CONTRASTING PERSPECTIVES BETWEEN THE TEACHER AND STUDENTS: A REFLECTION ON THE LEARNER'S PERSPECTIVE STUDY

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The Learner's Perspective Study (LPS) has provided a vehicle for the work of an international community of classroom researchers. The distinguishing characteristic of the research design for the LPS is the inclusion of complementary accounts of classroom events. Among the different levels of complementarity, the current paper focuses on the various accounts of classroom participants to discusses how we take the teacher and students perspectives together by contrasting and juxtaposing them to explore the co-constructed nature of mathematics classroom. By referring to previous studies in the LPS as well as to a "spin-off" study, the author argues that examining participant perspectives on the same classroom event provides better understanding of and insights into classroom practice with expanding the researcher's perspective.

INTRODUCTION

The ultimate goal of any research on classroom practice is to improve teaching with the enhancement of students' learning. For this goal, various approaches and methodologies have been adapted in the international comparative studies on classrooms. In the earlier years, among others, international comparative studies of mathematics classrooms were conducted based on the classroom observations and interviews with the teachers and students to reveal similarities and differences found in the classroom in different cultural/social background (e.g. Becker et al., 1990; Stevenson & Stigler, 1992). Since mid-1990s, with the advancement of digital technology and facilities available for collecting and analysing classroom data including videos, large-scale international studies of classroom practice have been conducted. In particular, the video component of the Third International Mathematics and Science Study (TIMSS1995 Video Study) was the first attempt ever made to collect and analyse videotapes from the classrooms of national probability samples of teacher at work (Stigler et al., 1999). The study was a breakthrough as a scientific exploration into the classroom practice, showing the feasibility and the potential of applying videotape methodology in wide-scale national and international survey of classroom instructional practice, and it was followed by an extension of the study with the same research design (e.g. Hiebert, et al., 2003).

These international studies have tried to identify coherent sets of actions, and associated attitudes, beliefs and knowledge, that appear to constitute culturally-specific *teacher practices* (Stigler & Hiebert, 1999). On the other hand, we certainly need to hypothesize that there is also a set of actions and associated attitudes, beliefs, and

2022. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 51-66). PME.

knowledge of students that constitute a culturally-specific coherent body of *learner practices*. Research on classroom practice needs to focus on both teacher practices and learner practices as produced by co-construction of those practice through the activities of all the participants.

The Learner's Perspective Study (LPS) was designed to examine the practices of eighth grade mathematics classrooms in a more integrated and comprehensive fashion than had been attempted in previous international studies such as TIMSS Video Study. An essential thesis of the LPS is that international comparative research offers unique opportunities to interrogate established practice, existing theories, and entrenched assumptions (Clarke, 2017; Clarke et al., 2006; Clarke, Keitel, & Shimizu, 2006). The findings of the study included rich descriptions of the practices of participants in eighth grade mathematics classrooms in the participating countries, predominantly from the perspective of the learner, supplemented by the perspectives of the teacher and the researcher (Clarke, Keitel, & Shimizu, 2006; Clarke et al., 2006; Shimizu et al., 2010; Kaur et al., 2013; Leung et al., 2014).

With a reflection of the earlier studies in the LPS, this paper reconsiders how we can take the teacher and students perspectives together in classroom research for understanding complex phenomena of practice in the mathematics classroom. The findings from the research in the LPS and its "spin-off" project are briefly reviewed with an intention of examining the significance of contrasting and juxtaposing the teacher' and students' perspectives. The author argues that contrasting those different perspectives provides us an opportunity to explore the co-constructed nature of quality instruction in mathematics and that an integration of those different perspectives can offer better understanding of complex phenomena in the mathematics classroom as well as implications for improving classroom instruction.

LEARNING FROM THE FINDINGS OF TIMSS VIDEO STUDY

The TIMSS1995 Video Study collected and analysed videotapes from the classrooms of national probability samples of teacher at work (Stigler & Hiebert, 1999; Stigler et al., 1999). Focusing on the actions of teachers, it has provided a rich source of information regarding what went on inside eighth-grade mathematics classes in Germany, Japan and the U.S. with certain contrasts among three countries. In addition, objective observational measures of classroom instruction were developed to serve as valid quantitative indicators, at a national level, of teaching practices in the three countries. It was interesting to learn that the findings of the study included aspects of mathematics lessons as identified with a strong resemblance between Germany and the U.S. with Japan looking differently (Shimizu, 1999). Before looking back the findings from the study, an episode is provided.

The Didactic Triangle Revisited

Stigler and Hiebert (1999) reported on a meeting in which 'distinguished researchers and educators from Germany, Japan, and the United States' (p. 25) were invited to

review and discuss the classroom recordings made for the TIMSS video study. I had an opportunity to participate in this particular meeting as one of the Japanese consultants. Stigler and Hiebert (1999) noted that one participant shared his reflections after viewing video recordings made in Japanese, German and US mathematics classes as follows:

> In the Japanese lessons, there is the mathematics on one hand, and the students on the other. The students engage with the mathematics, and the teacher mediates the relationship between the two. In Germany, there is the mathematics as well, but the teacher owns the mathematics and parcels it out to students as he sees fit, giving facts and explanations at just the right time. In the U.S. lessons, there are the students and there is the teacher. I have trouble finding the mathematics; I just see interactions between students and teachers. (pp. 25-26)

The reader may recognize the didactic triangle in which student, teacher, and content form the vertices (or nodes) of a triangle that is the classical trivium used to conceptualize teaching and learning in mathematics classrooms in three countries. By quoting the episode above, Goodchild and Sriraman (2012) suggest that the observations made over a decade ago are still relevant today.

There also appears to be some differences among the lessons coded in the study. For example, a Japanese classroom would never have someone come into the lesson while it was under way. It would also be very rare for a lesson to be interrupted by a public announcement in the school. These events certainly took place in the lessons coded in the U.S. classrooms and to some extent in German classrooms. As a natural consequence of such events in the classroom, students' views on a lesson should be quite different among three countries. Stigler and Hiebert (1999) provide another episode from the meeting of TIMSS Video Study where a Japanese researcher incredulously asking about the moment of sudden interruption by the public announcement in the middle of the lesson (Stigler and Hiebert, 1999, pp. 55–56).

Although we could reconceptualize teaching and learning in mathematics classrooms by extending the model of triangle to a model of tetrahedron by adding one more vertex or node (e.g. technology or artifact (Goodchild & Sriraman, 2012), I reflect on the episode of watching the videos from mathematics classrooms with the recognition that the typical framing of the didactic triangle is narrower than it should be and that it should be broadened to view classroom activities from a more social/cultural perspective (Schoenfeld, 2012).

A Japanese Perspective on the Findings from the TIMSS Video Study

When I watched the videos from three countries, lessons from each country seemed quite differently. German teachers, for example, seemed to teach mathematics in a "one-to one" question and answer mode with an authority of mathematics behind them. On the other hand, Japanese teachers seemed to behave keeping their position at somewhere in between mathematics and students without authority of mathematics

behind them. Part of my early impressions seemed to be confirmed by the objective observational measures of classroom instruction developed by the TIMSS Video. I thought that we need to have a framework with which we describe co-constructed nature of classroom activities by integrating the teacher's and the learner's perspective.

It should be noted that the focus of the TIMSS Video Study was basically on the actions of teachers. A more complete description of the practice of mathematics classrooms would be obtained if learner's perspective was incorporated into the research design. Student interpretations of mathematics classrooms are necessary for a detailed account of classroom practice.

The analysis of video data collected in the TIMSS Video Study, as reported by Stigler and Hiebert (1999), centred on the proposition that the teaching practice of a nation could be explained to a significant extent by the teacher's adherence to a culturally-based "teacher script" at least in the case of mathematics. Central to the identification of these cultural scripts for teaching were the Lesson Patterns reported by Stigler and Hiebert (1999) for Germany, Japan and the USA. Their contention was that at the level of the lesson, teaching in each of the three countries could be described by a "simple, common pattern" (Stigler & Hiebert, 1999, p. 82).

In the TIMSS Video Study, using the sub-sample of 90 lessons coded by the Math Content Group, explicit linking was coded that the teacher wants students to understand in relation to each other (Stigler et al., 1999, p.117). Two kinds of linking were coded. Linking across lessons and linking within a single lesson. In the study, linking was defined as an explicit verbal reference by the teacher to ideas or events from another lesson or part of the lesson. The reference had to be concrete (i.e., referring to a particular time, not to some general idea); and the reference had to be related to the current activity. The results of this coding show a resemblance between Germany and the U.S. with Japan looking differently. The highest incidence of both kinds of linking, across lessons and within lessons, was found in Japan. Indeed, Japanese teachers linked across lessons significantly more than teachers of German lessons, and made significantly more linkings within lessons than teachers of both German and U.S. lessons.

Table 1 shows percentages of lessons that include explicit linking by the teacher to ideas or events in a different lesson, and to ideas or events in the current lesson (Stigler et al., 1999, p.118).

	Germany	United States	Japan
In a different lesson	55	70	91
In a current lesson	41	40	96

Table 1: Linking to ideas or events in a different/current lesson (%)

Japanese teachers usually plan a lesson as a part of a unit, a sequence of several lessons. This means that each lesson in a unit has a different purpose for attaining the goals of the unit, depending on the phase in the unit. At an introductory phase in a unit, for

example, the lesson will be a concept-oriented for introducing a new idea or concept. On the other hand, at the summative phase, the lesson may be more practice or skill oriented. Thus, lesson script for lessons at each phase can be slightly different from each other even in the same country. Collecting and analysing data over a lesson sequence are needed for clarifying a diversity of lessons at different phases in the unit.

THE LEANER'S PERSPECTIVE STUDY: AN INCLUSIVE METHODOLOGY

The methodology of survey-style TIMSS Video Study can be complemented by research techniques intended to give prominence to the perspective of the learner (Clarke, 2001). Among the methodologically most interesting aspects of the LPS has been the collaborative negotiation of the study design, the method of data generation, the general and local analyses, and the process whereby the various complementary accounts can be integrated into a rich and useful portrayal of mathematics classrooms internationally. Inclusivity as a methodological principle was as pervasive in the LPS research design as complementarity (Clarke et al., 2006; Williams, 2022). The inclination to integrate rather than segregate is at the heart of the LPS, since it was intended from the project's inception that any documented differences in classroom practice be interpreted as local solutions to classroom situations and, as such, be viewed as complementary rather than necessarily oppositional alternatives, within a broadly international pedagogy, from which teachers in different countries might choose to draw in light of local contingencies. International comparative classroom research is viewed as the exploration of similarity and difference in order that our understanding of what is possible in mathematics classrooms can be expanded by consideration of what constitutes good practice in culturally diverse settings.

Data Generation in the LPS

The TIMSS Video Study and the LPS differ in their breadth of data capture, and thus in the number and nature of research questions they support. Data generation in the LPS used a three-camera approach (Teacher camera, Student camera, Whole Class camera) that included the onsite mixing of the Teacher and Student camera images into a picture-in-picture video record that was then used in post-lesson interviews to stimulate participant reconstructive accounts of classroom events. These video records were supplemented by student written material, and by test and questionnaire data from students and the teacher. These data were collected for sequences of at least ten consecutive lessons occurring in the "well-taught" eighth grade mathematics classrooms of teachers in participating countries and regions. Each participating country used the same research design to collect videotaped classroom data for at least ten consecutive mathematics lessons and post-lesson video-stimulated interviews with at least twenty students in each of three participating 8th grade classrooms. The three mathematics teachers in each country were identified for their locally-defined 'teaching competence'. In the key element of the post-lesson student interviews, in which a picture-in-picture video record was used as stimulus for student

reconstructions of classroom events, students were given control of the video replay and asked to identify and comment upon classroom events of personal importance. Teachers were also interviewed using a similar protocol.

Contrasting Perceptions of Lesson Events between the Teacher and Students

In this section, the findings from previous study (Shimizu, 2006a) is reconsidered. The methodology employed in the LPS allows participants to identify those events that were significant to them. Namely, in the post-lesson video-stimulated interviews, which occurred on the same day as the relevant lesson, the teacher and the students were asked to identify and comment upon classroom events of personal importance (Clarke, 2006, See Table 2 for the examples of prompt).

Prompt Four: Here is the remote controller for the video-player. Do you understand how it works? (Allow time for a short familiarization with the control). I would like you to comment on the videotape for me. You do not need to comment on all of the lessons. Fast-forward the videotape until you find sections of the lesson that you think were important. Play these sections at normal speed and describe for me what you were doing, thinking and feeling during each of these videotape sequences. You can comment while the videotape is playing, but pause the tape if there is something that you want to talk about in detail.
Prompt Seven: Would you describe that lesson as a good one for you? What has to happen for you to feel that a lesson was a "good" lesson? Did you achieve your goals? What are the important things you should learn in a mathematics lesson?

Table 2: Selected Prompts in Post-lesson Video-stimulated Interviews

The analysis of LPS data has revealed both patterns and variations in the ways in which the teacher and students perceived the lesson. The LPS design provides the researchers with the opportunity to explore the commonalities and differences in perceptions of mathematics lessons by teachers and students by means of juxtaposing their reconstructive accounts of the classroom. Although video-stimulated interviews have been used in other studies to examine teachers' and students' ideas and beliefs, earlier studies have not focused on contrasting teacher and student perceptions of the same lesson they have just experienced.

Perceptions of lesson events

A key result of analyses conducted on the LPS data has been the suggestion that "the lesson" is unsuitable as a unit of comparative analysis of classroom practice and that the "lesson event" (those recurrent activities from which lessons are constructed) is more suitable for the purposes of comparative analysis, corresponding more closely to the decisions made by each teacher regarding the structure of any particular lesson, and being more effective in distinguishing between the practices of particular classrooms (Clarke et al., 2008). In the classroom, teacher and student practices can be conceived as being in a mutually supportive relationship. This is not to presume that the teacher and students have the same goals or even that they perceive lesson events in the same

way. By contrasting their perceptions of particular lesson events, it is possible to identify the discrepancies between the teacher and the students in their perceptions of classroom practice. These discrepancies can help us to understand each participant's contribution to the activities of the mathematics classroom. Also, through the analysis of their perceptions of lesson events and the associated values held by the teacher and the students, the influence of both perceptions and values on learning mathematics can be explored.

Valuing Matome

The LPS approach of juxtaposing teacher and student perceptions of mathematics lessons has the capacity to bring out the symbiotic nature of the teacher's and the students' contributions to the teaching and learning process. Japanese teachers, for instance, often try to organize an entire lesson around a few problems with a focus on the students' alternative solutions to them. In this "structured problem solving" approach, the "summing up" (Matome) phase is indispensable to a successful lesson (Shimizu, 2006b). Students' solutions are shared and pulled together during the phase in light of the goals of the lesson. While Japanese teachers may devote considerable effort to planning and structuring their lessons around the "climax" of the lesson, the structure may not be perceived by the students or may be perceived differently.

Teacher	Student 1	Student 2
3:50		
		6:00
9:29	9:23	
14:00	14:25	14:22
	16:00	
16:50		
		17:30
		24:44
	27:09	
28:00		
29:30		
32:26		
33:30		
	34:30	
	37:45	37:26
		40:00
42:50	42:55	43:02
	45:20	

Table 3. Elements in the lessons felt to be significant: J1-L5

The comparison displayed in Table 3 of perceived events felt to be significant by the teacher and the students clearly shows that there are agreements and discrepancies

between them on what is important in mathematics lessons and what is not. This analysis raises the issue of the possible influences of such agreements and discrepancies in perceptions of lesson events between teacher and students on students learning and teacher planning. The result illustrated in Table 3 also raises the need to attend to the meanings constructed by the teacher and the students as the participants in the same lesson. By examining the post-lesson interview data closely, any differences can be understood as discrepancies between the teacher's and the students' perceptions of classroom events.

The purpose of this section has been to highlight the differences between the perceptions of what constitutes a significant lesson event, as held by the teacher, the student(s), and (implicitly) the researcher. In the next section, we see leaners' perspective on what constitute a good lesson in mathematics compared with the teacher's perspective.

"Good" Mathematics Lessons from the Learner's Perspective

The Japanese term for teacher's behaviour in the classroom related teaching is "Gakushushido" that literally means "Guidance of Learning". It should be noted that the term "Gakushushido" is used as one word. Here we see a tradition of recognizing that teaching and learning are interdependent activities within a classroom setting and that classroom practices should be studies as such. If we focus on the teaching and learning as interdependent activities, we need to look into what participants, both the teacher and students, value in the classroom and how they perceive the lesson with associated values embedded in activities in classroom. In the following part of this paper, associated values attached to a "good" lesson are explored from the learner's perspective with a reference to the findings of Shimizu (2009).

Data collection for the study was conducted at three public junior high schools in Tokyo. All of the three mathematics teachers had experience of teaching mathematics more than twenty years. Two of them were writers of mathematics textbooks widely and commercially available in Japan. The criteria for identifying them reflected a locally-defined "teaching competence". Namely, the three mathematics teachers were identified for their active roles in the study groups of teachers in Tokyo, and the recognition in the community of mathematics teachers as a teacher who teach mathematics in excellent ways.

During the data collection that followed the LPS methodology, semi-structured video-stimulated interviews with the students occurred on the same day as the relevant lesson. In each lesson two students sitting next to each other were selected as "focus students" for that particular lesson. These students were interviewed individually after the lesson. Among three of them, two teachers were interviewed three times, roughly once a week, during the period of videotaping and one teacher was interviewed twice. The post-lesson interviews with sixty students, twenty students from each of three schools, were transcribed and subjected to the analysis. For the analysis of the interview data, a

coding system was developed (Shimizu, 2009). Table 4 shows the description of each coding category with an illuminating example of students' response to Prompt Seven on good lessons.

Code	Description	Example
Understanding/ Thinking	Those responses that refer to their understanding and thinking in the classroom	*I can understand the topics to be learned*. (J2-03M)
Presentation	Those responses that refer to presenting their ideas in the classroom	*I can present my solution on the blackboard*. (J3-07I)
Classmates	Those responses that refer to other students' presentations and explanations	*There is an opportunity of listening to classmates.* (J1-09S)
Whole class discussion	Those responses that refer to the whole class discussion	*We all in the classroom exchange ideas actively*. (J1-06U)
Teacher	Those responses that refer to teacher's explanation	*I listen to teacher's final talk. I always take a note and check a point*. (J3-06S)
Other	Other responses	*By preview the topic at home, I attend the lesson with a preparation*. (J3-09K)

Table 4: The Description and Examples of Categories for Coding

The first five categories and one additional category, "other", had appeared from the initial analysis of transcriptions from one of the three schools (labelled as "J1"). Then all the students' response to the prompt were classified into six categories for coding by the author and research assistant. When discrepancies in coding between coders appeared, they were resolved by discussions. It should be noted that these codes do not constitute a mutually exclusive coding system.

Codes	Responses
Understanding/Thinking	27 (45.0) *
Presentation	10(16.7)
Classmates	4(6.7)
Whole class discussion	16(26.7)
Teacher	10(16.7)
Other	10(16.7)

Table 5: Students' Response to the Prompt Seven in Video-stimulated Interview * Numbers in parentheses denote the percentage of each category.

Table 5 shows the result of the analysis as a whole of students' response to the prompt seven in video-stimulated interview. It is noted that the percentages do not add up to 100, because the coding system is not mutually exclusive.

As Table 5 shows, nearly half of the students interviewed (45.0%) described "understanding" or "thinking" to be happened in a "good" lesson. As the example in the Table 3, "I can understand the topics to be learned", illustrates, the students in this category regarded a lesson as "good" one if he can have a clear understanding of mathematical topic taught. Those students who mentioned to "understanding/thinking" seemed to attach values directly to the importance of their own understanding and thinking in the lesson. Some students in this category also referred to other activities in the classroom. MANA, a student from J2, for example, mentioned to teacher's explanation as the object of understanding: "*Even if your answer is wrong…to be able to understand what the teacher explained. If that happens, I think//that it was a good lesson*." Roughly a quarter of the students (26.7%) identified "whole class discussion" as the "component" of a "good" lesson. Then, two categories "presentation" and "teacher" follow the "whole class discussion". Only four students (6.7%) explicitly described the activities related to their "classmates" in mathematics classroom.

There is a difference between the first four categories and "teacher" category in terms of types of the activities referred by the students. That is, first four categories are directly related to students' own learning activity, while "teacher" category is related to both students' learning and teacher's instructional activities. The example of "teacher" category in Table 4, for instance, is the one that referred to "teacher's final talk" (highlighting and summarizing the main point), taking a note, and checking the key point of lesson. This example illustrates that teacher's instructional activities can also be a component of a "good" lesson.

Relating teacher and learners perspectives

To understand the characteristics of a "good" mathematics lesson, a detailed analysis was also conducted with an eye of relating students' responses to Prompt Seven to those by the teacher who taught each classroom.

Suzu, a student from the school J3, for example, responded to the questions, "When you think it's a good class?" and "What should happen in the class?", as follows.

01. INT: When you think it's a good class,
02. SUZU: Yes.
03. INT: What should happen in the class?

….

04. INT: Do you have anything that you think is a good class?
05. SUZU: I can present my answer, and then listen to my friend's way as well,
06. INT: Yeah?
07. SUZU: The teacher's final comment, or answer,

08. INT: Yeah?

09. SUZU: Listen to it carefully, and to make a good note from it.

The student clearly mentioned to the importance of presenting his answer to the problem to the class and of listing to his classmates' method to solve the same problem. He also referred to listening to "*The teacher's final comment, or answer*" carefully and of "*making a good note from it*". These comments suggest that, students' views on a "good" lesson are shaped through the classroom practices co-constructed by the teacher and the students. If the teacher keeps summarizing and highlighting the main points of the lesson as a daily routine, the students may become aware of the importance of the particular lesson event which tends to come on the final phase of lesson in the form of teacher's public talk together with time for note-taking. Then he or she will "listen to it carefully" and try to "make a good note from it". The teacher's summarizing and highlighting, in turn, have to rely upon students' understanding of the mathematical topic taught to be summarized and highlighted.

Teachers' comments on what constitutes a "good" lesson also suggest the co-constructed nature of a "good" mathematics lesson. Mr. K, the teacher of JP3, in the second interview, for example, mentioned to the importance of students thinking on alternative solutions and their understanding in a "good" lesson as follows: "*practically, what I think is that the students think in many ways...and they understand it well ...The students can ask me or each other where they can't understand*." Here, Mr. K expressed that he valued to have his students think in many ways and understand the topic well through the interaction with him and classmates.

The comment suggests that in a "good" lesson teacher and student practices can be conceived as being in a mutually supportive relationship. This is not to presume that teacher and students have the same goals or values, or even that they perceive the importance of particular classroom activities in the same way. The analysis described above suggests that a "good" lesson is a co-constructed classroom practices by the teacher and the students.

Unpacking the Technical Vocabulary of Japanese Mathematics Teachers

In this section, the findings of recent study on Japanese lexicon (Shimizu, et al., 2021) is shared and discussed. The study is as a part of the International Lexicon Project which is a "spin-off" project of the LPS to document and compare the naming systems of mathematics teachers on phenomena related to teaching in ten countries: Australia, Chile, China, the Czech Republic, Finland, France, Germany, Japan, South Korea, and the USA (Mesiti, et al., 2021). A study of teachers' naming systems is seemingly related to teachers' perspective but I argue that Japanese lexicon includes terms and phrases based on the teacher's reflections on learners' perspective.

The Japanese tradition of Lesson Study has created a teaching community in which observation and discussion of teaching are integral parts of professional practice with particular lexical terms of specific significance. The study aims to explore the

constituent elements and structural characteristics of the Japanese lexicon. Documenting and comparing teaching vocabulary will provide an opportunity for us to understand the nature of teaching and facilitates international comparative research (Mesiti, et al., 2021).

In the Lexicon Project, local teams of researchers and experienced teachers in each country, classify a common set of video records of mathematics lessons, drawn from all participating countries, in order to identify those terms in their local language that in combination constitute the national pedagogical lexicon, by which we discuss, analyse, reflect upon and theorize about the mathematics classroom.

Data generation was undertaken simultaneously in Japan and other participating countries. Each participating team contributed a videotaped lesson which was included in a stimulus package. This stimulus material was viewed by team members in each country to identify the well-established pedagogical terms or phrases of used in the communities. These terms are supplemented with the clearest possible operational definitions in English, describing both the form and function of each named term. The combined classroom video material from the participating countries then becomes a source of video exemplars of each of the named terms.

The Japanese Lexicon team members took part in the video viewing process to identify the terms and phrases used by Japanese mathematics teachers. The Japanese team consisted of two researchers, two experienced teachers and two doctoral students. The Japanese Lexicon was constructed by watching video material, time-stamped transcripts and classroom supporting material for one lesson of mathematics at an eighth grade classroom in a public school in Ibaraki prefecture.

An electronic survey was conducted for a national validation to examine how familiar the terms were for the mathematics teacher in Japan. Overall, the terms were very familiar to the respondents, although some terms were somewhat less frequently in use. A total of 70 terms or phrases are identified as the Japanese Lexicon (Shimizu, et al., 2021). Characteristic of the relationships among terms within the Japanese Lexicon were the multi-layered intentions of Japanese teachers as these were inferred by the Japanese team and represented in the constituent elements of the Lexicon.

For example, the term “hatsumon”, asking a key thought-provoking question, has a specific meaning within the system of terms and phrases related to teaching through problem solving. Teacher’s question with a particular intention should be effective in relation to the goal of today’s lesson and to the status of students’ understanding. It is considered as embedded in the system of particular style of teaching mathematics. In planning a lesson, for instance, the teacher anticipates alternative solutions to the problem and identifies possible students’ misunderstanding and mistakes. Thus, an enactment of “hatsumon” cannot be planned without the teacher’s views on and thinking about what students think and how they solve the problem.

Another example is "kikan-shido" which literally means "instruction between desks". This particular term refers to the teacher's behaviour during students' problem solving by their own. However, an important aspect of "kikan-shido" is in that the teacher scans students' work purposefully and uses the knowledge gained to select students to present solutions to the classmates (Hino, 2006). Also, the term has "evolved" to similar but different term "kikan-shien" that means "supports between desks" (Shimizu, et al., 2021). The terms "kikan-shido" and "kikan-shien" are constituted so as to incorporate what students expected to do and what the teacher anticipate and expect in students learning. In this sense, the Japanese Lexicon includes terms and phrases in which learners' perspective is "amalgamated" and integrated into the teacher's views on their teaching.

This study provides insights into the naming system employed by mathematics teachers in relation to their classroom practice by documenting and interpreting the constructs that are well-known and used among them. Documenting the Japanese Lexicon reveals that teachers' use of terms and phrases related teaching is profoundly influenced by the learner's perspective amalgamated with teacher's perspective.

DISCUSSION

Complementary Accounts of Classroom Events

The distinguishing characteristic of the research design for the LPS is the inclusion of complementary accounts of classroom events (Clarke, 2001; Ellerton, 2008; Williams, 2022). In the research in the LPS, four levels of complementary accounts can be identified (Clarke, et al., 2006): (a) at the level of data, the accounts of the various classroom participants are juxtaposed; (b) at the level of primary interpretation, complementary interpretations are developed by the research team from the various data sources related to particular incidents, settings, or individuals; (c) at the level of theoretical framework, complementary analyses are generated from a common data set through the application by different members of the research team of distinct analytical frameworks; and (d) at the level of culture, complementary characterizations of practice and meaning are constructed for the classrooms in each culture by the researchers from each culture and these characterizations can then be compared and any similarities or differences identified for further analysis.

Given the complementarity in the studies discussed in this paper is directly related to the level (a) and (b), the discussion can be examined and expanded further at the level of both theoretical framework and culture. Williams (2022) draws attention to the potential for the Complementary Accounts Methodology to contribute to support for researchers employing diverse theoretical frameworks. The "rich" data sets offer an opportunity for researchers not only to examine the nature of the classroom practice in details but also to expand researchers' perspective by interrogating their own implicit assumptions about classroom practices.

Significance of Generating Integrated Data Sets

With the reflection on the previous studies conducted in the LPS, it is safe to say that the strength of the project resides in the generation of integrated data sets of classroom practices. The collection of integrated data sets in the LPS enables the researchers to examine the questions related to the teacher and students practices and interrelatedness or mutually supportive relationship between them. As the name of the project suggests, the inclusion of students' interviews with a direct connection to their specific classroom activity they just experienced is crucial to the methodology. Furthermore, connecting the learner's perspective with the teacher's perspective facilitates our understanding of the complexity of classroom practices.

Focusing on the frame for comparisons in the LPS, Schoenfeld (2022) points out as "when people think about teaching, they think about the teacher; it's easy for the learner to get lost". The original idea of having the focus on "the learner's perspective" derived from the accumulation of David Clark' works on classroom practice and his ideas (Clarke, 2001) as well as our reflections on the significance and limitations of earlier studies of classroom practice such as TIMSS Video Study.

The characteristic of the research design of the LPS can be found in its focus on the practice of competent teachers. Instead of gathering statistically representative sample for finding an "average" lesson, the LPS methodology generates integrated data sets from the classroom taught by experience teachers. This research design raises researchers an opportunity to examine the cultural specificity of "good" classroom practice and opens the door to thinking about improvements of teaching.

CONCLUDING REMARKS

Accumulated international comparisons under the umbrella of the LPS have made clear how culturally-situated are the practices of classrooms around the world, and the extent to which students are collaborators with the teacher, complicit in the development and enactment of patterns of participation that reflect individual, societal and cultural priorities and associated value systems.

I have discussed how we take the teacher and students' perspectives together for understanding the classroom practice in mathematics by referring to the previous studies in the LPS and a related project. Contrasting and juxtaposing the teacher and students' perspectives provide us an opportunity to explore the nature of quality instruction in mathematics. The complex phenomena in the mathematics classroom can be understood better by taking both the teacher and students perspectives into considerations in research on classroom practice that describes both similarities and differences in participants' perspectives on the same classroom event.

Acknowledgement

I would like to express my sincere gratitude to co-founders of the project, Professor David Clarke and Professor Christine Keitel, from whom I have learned much about the significance and excitements of international collaborations in educational research. Also, I would like to express my gratitude to all of the colleagues in the

community of the Learner's Perspective Study. This study was supported by Grants-in-Aid for Scientific Research (B), Grant Number JP19KK0056, by the Japan Society for the Promotion of Science.

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