUCING THE TOPIC AND FOCUS OF THE RESEARCH FORUM

Research on resources in/for mathematics teachers' classroom work and/or professional development is a vibrant domain that has been addressed through a number of diverse approaches and emphases (e.g., Adler, 2012, 2021; Barwell, 2018; Gueudet, Pepin, & Trouche, 2012; Planas, 2018; Remillard, 2019; Ruthven, 2019). In this domain, the scope of a culturally-driven concept of resource has been expanded and used or contrasted to explain possibilities and challenges of mathematics teaching and professional development. Despite the ambitious agendas, the domain has not yet been placed in relation to advances in mathematics education research with a language lens, that is, field research that takes the study of mathematics education processes as integral to the study of the language underlying these processes.

We (a diverse group of domain researchers) believe this domain of research would benefit from discussion on the extent to which and how a language lens differently traverses and shapes or could shape different domain approaches across cultures and theoretical traditions. While much of the emphasis in the Resource Approach to Mathematics Education (e.g., Trouche, Gueudet & Pepin, 2019), for example, has been on material resources such as textbooks and digital technologies, the use of language in the interaction with these resources has hardly been explored. In a similar vein, much of the emphasis in the Language as a Resource Approach (e.g. Barwell, 2018; Planas, 2018) has been on symbolic resources such as discourse practices and linguistic moves, with scarce attention to their interaction with teaching and/or developmental material resources. Still drawing on one more example, the Mathematics Discourse in Instruction Framework (MDI, e.g., Adler, 2021) has placed emphasis on specific resources like examples and explaining, and their connections within language in use. In all these approaches, the concept of resource is a central focus for research and thus more interaction between them could have been expected.

Complementarily to the research forum in construction in the form of a 2023 ZDM Special Issue with L. Trouche, J. Adler and J. Remillard as guest editors, this Research Forum aims at putting forth a language-based discussion within the research domain on mathematics teachers' interactions with resources in school teaching and/or in professional development. We seek to provide newer understandings of and synergies around: i) how language is or can be a resource for mathematics teaching and developmental practice, and ii) how it interacts or can interact with other resources

towards their realization for mathematics teaching and teacher developmental practice. We hope to give focus and direction to two questions, each of which is connected to the major goals of learning from and expanding the discussion amongst frameworks:

- RF-Q1. How do we (as a mathematics education research community) understand language as a resource in our studies with curriculum, mathematics teachers and teaching?
- RF-Q2. How do we understand teachers interacting with resources in crossing languages and contexts?

Rather than summarizing findings in the domain over the last years, we choose to challenge ourselves to think beyond the boundaries of our apparently disconnected frameworks by trying to discern what could be gained, refined, or added through the introduction of either a language lens and/or the study of effects of the interaction of language with other resources at play in our settings. In doing so, we hope promising directions for future research will emerge, as we build more interconnected frameworks. Importantly, the risks of building the research domain in parallel to advances of mathematics education research with a language lens will be reduced.

SESSION 1 – FOCUS ON RF-Q1

How do we (as a mathematics education research community) understand language as a resource in our studies with curriculum, mathematics teachers and teaching?

In Session 1, we will discuss some of the theoretical-analytical approaches to language as a resource in research work on curriculum and teaching, and on the ways in which mathematics teachers meet language in their interactions with other curricular and developmental resources. While the focus on language as a resource in mathematics education research has mostly been developed with respect to students' home languages and learning, this field-based focus was importantly prompted by the analyses of mathematics teaching in the seminal work of Adler (2001). A number of complementary or alternative theoretical-analytical frameworks have progressively emerged to capture different aspects of the complexity involved in language use, and to differently account for how language intersects other resources such as knowledge, but also textbooks, digital tools, lesson plans, classroom tasks or developmental sessions. Following the introduction to the RF and a brief explanation of how its two sessions relate to each other and to mathematics education literature on resources and language, this session will focus on the presentation of three particular approaches that allow us to see a dynamic scene of mathematics teacher education research in which a language lens is fundamental.

The first contribution (N. Planas, J. Adler, & L. Mwadzaangati) will present theoretical-analytical tools originated in sociocultural (MDI) and sociolinguistic (SFL - Systemic Functional Linguistics) frames. These are tools oriented to use language with mathematics teachers for the design and promotion of mathematical discourse

practices in their content teaching. Situated insights within country contexts of Malawi and Catalonia-Spain will provide the basis for exploring challenges around the realization of language as a resource in the thinking, preparation and implementation of content mathematics teaching. The second contribution (H. Sabra & J. Alshwaikh) will present theoretical-analytical tools from the MDI and the Documentational Approach to Didactics (DAD, Trouche, Gueudet, & Pepin, 2020), approaches as applied in their study of the use of the mother tongue (Arabic) of mathematics teachers in Palestine and its realization as a resource to support their classroom teaching. Issues on how the teachers interact with their language in the use and interpretation of other teaching resources available (e.g., textbooks), as well as the eventual modification of the languages (e.g., naming of mathematical objects) in them, will be addressed. Adopting a language lens through a social semiotic framework, the third contribution (H. Van Steenbrugge & J. Remillard) will present theoretical-analytical tools designed to explore multimodal modes of communication put into use in the design of mathematics curriculum resources. Printed lesson guides and digital platforms for elementary mathematics education in Sweden, USA and Flanders-Belgium will illustrate the discussion of how images in/and written texts, in both printed and digital resources, communicate representational meanings about how the mathematical content is taught and learned, and how the relations between teacher and students evolve around encounters with curriculum resources.

A commentary and provocation by R. Barwell reflects on the approaches presented, pointing to possible directions of convergence and divergence and raises critical questions for engagement in this session in the forum.

SOCIOCULTURAL FRAMES FOR A FOCUS ON THE RESOURCE OF MATHEMATICS TEACHING TALK

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Introduction

In our studies with secondary school mathematics teachers in Spain, South Africa and Malawi, we share sociocultural frames in the approach to the mediational role of language and to its potential role as a resource in/for mathematics teaching and learning. At the intersection of Vygotskian sociocultural theory and Hallidayan functional linguistics, and with different emphases in our respective research contexts, we view language as an integral aspect of what makes teaching and learning possible, whose use in interaction with other resources (e.g., time, knowledge, curriculum texts) can be investigated and supported in developmental work with teachers. Related to our interest in the resource of language, we share the interest in the more particular resource of mathematics teaching talk as a means to enable and support learners' participation in the mathematical discourse.

We consider the focus on mathematics teaching talk very timely in the current moment of mathematics teacher education research and practice that is language related. Despite the value given to this talk, it is often subordinated to the mathematical discourse practices of reasoning and argumentation, and learners' productions of these, and remains under-researched as an object on its own. In our collaboration, we argue for mathematics teaching talk as an equally prominent focus of research about language-responsive mathematics classrooms, that does not compete with the focus on mathematical discourse practices and learner participation in mathematical discourse. It was a focus on mathematical teaching talk that led to substantial progress in the early years of mathematics education research on language (e.g., Pimm, 1987; Lampert, 1988). That said, the more recent developments of this research stimulated by the influential construct of the mathematical discourse practices (e.g., Moschkovich, 2007) requires renewing the debates around what kind of resource is mathematics teaching talk, or what is it for.

What mathematics teaching talk is for?

With respect to the question of what mathematics teaching talk is for, we find inspiration in the example of lesson work in Lampert (1998), with the blocks of a tangram and the task, "Can two of them be joined to make a hexagon?" (p. 1). In the middle of disagreement about whether the angles in one of the figures proposed should be measured with respect to the "inside" or the "outside" of the figure (p. 3), the teacher explicitly talked about these angle types and related this to whether it was or was not a hexagon. Mathematical discourse practices throughout the lesson with reasoning challenges such as "Does every figure that has six sides also have six angles?" (p. 4), developed with moment-to-moment teaching talk in which learner expressions such as "inside and outside angles", "equal sides" or "two of the same shape", were discussed. Importantly, the teacher inserted "relationship" in her talk ("So the fact that a hexagon has six sides that you started out saying there, and the relationship between these shapes...", p. 2), and by doing so she offered and connected vocabulary and reasoning. Mathematics teaching talk is here a resource that draws on word use and reasoning, and the relationship between these, capturing the learning challenges that evolve out of the participation in the task at play.

In our research strategy around a focus on a notion of mathematics teaching talk that aims at enabling and supporting learner participation in mathematical discourse, we build on two theoretical-analytical tools at the intersection of Systemic Functional Linguistics (SFL) (Halliday, 1978) and the Vygotskian-informed Mathematics Discourse in Instruction (MDI) (Adler, 2021). Both SFL and MDI refer to the meaning potential of linguistic interaction and word use in talk. Specifically, SFL argues the meaning potential of lexicon (e.g., "relationship" in Lampert's example) and grammar (e.g., "two figures of the same shape") in any language, and how it can be realised within concrete registers (e.g., school geometry) in communication. MDI considers the lexicogrammar level when examining word use in mathematics teaching and the role

played by naming (how mathematical objects, processes and procedures are referred to e.g. a figure with six sides and six angles is a hexagon) and explaining (how these are reasoned about, or given legitimacy as to what counts as mathematical e.g. that ‘joining’ shapes to make a hexagon is ‘relating’ them) within mathematical discourse. We thus zoom in from language to mathematics teaching talk, and then to naming and explaining as resources in/for mathematics teaching.

Two sites of professional development practice

We need to state that whether in relation to SFL or MDI, our attention to teaching talk, as it focuses on word use, and brings the teacher into focus, is frequently interpreted as concerned only with vocabulary or technical language and/or promoting “teaching as telling” through what comes to count as an explanation. Lampert’s example hopefully counters these interpretations. At the same time, we acknowledge that the study of naming, explaining and more generally word use is not new in the community of mathematics education research on language, indeed this goes back to Pimm (op. cit.) over 30 years ago now. However, we contend that more remains to be done in the theorization of word use that can inform work with teachers on language-responsive mathematics teaching (Prediger, 2019).

From the perspective of the interplay between theoretical and practical work (and of the scope of application of the theory in professional development practice), our conceptualizations of the tools or resources of lexicon and grammar, and naming and explaining continue to evolve, and remain challenged by numerous tensions. Some recent insights come from workshops conducted with secondary school mathematics teachers in Malawi and Spain, with a focus on word use in the teaching of angles.

In Spain, the tools of naming and explaining, in mathematics teaching talk, are being approached with respect to mathematical contents of the secondary school curriculum and content learning challenges faced by many learners as reported in the field literature. Naming and explaining are then operationally linked together and defined as: words and sentences with the potential to communicate meanings and induce reasoning or discourse practices to support the overcoming of learning challenges whose experience can easily evolve out of the learners’ participation in a concrete mathematical task. Given the task of replacing the machine that rotates the pieces in an image, and the widely documented challenge of the static thinking of angles, naming and explaining the centre of the rotation angle, during the task resolution, are important resources in mathematics teaching that can, in turn, enable and support learners in moving out of methods of ‘guessing’ the place for the machine.

In Malawi, in a lower secondary school lesson focusing on the meaning of an ‘exterior’ angle of a triangle, and its relationship to interior opposite angles, the teacher asked learners what they thought an exterior angle of a triangle was. Using their knowledge of interior angle as angle inside a triangle, learners referred to exterior angle as angle outside the triangle. This latter informal naming was reflected in some learners pointing to the reflex angle outside the triangle, others drawing a line intersecting a vertex of a

triangle and pointing to angle between the intersecting line and side of a triangle as exterior angle, and others extending a side of the triangle and marking the exterior angle formed without describing it. This unfolding was in the enactment of a first lesson plan in a lesson study, and as participating teachers reflected together on the lesson, they discussed the interaction of the informal talk with learners' thinking. They replanned the lesson so as to enable and support the naming and explaining of the exterior angle by linking learners' informal association of 'outside' with its specific mathematical meaning as the adjacent angle formed by extending a side of the triangle.

The numerous tensions at the research and theoretical levels of a focus on word use in mathematics teaching talk appear recreated at the practical level in the work with the teachers in Spain and Malawi. These are versions of well-known and ever-present tensions – but even more reason that they are included in teacher education practice on language-responsive mathematics teaching, and related research.

COMBINING MATHEMATICAL ARABIC AND THE TEXTBOOK FOR TEACHING THE SIGN OF QUADRATIC FUNCTIONS

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Introduction

In our study, we present the use of the mother tongue (Arabic) of mathematics teachers in Palestine and its realization as a resource to support their teaching. We focus on how Arabic language and the textbook interact for teaching “functions” in Grade 10. To better understand the current challenges of teaching mathematics in Arabic, a historical overview seems essential.

The movement to translate Greek mathematics into Arabic began towards the end of the 8th century. It was accompanied by the foundation of institutions to organize research activities and to produce new scientific knowledge; and led to developments in the Arabic language itself (Rashed, 2019). In the 19th century, another movement of translation happened to adopt the new science from Europe to different regions of the Arab world (Crozet, 1999). At this period, scientists in Egypt adapted the “new knowledge” by translating texts into Arabic. Educators sought to adapt new knowledge to develop course materials for higher schools. The products of these processes have mainly served as resources for teaching ‘translated mathematics.’

In the 1950s and 1960s, there were attempts to look at mathematics curricula to promote “uniformity in Arab education systems” (Jurdak & Jacobsen, 1981). Common textbooks were designed and translated into Arabic, each country adapted it and modified it separately. In this period, education in Palestine was influenced by the different rulers, mainly Egyptian and Jordanian and the ongoing Israeli occupation. The Palestinian Ministry of Education became responsible for education in the West Bank and Gaza Strip in 1994. A unified curriculum of the two parts of the territory saw

the light only in 2000. The textbooks are designed and delivered by the Ministry of Education; they are the main resource for mathematics teachers.

Thus, Arab teachers, including Palestinians, were provided textbooks containing translated mathematics. The development of a suitable language for teaching was left to them. The main question we address here is: how Palestinian teachers combine the use of the textbook and the Arabic language in their teaching?

Theoretical framework and methodology

To designate what the teachers have to develop as language, we rely on Halliday's (1978) concept of *mathematics register*. Our interest is in the way that teachers develop their own 'new mathematics register,' which we call 'mathematical Arabic.' Hence, following Halliday (1978), we define *mathematical Arabic* as the set of meanings that involve "the introduction of new thing-names, the ways of referring to new objects or new processes, properties, functions and relations." Therefore, we want to understand the way mathematics teachers develop their own mathematical Arabic by interacting with what is available to them in terms of resources, especially the textbook. Mathematical Arabic resources the expression of textbook content and facilitates student's learning of it.

Adopting Mathematical Discourse in Instruction - MDI (Adler & Ronda, 2015), we focus on explanatory communication as a tool of exploration to better understand how mathematical Arabic is used as a resource. In addition, MDI looks at the types of language used within a lesson, whether it is colloquial language or mathematical language. Even for the latter, MDI defines three types of mathematical language; school, semi-formal, and formal mathematical language. Similarly, and in order to evaluate how teachers justify mathematically, MDI suggests three categories: non-, partial and full mathematical justifications.

In addition, we draw on the Documentational Approach to Didactics - DAD (Trouche, et al., 2012) to characterize the way the textbook influences teaching, and the way in which teachers' dispositions guide their use of the textbook. DAD also helps us to study the combination that teachers create between the use of textbooks and deployment of their mathematical Arabic. For our study, we consider the teachers' schemes of use of the textbook and mathematical Arabic, and the way the teachers justify their choices when interacting with the resources.

We hypothesize that the DAD and MDI are complementary and come together to allow us to understand how the combination of mathematical Arabic and the textbook is constructed. While MDI enables us to understand what mathematical Arabic is used and the degree of formality of mathematics presented, DAD explores the scheme of use and the components of these schemes.

Our field of study is based on three Palestinian teachers (T1, T2, and T3); and a specific teaching aim of "how to determine the sign of quadratic function" for Grade 10. The data collected is related to the Grade 10 textbook, audio-records of classroom sessions,

and interviews with teachers. We listened to all recorded sessions and selected one common episode among the three teachers, and we transcribed those episodes. We then interviewed each teacher to reflect on their mathematical Arabic and the way they used the textbook. The interviews contain two parts. The first was the teacher's profile and the characteristics of her own mathematical Arabic. The second part was related to the teaching aim; we asked questions that allowed us to refine our analysis of the scheme of use of the textbook and the mathematical Arabic to reach that aim.

Discussion and conclusion

It appears that the responsibility for developing a language for the dissemination of mathematics is beyond the responsibility of teachers. We noticed that teachers use formal mathematical terms mentioned in the textbook when they give formal justification through definitions, rules and laws. However, when teachers presented justifications for students during the lesson to explore the sign of the quadratic function, we observed more use of their own mathematical language. Some of those justifications were partially mathematical such as “it's a law, and I memorize it” (قانون) (أنا حافظاه) referring to the formula of the discriminant. Despite the fact that the three teachers mention three steps in teaching the sign of the quadratic function, we observed that each teacher has a different way of doing so.

We observed that Palestinian teachers develop their own mathematical Arabic, either by referring to their experience as students, or –if applicable– their teaching training. The teaching of mathematics in English at university seems to constitute a break in the process of maturing their own mathematical Arabic. The teaching experience is probably the main ground for shaping one's mathematical Arabic. In practice, teachers shape their mathematical Arabic according to the subject taught, the students' needs (e.g., possible difficulties), and the textbook.

Furthermore, we identified three different forms of combination of use of the textbook and the mathematical Arabic for each teacher: complementarity in the case of T1, tension in the case of T2, and pattern of equivalence in terms of “when the textbook is lacking, mathematical Arabic bridges the gap” in the case of T3. The degree of agency that teachers have toward the textbook seems to be correlated with the development of their own mathematical Arabic.

This study opens up avenues for investigating different issues in teaching mathematics in Arabic. For example, an area of research is curricular studies; investigating the sources of the choices made in the Palestinian curriculum seems to be crucial for defining an adapted language. Another issue is the need for further investigation for defining foundations to help teachers build their own mathematical Arabic.

THE MULTIMODALITY OF CURRICULUM RESOURCES AND THE COMMUNICATION OF SOCIAL RELATIONS

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Curriculum resources as multimodal

We understand language broadly and as a multimodal form of communication. Designers of contemporary curriculum resources use “an ensemble of semiotic features” or modes “that shape what learning is and how it may take place” (Bezemer & Kress, 2008, p. 168). Teachers are generally a primary audience for these features. Increasingly, curriculum resources are offered in digital formats, in addition to or in lieu of print resources. Digitalization extends the possibilities for how authors communicate with teachers and who else they communicate with (Pepin et al., 2017). Using a multimodal lens, we examine how designers of different types of mathematics curriculum resources from Sweden, USA, and Flanders, communicate with teachers, focusing on implicit messages about social relations between the teacher, students, artefacts, mathematics, and eventual other social actors.

Analyzing messages about social relations

Our understanding of the implicit messages about social relations in curriculum resources reflects nearly ten years of work analyzing how print and digital resources communicate with teachers. In the process of structuring mathematics learning opportunities, curriculum authors communicate implicit messages about the social relations at stake in teaching and learning mathematics, specifically with respect to positioning, authority, and agency. The move from print to digital resources expands the typical social relations at stake, requiring us to expand our analytical framework.

Social relations have received relatively limited attention in curriculum resource analysis, which tends to focus on mathematics content and explicit pedagogy (Fan et al., 2013). Our analysis builds on the work of Herbel-Eisenman (2007), which examined linguistic features in textbooks using discourse analysis to uncover messages about the relative positioning of the students, teacher, textbook, and mathematics. Many of these messages are implicit and subtle. By using a multimodal lens, we are able to uncover how these subtle messages are communicated, not just through written language, but also through modes such as image, layout, and connectivity.

Framework for analyzing print lesson guides

The multimodal framework we used to analyze print lesson guides focused on three modes: writing, images, and layout. Bezemer and Kress (2008) explain that modes of communication have different *modal resources*, which specify the possibilities for variation within each mode. Written communication, for example, has graphical resources, like font size, lexical resources, like content, and grammatical resources, that shape the style of communication. Images can vary in size, color, shape, and

content. Layout involves the arrangement of these resources on the page. In this way, modes and their modal resources are not discrete, but intersect and work together in multimodal artefacts. Images represent a prominent mode in lesson guides, yet the size and placement of images can also be seen as modal resources in the mode of layout. The modal resources we analyzed for each mode are summarized in Table 1.

Mode	Modal Resources
Written communication	Quantity of writing Focus of writing How written messages were communicated
Layout	Placement of components on page Visual markers and text to signal guide navigation of page
Image	Content of images Size of images Location of images

Table 1: Modes and modal resources focal in our analysis of print lesson guides

We found that layout, images, and written communication work in combination to structure teachers’ interactions with the guides and communicate messages about the process and source of mathematics learning. These modes a) structured teachers’ reading path through the guide, b) signalled the locus of instruction, and c) configured the relationship between teacher, students, guide, and other instructional materials.

The mode of connectivity in digital resources

Digitalization allows curriculum resources to connect to other resources, people, and to make connections within the resource. Drawing on Akkerman and Bakker’s (2011) depiction of boundary objects and boundary crossing, we conceptualized the mode of connectivity as having three modal resources, shown in Table 2. Applying the framework to two digital curriculum resources’ (DCRs) adaptability and networking features, we surfaced implicit messages about a) the relations at stake in a typical classroom, b) those that expand beyond this typical web, to include outside actors, and c) the agency implied in these relations.

Mode	Modal Resources
Connectivity	People and objects being connected (e.g., teacher, students, parents, material and semiotic resources, content, units) Domains at stake (school, everyday, virtual, policy) Visibility of connections

Table 2: Modal resources of the mode of connectivity in digital curriculum resources

We found that the two DCRs differed in visibility of adaptability, which we related to messages about agency between the teacher, students, and the DCR. In one DCR, for instance, learning trajectories were made explicitly visible to both the students and the teacher. This visibility positions students and teachers as having control over learning and teaching, in relation to the DCR. We also noted how the visibility of the DCRs’

networking between teacher, colleagues, principal, and parents communicated messages about teacher agency.

Conclusion

Our findings illustrate ways that, in their design, curriculum resources communicate subtle messages about social relations that can either reproduce or challenge typical lines of authority and visions of mathematics teaching and learning. We assert that multimodal frameworks and comparative analyses are especially adept at uncovering these messages. As such, these analyses might uncover some of the covert elements of culture in these resources.

SESSION 1. COMMENTARY AND PROVOCATION

UNDERSTANDING LANGUAGE AS A RESOURCE IN MATHEMATICS EDUCATION WITHIN A DIALECTIC-DIALOGIC TENSION

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In this commentary, I frame my remarks with a broad characterisation of how the role of language in mathematics classrooms is theorised in mathematics education. There is, I propose, a long-standing tension in this area of research between a modernist, dialectic orientation, and an alternative, dialogic orientation (Barwell, 2016). In the former, associated with Vygotsky, language, including classroom talk and written or printed texts, is a cultural tool through which learners are initiated into socially established scientific concepts and forms of reasoning. In this perspective, differences in interpretation that arise in mathematics classrooms are to be resolved or overcome so that successful learners internalise established forms of mathematics knowledge, including standard or formal mathematical discourse. In the dialogic orientation, associated with Bakhtin, language is a kind of living social relation in which learners and teachers participate and which mutually defines them. The pre-given nature of language means that meaning precedes participants' intentions, so that they must grapple with the entire history of language to make mathematical meaning. For Todorov (1984), understanding is ultimately, not only an interpersonal process, but also "a relation between two cultures" (p. 109). From a dialogic perspective, then, differences in interpretation are not seen as problems to be resolved, but as fundamental to mathematical meaning-making (Wegerif, 2008). Rather than a focus on a goal or endpoint, in which the teacher passes on approved knowledge to learners, a dialogic perspective acknowledges the mutually constitutive, relational nature of mathematical meaning-making.

The (dialogic) tension between these two orientations is not so much apparent in the theorists cited or positions explicitly adopted by researchers. A quick look at the research literature would reveal a much greater prevalence of references to Vygotsky or authors working in the sociocultural tradition. Rather, this tension links the most

common goal of mathematics education to impart, transfer, or transmit an established body of mathematics knowledge on the one hand, and the progressive desire to give space to students' voices and involve them in charting their way in their mathematics learning on the other. This tension is often managed by incorporating dialogic ideas about voice, for example, into a broader dialectic, sociocultural framework (in an attempt, perhaps, to resolve or overcome the differences). In reality, the tension is irresolvable.

Language as a resource in mathematics education emerged as a response to deficit perspectives, such as the idea that bilingualism causes confusion or that language diversity creates obstacles to learning mathematics. In the literature, this metaphor is largely aligned with a sociocultural framework. The word resource evokes a substance or material to be used in order to complete a task. In mathematics education research, the idea of language as a resource generally means that learners use language to acquire the desired mathematical knowledge. In many studies, learners' multiple languages are indicated as the resource(s); in others, particular language practices are indicated, such as code-switching or use of narrative; in yet others, it is the various mathematical meanings or interpretations at play that are seen as the resource (see Barwell, 2018). The first three contributions to this research forum contribute to a recent development in work on language as a resource in mathematics classrooms by adopting ideas from Halliday's systemic functional linguistics or related work (although Halliday's work has informed mathematics education in other ways since the 1970s) (Planas, 2018). A key idea in this approach is that of 'meaning potential'. Language is understood as a semiotic system from which speakers choose among multiple alternatives to realise specific meanings.

The introduction of Halliday's notion of meaning potential is, I suggest, an extension of dialectic sociocultural perspectives on language as a resource. In effect, the material idea of resource has been interpreted as a form of potential (like oil reserves, perhaps). The task of learners is to learn how to exploit this potential (resource) to make prescribed mathematical meanings. In Planas, Adler and Mwadzaangati, the focus is on the role of the teacher in this process. The teacher's role is to use language as a 'mediational means' to guide learners to the correct mathematical meanings. In the illustrative example from a classroom in Malawi, we see the heteroglossia of students' everyday language and multiple meanings, and the teacher's efforts to mediate between their everyday language and meanings and the required mathematical meanings. In their contribution, Sabra and Alshwaikh provide a fascinating account of how mathematics textbooks play a mediating role between a formal Arabic mathematics register and the Palestinian teachers' (and presumably learners') individual versions of that register. Finally, Van Steenbrugge and Remillard examine how choices among "modal resources" in digital curriculum resources produce social relations in relation to mathematical meaning making. These texts are seen as mediating teachers' and learners' mathematical activity in terms of how they navigate the text and the authority relations such activity entails.

The empirical examples given in the three contributions illustrate well the dialectic-dialogic tension. In particular, in each case, there is the strong goal of the mathematics that students are required to learn, leading teachers or textbook authors to search for ways to guide them. At the same time, we see the inevitable heteroglossia of learners' or teachers' diverse ways of participating in language, including mathematical language, and we see hints of how their participation in language mutually defines them. Van Steenbrugge and Remillard refer to Bezemer and Kress (2008) who, in other work have noted a change in the organisation of textbooks since the 1930s, and a shift from a standpoint of 'vertical' authoritative relations, to one of 'horizontal' participatory relations (Bezemer & Kress, 2010). Nevertheless, textbooks are, in some sense always authoritative and so we see again the dialectic-dialogic tension at work. If the empirical examples illustrate the dialectic-dialogic tension, the dialectic, sociocultural theorisation of language as a resource is based on or creates various orderings, often in the form of binaries: formal mathematical discourse—everyday language; teacher—learner; official curriculum—local variation; official language—heteroglossia; semiotic system—participants' utterances. From a dialogic perspective, such orderings serve to define the relation between participants as a form of alterity. The teacher exists in relation to the learner, mathematical discourse exists in relation to everyday language, not as abstract entities, but as entirely relational. One cannot exist without the other. The way in which we theorise these relations is, therefore, crucial since these theorisations serve to structure and organise them.

The etymological roots of the word 'potential' are derived from power (as in 'potent') and resources are often implicated in power relations: humans fight to control resources or to have access to them. In language policy research, Ricento (2005) has argued that the discourse of language as resource often constructs marginalised languages as subservient to dominant ones. We can ask 'whose resource? Who controls this resource? Who consumes it? For what purpose? What kinds of relations are produced?' In mathematics education, research on language as a resource is mostly conducted in socially stratified contexts: children in Malawi or South Africa from African language backgrounds learning mathematics in English; Latinx students in the United States; or students from immigrant backgrounds in several parts of the world. Does the idea of language as a resource mean that learners' diverse language repertoires are to be harnessed just until they can do mathematics in the official language of instruction? Does the idea of language as a resource in relation to textbooks or curriculum materials mean that learners' own diverse ways of talking about mathematics should be harnessed just until they have mastered the desired form of mathematical discourse? Does this theoretical approach not risk reproducing in a more subtle way the social stratification that the more progressive goals of mathematics teaching might hope to dismantle? What alternative theorisations might we consider? There is no tidy answer to these questions but the tensions I have discussed are necessary if we are to think in new ways.

SESSION 2 – FOCUS ON RF-Q2

How do we understand teachers interacting with resources in crossing languages and contexts?

In Session 2, we will focus on the *words* used by teachers as well by researchers when describing/analyzing these interactions with resources, and the correspondence between these words when moving from one language to another one. It constitutes a zoom in perspective towards words, their structure in a language, and their association across languages. Considering words as “saturated with sense” (Vygotsky), we hope that this perspective will allow us to deepen our understanding of mathematics teachers’ interactions with resources.

The first contribution (M. Artigue, C. Knipping, J. Novotná, & B. Specht) comes from the International Classroom Lexicon Project which set out to document the terms and the professional vocabulary that teachers use for describing the phenomena of middle school mathematics classrooms around the world. The study of this vocabulary leads to evidence of different *naming systems* on which teachers’ discourse is based in different cultures. These naming systems constitute then windows into teachers’ resource systems, revealing part of their content and structure. The second contribution (M. Shao, I. Kayali, I. Osta, G. Gueudet, B. Pepin, & L. Trouche), using again the DAD, analyses in Arabic, Chinese and French the same episode of an English mathematics teacher interacting with resources. It leads the authors to think about the instantiation of the same concepts in three languages, and then to rethink the concepts themselves. The third contribution (C. Wang, Y. Shinno, B. Xu, & T. Miyakawa) questions also the process of translating a theoretical framework dedicated to analyze teachers’ interactions with resources, here from an anthropological point of view. It identifies different factors (linguistic, cultural, and social) that cause the difficulties or confusions encountered during the translation work: e.g the different status of teachers’ collective work in the West and the East. The issues of translations are explored from two levels of the cross-cultural perspective: between the West and the East, and between China and Japan. The provocation, by L. Radford, who with others, has analyzed resource-use crossing theoretical views, and how these refract cultural aspects of teaching and learning, will offer commentary and through this open up discussion.

NAMING SYSTEMS AS A WINDOW INTO TEACHERS’ RESOURCE SYSTEMS

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Documenting and understanding naming systems

The International Classroom Lexicon Project set out to document the terms and the professional vocabulary that teachers use for describing the phenomena of middle school mathematics classrooms around the world (Mesiti et al., 2021, Mesiti et al.,

2022). When identifying teachers' naming systems for classroom phenomena in the Lexicon Project, important differences appeared (*ibid.*). The term 'milieu' from the French lexicon, for example (Artigue et al., 2021, p. 195), has no English translation and no obvious equivalent in other lexicons. French teachers use this professional term to describe: "The element of a learning situation with which the students interact, and provide them objective feedback." (Artigue et al. 2021, p. 208). This French term has its origin in mathematics education research and is adopted by teachers to describe classroom situations. The generated lexicons, like this French one, indicate forms of conventionalized classroom practice in ten different countries (Mesiti & Clarke, 2018; Artigue et al., 2017). But it turned out that the naming systems in the lexicons per se were too limited to understand how they orient teachers' visions, analyses of mathematics classrooms, and their complex interactions with resources in the class.

For capturing these multifaceted visions, we created narratives, methodological artefacts, based on the lexicons and classroom lessons of the Czech Republic, France and Germany. These narratives and their comparisons turned out to be promising for understanding how the lexicons, seen as cultural objects, shaped visions of the classroom and of associated phenomena (Artigue et al., 2017). Approaching lexicons in this way, understanding them as cultural artefacts and using them as windows into teachers' resource systems (Trouche, Gueudet, & Pepin, 2020) is an enlargement of our lexicon based comparative perspectives so far.

Teachers' resource systems

Nevertheless, also this approach does not excavate the full range of teachers' resources related to classroom situations. The videos used as spurs in the Lexicon Project show that the teachers involved use a diversity of material resources and tasks in their lesson. All of them make extensive use of the board, and often more or less sophisticated projection devices. They use calculators, mathematics software and physical models, together with diverse tasks from textbooks, school platforms and created by themselves. However, the lexicons mirror this diversity in a very limited way. Some reasons may explain this situation: the fact that the lexicons document key pedagogical and didactical terms used to denote classroom phenomena, not classroom tools; the choice not to include names denoting mathematical software or tools such as rulers and protractors in the lexicons.

The focus in our comparative analyses is on narratives about classroom situations and the lexicon terms used in these. This approach by its own is already promising and reveals the perspectives concealed in the lexicon terms on classrooms, teaching and learning. So, language as a resource for teachers is discussed in this contribution primarily in respect to teachers' conceptualizing of classroom practice, instruction and learning, across different cultural contexts and linguistic traditions. Talking about classroom practice and describing teaching and learning, is a common resource that mathematics teachers and researchers use to get a better and shared understanding of classroom instruction and learning. Documenting and comparing naming systems that

teachers use, reveals how the different cultural naming systems capture disparate views on classroom practice, teaching and learning (see Mesiti et al., 2022).

Comparing lexicons and narratives - First results

Despite differences observed, comparing lexicons and narratives also revealed undeniable common linguistic resources for describing different types of tasks proposed to students as learning progresses, between different pedagogical methods, to describe the pedagogical and didactic management of the class by the teacher and to evaluate learning. However, there are also striking differences. The French lexicon and narratives differ from the other two in its mathematical and didactic focus. The Czech lexicon, which is clearly more pedagogical and its vocabulary is much less technical, is more descriptive and closer to ordinary language. The German lexicon and narratives occupy an intermediate position and exploit the possibilities by the German language to create compound words that are richer in meaning than their constituents, and may combine the concrete and the abstract, which results in a technical language that is both concise and accessible. The French narratives describe the mathematical-didactic management of the classroom by the teacher, going into great detail about the mathematical activity. The mathematical contents at stake are also well present in the German narratives, combined with a marked attention to the structure of the lessons, the teaching methods and the way in which teachers exploit learning opportunities. The Czech lexicon and narratives draw our attention to several possible forms of explanations. They also demonstrate that the pedagogical interactions described are closely bonded to the mathematical content at stake.

Even if limited to three lexicons and nine narratives produced by the same teams that produced the lexicons, our research tends to show that the cultural comparative approach of the lexicons and narratives can help to understand language as a resource for teachers describing mathematics classrooms teaching and learning, through the regularities and differences it reveals. However, this approach also has some limits. In approaching teachers' linguistic resources, the lexicons were subjected to a rigorous selection process, privileging terms reasonably shared by teachers. Finally, the nine videos that served as stimuli may also have more or less limited the repertoire of terms considered by the different teams.

DEEPENING THE CONCEPTUALIZATION OF TEACHERS INTERACTIONS WITH RESOURCES BY TRANSLATING A CASE ANALYSIS IN DIFFERENT LANGUAGES

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Introduction

The Documentational Approach to Didactics (DAD) is a theoretical framework that investigates mathematics teachers' professional development through the lens of their interactions with resources (Trouche, Gueudet, & Pepin, 2020). The theory was originally conceptualized in French and developed mainly in English. How can we properly translate DAD into other languages? How could the translation processes help to deepen the approach itself?

Methods

In this contribution, we try to respond to these questions by translating DAD in 'situ' (meaning across a case analysis). Our contribution is based on the DAD-Multilingual project (Trouche, 2020; Shao et al., submitted) and we focus on two languages involved in it: Chinese, and Arabic. They are far from English and both have ancient and rich cultural and curricular traditions related to mathematics teaching resources.

In terms of the DAD concepts to be translated, we focus on two dualities: *resource – document* and *instrumentation – instrumentalisation*. A *resource* is defined as everything that can 're-source' (Adler, 2000) the teacher's activity; a *document* is a hybrid entity composed of a set of resources and a scheme of usage of these resources for facing a given situation. *Instrumentalisation* is the process in which a teacher adapts the resources to his/her didactical goals; *instrumentation* is the process in which the resources, with their affordances and constraints, influence the teacher's activities.

In terms of the 'situ', we select one case of an experienced mathematics teacher (named George). The case consists of three lessons he taught to grade 13 students (aged 17-18 years) in England on the topic of 'volume of revolution' (Kayali, 2020). George expressed confidence and willingness to use a 3D visual software – Autograph – and a variety of other resources in the lessons. We identify, in this case analysis, the instantiations of the DAD concepts, and translate both the concepts and their instantiations from English into the two target languages. We discuss issues arising from this process, taking into account whether the instantiations would also emerge in a similar 'situ' in other educational contexts corresponding to the target languages.

Reflection on the translation process

Due to space limitations, we will not elaborate on the case, but directly present the instantiations of DAD concepts and discuss the related translation issues.

Duality resource-document

The *resources* used by George in the case include: textbooks, past-examination questions and grading standards, formulae cards, students' discourses in class, George's mathematics knowledge about volume of revolution and so on. All the resources, together with a global *scheme of use* attached to them, are considered as a document. The scheme corresponds to a class of situations involving two main goals: introduce the volume of revolution and prepare students for the exam.

The Arabic translation for the global notion of *resources* – مَوَارِد (mawaarid) – holds a historic connection with the vital need for water in the desert; the singular form of the term – مَوْرِد (mawrid) – originally means water spring (source of water) and is now extended to mean ‘the place to go for getting informed or inspired’. With this term, the idea of ‘re-source’ in the concept of *resource* is more evident. For the notion of *scheme*, the two potential Arabic terms reveal two different interpretations of this concept: مَخَطَّط (moukhattat) (a plan with a static and linear structure) versus صِيغَة (sighah) (an intertwining and flexible structure open to redesign and modifications). Obviously, the latter term is closer to the notion of scheme used in DAD.

In Chinese, *resource(s)* is widely translated as 资源 (zī yuán), and here we mainly discuss the translation of some concrete resource – *textbook* – 课本 (kè běn). The 课 and 本 respectively indicate *lesson* and *book*, suggesting that mathematics teachers in China need to closely comply with the textbook and preview all its content carefully to teach a similar lesson, but it is not the case of George, who only noticed an illustrative figure in the textbook towards the end of the first lesson.

Duality instrumentation-instrumentalisation

We also notice many instantiations of *instrumentation* – *instrumentalisation* in the case. George instrumentalised Autograph by creating a solid of revolution of graph $y=x(3-x)$ in it (Fig. 1). He was *instrumented* by the textbook which provided an illustrative image (Fig. 2) and stated that the students must be able to use definite integration to find the area under a curve. George *instrumentalised* the textbook by: deciding to use its image to explain the formula of volume of revolution, selecting the textbook questions for student practice and connecting them with the exam standards.

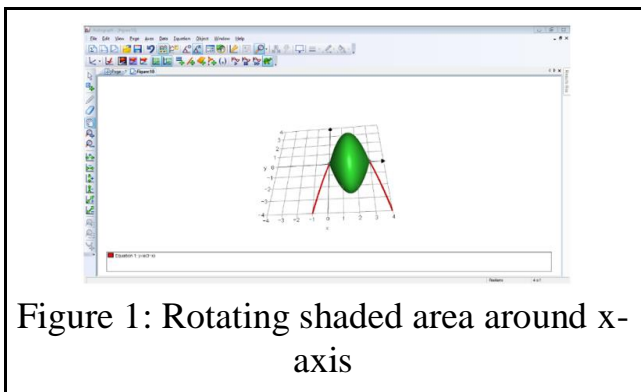


Figure 1: Rotating shaded area around x-axis

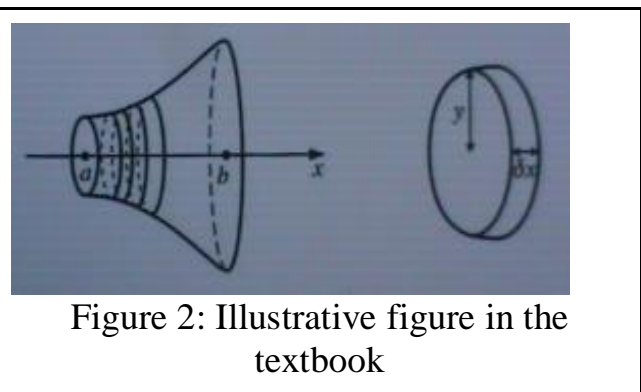


Figure 2: Illustrative figure in the textbook

In Arabic, *instrumentation* and *instrumentalisation* correspond to two nouns: إمداد (imdaad), which means *supply*, and تَسْخِير (taskhiir), which means adapt according to one’s needs and practices. The most significant example in George’s case showing the differences between the two terms is related to the use of the textbook figure (Fig. 2):

لم يبحث جورج في البداية عن الصورة التوضيحية. إِنَّمَا أَمَدَّ الكِتَابُ عَمَلَهُ بِالصُّورَةِ وَبِفِكْرَةِ اسْتِخْدَامِهَا. ثم سَخَّرَ جورج تلك الصورة لتقديم إيضاح تمثيلي للقاعدة

which respectively mean: The book instrumented his work with the figure and the idea of using it; then, he instrumentalised that figure to illustrate the idea of the formula.

In Chinese, we focus on the translation of *instrumentation*. The corresponding Chinese term is 工具化, which has a connection with the philosophical field implying the deviated usage of an object leading to its impoverishment and/or enrichment; inspired by this we could consider that George's *instrumentalisation* of Autograph involves an impoverishment as he only mobilized a limited set of functions of the software (constructing functional graphs and creating their solids of revolution).

For a similar 'situ' in China, teachers' *instrumentalisation* of a software similar to Autograph (e.g., GeoGebra) could also happen, but Chinese teachers seldom instrumentalise a textbook by connecting textbook questions with exam standards as the former questions are often basic and much easier than the standards of the Chinese college entrance exam (高考, gāo kǎo).

Conclusions

As can be seen, the translation 'in situ' affords a bridge to communicate instantiations of the DAD concepts in the English educational context and those 'potentially' existing in the educational context corresponding to a target language. Some instantiations in the English educational context are not totally equivalent to their linguistic counterparts in another context, like textbook versus 课本. Even if we can properly express an instantiation in a target language, the instantiation itself (e.g., the instrumentalisation with respect to a textbook) may not exist in the corresponding educational context. These linguistic non-equivalences open up a perspective for contrasting teachers' interactions with resources in the crossing educational contexts and cultures.

In terms of deepening DAD concepts, the translation process in section 3 shows that we can draw inspiration from the educational, cultural, theoretical traditions in other cultural spheres to enrich the connotation of the theoretical concepts. Also, contrasting the different possibilities of translation for the same concept, in connection with their use 'in situ', can help clarify critical aspects of the concept.

Above all, the more one goes into the translation of details of DAD, the more one can deepen the comparison of teachers' interaction with resources across cultures. The theory itself will also be 'enlightened' by the different cultural analyses and nuances in the target language that needs to be considered during the translation process.

TRANSLATION WORK FROM AN ANTHROPOLOGICAL PERSPECTIVE

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Translation of a theoretical framework

The DAD Multilingual Project (Trouche, 2021) was launched in 2020 to gather translations of the English article that introduces the theoretical framework, *the Documentational Approach to Didactics* (DAD). We were involved in this project and worked on Chinese and Japanese translations. Our translation work entailed various difficulties and issues due to the linguistic and cultural distance between the West and East. We explored these difficulties and issues and attempted to reveal cultural specificities of teachers' and researchers' work related to the resources.

Our study adopts a perspective of *the Anthropological Theory of the Didactic* (ATD), specifically the concepts of *praxeology* and *transposition* (Chevallard, 2019). In general, praxeology is a tool to model the knowledge and practice related to teaching and learning. This notion may be applied to different practices. Regarding our study, teachers' work with resources can be modelled in terms of the *didactic* or *paradidactic praxeology*, and DAD developed to investigate and understand this praxeology can be considered as an element that constitutes a *research praxeology*, which models researchers' practice and knowledge (Artigue et al., 2011). Furthermore, the translation work of a theoretical framework is considered as a process of *transposition* of a part of research praxeology (*logos* block) from a research *institution* (e.g., French or English) to another (e.g., Chinese and Japanese). Taken together, the overall structure of our translation work can be outlined like Figure 1 in terms of ATD.

Another critical hypothesis of ATD is that any praxeology cannot survive in an empty society but in an *institution*. In the institution, a praxeology is always subject to *conditions* supporting it and *constraints* hindering it. In our case, the transposition process or translation work is exposed to the conditions and constraints entailed in the target institution, which presumably produce the difficulties of translation. Thinking upside down, we investigate the difficulties and issues of translation to identify linguistic and cultural elements that constitute the system of conditions and constraints, which is called *ecology* in ATD.

Some linguistic and cultural issues of translations

We faced several challenges in the translation work, which appeared at least at two levels, linguistic and cultural, which were sometimes intertwined.

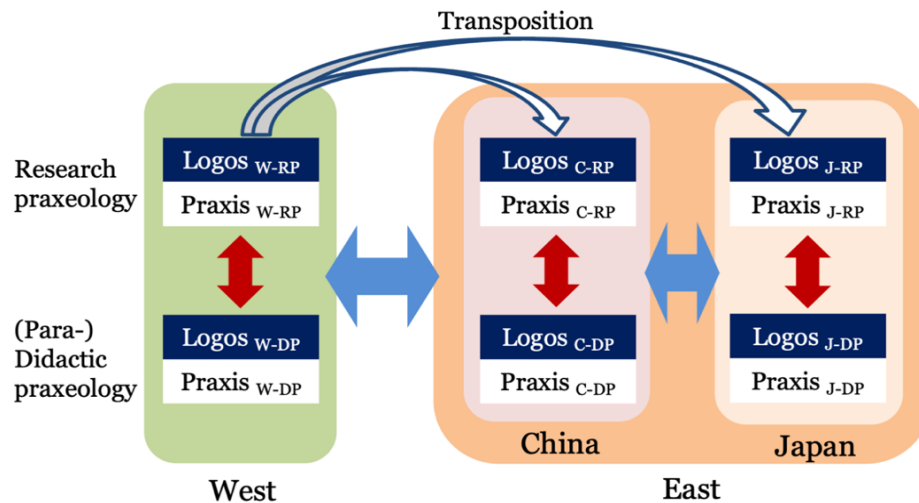


Figure 1: The overall structure of our work in terms of ATD

The distance in terms of the language between the West and East is not limited. This can be firstly attributed to the non-use of alphabets, but the use of Chinese characters and/or other letters. One important issue for us was to create new terms for technical terminologies, due to the linguistic distance such as the inexistence of one-to-one correspondence between Western and Eastern terms, the grammatical difference of the rules of derived words, and so forth. For example, we struggled with the translation of the term *documentation*, which seems easy to be translated within Romance languages. We had multiple candidates for translating the term *document*, and an additional term was necessary to express the meaning given by the suffix “-ation”. Considering the literal or contextual meaning of the original term, we finally arrived at the Chinese and Japanese translations, 文献纪录 (*wén-xiàn jì-lù*) and 文書活動 (*bunsho katsudō*), as a new term with two terms that already exist in the respective language.

Even if we found a word (at literal or contextual level) of a given English (or French) word, we sometimes faced other issues owing to cultural differences, such as teachers’ terminologies, educational researchers’ terminologies, and researchers’ perspective. For example, our translation of the key word *resource* was 资源 (*zī yuán*) in Chinese and リソース (*risōsu*) in Japanese, by taking into account the definition given within DAD. We were wondering to what extent these terms are appropriate and will be used for studying Chinese and Japanese teachers’ work. In the educational context, most teachers in both countries are not very familiar with these terms in their professional communities. Instead, teachers use certain specific terminologies for describing and discussing mathematics teaching including resources. In China and Japan, the teachers use the terms 教材 (teaching material; *kyōzai* in Japanese and *jiào-cái* in Chinese) and 教具 (teaching instrument: *kyōgu* in Japanese and *jiào-jù* in Chinese), which are similar but not identical to the term *resource* in DAD. From a scientific perspective, the evolution of science requires the development of new theoretical concepts and terminologies to better understand the object of study. However, in the Chinese and Japanese communities of mathematics education, teachers and researchers often share

terminologies for their collaborative work. The terms 教材 and 教具 mentioned above are used not only by the teachers but also by the researchers. Thus, the choice of translation was made according to teachers' and researchers' terminologies as well as researchers' interpretations of the theoretical framework and expectations of how the theoretical term could be received in their local contexts.

Conditions and constraints in transpositions

Language was one of the biggest constraints that causes difficulties of translation and shapes the theoretical concepts of DAD. The linguistic distance between West and East was greater than between China and Japan. Exploring translation issues leads us to determine cultural similarities between our two countries rather than the differences, such as teachers' terminologies related to the resources, the critical role of textbooks, teachers' individual/collective work like Lesson Study and Teacher Research Group (TRG); and researchers' work in mathematics education.

A critical aspect highlighted in our study was the nature of research praxeology in the East, specifically the close relationship between research praxeology and (para-) didactic praxeology. Teachers and researchers often share terminology for their collaborative work as mentioned above. Related to this, the multiple roles played by researchers were also highlighted: conducting scientific research; working with teachers and playing the role of a "knowledgeable other" in Lesson Study or TRG; and the transposition of research praxeology from the West to the East.

Further, the discussion on the translation in terms of the transposition also questions the viability of the Western theoretical framework (or research praxeology) in the East. A "theoretical framework" is often received in the East as prescriptive or normative, which can be used for developing, designing, and improving educational practices. This would be a crucial question for the further development of DAD.

SESSION 2. COMMENTARY AND PROVOCATION

THE CHALLENGES OF TRANSLATING AS A CULTURAL ENCOUNTER

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Laurentian University

In dealing with RF-Q2, one of the central themes that arises is the one of translating teacher resource use from one cultural context to another. In fact, two of the three papers of this RF session deal with the difficulties that researchers have found in translating the ideas of the DAD into other languages. Before I comment on the challenges of translation, and using the DAD as an example, I present a short overview of what we may term the epistemological apparatus of the DAD.

The epistemological apparatus of the DAD

At the epistemological core of the DAD lies the concept of resource—a concept that appears to have different meanings in current mathematics education literature. An encompassing meaning was suggested by J. Adler: everything that can re-source the teacher activity (see Shao et al.). In the DAD, however, for something to be a resource, a *schema of usage* is required. The alluded schema must be understood in the sense of Vergnaud’s reformulation of Kant/Piaget’s idea, namely in terms of *psychological operative invariants* that organize human behaviour for a given class of situations. As we can see, there is, in the concept of schema, a removal of materiality—an abstraction in Aristotle’s sense—that gives the schema its invariance and readiness to be used in front of similar situations. And this is how the schema appears in the DAD (see, e.g., Trouche, 2004).

The ergonomic approach that runs underneath the DAD brings to the fore the need to somehow reverse the Aristotelian abstraction in the didactic cogitations and to return to the materiality of the world. This materiality did not seem to have been relevant to Kant who found in the faculties of the mind (e.g., the faculties of understanding and imagination) enough ingredients to account for its functioning, or to Piaget for whom the objects of his experiments were instrumental means to elicit the logical-mathematical children’s schemes. Materiality is the substance of the ergonomic approach, which is a response to late modernity: precisely, a response about our dealings with concrete objects; it is about the interface between body and matter—matter seen Piagetianly; that is, as we accommodate it to our ends (instrumentalization), and, following a Vygotskian thread, matter as it affects us cognitively (instrumentation). Thus, in Shao et al.’s paper we see how the teacher “was instrumented [affected] by the textbook” but also how he “instrumentalised the textbook”; that is, how he accommodates the book to his thinking and needs.

Translating the DAD as a cultural encounter

In the human sciences, a theory is a complex cultural artefact that intends to explain something while at the same time bearing and conveying a specific outlook of reality. There is no exception when the something is mathematics education. The DAD, as any other Western educational theory, has been shaped by a series of conceptions about learning, knowing, knowledge, the teacher, the student, etc. Its main concepts arose in specific historical conditions and have been refined, modified, and adjusted, as new circumstances have required. This is why the DAD, as any other theory, cannot be neutral. It makes assumptions about the very fabric of the educational world. In other terms, the DAD and any other educational theory is unavoidably ideological (it unavoidably conveys a specific cultural system of ideas). Thus, drawing on its assumptions, the DAD sees things as occurring in certain ways: George, the teacher, is instrumented by cultural objects; he acts following some Piagetian schemas, etc.

The fact that theories are ideological invites us to consider translation as a delicate process. For one thing, it would be perilous to consider translation as ideologically free.

To do so would amount to adhere to the view that the earth's various cultural forms of life are in the end all the same—even worse that they are the same as *ours*, which is nothing less than adopting an ethnocentric view of humans and, in the case of educational theories, of how humans learn.

It is precisely the dissonances between various forms of life that surface in the process of translation. Confronted by these dissonances, Shao et al. remark that “Even if we can properly express [a DAD's concept-word, e.g., ‘resource’] in a target language, the [concept word] . . . may not exist in the corresponding educational context.” The same remark is made in Knipping et al.'s and Wang et al.'s papers. The target language, indeed, responds and co-responds to an altogether different cultural view with its own history and its own political, economic, and social conceptions of the school and learning. The conceptual Kantian schema and the material resources the DAD brings to the fore are foreigners to the Asian cultural views where translation tries to find its niche. The DAD's concepts of schema, resource, language, etc. are part of an ideological apparatus of the Western world through which such a world intended to respond to its own culturally situated needs. There were, in particular, the need to shape a new Western conception of the modern subject (Radford, 2021), the need for a rationality understood instrumentally (Bohy-Bunel, 2022), and the need to come to grips with the question of materiality in face of the Western world's understanding of progress as a technological event (Radford, 2004). These three needs find an answer in the Kant-Piaget-Vergnaud lineage of ideas as challenged by the conception of matter of late modernity.

We see hence that a great deal of the difficulties of the process of translation rests on translating a cultural form of life into a different one. These difficulties do not prevent one from translating one cultural theory into the language of another culture. The problem is not (or not only) a question of language. The problem is to find one's way into the practice of what I want to term a post-colonial, culturally responsible translating practice; that is, one that emphasizes the aesthetics of cultural pluralism; one that places the translated ideas in the web of metaphors and cultural significations of the target culture; one which, for example, makes room to understand the Chinese textbook not as a mere technical tool but as an artefact imbued with the meanings of its own culture and ways of conceiving of the teacher and teaching and learning (see Shao et al.). A post-colonial, responsible translating practice should also be one that is not unidirectional, but *dialogical*. Shao et al. contend that the impossible matching of the DAD terms in the language of the target culture “open[s] up a perspective for contrasting teachers' interactions with resources in the crossing educational contexts and cultures.” They go on to say, “We can draw inspiration from the educational, cultural, theoretical traditions in other cultural spheres to enrich the connotation of the theoretical concepts.” Whose concepts? DAD's? What about the other cultures and their indigenous ways of conceiving of learning, knowledge, the teacher, and the student? What about their *influence* on the DAD's theoretical assumptions? How do

the indigenous philosophers and educators challenge Kant, Piaget, Vergnaud, and all those that inform the DAD?

RF-Q2 points to a profound problem that is always present in the encounter of cultures, namely, that the theories to which we resort in our work are carriers of historically produced ideological stances. These stances surface when we encounter the Other. Taking into account these ideological stances, it seems to me, is a prerequisite to the practice of genuine translating. RF-Q2 moves us beyond the possibilities of language and brings us into the domain of culture, power, and ethics.

CONCLUSION AND INVITATION TO PARTICIPANTS

To summarise, this Research Forum aims to place the longstanding domain of research on resources in mathematics teaching in conversation with the longstanding domain of research on language as an inherent shaping process in the mathematics classroom. These domains have a number of natural intersections, including shared commitments to understanding mathematics teaching and learning within and across differing cultural contexts. At the same time, neither domain embodies a singular theoretical perspective and builds on differing epistemological underpinnings. Moreover, within each domain, digital evolutions and cultural boundary crossing have necessitated expansion and re-conceptualisation of key constructs and processes. Bringing the two bodies of work together invites additional complexities and debates. Still, we find doing so to be crucial and productive if we seek to understand the work of mathematics teaching in the current digital, connected, and translanguaging world.

We have organized the forum to open discussion on two focal questions repeated here:

- RF-Q1. How do we (as a mathematics education research community) understand language as a resource in our studies with curriculum, mathematics teachers and teaching?
- RF-Q2. How do we understand teachers interacting with resources in crossing languages and contexts?

To explore these questions, we have brought together six contributions from mathematics education researchers around the globe, seeking to intertwine research on language and teachers' interactions with resources. These contributions offer and make use of differing lenses and empirical approaches for uncovering and interpreting the use of language in mathematics classrooms and resources and their consequences for how mathematics learning is framed and understood. The presentations in Part 1 offer theoretical-analytical frameworks for examining the intersection of language and resources, particularly in teaching and teachers' curriculum work. The presentations in Part 2 focus specifically on language in the form of words, their translations, and their related meanings across cultural and linguistic boundaries. In addition, two well-versed scholars offer commentary and provocations from differing theoretical perspectives, inviting contributors and members of the audience to explore critical

questions. Richard Barwell, asks us to consider the potential consequences of framing language as a resource when used in relation to mastery of “mathematics in the official language of instruction.” Luis Radford raises questions about what he calls “post-colonial, culturally responsible translating practice”, and specifically the cross-cultural applicability of a framework like DAD informed by Western-world philosophies.

Crucially, this research forum invites participants to bring their perspectives, questions and critique to the above issues. Additional questions readers might consider include:

How might we make use of the issues emerging as we put these two domains in conversation with one another to examine our own assumptions?

How do we navigate the tension between merging/building new frameworks and the need to dismantle existing structures in order to take seriously issues of power and reproduction in cross-cultural, cross-linguistic research?

What might we look for in our initiated collaboration for continuing the investigation of how a language lens can support our understanding of mathematics teachers’ interactions with resources?

The co-ordinators and all authors look forward to what we hope will be fruitful engagement with these questions and issues over the course of the conference sessions for this Research Forum.

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