EXPLORATIONS ON VISUAL ATTENTION DURING COLLABORATIVE PROBLEM SOLVING

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In this plenary, I reflect on the MathTrack reseach project that examines the role of visual attention in the multimodal social interaction in the classroom contexts of collaborative non-routine problem solving. The project is using multiple mobile eye-tracking devices to record teacher and student visual attention when students work in groups solving a non-routine geometry problem. Project outcomes include methodological innovations for working with eye movement data, findings about joint representational attention, and the importance of eye contact in teacher-student interaction. Our experience suggests that eye movement research in classrooms should focus on analysing visual processes and making within-person analyses.

INTRODUCTION

In my plenary, I reflect on a research project, MathTrack (Mobile gaze tracking for the study of attention and emotion in collaborative mathematical problem solving). The pilot phase of the project begun in 2014, a grant from the Academy of Finland funded the main project 2016-2020, and the analysis of the collected data is still ongoing. In MathTrack we used multiple mobile eye-trackers in Finnish grade 9 mathematics classrooms to study teachers' and students' visual attention. The project extended eye-movement research into new areas and required methodological innovation.

Representations are essential in mathematics education (e.g. Kaput, 1987). Multiple representations may exist, for example, on board, in textbook, on computer screens, and as gestures and spoken words. In addition to these representations, the classroom is a social setting, where both teachers and students continuously observe and react to what others are doing. Such richness of information raises a question regarding how teachers and students navigate between the multiple channels.

Until recent decades, research on social interaction had focused on verbal communication. More resent research has started to acknowledge that communication is not just words; gestures, glances, body movement, and prosody are also important aspects of it (Radford, 2008). For example, signalling one's own attention and reading the attention of others is essential for fast-paced interactions when people collaborate (Clark & Schaefer, 1987). The facial region is especially important, serving to regulate the flow of conversation, to provide feedback on the reaction of others, to communicate emotions, and to communicate the nature of relationships (Argyle & Cook, 1976). In mathematics education, research on multimodal communication between teachers and their students is relatively recent (Radford, 2008; Arzarello et al., 2009).

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Recent reviews have summarized the research on eye movement related to mathematics education. A review on eye tracking research on learning (Lai et al., 2013) identified seven main areas of research: patterns of information processing, effects of instructional design, re-examination of existing theories, individual differences, effects of learning strategies, patterns of decision making, and conceptual development. From their review within mathematics education, Strohmaier et al., (2020) concluded that eye tracking seemed particularly beneficial for studying processes rather than outcomes, for revealing mental representations, and for assessing subconscious aspects of mathematical thinking. Furthermore, in a review of PME proceedings until 2018, Lilienthal and Schindler (2019) found altogether 33 papers using eye tracking, the earliest appearing in 2013. Six of these papers came from the MathTrack project. Other active PME researchers in this area are Lilienthal and Schindler themselves (6 papers) and Shvarts (e.g. 2018) with her collaborators (10 papers). The trends indicate an increasing interest in eye tracking, especially on dual and mobile eye tracking.

I will next do a brief review of the research on visual attention, focusing on problem solving and social interaction. Then, I will give an overview of the MathTrack project that examines the role of visual attention in the multimodal social interaction in the classroom contexts of collaborative non-routine problem solving. Finally, I will make recommendations for future research on visual attention in mathematics classrooms.

VISUAL ATTENTION AND VISUAL INTERACTION

The study of eye movements is based on the premise that gaze and thinking are related (eye-mind hypothesis, Just & Carpenter, 1980). Studies on human perception show that we can identify finer structures such as letters and the fine articulation of gestures only in the fovea of the eye, which spans less than two degrees of our perceptual field (Gullberg and Holmqvist, 1999). From a distance of 5 meters, that is about 17 cm in diameter, and for normal reading distance (40 cm) the diameter is about 1.4 cm. This means that we cannot recognize symbols, facial expressions, or finer gestures unless we look at the target. Outside the foveal area, light and motion recognition is good, (Gullberg and Holmqvist, 1999), allowing us to direct our gaze at interesting targets originally observed in our peripheral vision.

The nature of visual attention

Most important elements of eye movement are fixations (brief pauses when the eye is immobile) and very fast transitions called saccades to next fixation (Gullberg and Holmqvist, 1999). Perception takes place during fixations, which vary from some tens of milliseconds to a few seconds in duration. Fixations are typically around 250 milliseconds when reading, while in a natural situation (making tea) they vary more and are longer, being on average about 400 milliseconds (Land et al., 1999).

Different methods have been developed to analyse eye movement behaviour. As in all areas of research, a phase of qualitative research has been necessary to get a basic understanding of the eye movements in a specific task (e.g. reading, social interaction).

On this foundation, research has developed into a more systematic quantitative research with mostly experimental designs in laboratory settings. Some eye-movement analyses require pre-defining areas of interest (AOI) while some allow the areas to emerge trough the analysis. The data may include number and durations of fixations on AOI, sequence of transitions between different AOI, distance and speed of saccades, but also blinks and pupil dilation. Typical analysis is based on the knowledge that higher fixation frequency or longer duration of fixation often mean either greater interest in the target or that the target is complex and difficult to encode.

While eye-movement research is based on the assumption that what we look at tells something about our thinking, the relation between eye movement and cognition is not straightforward. One key issue is to make a distinction between more automatic (involuntary or bottom-up) attention regulation and more conscious (voluntary or top-down) attention regulation (Noudoost et al., 2010). This is well illustrated in the seminal experiment by the psychologist Afred Yarbus, where people were looking at the same painting with different instructions, each instruction leading to different viewing pattern (Tatler, et al., 2010). Moreover, in a social setting, the attention is often directed through the interaction with others. For example, pointing gestures (McNeill 1992) and gaze (e.g. Gullberg & Holmqvist, 1999) are important for directing attention, to the extent that they are typical means that magicians use to misdirect audience attention, when performing magic tricks (Kuhn et al., 2014).

Even when one fixates on a target, it is possible that one’s attention is on something else, leading to inattentional blindness (Memmert, 2006). A classical example of this is when research participants watch a video and try to count how many times the players with white shirts pass the ball, they don’t notice that a research assistant in a gorilla suit walks amongst the group of players, pounds her chest, and walks away (Simons and Chabris, 1999). Eye tracking (Memmert, 2006) confirmed that those who noticed and those who failed to notice the gorilla had equal amount of fixations on the gorilla. Hence, the blindness was not due to not seeing the gorilla, but because of not attending to the gorilla. Moreover, Memmert (2006) found out that expertise with basketball increased the likelihood to notice the gorilla. This suggests that those who have more automatized processes for the task can better notice unexpected stimuli.

Eye-movement research in natural contexts is more difficult and has evolved more slowly than research in laboratory settings (Tatler et al., 2019). Important areas of more recent advance in eye movement research have been in studying social interaction and perception in action (Foulsham, 2015), both relevant for classroom research.

Visual attention in problem solving

Regarding insight problems, experts (i.e., high-performers) are known to find the task-relevant features of the visual information faster than novices and their visual attention is focused more on the relevant areas of the visual stimulus (Gegenfurtner et al., 2011). Novices display more attentional transitions than experts while they also use longer gaze sequences for each task, compared with more expert counterparts (Kim et al., 2014). Knoblich et al. (2001) add that even novices are more likely to solve a task requiring insight successfully if they attend to the relevant areas.

The value of attentional transitions (i.e., switches between regions of interest) has been highlighted for geometry (Kim et al., 2014). A scanpath analysis (i.e. sequence of fixation targets) indicated that successful and unsuccessful solvers have mutually inverse direction of fixation targets. Specifically, unsuccessful solvers struggle both with decoding the problem and in locating relevant information (Tsai et al., 2012). Even teachers demonstrate the same expert—novice contrast in a concept-mapping task measuring subject-knowledge (Dogusoy-Taylan & Cagiltay, 2014). Though both teacher groups followed the same overall strategy in solving the problem, expert gaze focused more on relevant regions than novices.

Eye movement behaviour in a social context

Social interaction includes many processes related to eye movement. Skarratt et al. (2012) summarize earlier research to show that humans “prioritize other humans, their faces and, in particular, their eyes when viewing natural scenes” (p. 3). They conclude that eye movement behaviour is different when there is an actual person to look at compared to watching a video of a person. The potential for interaction seems relevant to eye movements. In social interaction, gaze can be used to make or avoid eye contact (Laidlaw et al., 2011), to communicate the direction of attention (Gullberg & Holmqvist, 1999; Skarratt et al., 2012), and more specifically to build joint attention (Pfeiffer et al., 2013).

The teacher has an important role in the classroom social interactions. There are two main functions for teacher gaze. One is the attentional (information seeking) function and the other is the communicative (information giving) function (McIntyre et al., 2017). An important foundation for successful collaborative work is socially shared regulation of learning (joint regulation of cognition, metacognition, motivation, emotion, and behaviour; Panadero & Järvelä, 2015). As gaze is important in social interaction, it is quite likely also involved in socially shared regulation of learning.

MATHTRACK RESEARCH PROJECT

The motivation for MathTrack was threefold. First, we wanted to learn about student visual attention when they solve mathematics problems as a group. After all, few studies have examined the visual attention during collaborative processes. Second, we wanted to learn about teacher visual attention when they observe and facilitate such problem solving activity. The classroom is rich in rapidly changing visual information, requiring efficient navigation across potential targets while teacher’s gaze is also an important communicative tool, having potential to direct student attention. Third, the short history of earlier work doing multiple person mobile eye tracking in natural context challenged us to develop new methodological solutions.

Methods

The MathTrack project used mobile eye-tracking devices and the algorithms and software developed in the Finnish Institute of Occupational Health (Toivanen et al., 2017). Toivanen worked for the MathTrack project and manufactured the eye-trackers. The method utilizes a 3D model of the eye, making the trackers robust to motion. The accuracy of the device was approximately 1.5 degrees of the visual angle, which is comparable to or better than commercial alternatives. The device consists of a glasses-like frame equipped with some electronics and three mini-cameras connected to a computer that was carried in a backpack (see Figure 1), allowing the participants' freedom to move. The software on the computer recorded the video frames and produced a video of the scene camera, superimposed with a gaze point. The frame rate of the video varied according to the amount of light; optimally, it was 30 fps.

For the main study, we collected data from seven ninth-grade mathematics classrooms. When recruiting participating students among volunteers we including both male and female students with both positive and negative affect towards mathematics. Moreover, in three of the classes students used GeoGebra for solving the problem, while in the other four they worked with pen and paper.

Three stationary video cameras and several microphones recorded the actions and conversations of the students and the teachers. Smartpens recorded students' writing and screen capture videos recorded students' work on computers. Most importantly, five sets of wearable eye-tracking devices recorded the eye movements of the teacher and the four focus students. We synchronized the camera clocks before each recording, but also used a physical clapperboard to signal the beginning of the recording to be able to synchronize the multiple channels of data that we collected.

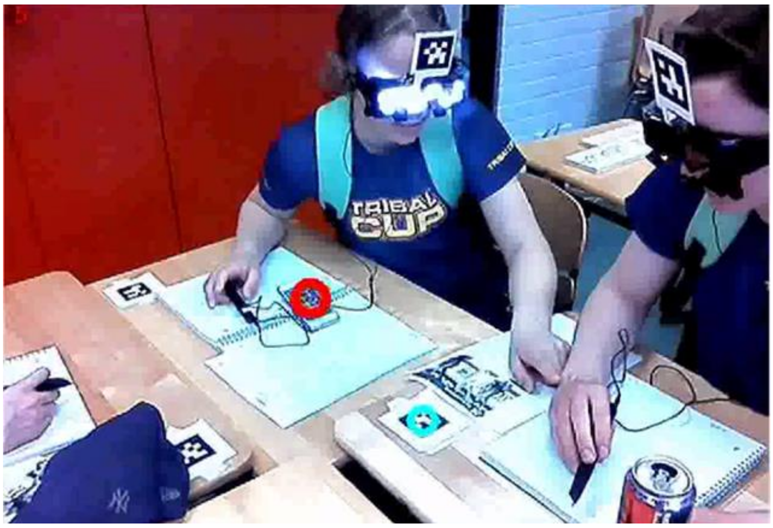

Figure 1. A frame from eye-tracking video showing students wearing the trackers. The red circle indicates the computed location of teacher gaze and the blue circle indicates the visual marker that is closest to gaze target. The bright light around student eyes is infrared, which is invisible to the naked eye.

For each class we recorded two mathematics lessons. During the first lesson, we calibrated the devices and let the teacher and the students get used to the equipment and researchers. The actual research data was recorded from the second lesson. For this lesson, the researchers gave a non-routine task (a four point Steiner tree problem) to the teacher in advance and instructed them to organize the lesson in a certain way. The students first worked on the task individually, then in pairs, then in groups of four, and finally there was a whole class discussion. The researchers instructed the teacher to engage in activating guidance, using questions and not revealing the key idea of the problem (Hähkiöniemi & Leppäaho, 2012).

After the second lesson, we individually interviewed the teacher and the four focus students with the lesson video as a stimulus. Specifically, we asked the students about the moments when they experienced curiosity, frustration, flow, anxiety, or boredom. The focus on teacher interviews was their observation of the focus students' progress and their decisions regarding when and how to intervene with different groups. We also collected questionnaire data from target students and teachers.

RESULTS

Some of our early observations related simply to the overall nature of teacher and student eye movements in mathematics lesson. One thing that we observed, but have not reported until now, is that teacher eye movement is much more volatile than student eye movement. Teachers really seem to pay attention to everything. We also noticed that both students and teachers pay a lot of attention to others' faces (e.g. Haataja et al., 2019). These early observations paved way for more focused research questions.

Our first analyses of data were qualitative case studies. This was a way to start making sense of the complex data we had generated. We learnt things that in retrospect seem expected and even trivial. For example, we witnessed a student to observe teacher's gestures and gaze cues quite closely to follow and even predict where the teacher wanted her students to focus next (Garcia Moreno-Esteva & Hannula, 2015), and how a silently gazing student's eye movements indicated interesting cognitive activity (Hannula & Williams, 2016). As we progressed, we begun to apply mixed methods, using quantitative analysis to report the patterns of eye movement and qualitative analysis to give meaning to it (Haataja et al., 2019, 2021; Määttä et al., 2021).

Because earlier eye movement research in natural social settings was quite limited, we had to look at some fundamental methodological issues. One of these was how students and teachers experienced the data collection with all the extraordinary equipment around. Fortunately, none of them reported that the equipment or the presence of the researchers affected their behaviour or learning significantly. Some compared the experience to watching a 3D-movie: as soon as the action began, they forgot the goggles. However, we noticed that the device was sometimes inconvenient, leading to some students repeatedly adjusting them. We also noticed students paying special attention toward the devices, illustrated as jokes about the video recording of peer's behaviour or examining where the smartpen's camera is.

One hindrance for making progress with eye movement research in natural environments has been the slow manual annotation of fixations. When we started our project, it took us two hours to code one minute of eye movement data. While we got twice as fast with more experience, it was still slow and prone to errors. During the project, we developed ways to use visual markers (see Figure 1) in the learning environment to identify fixation targets automatically (Hannula et al., 2019).

We also had to resolve what measures to use for eye-movement data. Depending on the research question, we reported, for example, number and average duration of dwells (Haataja et al., 2019) or the distribution of fixation durations (Hannula et al., 2019). We also developed our own methods for analysing eye movement data.

For eye movement behaviour, the sequence of fixation targets is important. To compare long sequences of eye movements, we developed a method to synthesize the information from hundreds of fixations as a *scanning signature* (Garcia Moreno-Esteva et al., 2020). A scanning signature gives a visual representation of a person's eye-movement behaviour across targets. Figure 2 shows an example how these can illustrate the different eye movement patterns, in this case just before the key insight (Garcia Moreno-Esteva & Hannula, 2021). This method even computes values for the number of fixations on different targets and transitions between them, as well as their temporal average occurrences all of which can be used for quantitative analyses.

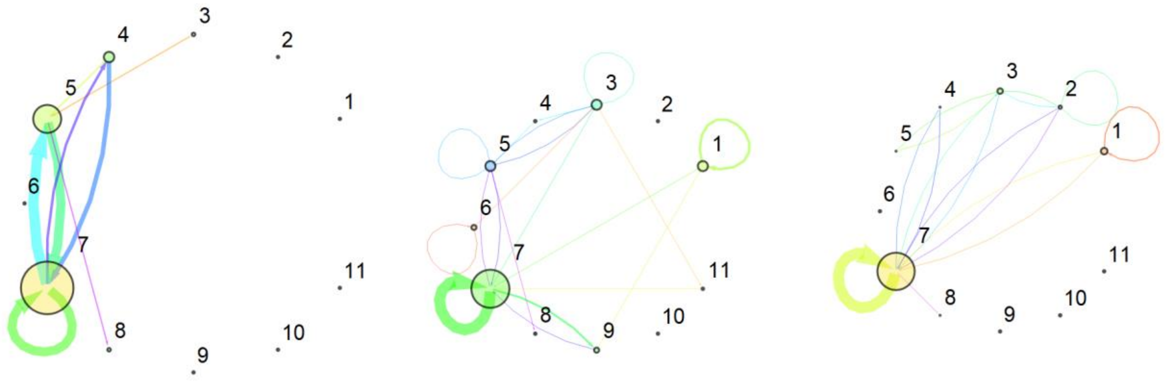

Figure 2. Scanning signatures for three students before an insight leading to the optimal solution (Garcia Moreno-Esteva & Hannula, 2021). The node 1 is the problem, nodes 2 to 10 are different solutions, and the node 11 is the area with computations. Circle diameter reflects the number of fixations on that target. Arrow thickness reflects the number of respective transitions. Temporal sequences are colour coded (red-yellow-blue-purple).

For our study on student visual attention during group problem solving (Salminen-Saari et al., 2021), we developed a novel method to measure the level of synchrony between two or more eye-movement patterns. Moreover, we extended the idea of joint attention (e.g. Tomasello 1995) to cover also episodes when students were looking at different representations of the same idea. Such *joint representational attention* was common, and it happened, for example, when students were watching similar solutions each on their own notebook. We found joint attention to be most frequent when students were verifying a solution and the moments of joint attention usually led to making progress in solving the problem. While our study was limited in scope, it suggests that joint attention is beneficial for collaborative problem solving process and that the moments of verification have strong potential to bring students attention together. Further work on joint visual attention, especially in mathematics, should pay attention to joint representational attention.

Our research on teachers' eye movement behaviour was the first one recording how teacher visual attention related to their scaffolding intentions and their interpersonal behaviour when facilitating group work. We found out that during cognitive scaffolding, teacher's visual attention was mostly on student written products, even though the student was then often trying to make eye contact with the teacher, while during affective scaffolding the teacher was more frequently watching the students' faces (Haataja et al., 2019). For the interpersonal behaviour, we used Kiesler's (1983) interpersonal theory, where communication is identified along two dimensions: communion (warmth) and agency. We found out that moments of higher teacher communion were often characterized by teacher-initiated eye contact, and related to more and longer student fixations on the teacher (Haataja et al., 2021). Our results highlight the importance of eye movement behaviour in teacher-student interaction. Specifically, moments of making and avoiding eye contact seem important communicative acts in this interaction.

LESSONS LEARNED IN THE PROJECT

Reflecting back our research so far, I will now summarize some lessons learned. I will first discuss the methodology of eye movement research in classrooms. Then I will reflect the research designs that are suitable for this approach.

Methods

There is extensive research done on mathematical thinking of students based on observational data and self-reporting. However, clinical interviews or think-aloud protocols distort the nature of social interaction and thus lack ecological validity. Interviews done afterwards – including stimulated recall – only have access to the student's post hoc reconstructions. Hence, these approaches have a limited possibility to access the automatic level of cognitive processes, which include both navigating in social interaction as well as interacting with the physical world. The automatic processes are typically very fast, fleeting, and inaccessible to introspection. Eye tracking data opens a new window to explore them and to contrast with earlier findings.

The eye movement research has developed into a paradigm that forefronts experimental research designs with carefully controlled stimulus and environment, typically in a research laboratory, watching a computer screen. Until recent decades, a major challenge for studying eye movements in natural settings was the lack of affordable and reliable equipment. While we now have the technology, the methods for data pre-processing and analysis developed for laboratory settings often do not work in natural settings. For example, in order to measure pupil dilation, it is important that the lighting is controlled, which is not possible in classrooms. Moreover, study designs in natural context are often explorative in the beginning, and as such, not always considered relevant by those entrenched in the experimental paradigm. This may lead to difficulties in publishing research.

With a novel and highly technical research method, we encountered several issues relating to research ethics (for more thorough reflection, see Hannula et al., 2022). The first issue concerned the nature of video research, which, according to Everri et al. (2020) has still surprisingly little ethical guidance. As people are recognizable from the video, the data becomes effectively a personality register, setting constraints for storing and sharing video data for research purposes. Moreover, video data may reveal 'special categories of personal data', such as racial or ethnic origin or religious beliefs (Finnish Social Sciences Data Archive, n.a.), making the personal data sensitive. As video data can be used to address questions not foreseen at the time of data collection, it is challenging to describe the intended research to participants, their guardians, and ethics review boards in a way that is at the same time informative and not unnecessarily restrictive. This applies especially to eye tracking in naturalistic settings, where participant gaze may reveal more than they expect. This has made us aware of our responsibility for being sensitive to what we analyse and report.

Another ethical issue for the project was the definition of the physical integrity when wearing the eye tracker (Hannula et al., 2022). When new technology is used for social and behavioural research, it requires revisiting old definitions as some may be more invasive in terms of privacy or bodily experience than others (e.g., Duru, 2018). In Finland, only studies involving deviation from informed consent or risks—such as a violation of physical integrity—must be reviewed by the ethics review board. It was not clear whether the wearable trackers would be considered to fall into this category. Our study actually became a precedent for re-defining 'intervention in physical integrity'. When the Finnish National Board on Research Integrity (2019) revised the national guidelines for ethics review for non-medical research involving human participants, they added a new definition for the intervention in the physical integrity to happen if participants cannot free themselves from the devices within a reasonable time.

There were several lessons related to management of staff and data. Having a novel, highly technical setting requires specialized technical staff. MathTrack would not have been possible without Miika Toivanen. He had been one of the developers of the mobile eye-tracking glasses and the related software before working in the MathTrack project. Collecting and post-processing the eye-tracking data was his special expertise. The necessity for highly specialized technical expertise, embodied in a single person, makes such research vulnerable. Therefore, one of the priorities in his work was to document the method and to teach it to other researchers in the project

Data management is a particularly complex issue in a project like ours. Because of the large number of video files, such projects need more data storage space than usual in educational sciences. In MathTrack, we stored for each lesson altogether 28 video files, including stimulated recall videos, screen captures, and the raw data from ten cameras recording eye movements. During the project, we realized how important it is to have a good metadata for each file and a clear structure in the data archives. This was highlighted because of several researchers and research assistants doing different analyses on different parts of the data. While we had a master document for each file in a joint network folder, doing the actual analysis required downloading the files on one's own computer. Keeping track of these processed files and deciding when and which ones to upload in the network file became a non-trivial task.

Some wearable eye-tracking devices are sensitive to movement. In natural settings with longer data collection (e.g. full lesson) it is likely that the device will be touched. Even a slight movement (called "slippage") will reduce the accuracy of data. On the other hand, our device was robust against movement to the extent that the participant could remove the device and put it back on later without a need to calibrate again. This allowed us to calibrate the equipment one day and to focus on uninterrupted collection of data on another day. When doing eye tracking in natural settings, the robustness against slippage is an essential feature.

To avoid laborious and error-prone human annotation of fixations, I warmly recommend using visual markers in the environment. In our context, we were able to identify the location of student gaze automatically for 74% of their fixations. Moreover, this allows also generating heat maps, a useful way to illustrate how visual attention is distributed. While automatic object recognition is rapidly advancing and is successful in some contexts (e.g. Jongerius et al., 2021), its performance seems not yet sufficiently reliable for eye movement research in real classrooms.

For data analysis, we point out the potential of methods based on graph theory, which underlie both our method to recognize moments of joint attention (Salminen-Saari et al., 2021) and the method to synthesize fixations and saccades as a scanning signature (Garcia Moreno-Esteva et al., 2020).

The basic assumption of eye movement research is that the fixation on a target is informative about our visual attention. It seems that peripheral vision is sufficient for some elements, such as large gestures. We also believe that in a natural context teacher and students may sometimes rely on their memory of something they have looked at. Such peripheral vision and memory may be sufficient in some situations. Yet, based on our experiences, the basic premise of attention correlating with fixations seems justified for research even in natural contexts. However, making conclusions based on fixation duration is more difficult. When reading, longer fixations typically indicate more demanding cognitive processing. However, in our study the long fixations when working with GeoGebra were more commonly related to the difficulty of moving the cursor to an intended location on the screen (Hannula et al., 2019). The use of gaze to define a target of action is something that seems to include long fixations (see also Land et al., 1999).

Research design

Eye movement data is continuous, providing hundreds of data points each minute. While eye movement data can be used to analyse how much different targets receive attention overall, this type of dense data has a specific potential for analysing processes. Combining it with other continuous data seems especially fruitful (e.g. Haataja et al., 2021). Transitions between targets inform how things are connected. The sequence of events can be used to examine which ideas were attended to before coming up with a new idea (e.g. Garcia Moreno-Esteva & Hannula, 2021).

Eye-movement data collected in natural contexts is not well suited for between-person comparisons. While eye movements have some universal characteristics, there is also significant idiosyncracy, i.e. each person has their personal pattern of fixations and saccades (Poynter et al., 2013). Our data showed significant variation in the distribution of fixation durations, specifically between teacher and the students but also between different students. The individual idiosyncrasies led us to leave out several students from our analysis comparing eye movements while using GeoGebra vs. using pen and paper (Hannula et al, 2019). In principle, this could be handled statistically if samples were sufficiently large, but that would significantly extend the data collection in scope. On the other hand, eye-movement data suits very well within-person analyses, providing convincing evidence even with small samples (e.g. Määttä et al. 2022).

One of the benefits for research in natural classrooms is the ecological validity of the data. However, we found it useful to control the learning situation by asking the teachers to conduct a lesson around the same task and using the same instructional approach. This way, we could pool data from different lessons for a meaningful analysis. Yet, we also made systematic variation to the context by asking some of the teachers to have their students solve the task using GeoGebra. As it is likely that the amount of data researchers can collect in natural contexts will be quite limited also in the future, we recommend reducing the variation of contextual variables. You should be clear to identify what you wish to vary and try to limit the variation of other features.

CONCLUSIONS

One of the key things to know about eye movement research “in the wild” is that it is not easy. The eye-tracking methodology requires investing in devices and technical expertise. Hence, eye-movement research in classrooms should focus on such questions that are difficult or impossible to study with other approaches.

One obvious area to continue exploring is teacher eye-movement behaviour in the classroom. While we have used the eye-movement data to examine teacher-student interaction (e.g. Haataja et al., 2019, 2021), such data could inform also research on teacher's professional vision and decision-making (Stahnke et al., 2016). We know that eye movement behaviour watching live people is different from eye movements when watching a video. Therefore, mobile eye tracking can access such aspects of teacher visual attention that other methods cannot.

Because of the importance of gaze in social interaction, collaborative work is another area where eye movement research has potential. We have examined joint visual attention during problem solving (Salminen-Saari et al., 2021), which is definitely an area worth further examination. Moreover, studying the student eye movements in their multimodal communication would inform about when and what visual information (diagrams, gestures, facial expressions) students attend to in such communication.

Another promising avenue for research on visual attention in classrooms is to examine eye movements of those students whose attention deviates from average. In a meta-analysis Armstrong and Olatunji (2012) synthesized research on how affective disorders bias attention towards emotional stimuli. As mathematics anxiety is an enduring problem in mathematics education (e.g. Hannula, 2018), studying anxious students' eye movements in classrooms is another valid venue for mobile eye tracking. Furthermore, eye tracking has been used to study the attentional processes of people with attention disorder (Maron et al., 2021) or those on autism spectrum (Laskowitz et al., 2022). So far, this has been done almost exclusively in laboratory settings. As the trend in education is to integrate students with special needs into ordinary classrooms, it is of utmost importance to study their attention in these natural contexts.

As an overall conclusion, it is clear that mobile eye tracking in real classrooms is a viable research approach. It provides a unique approach to studying the visual attention of teacher and students. Specifically, it captures the visual processes as they unfold during the lesson – rather than studying them in retrospect. Moreover, while eye tracking is not mind reading, it provides new information about the automatic and non-conscious processes in mathematics teaching and learning. Hence, it should be an essential part of the research agenda on mathematical thinking and interaction.

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