# INNOVATIVE PERSPECTIVES IN RESEARCH IN MATHEMATICAL MODELLING EDUCATION

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*The research forum shares and discusses innovative perspectives in research on mathematical modelling education. Specifically, the proposed research forum intends to give an overview of current perspectives from different research strands (amongst others, psychologically and pedagogically oriented research) and from different social-cultural contexts (including Eastern and Western contexts). Finally, the research forum aims to develop prospects for further developments in modelling education research.*

## AN OUTLINE OF THE THEORETICAL BACKGROUND OF THE RESEARCH TOPIC

Research on mathematical modelling education, as well as its orientation and relationship to various neighbouring disciplines, has become a prolific and productive field that is growing with enormous speed especially in the last decade. Various special issues on the field have been published by high-ranking mathematics educational journals (see, for example, the recent special issue on mathematical modelling competences in *Educational Studies in Mathematics* (Kaiser & Schukajlow, 2022) and the special issue on psychologically influenced approaches in *Mathematical Thinking and Learning* (Kaiser et al., 2022). In addition, rigorously peer-reviewed proceedings on the topic have been continuously published for several decades.

For decades, research on mathematical modelling education has had pedagogical goals; in other words, it has aimed to improve mathematics education with empirically developed and evaluated examples of mathematical modelling (Kaiser & Brand, 2015). The design of innovative teaching methods and use of technology are two central areas of research with high relevance to the learning of mathematical modelling. Psychological topics, such as affect, intuition and creativity, have been recently introduced to the research discourse on mathematical modelling education (Schukajlow et al., 2018). Affect and intuition were demonstrated to be critical for learning in earlier research, whereas creativity when solving modelling problems is an emerging topic of research. The relevance of socio-cultural perspectives on modelling research has been emphasised more strongly in the last few years, especially by performing East–West comparisons and ethno-mathematical studies. Overall, the current field of mathematical modelling education research can be described as experiencing a diversification of dominant approaches and the introduction of new



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research perspectives, such as new media/technology and its usage in education, especially during the COVID-19 pandemic.

In the research forum, these perspectives will be presented in more detail, focusing on their novelty and potential to advance the current research discourse on mathematical modelling and, more generally, mathematics education. In his commentary, Wim Van Dooren addresses new developments in research on modelling that arose after the 2014 PME research forum in Vancouver (Cai et al., 2014).

## GOALS AND KEY QUESTIONS OF THE PROPOSED RESEARCH FORUM

This research forum addresses three strands of research on mathematical modelling education:

### Pedagogically oriented research perspectives

- Innovative research approach used to explore teaching approaches and their role in the promotion of modelling competences: Werner Blum, Berta Barquero, Rina Durandt
- New media and technologies and their role in modelling education research: Stefan Siller, Mustafa Cevikbas, Vince Geiger, Gilbert Greefrath

### Socio-culturally oriented research perspectives

- Cultural and socio-cultural influences on the implementation of mathematical modelling education and consequences for mathematical modelling education research including the ethno-mathematical perspective: Xinrong Yang, Björn Schwarz, Milton Rosa

### Psychologically oriented research perspectives

- The influence and role of affective aspects within mathematical modelling activities: Stanislaw Schukajlow, Janina Krawitz, Susana Carreira
- The influence of creativity on mathematical modelling and its role within mathematical modelling activities: Xiaoli Lu, Gabriele Kaiser, Roza Leikin
- The role of intuition within mathematical modelling: Rita Borromeo Ferri, Corey Brady

Discussion of the perspectives: Wim Van Dooren

## FORMAT OF THE RESEARCH FORUM

The format of the research forum will integrate brief formal presentations, small group discussions, pre-prepared commentary and coordinated Q&A sections. Each of the two 90-minute sessions of the forum will start with formal presentations that introduce the research topic by sharing existing research and perspectives on mathematical modelling education. The participants will then be invited to join small group discussions, which provide a good opportunity to ask questions and learn more about research approaches in mathematical modelling education. During these discussions, participants may be invited to share what they know about these approaches and/or further perspectives. A summary of the information shared during the discussions and further explanations will be prepared and used as a commentary to finish the first 90-minute session. In the second 90-minute sessions, the contributors will present innovative psychologically oriented perspectives to research on mathematical modelling education. These presentations will be followed by a commentary led by the coordinators of the research forum. The session will end with a Q&A involving the audience, contributors and discussants.

# INNOVATIVE RESEARCH APPROACHES FOR EXPLORING TEACHING ENVIRONMENTS DESIGNED TO PROMOTE MATHEMATICAL MODELLING COMPETENCY

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*The paper addresses the question of which teaching designs can advance students' modelling competency. After some general results of empirical investigations, five examples of research studies on the secondary and tertiary level are described in which teaching environments for modelling have been constructed and investigated which proved to be effective.*

## MATHEMATICAL MODELLING COMPETENCY

There is a broad consensus in the educational debate that mathematical modelling has to be an integral part of mathematics teaching on all educational levels. One essential aim of teaching modelling is to advance the *competency of mathematical modelling*, that is the ability to deal with extra-mathematical problems by using or creating suitable mathematical models, working within these models, and interpreting the obtained results for the solution of the problems.

We know from research that mathematical modelling is cognitively demanding because it usually requires several skills and abilities as well as mathematical and real-world knowledge. Each step in the modelling process may be a cognitive hurdle for learners (Blum, 2015). So, an important question is: How can students' mathematical modelling competency be advanced? In this paper, we focus on the following, slightly more specialised question: Which teaching designs promise or have proven to enhance progress in school or university students' ability to solve modelling tasks?

## ADVANCING MODELLING COMPETENCY

To advance modelling competency requires well-aimed teaching methods which are designed so as to fulfil criteria of quality teaching. Several studies report on encouraging results which show that it is indeed possible to advance the ability to solve modelling tasks by means of suitable learning environments (for an overview see Niss & Blum, 2020, chapter 6; Cevikbas et al., 2022). An analysis of these studies reveals that, globally speaking, those teaching designs are particularly effective which contain, on the one hand, *instructional* elements (well-designed teaching material, with appropriate tasks, and adaptive teacher guidance) and, on the other hand, *constructional* elements (students' self-directed activities in solving modelling tasks and students' use of suitable strategies). Crucial seems to be a permanent balance between these elements, that means students ought to work as independently as possible, supported by minimal teacher interventions when necessary.

## RESEARCH INTO ADVANCING MODELLING COMPETENCY

In the following, we refer to five studies which can be regarded as examples of innovative research into possibilities of advancing the competency to solve modelling tasks. In all these studies, teaching environments have been constructed which constitute a certain blend of constructional and instructional elements as outlined above.

*Example 1:* In the German interdisciplinary research project DISUM (see Blum & Schukajlow, 2018), the effects of a more independence-oriented teaching style, called "operative-strategic", was compared in a ten-lesson mathematical modelling unit in altogether 26 grade 9 classes (14-15-year olds) with the effects of a more teacher-guided style, called "directive", and with an improved version of the operative-strategic style, called "method-integrative". The same 14 modelling tasks were treated in the same order in all designs. In both the operative-strategic and the method-integrative design, the major aim was maintaining a permanent balance between teacher's guidance and students' independence, and encouraging individual solutions, whereas in the directive design, the teacher guided the students and developed common solution patterns. In the method-integrative design, a meta-cognitive aid called "solution plan" (essentially a four-step modelling cycle) was in students' hands, and the teacher introduced its use by demonstrating in the fourth lesson how modelling tasks may be solved. It turned out in a pre-/post-test research design that all teaching styles had significant and similar effects on students' technical mathematical skills, but only the two independence-oriented styles had significant effects on students' modelling competency. Moreover, the method-integrative classes outperformed the operative-strategic classes in mathematical modelling.

*Example 2:* In an Australian study (see Galbraith, 2018), the effects of a systematic teaching program in grade 8 (12-year olds) over a year was investigated in which the development of concepts and skills as well as the ability to apply mathematics, was pursued by means of mathematical modelling focused around a sequence of carefully selected problems. The grade 8 medium ability group of a school was chosen as the trial group whereas the high ability group followed a conventional teaching programme involving exposition of ideas, techniques, and worked examples by the teacher followed by consolidation exercises. In the trial group, mathematical concepts and skills were introduced and developed through application contexts, while simultaneously the process of modelling itself was practised, but also traditional homework exercises were set for concept reinforcement and skill practice. A six-step modelling cycle served as a set of meta-cognitive prompts for the students. Remarkably, the trial group outperformed the conventional group in a standard grade 8 mathematics test at the end of the school year. In addition, the trial group showed substantial progress also in modelling competency, assessed by qualitative data, taken from students' journals and from oral interviews.

*Example 3:* There is broad empirical evidence that the use of meta-cognitive strategies can be a substantial support in solution processes. This holds also for modelling processes. In the LIMo project, a five-step “Solution Plan” with strategical hints was used as a meta-cognitive tool, and its effects were controlled in a comparative study in 29 grade 9 classes with a pre-/post-test design (see Beckschulte, 2020). Both the experimental group and the control group were exposed to a four-lesson mathematical modelling unit with the same four tasks. In the experimental group, the Solution Plan was introduced at the end of the first lesson. The tests revealed significant progress in students’ modelling competency in both groups, with a significant advantage of the experimental group in the sub-competencies of Interpreting and Simplifying, especially strong in the follow-up test. So, the researchers conclude that working with a meta-cognitive instrument has advantages particularly in the long run.

*Example 4:* In line with the research approach of the study and research paths (SRP) for the teaching of modelling, Barquero et al. (2011) describe the design and analysis of an SRP about population dynamics which was tested with first-year engineering students over six consecutive years. The implementation took place in a “mathematical modelling workshop”, in parallel with regular lecture sessions, facilitating a mixture of more constructional with more instructional teaching elements. Throughout the entire year, students received different sets of population data and were asked to develop models to forecast the size of the population. To align the workshop to the standard syllabus of a first-year mathematics module, the SRP was divided into three branches: considering time as discrete and a population with independent generations; forecasting in discrete time while distinguishing groups in a generation; considering time as continuous with one or more generations distinguished. Despite the expected variation among the implementations, students’ submissions (weekly teams’ reports, individual reports at the end of each branch, and three individual tests) provided substantial empirical evidence about their modelling competency progress. This research also provided robust designs which have been later transferred to teacher education and to secondary education.

*Example 5:* In a study also at the tertiary level, two teaching designs similar to those used in the DISUM study were implemented in a five-lesson mathematical modelling unit with three groups of first-year engineering students in South Africa (see Durandt et al., 2022). Two groups were instructed according to the teacher-directive design, and one group followed the method-integrative teaching design with its characteristic mixture of constructional and instructional teaching elements. The same ten modelling tasks were treated in the same order in all three groups. The students' progress in mathematical modelling and in mathematical topics underlying the modelling tasks was measured by a pre-/post-test design. It turned out that, like in the DISUM project, the group taught according to the method-integrative design had the biggest competency gains in modelling, while the progress in mathematics was the same for all groups. There were also differences between the two directive groups, presumably due to the fact that two different lecturers taught these groups, thus pointing to the high importance of the teacher variable in such investigations.

Several more such studies can be found in recent special modelling issues of journals: Carreira & Blum (2021a,b); Kaiser & Schukajlow (2022); Kaiser, Schukajlow, & Stillman (2022).

# THE ROLE OF DIGITAL RESOURCES IN MATHEMATICAL MODELLING RESEARCH

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*The integration of mathematical modelling into instruction can promote learners' understanding of mathematical content, ideas, and concepts as well as offering an approach to solving real world problems. Resources such as digital tools, media, and simulations hold great potential for the implementation of mathematical modelling in school classrooms. The use of digital resources can be used to support the generation of solutions to real world problem. In this paper, we present a concise synthesis of research regarding the role of digital resources and their potential for promoting learners' capability with mathematical modelling. We conclude the paper by identifying future directions for research into digital resources enhanced mathematical modelling instruction.*

## MATHEMATICAL MODELLING VIA EMERGING DIGITAL RESOURCES

Emerging digital resources are becoming increasingly important in the context of mathematical modelling instruction in school classrooms. These emerging digital resources open new possibilities for exploring mathematical situations. Research on mathematical modelling has been consistently extended to include the use of digital tools (e.g., Geiger, 2011; Greefrath et al., 2018), although its integration has attracted discussion within academic discourse (e. g., Doerr et al., 2017; Monaghan et al., 2016). Research into digital resources enhanced mathematical modelling has focused on its affordances when solving with real world problems and how it can best be used to enhance and support classroom instruction. Most of this research, however, has focused on the use of the digital tool itself rather than how digital resources can be integrated into thinking about the strategies needed to solve a problem in a real-world context. Considering digital resources as thinking tools when dealing with real world problems, however, has been largely theorized rather than the focus of empirical research. Further, there are a limited number of studies that have investigated the effective integration of digital resources into learning environments, for example, the use of simulations of real-world scenarios based on mathematical models as in the case of computer apps or virtual reality technologies. With this paper, we make it clear that despite the limited availability of findings, there is clear research potential inherent in the underlying approach. To explore this potential, we first present a perspective on the role of digital tools in mathematical modelling, and then present open questions based on (current) empirical studies on modelling with digital resources.

## PERSPECTIVE OF THE ROLE OF DIGITAL TOOLS IN MATHEMATICAL MODELLING

A number of studies have pointed to difficulties learners may experience during the modelling process and how the use of digital tools can act as a bridge between the real model and the mathematical results (e.g., Galbraith & Stillman, 2006). Doerr and Pratt (2008) point to the potential of digital resources for providing different representations of a real-world problem adding that the technology-based model represents a new field of knowledge with its own learning opportunities. Consistent with this perspective, Confrey and Maloney (2007) describe a holistic approach to modelling with digital tools by positioning relevant digital resources as mediators of meaning through the different representation that can be generated. Other studies have complemented this perspective by providing evidence that digital resources can provide affordances that can support mathematical modelling throughout the process (e.g., Geiger, 2011; Siller & Greefrath, 2010). Greefrath et al., 2018, however, note that a holistic view on the use of digital tools during the modelling process, rather than focusing on specific sub-competencies, better describes learners' actual approach to modelling when using digital resources.

## RECENT EMPIRICAL STUDIES ON MODELLING WITH DIGITAL RESOURCES

To date, there have been only a limited number of systematic reviews of research into mathematical modelling (see for example, Cevikbas et al., 2022). In Cevikbas et al.'s (2022) review, the literature on conceptualizing mathematical modelling competencies and their measurement and fostering is described based on the analysis of the papers published between 2003-2021. However, none of the studies focused specifically on the role of digital resources in teaching and learning mathematical modelling. For this paper, we conducted a new systematic literature search by using three well-known electronic databases (Web of Science, ERIC, and EBSCO Teacher Reference Center). In this search, the Boolean string "(mathematical model*) AND (technology* OR digital)" was employed with a focus on titles and abstracts of the peer-reviewed journal articles and book chapters written in English. Our review encompassed studies published between 2012-2021. As the International Community of Teachers of Mathematical Modelling and Applications (ICTMA) conferences have been influential, we also conducted a manual search of ICTMA book chapters (1984-2021) and identified 30 eligible publications in total.

The results of this search reveal that a variety of digital resources can be used to promote the teaching and learning of mathematical modelling. Many studies focused on the role of digital resources in thinking in modelling, not simply as a means to complete computational tasks. The digital resources that were most often investigated across these studies was Dynamic Geometry Systems (DGSs) (37%), followed by Internet (33%), spreadsheets (27%), Computer Algebra Systems (CASs) (17%), mobile devices (20%), computers (17%), graphic calculators (17%), simulations (computer-generated representations of real world situations) (13%), specialized software such as 3D design software and Game Maker Studio (13%), videos and videogames (10%), motion detectors (7%), apps (7%), applets (7%), sensors (7%), smartboards (3%), programming languages (3%), 3D printers (3%), simulators (3%) and electric circuits (3%) and animations (3%). No studies were identified that related specifically to new pedagogical approaches (e.g., flipped classroom) or innovative technologies (e.g., augmented and virtual reality, artificial intelligence). An examination of the identified studies indicated that the above-mentioned digital resources were used for various purposes in the modeling process including: (a) finding information or data; (b) enhancing to explore possible solution pathways; (c) formulating problems, equations, schemas, or diagrams; (d) visualization; (e) calculation; (f) interpreting results; and (g) validating solution. Results suggest that the use of digital resources can be beneficial at different points in the modelling cycle, consistent with Geiger (2011) and Siller and Greefrath (2010). From a different perspective, a number of studies were concerned with the notion of the "black box" approach to the use of technology – this term refers to any complicated device whose inputs and outputs we know, however whose inner workings we do not know (O'Byrne, 2018). Concerning this issue, studies reported that the problems encountered in the solution processes of modelling, which are seen in digital group work, seem to be a direct result of the automatic calculation provided by the technology. In addition, there were a small number of studies that noted that an impediment to the use of digital resources by teachers and learners in the practice of mathematical modelling was a lack of experience.

## THE NEED FOR RESEARCH ON MODELLING WITH DIGITAL RESOURCES AND OPEN QUESTIONS

Our review of the literature identified only 30 relevant publications in total, indicating that digital resources enhanced mathematical modelling remains under researched, even though it is no longer a new area of interest to scholars in the field. Further, the number of publications in high-ranking journals, such as those indexed in Social Science Citation Index is especially limited, with chapters from ICTMA books making up the bulk of the identified literature. Our review also indicates that most studies were based on what might now be considered conventional digital tools that have an established role in teaching and learning modelling (e.g., computers), rather than new and emerging technologies (e.g., augmented and virtual reality) that may have great potential for instruction in responding to real-world problems. Overall, our review indicates that areas that require further attention in research include: How to improve the experience and knowledge of educators and students on the use of digital resources in modeling? What innovative technology active teaching approaches may be effective in supporting student learning in modelling? How can digital resources be used in the modeling process while avoiding black-box related issues? Ultimately, many interesting questions remain open for current research.

# CULTURAL AND SOCIO-CULTURAL INFLUENCES ON THE IMPLEMENTATION OF MATHEMATICAL MODELLING EDUCATION AND CONSEQUENCES FOR MATHEMATICAL MODELLING EDUCATION

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*In the paper, we first discuss main social cultural factors which influence the implementation of mathematical modelling education such as differences of theoretical perspectives of modelling or ways of teaching. We then review the differences of mathematical modelling competences between students from Western and Eastern contexts identified in available comparative studies in this field. We also discuss the approach of ethnomodelling to expand the understanding of social and cultural influence on mathematical modelling education. We close the paper with a few recommendations.*

## CULTURAL AND SOCIO-CULTURAL INFLUENCES ON THE IMPLEMENTATION OF MATHEMATICAL MODELLING EDUCATION

Mathematical modelling is a central part of mathematical education, which for example becomes obvious by its embeddedness into theoretical frameworks of studies on both,

students' competences (e.g. the concept of mathematical literacy in PISA, OECD, 2003) as well as (future) mathematics teachers' competences (e.g. TEDS-M, Blömeke et al., 2014). However, there is no joint understanding of mathematical modelling or mathematical modelling competences and instead, various approaches can be identified (Cevikbas et al., 2022). Kaiser and Sriraman (2006) distinguished various perspectives on modelling such as epistemological and realistic modelling.
Moreover, the teaching and learning of mathematical modelling in mathematics classroom is of course embedded into approaches of teaching and learning of mathematics in general. Thus, cultural and socio-cultural differences concerning the teaching and learning of mathematics in general also influence the teaching and learning of mathematical modelling in particular, becoming manifest for example in different accentuations in the curricula as well as different ways of teaching. A prominent distinction with regard to different approaches in East Asian and Western countries was formulated by Leung (2001). He formulates various dichotomies according to which the teaching and learning of mathematics differs between the two regions for example referring to rote versus meaningful learning or whole class teaching versus individualised learning. It is obvious, that respective differences can have a strong influence on how mathematical modelling is taught. However, also the analysis of processes of the teaching and learning of mathematics in research has to take social-cultural aspects into account (Lerman, 2001) as well as teacher education (Presmeg, 1998).

## CONSEQUENCES FOR MATHEMATICAL MODELLING EDUCATION

In the past years, researchers have started to compare students' and teachers' mathematical modelling competencies between different educational environments. For example, Ludwig and Xu (2010) compared the overall mathematical modelling competence levels of 1108 secondary school students (Grade 9 to Grade 11) from Germany and Mainland China and mainly found that the general performance of the participants was nearly the same, except that students from Mainland China were found to progressively improve their competencies from Grade 9 to 11.

Recently, Chang, Krawitz, Schukajlow and Yang (2020) compared a specific sub-competence of mathematical modelling, namely making non-numerical and numerical assumptions, between secondary school students from Germany and Taiwan. They found that the German participants performed slightly better for the making assumption tasks than their counterparts from Taiwan. Furthermore, it was found that if students in the two educational systems were on the same level of mathematical knowledge, the German participants were found to have higher modelling performance compared to the participants from Taiwan in solving the same modelling tasks. Similarly, Hankeln (2020) compared the mathematical modelling processes between 18 French secondary school students (from Grade 10 to 12) and 12 German students with the use of think-aloud methods. It was also found that even though none of the participants was familiar with open modelling problems, the French participants were hindered more by the underdetermination of the task, false assumptions and wrong representation of the situation. By contrast, the German participants were found to reflect upon the real-world situation rather superficially and to be hindered by difficulties in the calculation.

Quite recently, Yang, Schwarz and Leung (2022) compared pre-service mathematics teachers' professional modelling competencies between Germany, Mainland China and Hong Kong. It was found that pre-service teachers from Germany demonstrated the strongest MCK and MPCK of mathematical modelling, while those from Hong Kong demonstrated the weakest professional competencies, with pre-service teachers from Mainland China falling in between. Specifically, Bonferroni-adjusted post hoc tests showed that significantly more participants from Hong Kong and Mainland China displayed low or very low levels of MCK and MPCK of mathematical modelling, and by contrast, more participants from Germany were found to possess high or very high levels of MCK and MPCK of mathematical modelling.

Overall, such differences identified between different educational systems and countries may be explained by differences of tradition of mathematical modelling in mathematics curricula, mathematics textbooks, teacher education, and teaching culture in these systems.

## ETHNOMATHEMATICS AND THE SOCIOCULTURAL PERSPECTIVE OF MATHEMATICAL MODELLING

Historical evolution enabled the development of alternative mathematical knowledge systems that provide explanations of daily problems and phenomena, which leads to the elaboration of ethnomodels as representations of facts present in our own reality. Ethnomathematics helps members of distinct cultural groups to draw information about their own realities through the elaboration of representations that generate mathematical knowledge that deals with creativity and invention. According to D'Ambrosio (2006), ethnomathematics is a way in which people from particular cultures use their own mathematical ideas, procedures, and practices for dealing with quantitative, qualitative, spatial, and relational daily phenomena. This process legitimates and validates their own mathematical experience that is inherent to their lives. Similarly, it is important to argue that, in an ethnomathematical perspective, mathematical thinking is developed in different cultures in accordance with the common problems that are encountered within the sociocultural context of their members.

In this regard, D'Ambrosio (2006) has affirmed that in order to solve specific problems, members of distinct cultural groups develop non-generalizable solutions that cannot be adapted to other purposes. These members also create methods that are generalized to solve similar situations in their own contexts, and then theories that are developed from these generalizations so that they are able to understand these phenomena through the development of ethnomodels. In the ethnomathematics context, these members come to develop mathematical representations in ways that are quite different from school/academic/Western mathematics as taught in schools, which can be represented through the elaboration of ethnomodels.

**OUTLOOK**

During the past years, there has been an increasing interest to explore and investigate the social and cultural aspect of mathematical modelling, however, firstly, more empirical studies are needed, especially more cross-cultural comparative studies are needed which compare students' and teachers' modelling competencies between Western and Eastern contexts and which involve more countries and regions. At the moment, almost all the comparative studies mainly involve Germany as the typical Western representative. In addition, it will be also necessary to conduct more comparative studies within Western or Eastern context as well to understand more deeply about how a specific social and cultural context influences the development of mathematical modelling competencies.

Secondly, it is needed to develop more cross-culturally reliable and valid instruments in the field. At the moment, only one or two modelling tasks were employed in most of the available comparative studies to measure participants' modelling competences, therefore, it is very possible that their modelling competences were not fully measured. In addition, from the statistical point of view, it is impossible to make more advanced statics analysis such as causal inference analysis with the involvement of a wide range of other variables such as knowledge and affective factors.

## THE INFLUENCE AND ROLE OF AFFECTIVE ASPECTS IN MATHEMATICAL MODELLING

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*Affective aspects, such as motivation and emotions, are essential for the teaching and learning of mathematical modelling. However, research on students' affect in modelling is just beginning. In this contribution, we summarise the knowledge acquired in recent years about students' affect with respect to modelling problems, how instruction in mathematical modelling influences students' affective outcomes, and which affective constructs were found to be important for students' progress in mathematical modelling.*

**INTRODUCTION**

Affective aspects of students' learning are essential for their life-long learning, career choices, and future lives. For example, while choosing classes in high school, college, or university, students greatly rely on what they are interested in, what they like, and whether they consider themselves able to successfully face the demands of their mathematics classes. For a long time, research on modelling – similar to research on other competencies and content areas in mathematics education – was focussed on cognitive outcomes, whereas noncognitive outcomes were largely ignored. Broadly speaking, affect includes all noncognitive variables, such as motivation, emotions, attitudes, and beliefs. Important characteristics of affective outcomes are their valence (positive, negative, or neutral), temporal stability (stable traits vs. unstable states), and objects (e.g., learning, mathematics, strategies, or competencies) (Schukajlow et al., 2017). To adhere to the space restrictions, we focus on the roles that affect plays for school and university students but not for teachers. We review studies on students' perceptions of affect regarding modelling problems and the relationship between affect and performance, summarise findings on the effects of teaching methods on affective outcomes, and analyse research on affective variables as predictors of performance in modelling.

## STUDENTS' AFFECT REGARDING MODELLING PROBLEMS

Mathematics problems with a relationship to the real world are expected to be motivating for students and enhance their positive beliefs, attitudes, and emotions. These considerations are in line with theories in the area of affect (e.g., expectancy-value theory of motivation, interest theory or control-value theory of achievement emotions). In the expectancy-value and control-value theories, research has suggested that task (utility) value might be higher if a problem's solution is useful in real life. Theories of interest suggest that connections to reality might be an additional source of students' interest that adds to their interest in the underlying mathematical problem. On the basis of these theories, one would expect higher interest, enjoyment, and value for modelling problems compared with intramathematical problems (i.e., problems that are not related to reality). However, prior research has not supported these expectations and has indicated similar or even lower motivation and positive emotions while solving modelling problems than intramathematical problems in school students. One explanation for this result is that not all problems that are anchored in the real world are relevant for students. Moreover, as school tests rarely include modelling problems, their relevance for students whose goals are to improve their grades and pass exams might be low. Summarizing this line of research, we ask teachers to choose contexts that have relevance for students and encourage teachers to emphasize the relevance of the specific modelling problem while presenting it in the classroom. Ways to increase the relevance of problems include developing problems that refer to the local context or personalizing tasks with digital tools so that the context captures students' interests.

## RELATIONSHIP BETWEEN AFFECT AND MODELLING PERFORMANCE

Theories of affect assume a bidirectional relationship between affect and achievement, including modelling performance. Students with high initial motivation and positive emotions are expected to engage more deeply in solving modelling problems and to demonstrate better modelling performance. Students with high prior performance experience higher situational interest, enjoyment, autonomy, and competence while solving modelling problems and see increases in their self-efficacy expectations and positive emotions regarding modelling. On the basis of these considerations, researchers hypothesized the existence of a feedback loop between affective and cognitive variables. Some empirical studies have confirmed these expectations for some affective constructs. For example, enjoyment in solving modelling problems in mathematics classes was positively related to modelling performance assessed after mathematics classes. Self-efficacy in modelling was found to be positively related to modelling performance in university students and in school students. In one study, researchers asked students to report their interest and enjoyment in solving modelling problems prior to solving the problems. Higher prior interest and enjoyment in solving modelling problems was positively related to students' modelling performance. In a study with engineering students (Gjesteland & Vos, 2019), students also reported high flow (i.e., they forgot about time and experienced happiness) while solving modelling problems. The authors attributed these findings to task characteristics, such as the openness and accessibility of the task.

## INFLUENCE OF INSTRUCTIONS IN MODELLING ON AFFECT

In the last decade, an increasing number of studies have evaluated the effects of teaching methods for modelling problems on affect with mixed results. Studies that compared student-centred and teacher-directed teaching methods for modelling in ninth and tenth graders revealed positive effects on students' enjoyment, interest, and self-efficacy, whereas no differences were found for students' value of modelling and attitude towards mathematics, even though qualitative analyses of students' responses indicated that students preferred the student-centred teaching method and more specifically cooperative group work. In a study with engineering students in South Africa, a student-centred teaching method that was enriched with some directive elements showed greater development in students' interest, effort, and value than a teacher-directed teaching method, but the effects just missed significance (Durandt et al., 2022). In the framework of a mathematical modelling competition, solving modelling problems was demonstrated to improve self-efficacy in mathematics.

No differences in students' interest, enjoyment, or boredom were found between German school students who solved modelling problems in the classroom by paper and pencil and outside the classroom by using MathCityMap. Therefore, both teaching methods can be beneficial for students' affect. The authenticity of the problem seems to play a more important role than where the students are when solving the problems.

In order to uncover possible mechanisms behind how learning environments affect students' learning in the classroom and how learning in the classroom in turn affects modelling performance, several studies have addressed the specific characteristics of modelling problems in teaching interventions. Providing students with reading comprehension prompts (i.e., presenting questions about the situation described in the task in this study) improved students' situational interest in solving modelling problems in Germany and Taiwan (Krawitz et al., 2021). The authors attributed these positive effects to an increase in students' reading comprehension and their greater involvement in problem solving resulting from engaging in the processing of reading comprehension prompts. Further, a series of studies compared the effects of prompting students to develop multiple solutions for modelling problems with the effects of prompting students to find one solution on students' affect. Prompting students to develop multiple solutions for modelling problems that required them to make assumptions increased students' experiences of competence, autonomy, enjoyment, and interest and decreased boredom while solving modelling problems. Positive effects of prompting students to apply two different mathematical procedures while solving modelling problems were found on students' experiences of competence. Further indirect effects from this teaching method were found on students' self-efficacy in mathematics via experiences of competence and enjoyment as intervening variables. Consequently, affective aspects can explain how an intervention influences modelling and which affective aspects teachers should focus on in mathematics classrooms.

## AFFECTIVE ASPECTS AS PREDICTORS OF THE DEVELOPMENT OF MATHEMATICAL MODELLING

Another important line of research involves affective outcomes as predictors of students' learning. In a study on teaching modelling with digital tools, self-efficacy in using software but not attitude towards software predicted school students' development of mathematizing. In a study on students' drawing strategies, researchers assessed students' enjoyment of and anxiety towards drawings before problem solving. Students who enjoyed making drawings used the drawing strategy more often and more often solved the modelling problems; students who were anxious about using this strategy rarely made drawings and rarely solved the modelling problems. In an intervention study on knowledge about drawings, strategy-based motivation (self-efficacy and cost) at pretest were found to predict the quality of drawings and modelling performance at posttest (Schukajlow et al., 2021).

## SUMMARY AND FUTURE DIRECTIONS

Our analysis indicates that it is not always easy to improve students' affect. Increasing the relevance of the context might be a promising way to foster students' interest, motivation, and positive emotions regarding modelling. Future research should clarify which characteristics of modelling problems contribute to higher affect. Initial studies confirmed that some affective constructs are related to performance in modelling, and we call for more research to collect indications of the relationships between affect (e.g., self-efficacy, values, emotions, identities) and engagement, performance, and other achievement outcomes in the short and long terms. Intervention studies have indicated that student-centred teaching methods and specific teaching approaches, such as prompting students to develop multiple solutions or offering solution plans in the classroom, improved some students' affective outcomes. More research is essential to clarify which teaching methods are beneficial for which students' learning outcomes. Further, studies have revealed the importance of strategy-based motivation and emotions or self-efficacy regarding software as predictors of students' progress in modelling. We suggest that researchers target different objects of affect in the context of modelling. Teachers' beliefs about modelling and their judgements of students' affect are other important areas of research, even though we did not address them in this review due to the space limits.

# THE INFLUENCE OF CREATIVITY ON MATHEMATICAL MODELLING AND ITS ROLE WITHIN MATHEMATICAL MODELLING ACTIVITIES

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*Although mathematical modelling is playing an increasing role in mathematics education, only recently was this approach connected with creativity and its development in mathematics education. In this contribution we describe recent empirical studies connecting mathematical modelling and creativity, referring to long-standing approaches on the conceptualization and measurement of mathematical creativity. The empirical studies point out that mathematical modelling requires creativity at each step of the modelling process. However, the studies present somewhat contradictory descriptions of the relations between the components of creativity and the adequacy of mathematical modelling approaches.*

## CREATIVITY IN MATHEMATICS EDUCATION

Developing creativity is one of the major goals of mathematics education. Its importance is rooted in two main observations. First, the activity of professional mathematicians is directed at mathematical invention and leads to the development of mathematics as science. Thus, this activity is inherently creative. Second, following Vygotsky's approach, creative ability is one of the foundations of knowledge development, and knowledge development and creativity have a mutually supportive relationship (see Leikin & Sriraman, 2022).

Interest in creativity in the field of mathematics dates back to the mathematicians Poincaré and Hadamard, who analyzed creative processing among professional mathematicians. Poincare stressed the importance of intuition and a feeling that mathematics is beautiful for mathematical creation. Hadamard identified four stages of the creative process: preparation, incubation, illumination and verification. Later, it was argued that creativity is a critical component of advanced mathematical thinking related to mathematicians' ability to perceive original and insight-based solutions to complex mathematical problems. Connections between mathematical creativity and mathematical giftedness were pointed out.

At the school level, mathematical creativity in mathematics (education) was overlooked for several decades. From 1960 to 1970, scholars developed connections between mathematical creativity and psychological theories. Of specific importance to the current discourse is the model of creativity proposed by Torrance (1974), which posited that creativity is composed of fluency, flexibility, originality and elaboration. It was suggested that open-ended problems requiring divergent production are effective for the development and evaluation of creativity. Referring to Torrance's model, Silver (1997) proposed that creativity could be fostered by instruction rich in mathematical problem-solving and problem-posing. Later, Leikin (2009) suggested a model for the evaluation of mathematical creativity using multiple-solution tasks.

In their survey paper, Leikin and Sriraman (2022) reported that, during the past decade, there has been a meaningful growth of interest in research on creativity in mathematics education. Based on a systematic literature survey of mathematics education and creativity from 2010 to 2021, the authors identified three major lines of research: research examining the relationships between creativity in mathematics and other characteristics, research analyzing instructional practices and mathematical tasks, and research focused on teachers' creativity-related conceptions and competencies. Referring to the instructional practices and tasks used in these studies, Leikin and Sriraman (2022) report that there are hardly any studies examining creativity related to mathematical modelling. In this paper, we attempt to describe the relevance of the relationship between creativity and modelling in mathematics education.

## CREATIVITY IN MATHEMATICAL MODELLING EDUCATION

### Earlier research in mathematical modelling education

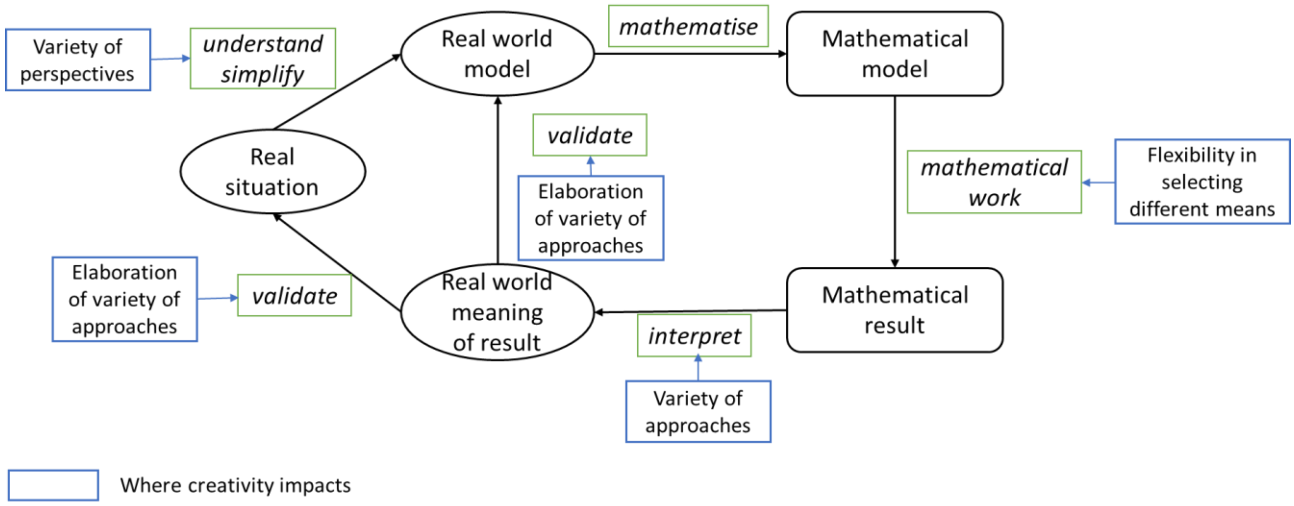
Mathematical modelling has gained increasing importance across the world in the last few decades, bringing real-life contexts to mathematics classes. Modelling practices in school have the potential to motivate students, help them develop appropriate views on mathematics, foster mathematical and extra-mathematical literacy, promote in-depth understandings of mathematical content, and, as a result, promote civic competences for which creativity is a crucial component (Maaß et al., 2019).

Until now, only a few empirical studies on modelling education have involved creativity. Dan and Xie (2011) measured university students' modelling skills and levels of creative thinking and found a strong positive correlation between modelling and creative competence. Chamberlin and Moon (2005) pointed out that complex, open, non-routine model-eliciting tasks could motivate learners to develop models and elicit creative applied mathematical knowledge. Based on this approach and Torrance's model of creativity, Wessels (2014) defined creativity in modelling as comprised of four components: fluency, flexibility, novelty (originality) and usefulness. Of these, usefulness is of specific importance for modelling, since modelling is characterized as applicable mathematics, unlike mathematics in general.

## Recent research in mathematical modelling education

In their studies, Lu and Kaiser (2022a, b) pointed out the necessity of creativity in all phases of modelling process. They argued that creativity allows for a rich understanding of real-world situations through their analysis. This is particularly important when developing mathematical solutions that reflect the value of varied mathematical content and when elaborating ideas for interpreting and validating mathematical results that link the results with new understandings of real-world situations. Overall, they proposed that creativity should be incorporated into the construct of modelling competences and the modelling cycle be enriched by creativity.

Fig. 1: Enriched modelling cycle




Based on Wessels' (2014) work and studies on creativity in problem-posing and problem-solving (e.g., Leikin, 2009), Lu and Kaiser (2022 a, b) further differentiated creative components in the modelling process through two empirical studies that focused on upper secondary school students and pre- and in-service mathematics teachers in China. These components are as follows:

- Usefulness, which describes the efficiency of a modelling approach for solving a task. Higher levels of usefulness are assigned when an approach has the potential to be applied to other situations.
- Fluency, which refers to the application of various solutions to the task.
- Originality, which describes the relative rarity of the modelling approach.

Lu and Kaiser developed a framework for measuring creativity in modelling that includes these components. This framework includes an independent measurement of modelling competencies based on an analysis of the adequacy of participants' modelling approaches, and it is enriched by evaluation of the three creativity-related components of participants' modelling approaches.

With this framework, Lu and Kaiser (2022a) evaluated the modelling approaches used by upper secondary school students and pre-service and in-service teachers from China. They found (1) a significant positive correlation between adequacy of the modelling approach and usefulness and fluency, (2) a negative correlation between usefulness and originality, and (3) dependency of the chosen modelling approach on the mathematical knowledge of the participants, although the influence was less strong than expected. In their second study on upper secondary school students, who had more experience in tackling modelling tasks than their peers, Lu and Kaiser (2022b) also identified significant positive correlations between fluency and originality, but inconsistent correlations between usefulness and fluency or originality.

Overall, the results of these studies indicate the importance of including creativity in mathematical modelling, as well as the relation of usefulness as part and characteristic of modelling problems. The components of creativity remain ambiguous.

## OUTLOOK

Given that existing studies have produced ambiguous results, further empirical work is needed. It is especially important to investigate the role of the adequacy of the modelling approach and the relation of adequacy to the components of creativity. In addition, the role of culture in mathematics education must be considered. Thus, studies should be conducted in other parts of the world. Furthermore, the characteristics and complexity of the modelling task strongly influence the originality of the modelling solution and chosen approach; future studies must examine the influence of the kind of modelling tasks and its complexity. Overall, it seems necessary to develop refined, and partly standardized, measurement instruments.

# INTUITION AND INNOVATION IN MODELLING

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*In this paper, we aim to motivate the study of intuition in the field of mathematical modelling. Experiences of intuition and of "a-ha" moments can be significant episodes in the development of mathematical dispositions and identities. We thus argue that research on intuition serves an important equity goal: grounding an approach to mathematics education that assumes students are capable of innovations in mathematics—of creating mathematics that is new to them. In such an approach, intuition plays a primary, active role, along with other, less well-studied "ways of knowing." We close with a call to study intuition alongside these other facets of mathematical knowledge, in a shared to construct a modelling education that more adequately engages the full range of student experience.*

## FRAMING THE STUDY OF INNOVATION

Researchers have tended to characterize intuition in contradistinction to processes of ratiocination, and to contrast intuitive knowledge with knowledge that the knower can articulate explicitly. Building on this tradition, Sinclair (2009) groups intuition with aesthetics, gesture, and embodied cognition under the heading of "covert" ways of knowing mathematically, as opposed to the "propositional" forms that dominate externalized manifestations of mathematical thinking and knowing.

The challenges of characterizing and studying intuition appear concretely in the context of modelling. Observing individuals solving modelling tasks, for example, one asks the questions, whether the solution already exist in the unconscious mind and merely took time to rise to conscious awareness? Or, alternatively, is there a parallel and unconscious problem-solving process taking place, whose results then enter consciousness in the moment when intuition is experienced? For example, Davis et al. (1998) argue that unconscious perception and action can be recognized by individuals, but that it is difficult for those individuals to describe such states or connect them with intuition.

## INTUITION'S ROLE IN MATHEMATICAL INNOVATION

At the same time, researchers have recognized that intuition and intuitive knowledge do in fact play vital roles in modelling. Borromeo Ferri and Lesh (2013) distinguish between implicit (intuitive) and explicit worlds of modelling. Moreover, they hypothesize that when modellers are provoked into conscious reflection on their interpretation systems, they may begin to articulate explicit models that are based in or concordant with their intuitive models. Otherwise, their externalized work may represent only the 'tip of the iceberg' of their implicit models.

Intuition thus plays a role that should not be underestimated in modelling and in other creative mathematical work. An ample literature (e.g. Fischbein, 2002), affirms that intuition can be central as a trigger or even a driving force in mathematical learning processes. Moreover, it plays a prominent role in famous 'a-ha moments' of mathematical discovery (Liljedahl, 2005).

## WHO CAN INNOVATE?

Thus, one may ask: How central should intuition and the "a-ha" experience be in our designs of learning environments for mathematics education, and in our expectations about what students are capable of? Liljedahl (2005) found that the a-ha experiences that pre-service teachers recalled were disappointingly shallow, but that they were nevertheless very significant moments for developing mathematical identities and dispositions. These findings are ambiguous: one's interpretation of it depends on one's beliefs and values. We argue that the positive impact of a-ha moments urges us to identify opportunities for our students to develop and use intuition, and to create occasions for them to see themselves as innovative makers of mathematics.

The importance of intuition and innovation implicates our beliefs and values, since it causes us to ask, what proportion of the population do we expect are capable of producing and experiencing innovation in mathematics? If we believe this is a *small* proportion of the population, then we are likely to view the support and study of intuition and innovation as a subfield of 'gifted and talented' education. If, on the other hand, we believe that *every* student has the capacity to make original mathematics, we will feel obligated to provide all students with opportunities to engage in mathematical innovation, throughout their educational lifespan.

## INTUITION AND INNOVATION IN THE PHILOSOPHY OF SCIENCE

The question of whether intuition and innovation are focused in an elite few or are resources for all people extends well beyond Mathematics Education. It has been central as well in debates on the philosophy of science. In particular, in his account of the nature of science, Kuhn (2012) depicts genius and originality erupting in discontinuous innovations that transform fundamental paradigms. Kuhn's division between 'normal science' and 'revolutionary science' treats creativity and intuition as rare and mysterious phenomena. Under this perspective, the study of scientific discovery is the province of exceptional psychology.

In contrast to this 'irrationalist' view, Lakatos (1976) argues for studies of the "*logic* of discovery" that emphasize collective and discursive interactions as a source of innovation and creative power. Lakatos paints a picture of mathematical work that features bold conjectures made by ordinary participants, along with a collective discursive process that struggles to 'prove' and foregrounds collective efforts to 'improve' these fallible conjectures.

Mathematicians as a group are not unified in endorsing either view. On one hand, Hadamard's (1945) study of "the psychology of invention in the mathematical field" can be seen as contributing to a Kuhnian perspective, as it focuses on exceptional names in history; on the other hand, the account of innovation humanizes these historic figures. Thom (1971), the famous topologist, wrote about the importance of cultivating "intuition" in *all* mathematics students; but Dieudonné opposed this perspective, arguing that only "four or five men in the eighteenth century, about thirty in the nineteenth, and not more than a hundred" in the twentieth had mental faculties that he would describe as valuable "intuition." In particular, Dieudonné argued that *teachers* of mathematics should instead focus on becoming "adequately educated" in correct formalism and use that as their guide.

Henderson and Taimiņa (2005) argued that viewing intuition as the property of the few can be damaging. A heavy focus on formalism can be alienating and can separate mathematics from learners' lived experience. Instead, they described geometry courses with pre-service teachers, in which embodied, intuitive, and aesthetic ways of knowing contribute to 'alive mathematical reasoning' being described by Hilbert described as 'intuitive understanding', which is offering a more immediate grasp of the objects one studies, a live *rapport* with them, which stresses the concrete meaning of their relations.

## INTUITION IN THE MODELLING CURRICULUM

The success of the international research agenda in modelling makes it vital to consider the role of intuition and innovation. The discussion above underlines the ethical and philosophical stakes, but there are also exciting opportunities for cooperative research. Studies on the relationship between creativity and modelling have already made theoretical and practical progress (see this Research Forum and Lu & Kaiser, 2022a. b). Moreover, research suggests that intuition lies at the intersection of consciousness and creativity, and the interaction between these cognitive functions can have a strong influence on modelling. Creativity and intuition can therefore also be mutually dependent, and at the places where Lu & Kaiser (2022a, b) locate creativity in the modelling cycle, intuition may also play a role. More generally, with a greater appreciation of the value and interconnections among what Sinclair (2009) has described as 'covert' ways of knowing and their role in modelling, we will be better able to position *all* students to develop these facets of mathematical identity.

## DISCUSSION: MATHEMATICAL MODELLING AS EMBLEMATIC FOR RESEARCH IN THE PSYCHOLOGY OF MATHEMATICS EDUCATION

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*The Research Forum discusses important developments in research on mathematical modelling along three different strands: (1) teaching approaches, (2) socio-cultural approaches and (3) psychological aspects. In this discussion, I point at several links between these strands, indicating how more insight is needed into the implications of research on socio-cultural and psychological aspects for the design of mathematical modelling tasks and learning environments in which they are used.*

Mathematical modelling is not only being more and more acknowledged as a major and essential part of the mathematical curriculum (thereby also providing opportunities for STEM teaching in which other scientific disciplines, technology, and engineering approaches are integrated); it also has become a mature field of research in its own right. The current Research Forum brings together some recent lines of research on mathematical modelling, organised in three different strands: (1) teaching approaches for enhancing a mathematical modelling competency (including the use of technological tools) and (2) socio-cultural issues and (3) psychological aspects of what it implies to be or to become competent in mathematical modelling. I will discuss what I see as major remaining challenges in these areas, and I will try to show that these challenges might be met by looking into the insights gained in the other strands.

Blum, Barquero, and Durandt rightfully describe mathematical modelling as a very demanding competency, in which many skills and abilities come together (and – as we see from the other contributions in the Research Forum – also many affective aspects play a role). Blum et al. review studies that show which teaching designs seem promising for enhancing students' ability to solve modelling tasks. It is very nice to see that in recent years, such teaching designs have been studied empirically, partly also by means of experimental studies with a systematic measurement of learning gains. And even better: Findings seem to converge to the importance of a balance between instructional elements such as teacher/material guidance on the one hand and constructional elements including student's self-directed activity on the other hand. Still, I missed a theoretical elaboration as to why this importance of a balance between instructional and constructional elements would be specifically important for the teaching of mathematical modelling. What is specific in a mathematical modelling competency that necessitates the teaching of it to have such a balance? And importantly: what is the optimal blend? For what aspects of mathematical modelling and at which moments in the sequence of teaching and learning activities is it important to be more teacher directed and when is it crucial to be more student initiated? And is the answer to these questions the same for students of all expertise levels? Is it similar across cultures, given the insights of crosscultural studies reported by Yang et al.? Is the balance essential to elicit some of the desirable affective processes and outcomes (as discussed by Schukajlow et al.), and what do the insights in the teaching and learning of creativity and the stimulation of intuition tell about the importance of a balance between instructional and constructional elements?

Also, Siller, Cevikbas, Geiger and Greefrath consider the ways in which teaching of mathematical modelling can be done, with a specific focus on the potential of digital resources. They report a systematic review revealing that there is research on a wide range of digital resources, ranging from tools such as dynamic geometry systems, computer algebra systems and spreadsheets to technology that allows to bring complex reality to the classroom, such as simulations, videos and video games. Importantly, the review indicates that these technologies can serve different purposes in the modelling process and thus have great potential, but it also points at potential fallacies, for instance in using technology that automatically provides the result of calculations. It seems that besides the open questions that Siller et al. raise after their review, future research may also benefit from focusing on the psychological aspects of the acquisition of a modelling competency and the (theoretical) affordances of specific types of technology: As Schukajlow et al. suggest, authentic tasks may be more motivating, and AR/VR technology may be used in increasing the authenticity of modelling tasks. Still, research may need to show whether students indeed consider tasks offered in such technology as being authentic and sufficiently competitive with "real" problems. Technology may also be used to act on the self-efficacy of students: Certain tools like dynamic geometry systems, excel sheets and computer algebra systems may take away the burden to work through formal mathematics, and to focus on the mathematising and interpretation phase of modelling. As such, this kind of tools may allow students to try out many solutions to a problem, to finetune them, to make predictions and check conjectures without the burden of calculating, manipulating expressions and drawing geometric constructions. This may make room in students' minds for creativity to occur (see Lu et al.), or for some intuitions to arise (Borromeo Ferri and Brady). But such effects cannot be taken for granted. The modelling tasks and the learning environment may need to be designed to facilitate that, and there may be an important role for the teacher to direct students to these processes that are deemed important in modelling, which otherwise may still not occur.

Yang, Schwarz and Rosa convincingly show that mathematical modelling is a socio-cultural construct, and that various approaches can be identified. Cultural and socio-cultural differences regarding mathematical modelling can be related to – but certainly do not completely coincide with – cultural and socio-cultural differences in the teaching and learning of mathematics more generally. Yang et al. refer to a number of studies that typically compare students' and teachers modelling competencies in two countries, often a European and an Asian country. And differences are indeed found. One can argue that mathematical modelling may be more susceptible to cultural and socio-cultural differences than other aspects of mathematics education. Modelling tasks are often quite complex and open tasks, susceptible to multiple solution approaches, the adequacy of which need to be considered, discussed and negotiated. Classroom norms – which often remain implicit – play an important role in such situations: Students and teachers need to negotiate and come to an agreement about what constitutes as a good solution to a complex and open modelling tasks, what can be considered as valid arguments and considerations in proposing a certain solution, how a solution needs to be communicated, and so on. While the research reviewed by Yang et al. convincingly shows the cultural and socio-cultural embeddedness of mathematical modelling, it remains far from clear what the implications for teaching mathematical modelling are. Should learning environments and modelling tasks be designed very differently, taking into account the socio-cultural context? And if so, does that imply that the final goal of such learning environment, i.e. the mathematical modelling competency that one tries to establish in learners, is also different across contexts?

Given that mathematical modelling is an activity that has a strong socio-cultural embedding, it is not surprising that affect plays an important role. Schukajlow, Krawitz and Carreira provide an overview of the main affective constructs that play a role in the acquisition of a mathematical modelling competency, and also clearly argue why affective outcomes are also part of the learning outcomes of teaching and learning activities around modelling. The main kinds of affect that they address in their review relate to motivation (including interest and enjoyment) and self-efficacy. Importantly, they do not only show the (recursive) correlation between such constructs and modelling achievement; they also show that affect can be influenced, although – as explained above – this line of research be deepened specifically in relation to the insights of the need for a balance in instructional and constructional teaching approaches. An affective aspect that may also deserve some attention in this respect relates to learners' goal orientation, which can be performance oriented or learning oriented (Dweck, 1990): Modelling tasks are often complex and open, and the assessment is not straightforward. Unlike for many other mathematical tasks, there often is no simple distinction between a correct and an incorrect answer. Learners who have a strong performance-oriented goal orientation may feel insecure when involved in assessments with such open tasks, as opposed to learners with a more learning-oriented goal orientation.

Lu, Kaiser, and Leikin elaborate on the construct of creativity, how it is essential in the activity of mathematics, and particularly in mathematical modelling. While being overlooked for a long time, creativity has taken an important place in research on mathematics education. The theoretical construct has been operationalized, and in their contribution, Lu et al. propose an enriched modelling cycle in which it is shown that in the various stages of the modelling cycle, creativity plays a role. They also review the first studies that link creativity to performance on modelling tasks. However, they also clearly indicate that much further work int his field is needed, for instance in theorizing and in developing adequate measurement instruments. I wish to add that there is a need to come to a deeper understanding of whether and how creativity in mathematical modelling activities can be enhanced in instruction. If teaching is conceived as the systematic, methodical design and organization of certain teaching activities in order to elicit specific learning activities in learners for them to achieve pre-specified learning goals, teaching for creativity seems almost a *contradiction in termini*. Still, if we acknowledge the important role of creativity, this will be the challenge: How can we design tasks and organize tasks in a learning environment so that learners can experience the importance of being creative, and their creativity is stimulated.

The contribution of Borromeo Ferri and Brady on the role of intuition is somewhat on the same line as that of Lu et al. on creativity. Creativity and intuition share some important characteristics, be it that intuition has a more controversial history in mathematics education. Mathematics is often seen as a purely rational, deductive activity in which reasoning relies on consciousness and logic. However, mathematical problem solving – including mathematical modelling – often is not, and intuition seems a particularly fruitful pathway. Once more the challenge is how one can make room for intuition – and even stimulate it – in teaching/learning environments that are focused on mathematical modelling, and what role a teacher can play in it. Making the link to the contribution of Blum et al. showing the importance of a balance between instructional and constructional approaches in education, teachers may at some occasions act as a "model" in solving mathematical modelling problems, verbalize their thinking processes, thoughts, heuristics, considerations, etcetera, but also make explicit their intuitions, their search and hope for an "aha" experience, thereby revealing that also expert mathematical modellers do rely on it. They may show that the use of certain technological tools, as described by Siller et al., may shed a different light on a modelling problem, thereby potentially facilitating – but not guaranteeing – such an "aha" experience.

Based on the above considerations, a tentative conclusion that I want to draw is that the perspectives across three major strands of the Research Forum need to be integrated. I want to argue that exactly this is the essence of what research in the Psychology of Mathematics Education should do: Based on research that investigates the psychological processes of what it implies to learn a mathematical skill and/or to acquire a specific mathematical competency (in this case: modelling) and the understanding of its socio-cultural embeddedness, the field should aim to unravel the principles that would guide the teaching of these competencies, and thus the design of learning environments (including the technology used in them). As I see it now, the contributions that focus on the teaching approaches aimed at enhancing a mathematical modelling competency would benefit from a close consideration of the socio-cultural and psychological issues that are involved in acquiring a modelling competency, while the contributions that review the insights from research on socio-cultural and psychological aspects may need to go more deeply into the implications for the teaching of mathematical modelling and into the principles underlying the design of learning environments.

## OVERALL SUMMARY AND CONCLUSION

The present research forum demonstrates impressively that mathematical modelling is a dynamic research field with diverse research topics, theories, methodologies, and practical implications. While preparing this research forum, we build upon the previous research forum (Cai, 2014), recent special issues in research on modelling in in *Education Studies in Mathematics* (2022), in *Mathematical Thinking and Learning* (2022) and in *ZDM – Mathematics Education* (2018), as well es overviews about empirical research on teaching and learning of mathematical modelling (Schukajlow, Kaiser, & Stillman 2018), about the current discussion on mathematical modelling competencies (Cevikbas et al., 2022), and about research on modelling from a cognitive perspective (Schukajlow et al., 2021). The research forum provides a unique opportunity in presenting and discussion of innovating perspectives in research on mathematical modelling education. As a result of the synthesis of the contribution of the research forum we call for (1) integration of various theoretical perspectives such as social-cultural approaches into research in mathematical modelling, (2) taking into account theoretical foundations from other research areas such as teacher education, intuition, creativity or technology for the development of teaching methods for improving modelling competencies, and (3) considering social, cognitive and affective process and outcomes in research on modelling.

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