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Nousiainen, Maija

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Pre-Service Teachers' Declarative Knowledge of Wave-Particle Dualism of Electrons and Photons: Finding Lexicons by Using Network Analysis

Maija Nousiainen * and **Ismo T. Koponen**

Department of Physics, University of Helsinki, 00014 Helsinki, Finland; ismo.koponen@helsinki.fi
* Correspondence: maija.nousiainen@helsinki.fi

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Abstract: Learning the wave-particle dualism of electrons and photons plays a central role in understanding quantum physics. Teaching it requires that the teacher is fluent in using abstract and uncommon terms. We inspect the lexical structures of pre-service teachers' declarative knowledge about the wave-particle dualism of electrons and photons in the context of double-slit interference. The declarative knowledge is analyzed in the form of a lexical network of terms. We focus on lexical structures because, in teaching and learning, knowledge is communicated mostly through lexical structures, i.e., by speaking and writing. Using the lexical networks, we construct the lexicons used by pre-service teachers to express their knowledge of electrons and photons in the context of double-slit interference. The lexicons consist of eight different key terms, each representing a set of closely-related or synonymous terms. The lexicons by 14 pre-service teachers reveal remarkable variation and differences, and are strongly context-dependent. We also analyzed lexicons corresponding to two didactically-oriented research articles on the same topic and found that they also differ. Lexicons paralleling both texts are found among the pre-service teachers' lexicons. However, only some of the pre-service teachers use such rich vocabulary as would indicate multi-faceted understanding of quantum entities.

Keywords: declarative knowledge; lexical network; networks; pre-service teachers; wave-particle dualism

1. Introduction

Recent science education research has placed a lot of attention on learning the wave-particle dualism of quantum entities [1–5]. This is partly due to the central role of wave-particle dualism in learning quantum physics, but also because it concerns the very basic phenomenon of how a learner constructs mental models of a new ontological category (i.e., categories of existing entities) in the absence of direct perceptions or experiences of that category. First, it demands a shift in the ontological nature of the basic entities of particles and waves (or fields) as they are familiar from classical physics. Second, it demands an epistemological shift (i.e., change in our conceptions of the justification of our knowledge claims) regarding causal determination and the probabilistic determination of events [1,2,6]. The research concerning learning wave-particle dualism has recently been comprehensively summarized in studies by [1–3,6] whose main findings are not reiterated here; only the aspects relevant to the present study will be discussed in what follows.

Research on learning wave-particle dualism, or more generally, on learning quantum physics, has focused on discovering students' mental models, which are taken as structures of mind that allow cognitive simulation of reality, for example in making predictions, providing explanations and evaluating outcomes [1–3]. Many research approaches attempt to reveal students' mental models using interviews and their interpretative analysis (see e.g., [1] for detailed discussions). Such research approaches have revealed certain apparently robust mental models that recur in several situations and contexts, and thus provide a basis for understanding the dynamics of learning when students' conceptions change. However, the mental models found in this manner appear to be sensitively context-dependent [1,2,5], thus demonstrating plurality instead of parsimony.

Some researchers have noted that attempts to discover mental models necessarily need to approach their targets of investigation indirectly; through the verbal and linguistic expression of students, which only indirectly provides information about the underlying mental models [2,7]. The mental models may thus not be as directly accessible through students' expressions as assumed by interpretative analyses of students' explanations, in the form of declarative (i.e., written or spoken) knowledge [8]. Therefore, some recent research in science education has turned to investigating students' linguistic expression of their knowledge [7,9–13]. This "linguistic" turn is supported by cognitive linguistics, which provides support for making a sharper distinction between accessible targets of research and their underlying deeper structures. For example, Evans [14,15] has made a clear distinction between lexical, semantic and conceptual levels, positing the cognitive structures at the deepest level of cognitive simulations. That deepest level of conceptual (and mental) models is, however, inaccessible, and the accessible lexical level only touches on it [14,15]. In such views, the underlying mental models may well be parsimonious but lexical expression (i.e., in level language) provides a plurality of context-dependent expressions instead of parsimony.

In the case of wave-particle dualism, the role of lexical, declarative knowledge in learning is particularly clear, because learners must acquire the meaning of relevant words and terms, and the admissible ways to use them, from teaching rather than from experience or perception. Teaching and learning such scientific knowledge involves essentially learning the vocabulary and syntax of terms and their relations; to learn the lexicons of science. Lexicons, as systems of terms and words, unlike conceptual or mental models, are directly accessible in students' writing, speaking and other ways of explicating their knowledge [14–18].

We analyze here university students' (pre-service physics teachers') knowledge of the wave-particle dualism of electrons and photons as quantum entities by analyzing the lexicons they use in writing about electrons and photons in the context of the double-slit interference phenomenon. The specific phenomenon we study here is the double-slit experiment with dim light (single photons) and with weak electron beam (single electrons). In both experiments, an interference pattern emerges, but gradually as consisting of single hits. In such an experiment the dualistic particle and wave nature of light and electrons becomes visible. This is the same context as discussed in several previous studies [1,3–6]. In science education, there are two well-known but differently biased approaches to learning wave-particle dualism. Our stance is that we need to understand how the different views on wave-particle dualism in the context of the double-slit experiment are related in terms of lexicons, and to what extent the terms of the lexicons overlap to provide a basis to communicate different interpretations and ideas. The empirical sample we analyze here consists of pre-service teachers' written explanations of the formation of interference patterns in the context of the double-slit experiment. In their explanations, the pre-service teachers use several sub-contexts, and represent their relations as graphs. Based on that data, we construct the lexical network that each pre-service teacher attaches to electrons and photons. The lexicons, which condense the relevant and nearly synonymous terms found in lexical networks, are then formed by applying network analysis [9–11]. The most basic lexicon needed to discuss electrons and photons can be constructed from terms that can be categorized into eight different property classes which are relevant in the context of wave-particle dualism. Each of the property classes can be considered a dimension of the lexicon, thus forming an eight-dimensional lexicon.

The results of the study show that pre-service teachers use very different lexicons so that in some cases certain dimensions are underrepresented or missing altogether. In such cases, with the given lexicons, communication about the wave-particle dualism is limited to only those properties for which the lexicons overlap and are shared by the students. We refer to such shared lexicons, which consist of *L* shared dimensions out of eight, as *L*-lexicons. The most common *L*-lexicons shared by the pre-service teachers can be classified in seven to eight classes. Some of the classes resemble the classes found in qualitative, interpretative studies attempting to construct pre-service teachers' mental models, but, in general, exhibit greater diversity than the mental models reported in research literature.

Finally, the implications of the lexicons and diversity of lexicons on teaching and learning about wave-particle dualism are discussed. The most important conclusions concern the clear inadequacy of some lexicons in the context of wave-particle dualism of electrons and photons, thus preventing the explication of key ideas. A consequence of the restricted lexicons is also the rarity of sufficiently rich *L*-lexicons, needed in mutual communication (either written or spoken) of the ideas. It is important for teacher educators to focus on such deficiencies deriving from the vocabulary and use of terms, and direct attention to the proper use of scientific language and terminology in teaching. The current study provides research tools based on analysis of lexicons to help in monitoring the use of scientific language.

2. Theoretical Underpinnings: Lexicons of Declarative Knowledge

In the philosophy of science, Kuhn [16] proposed that scientific knowledge is essentially built in the form of lexical networks, or a lexicon of terms, in which the connections between the terms derive from contextualized instances of how the terms are used [16]. This "linguistic" move was taken by Kuhn as an attempt to clarify his conception of paradigms, with the purpose of replacing the notion of paradigms with the notion of lexicons as a key component of his conception of science [17,18]. For Kuhn, lexical networks were interrelated systems of terms, which must be learned and acquired together; furthermore, the lexicons also define the scientific communities, because the individual members of the communities must share substantial parts of the lexicons to be able to communicate and provide identity to the community [16–18]. Such overlapping lexicons of science are shared lexicons, which contain the shared terms and their meanings.

The focus on lexical networks also finds support from the research on how the meaning of words is learned. It has been pointed out that learning the meaning of words involves lexical networks made of names and words and the semantic connections between them, which build upon the conceptual system but are different from it. Thus, the conceptual system is not directly accessible in communication, in the form of declarative knowledge (i.e., through e.g., speaking or writing) but lexical networks and semantic connections provide access points to it [14,15]. In this view, the three levels of knowledge—lexical, semantic and conceptual—are understood as being distinct but related.

Students' declarative knowledge is often approached by examining the structure of networks formed by terms that stand for concepts, often referred as networks of words [7,12,19–21]. Some researchers maintain that representations encoded by language can be equated with semantic meanings [22], while some others see these linguistic and semantic structures as being different [14]. From the viewpoint of lexicons and the role of lexical networks [14,16] as adopted here, several studies that have framed their targets as students' semantic networks have in fact focused on lexical networks rather than on semantic networks [7,12,13,19–21]. The difference between the lexical and semantic networks, however, is not crucial here, and there are different views on how lexical and semantic networks are related (see e.g., [14,15,22]). Here, for reasons of clarity and not to imply more than is securely contained in the data sample, we have chosen to retain the distinction and frame our target as the analysis of lexical rather than semantic networks. The lexical network approach to declarative knowledge provides practical tools to develop well-defined operationalizations for the structure of students' lexicons of declarative knowledge by using network analysis to measure the connectivity of nodes in the lexical networks. We focus here on pre-service teachers' declarative knowledge of electron and photons as quantum entities, simply by focusing on how they relate different terms and words when they express in writing their knowledge of how these entities behave in different contexts.

The lexical networks provide us direct access to construct lexicons, which condense the key terms found in lexical networks. Since many expressions used by students can be taken as nearly synonymous, the relevant terms can be brought under a key term, which describes many similar terms that pre-service teachers use for electrons and photons in the context of the double-slit experiment. Such lexicons can be also compared indirectly with the mental models reported in literature, as sets of terms that are needed to express and communicate ideas about a given type of mental model; lexicons are not mental models themselves but relate to mental models as vehicles to express knowledge.

3. Research Design and Sample

The participants (N = 14) in the study are pre-service teachers who are studying physics to obtain the license to teach physics in upper secondary level. All have a background of basic physics studies including elementary and first intermediate level courses in quantum physics. The data was collected as a part of the physics teacher preparation course (intermediate-level course during the third or fourth year of studies), which focuses on the organization of physics content knowledge for teaching purposes. The context of the course was introductory quantum physics at the level taught in upper secondary school.

3.1. Tasks and Their Contexts

The basic data of this study comes from two separate but connected tasks, in which pre-service teachers expressed their understanding of how the terms "electron" and "photon" are used in describing two parallel phenomena (contexts) and their properties. The contexts were: double-slit experiment with single photons and double-slit experiment with electrons. The pre-service teachers did not carry out the experiments themselves, but were asked to explain the phenomena in these well-known experiments. In both experiments, we can detect an interference pattern which consists of single hits. In the case of photons, the experiment demonstrates the particle nature of light, and for electrons, the experiment demonstrates the wave nature of electrons. The pre-service teachers had previously read research articles on both of these phenomena and their task was to express how the most relevant aspects of these phenomena (from the point of view of physics) could be expressed in upper secondary school teaching. The instructions for completing the task were designed so that the pre-service teachers needed to write down an explanation for the basic purpose, the findings of the experiments and the argumentation to support the findings. The length of the texts were usually 1–2 pages. Both contexts consisted of several sub-contexts K related to certain specific aspects of the experiment, measurement, or description or interpretation of data. The pre-service teachers also drew graphs to show how different sub-contexts are related within each main context (task) and between them. In the graphs, the pre-service teachers indicated how they thought the sub-contexts K are related.

3.2. Data Collection and Handling

Data was collected in the form of written reports. Both reports were completed prior to a weekly discussion session about the topic and returned in advance. The written reports consisted of eight sub-contexts K on average (but ranging from 4 to 21), each discussing relevant aspects and explanations.

Voluntary participation, informed consent, and anonymity of the participants were ensured during the research process. In collecting the data, the pre-service teachers were asked for permission to use their written reports and knowledge integration maps as research data. Consent forms, which explained the purpose of the research, were used to obtain their permission. The pre-service teachers were also given the option not to participate in the research. The pre-service teachers were given the opportunity to ask the researchers about the study, and received detailed answers to their questions. All data was stored in encrypted external storage devices and only accessible to the researchers. All researchers have agreed to follow the regulations conforming to the national laws for handling data. The research did not involve intervention in the physical integrity of the participants in any way and thus, according to the National Advisory Board on Research Integrity, did not require an ethics review.

3.3. Research Questions

The pre-service teachers used different terms in describing the given contexts, and how the experimental observations and outcomes of measurements in these cases can be understood. These terms and how they are related reveal the lexicons that pre-service teachers attach to electrons and photons in these discussions. The research questions posed in this study are:

RQ1. What kinds of lexicons do pre-service teachers use for electrons and photons?
RQ2. What are the lexicons shared by at least a pair of pre-service teachers?

4. Methods: Construction and Analysis of Lexical Networks and Lexicons

The lexicons are obtained from lexical networks, which are constructed from pre-service teachers' written reports and by using the information contained in graphs in which 14 pre-service teachers show how sub-contexts K are connected. In addition, for comparison, the text explaining the double-slit experiments in two didactically aimed research reports, one by Müller and Wiesner [23] and the other one by Hobson [24], are similarly analyzed. The construction of lexical networks resembles the analysis of written essays, where lexical networks are constructed based on co-occurrence of words and terms in sentences, thus constructing a proximity network of key words and terms [7,19,20]. However, in many cases the knowledge of interest cannot be assumed to be an outcome of associations, but instead of rule-based (law-like) connections. In such cases, counting which is more sensitive to global connectivity is needed [7,9–11]. The method used here to find the connectivity of terms is based on so-called communicability centrality [25], which has proven to be a relevant operationalization in the case of concept networks [9–11].

4.1. Text Analysis

The analysis of pre-service teachers' written texts is basically a grammatical sentence analysis in which special attention is paid to nouns (as subjects and objects) and verbs (active and passive). The analysis thus contains no interpretation of the meanings of sentences, only analysis of their grammar and syntax. Based on the text analysis, the descriptions written by pre-service teachers are simplified to simple sentences in which subject, verbs, objects and nouns are resolved. In the process, text that consists of e.g., subordinate clauses are transformed into main clauses, thus simplifying the text structure. In the next step, nouns are recognized and systematized so that a common tag word is used for closely synonymous nouns. Similarly, the root verb is identified for each main clause. An example of text analysis and how key terms and words are coded is provided in Appendix A. Since the text analysis is grammatical sentence analysis, for which the usual grammatical rules are applied, this part of analysis involves no interpretation of the meaning of the sentences. Therefore, it should be understood that cross-checking, which is usually needed in cases of interpretative text analysis, and thus analysis of agreement in interpretations, is irrelevant here.

4.2. Construction of Lexical Network

The next step of analysis consists of transforming the simplified text structure into networks in which nodes corresponding to tag words are connected to nodes representing root verbs. The nodes corresponding to the root verbs are finally connected to nodes that represent sub-contexts K as they are indicated by pre-service teachers in their reports. Examples of the sub-contexts are provided in Appendix A. When a term (word) referring to electron or photon appears in a sub-context K, we track all terms that are connected to it: (1) Through a root verb attached to sub-context K, or (2) through root verbs attached to other sub-contexts and connected to sub-context K. In their reports, the pre-service teachers themselves indicated the sub-contexts, with the result that the number of sub-contexts varies from 5 to 13. In the cases of texts by Müller and Wiesner [23] and Hobson [24], we identified the contexts on the basis of paragraphs and decided to use 11 contexts. It is assumed that the lexical meaning of the term "electron" or "photon" thus builds up through these connections to other terms and words (nouns), as schematically shown in Figure 1, which exemplifies how the network is constructed.

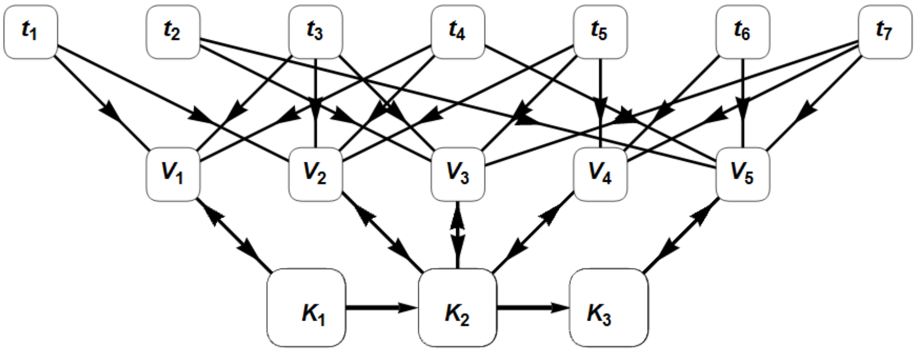

Figure 1. Schematic figure of lexical network construction. The terms of interest are denoted by t, root verbs by V and sub-contexts by K. The term $t = T$ of special interest, referring to either an electron or photon, the connection $t \rightarrow V$ is reversed in analysis. Note that in most cases, sub-contexts are sequentially ordered as shown, but not always.

4.3. Lexicons

The lexicon is based on the connections of terms in the lexical network and it contains a summary of the information about the most important connections that pre-service teachers attach to the terms "electron" and "photon" in the context of double-slit experiments. Starting from the lexical networks, we construct a representation of the lexical information attached to a target concept called term T. The target terms T are denoted by parentheses [. . .]. Here, the target terms of interest are T= [electron] and T= [photon]. The target terms T are connected to sets of other nouns (terms and words), some of them occurring in the same sentences, some of them in more distant sentences, as shown in Figure 1. This representation, which condenses the relevant lexical information extracted from the lexical network (which contains all information on the connections), we refer to as a lexicon for terms T.

To extract the relevant lexicon for T, the terms of interest in the lexical networks are classified into eight different categories P, corresponding to eight different types of properties, each consisting of key words $p(P)$. This classification is needed because different words (nouns) can be used to refer the target term T and are thus closely enough related and sometimes even synonymous (for examples, see Appendix A) to be included in the category connected to the same property class. Here we have used eight categories, *P1-P8* as summarized in Table 1, each of them with at most seven such key words for each category P, as summarized in Appendix A in Table A2. Note that the property classes are different enough that no major ambiguity arises in how different nouns are classified in the eight classes (see examples in Table A2). The set of key words thus forms the lexicon to describe the property category P.

Table 1. The eight properties forming the lexicon.

Property
Wave and/or field-properties (W)
Particle properties (P)
Dualism (D)
Energy related properties (E)
Quanta and quantization (Q)
Stochastic and probabilistic properties (S)
Properties related to double-slit experiment for photons $\left(X_p\right)$
Properties related to double-slit experiment for electrons (X_e)

In each case, the property is composed of several closely related words. Some of the words relevant for the property are shown as nodes in Figure 2 with symbols denoting the property class. Examples are given in Table A2 of Appendix A.

The key words attached to property P, when they are connected to the term T of interest, provide the lexicon for T. The complete lexicon, consisting of eight properties P and at most seven key words indicating those properties, thus forms an 8×7 dimensional matrix. In evaluating the importance of a given word for the lexicon, we pay attention to how distant that connection is. This is the usual choice in deciding which words and terms are closely related and support each other's lexical meanings [7,19,20]. To find the distances between words, whether they occur in the same sentence, in different sentences belonging to same sub-context or only in sub-contexts that belong to a given context (see Figure 1 for this stratification), we count all weighted walks of how key words $p(P)$ are connected to common target term T. To reduce the amount of data, and to weight the connections according to their length, we use the so-called communicability G_{pq} measure between each pair of nodes p and q. A detailed mathematical definition of communicability G is given in Appendix B. Roughly, communicability G_{pq} roughly describes how many ways (e.g., information) the content of node p flows to node q. In counting the flow, the flow is reduced with the increasing length of walk between the nodes, because with increasing length, more paths become available and the flow becomes distributed and divided. Communicability G is defined in detail in Appendix B. Communicability thus describes the support that the node q receives from node p by weighting the support so that nodes that are easier to reach provide more support than nodes that are distant or difficult to reach. Such formal quantification of the distance between words has large values when given words p and q occur frequently in the same sentence, and it becomes smaller the more distant the connection is. However, it also takes into account cases in which connections are made through more complicated syntax, but occur also frequently.

By using communicability G (see Appendix B) we can now obtain the total lexical support of node q from all other nodes p, which are taken to be relevant in providing the lexical meaning of node q. The lexical support that the term T (either electron or photon) receives from the key words $p(P)$ attached to a given property P is now defined as the sum of the logarithm of communicabilities $\log G_{pT}$, where $p \in \{p(P)\}$. The sum of logarithms of communicabilities corresponds roughly to the lexical information flowing from all key word nodes p of property P to target node T and is thus called component $\pi_T(P)$ of property P of term T. All eight property-components define the profile of the lexicon of terms T, which will be denoted $\boldsymbol{\pi}(T) = \{\pi_T(P1),\ \pi_T(P2), \ldots, \pi_T(P8)\}$ and is thus an eight-dimensional property vector of term T. This property vector $\pi(T)$ is now the operationalized form of the lexicon for term T, as operationalised through counting the paths of key words in P that are connected to T. The lexicon $\boldsymbol{\pi}$ now contains the information about the relative importance of words in property class P: The more closely and frequently connected are the words in P to target term T, the larger is the value $\boldsymbol{\pi}_T(P)$.

In addition to lexicons π, it is also important to define shared lexicons, which are those parts of lexicons common to at least two pre-service teachers. We use the term L-lexicon and denote it by $\Pi_L(g, g')$ to refer to the shared lexicons of L-shared features, which are formed on the basis of the similarity between pre-service teachers' g and g' lexicons $\boldsymbol{\pi}_g(T)$ and $\boldsymbol{\pi}_{g'}(T)$. The shared lexicons are found by comparing the similarities of lexicons based on thresholded cosine-similarity, as explained in Appendix B, Equation (A3). According to the lexical view on declarative knowledge, shared lexicons are the cornerstones of communicating and exchanging knowledge in spoken or written form (i.e., by words that belong to lexicons).

5. Results

The lexicons used by the 14 pre-service teachers to express their knowledge about electrons and photons as they appear in double-slit experiments were first constructed for each of the subjects, and for comparison, from texts by Hobson [24] and by Müller and Wiesner [23], referred to as texts A and B in the following discussion. Texts A and B are included in the comparisons because of what we know from the announced purpose of the texts. While text A attempts to build a genuine field-quantization based on understanding of electrons and photons, when they should appear and be discussed rather symmetrically, the text B emphasizes measurement, statistical interpretation of measurements and the particle-like nature of electrons and photons. In texts A and B this is explicitly stated by the authors, while in students' written explanations such explicit statements are rare.

We show in Figure 2 the network representing how the different terms are lexically related to the terms "electron" and "photon" in texts A and B. The students' lexical networks appear to be of similar type and the differences in them can be as large as in the lexical networks for A and B. The cases A and B are chosen as examples, because we have good reasons to expect that they should be different. The original lexical networks used to obtain the lexicons are very dense, and therefore in Figure 2 we show a pruned version, where only the most frequently occurring and relevant words and terms appear as nodes (but all key words among them). In the networks, links represent the nodes that have a mutual communicability G exceeding a threshold value $G^* = 0.25$. The size of each nodes in the lexical proximity network is roughly proportional to the total communicability of the node shown. More exactly, it is proportional to closeness centrality (for definition, see [25]) in the proximity network, but it has high correlation with total communicability. Note that the lexical proximity network is always specific to the term T of interest. The connections based on the communicabilities visualize how the terms are linked in the texts, at the level of sub-contexts K. The meaning of the symbols in Figure 2 are explained in Table 1, in the context of lexicon construction. Based on the lexical proximity network, we form the lexicon profiles that the texts A and B attach to "electron" and "photon". The lexicons are represented in form of eight-dimensional vectors, where each dimension collects closely related terms.

5.1. Lexicons

The lexicons for electrons and photons are here taken to consist of terms connected to eight different types of properties that are relevant in contexts in which the terms "electron" and "photon" are used to express declarative knowledge. The eight properties p of interest here are given in Table 1 below.

The lexicons that the pre-service teachers attach to the terms "electron" and "photon" are reduced to an eight-dimensional vector, where each dimension is denoted by one of the tags $p \in \{W, P, D, E, Q, S, X_p, X_e\}$ that represent the set p of key words listed in Table A1 in Appendix A. The 14 pre-service teachers' and the 2 lexicons didactic texts A and B for electrons are shown in Figure 3 and for photons in Figure 4 in the form of octagonal radar-maps of the eight-dimensional lexicons $\boldsymbol{\pi}$.

The lexicons shown in Figures 3 and 4 are different for different pre-service teachers. For electrons, the lexicons 1–7 for both electrons and photons cover many properties (the polygons are large, indicating that the corresponding key words are central), and, moreover, the quantum and quantization properties that are central to wave-particle dualism are well represented, as are the properties related to particle-nature but also field- and/or wave-nature. In some other cases (like 12 and 10) the polygons describing the lexicons are also large, but do not cover the properties related to quantization and dualism for both electrons and photons. The lexicons 1–7 for photons place substantial emphasis on particle nature and in cases 1–3 the quantum and quantization properties are also dominant. Thus, lexicons 1–3 represent lexicons which are relatively symmetric for electrons and photons, both emphasizing the particle-nature as well as the field- and/or wave-nature of quantum entities. In some cases, (e.g., cases 2 and 6), dualism is a strong component in the lexicon. These symmetric cases apparently represent genuinely dualistic, full-quantum lexicons. In this group, lexicon 1 is based on Hobson [24]. It is noteworthy that Hobson's goal as he states it is specifically to provide a symmetric, full field-based view of electrons and photons as particle-like quanta of fields. This is directly reflected in the corresponding lexicon. Interestingly, the pre-service teachers' lexicons 2–7 resemble Hobson's lexicon.

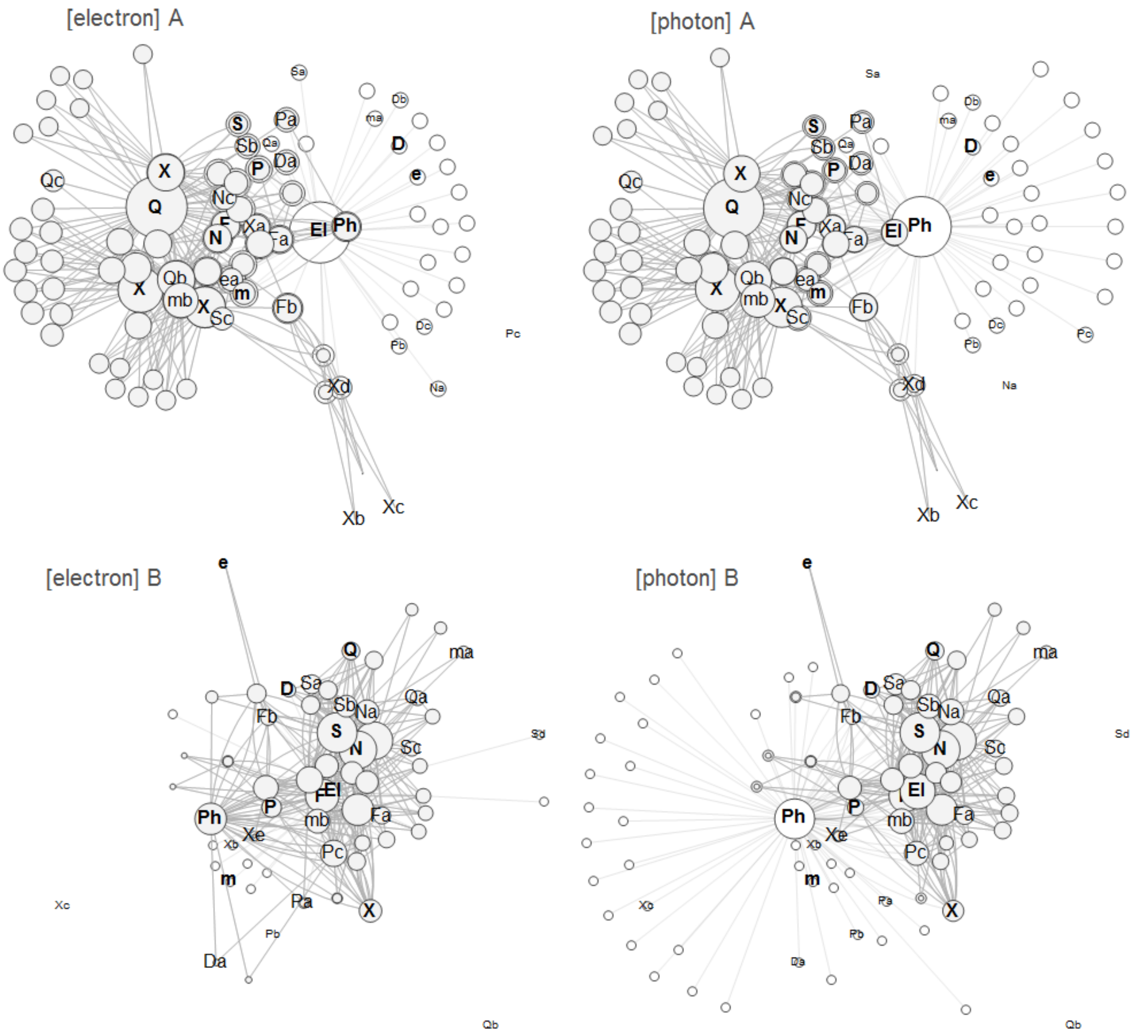

Figure 2. The lexical proximity network for terms T = [electron] and T = [photon] as they appear in texts A and B. The nodes corresponding to [electron] and [photon] are denoted by El and Ph, respectively. In the left column, lexical proximity networks for [electron] are shown in light grey and for the [photon] with white nodes. In the right column, [electron] is shown with white and [photon] with light grey nodes. The sizes of the nodes roughly correspond to their total centrality in the network (see main text). The meaning of symbols in the figure is explained in Table 1. The symbols for tag words shown in this figure are summarized in Table A2 in Appendix A.

Lexicons 9 and 10, on the other hand, are very different from lexicons 1–7. Lexicons 9 and 10 are dominated by stochastic and probabilistic properties and the next two most important properties are particle-nature and field/wave-nature, for both electrons and photons. Lexicon 9 is for Müller and Wiesner [23], who approach wave-particle dualism by emphasizing quantum probabilities as they are connected to measurement outcomes and as revealed in measurement statistics. They also often connect particle-like single-impacts and discussions of statistics and the role of wave functions in predicting the statistical distribution of single hits. This emphasis is again directly reflected in the lexicons, as expected. It is interesting that one of the pre-service teachers' lexicons (number 10) very closely resembles that of Müller and Wiesner (number 9), but also that some (like 3 and 6) contain strong *S* and *P* components, in addition to other strong components, for both electrons and photons.

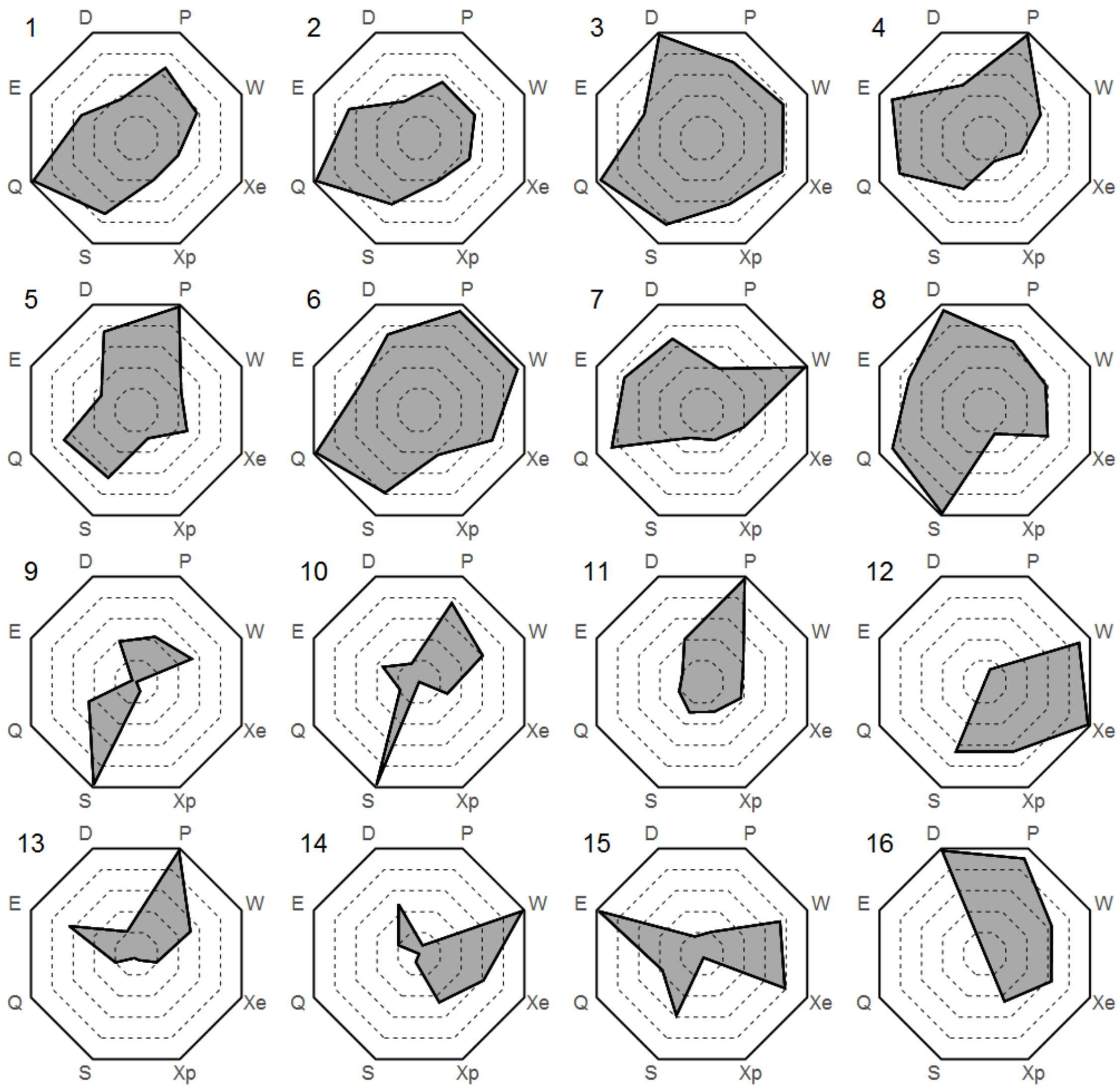

Figure 3. Lexicons $\{W, P, D, E, Q, S, X_p, X_e\}$ for electrons. The lexicons are normalized to maximum value of 1 in each case (denoted by outer boundary of octagons). The dotted lines represent values from 0.2 to 0.8, in steps of 0.2.

A comparison of the results in Figures 3 and 4 shows that the lexicons are in general more balanced for electrons than for photons. In fact, the logarithm of communicabilities in electron lexicons is in most cases nearly twice as extensive as in lexicons for photons, indicating nearly twice as many connections between terms referring to electrons in comparison to connections between terms referring to photons. This might reflect that pre-service teachers have better connected and more coherent declarative knowledge of electrons than of photons, or at least they express themselves with more connected vocabulary when writing about electrons than when they are writing about photons. Interestingly, for photons the double-slit experiment context provides more connections between terms in the particle-nature property class P. Similarly, as expected, the double-slit experiment for photons (X_p) is now more dominant than that for electrons (X_e). In some cases, X_e is also important for photons because the pre-service teachers have made use of the analogy between the experiments. In lexicon 9 for Müller and Wiesner, the terms corresponding to the experiment for electrons has a very marginal role in the lexicon because they discuss real experiments for photons only.

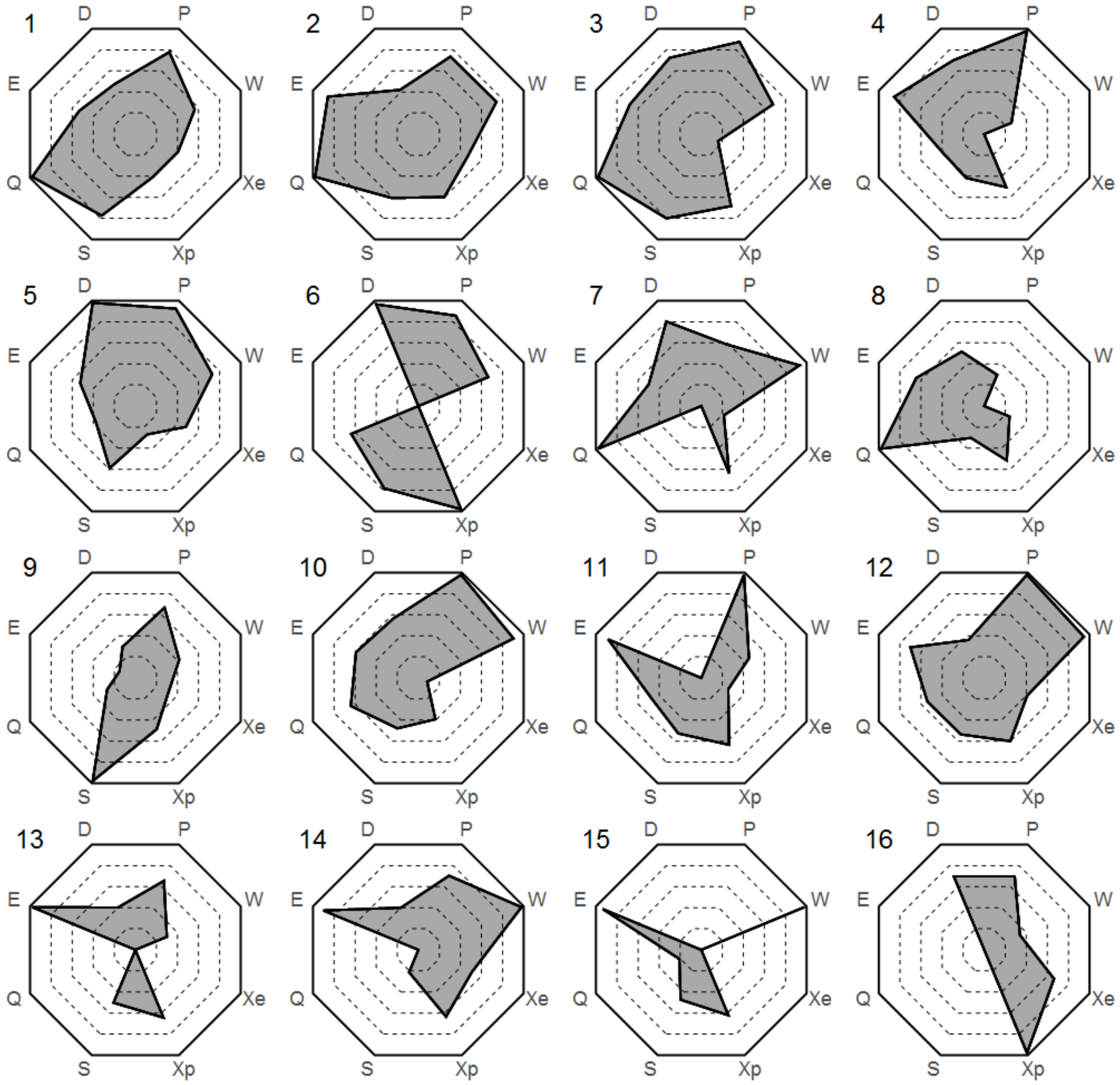

Figure 4. Lexicons $\{W, P, D, E, Q, S, X_p, X_e\}$ for photons. The lexicons are normalized to maximum value of 1 in each case (denoted by outer boundary of octagons). The dotted lines represent values from 0.2 to 0.8, in steps of 0.2.

5.2. Shared Lexicons

The different lexicons shown in Figures 3 and 4 form sets of shared *L*-lexicons with *L* shared features out of eight. The *L*-lexicons show how different features form a basis for shared conceptions and which of them are the most prevalent within the pre-service teacher group. Moreover, according to the lexical view on declarative knowledge, when that knowledge is explicated by speaking or writing, *L*-lexicons are also expected to surface in communication and exchange of knowledge; they form the basis of verbal interaction and sharing of knowledge (see Section 2). Simply, the *L*-lexicons contain the terms and words the student pairs can share if the lexicons that the analysis reveals remain intact in the communication events. Of course, now we only have lexicons based on written expression and situation may change in real communication events. Nevertheless, the lexicons that the students have chosen to use in written expression indicate what aspects they have found most important, even if they may possess broader lexicons.

The *L*-lexicons, because of their importance for communication, which requires shared expression, are more also interesting for teaching and monitoring of learning than individual lexicons. In what follows, we focus on *L*-lexicons shared by more than two pre-service teachers and with at least three shared features. The threshold for communicability to count as a shared property is set to value

$G = 0.5$. More stringent values lead to a diminishing set of shared properties, nearly none at $G > 0.75$, while smaller values $G < 0.3$ will lead to a rapidly increasing number of shared properties. At value $G = 0.5$ only electron L-lexicons have one $L = 8$ group (including four cases) of shared properties, while photons have none, and only one $L = 7$ L-lexicons (including five pre-service teachers). Therefore, threshold $G = 0.5$ has enough resolving ability but it is not too stringent a demand for sharing a lexicon. The L-lexicons are shown for electrons in Figure 5 and for photons in Figure 6. Figures 5 and 6 contain detailed information on how lower level L-lexicons are related to those of higher levels.

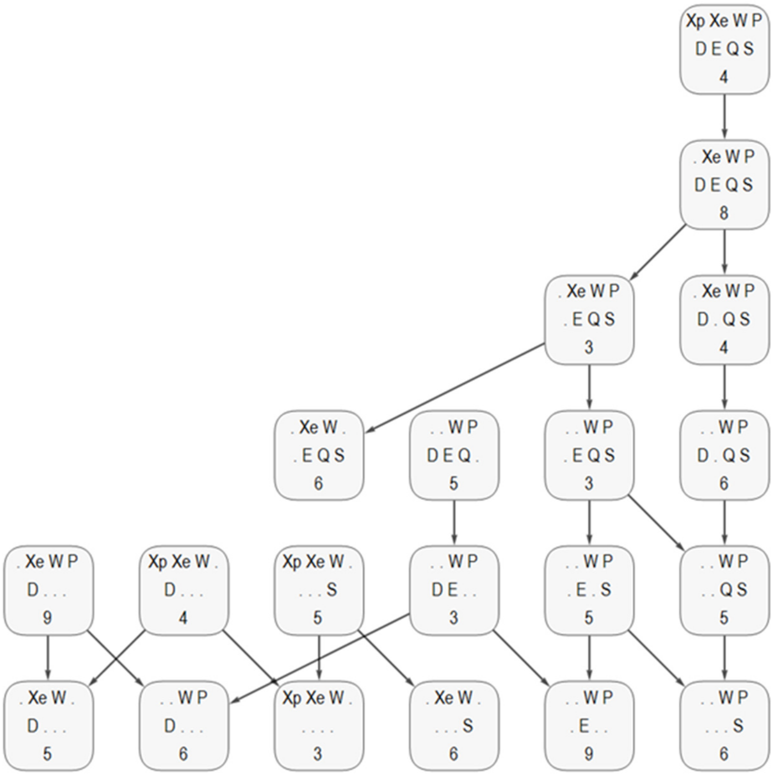

Figure 5. *L*- lexicons of electrons. Acronyms refer to combinations of features shown in Figures 3 and 4. The numbers of pre-service teachers who have the same *L*-lexicons are indicated. Arrows denote connections where only one feature is changed.

Figures 5 and 6 show several L-lexicons for electrons, some of them related so that only a single feature is different. These connected L-lexicons represent contiguous paths of changes (i.e., only one feature changed at a time) from a rich, advanced and sophisticated L-lexicon to less sophisticated lexicons. However, it is not always the same pre-service teachers who are involved in contiguous paths from L7 to L4. These results also show that L-lexicons are not easily reduced to a few characteristic classes as mental models or conceptual models. However, seven classes A–G can be formed based on the contiguous paths. The classes A-D and G for electrons are summarized in Table 1 and classes A, B, E, F and G for photons in Table 2. Note that in Tables 2 and 3 the energy category E does not appear although it is shown in Figures 5 and 6. The reason for this omission is that it turned out that category E does not make a difference for L-lexicon classifications and is therefore omitted in what follows.

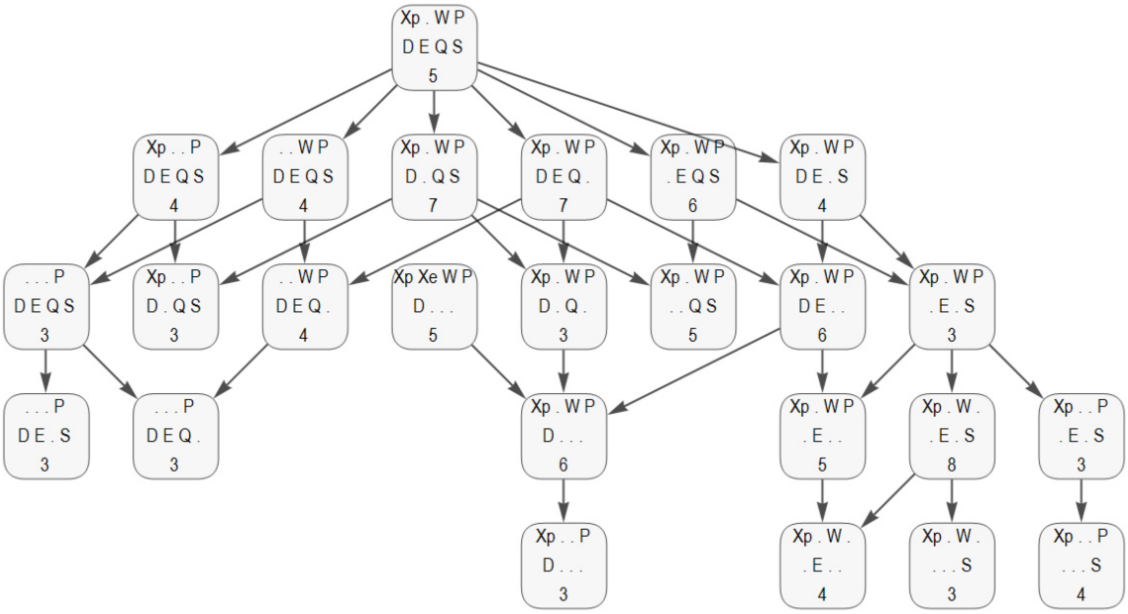

Figure 6. *L*-lexicons of photons. Acronyms refer to combinations of features shown in Figures 2 and 3. The numbers of pre-service teachers who have the same *L*-lexicons are indicated. Arrows denote connections where only one feature is changed.

Table 2. Pre-service teachers' *L*-lexicons of electrons.

Level	**A: L=8,7**	**B: L=6**	**C: L=5,4**	**D: L=5,4**	**G: L=4,3**
Name	QFX Quant. Full+X	QSX Quant., Stats+X	QS Quant., Stats.	FDQ Field, D+Quant	SC: SemiClas.
Type	X_eX_pWPDQS	$X_e.WPDQS$	$..WPDQS$	$..W.DQS$	$..WP.QS$
Cases	1-3,6	3,5,6,11	1-3,5,6,8,15	1-3,6,8,15	
Type	$X_e.WPDQS$	$X_e.WP.QS$	$..WPDQS$		$X_eX_pWP...$
Cases	1-8	4,5,15	1-3,5,6,8,15		
Type			$..WP.QS$		
Cases			1,2,4		

The number of shared features *L* is given and types are denoted by tags W, P, D, Q, S, X_pandX_e The naming of different classes is provided by acronyms explained in the text. Numbers 1–16 refer to Figure 3. Lower dot denotes a missing category.

Table 3. Pre-service teachers' *L*-lexicons of photons.

Level	**A: L=8,7**	**B: L=6,5**	**C: L=6,5**	**D: L=6,5**	**G: L=5-3**
Name	**QFX** Quant. Full+X	**QSX** Quant, Stats+X	FPDQ Quant, Dual.	PDQ Quant, Particle	**SC** SemiClas.
Type	X_eX_pWPDQS		$..WPDQS$	$X_e..PDQS$	$\boldsymbol{X_e.WP..S}$
Cases	1-3,10-12		1-3,5,10	2-4,8	
Type	$\boldsymbol{X_e.WPDQS}$	$\boldsymbol{X_e.WP.QS}$	$.X_pWPDQ.$	$...PDQS$	$\boldsymbol{X_eX_pWP...}$
Cases	**1-3,5,6,**10,12	1-3,6,9-12	1-4,7,10,12	2,4,6,8,14	
Type			$..WPDQ.$		
Cases			4,5-7,9,10		

The number of shared features *L* is given, and types are denoted by tags W, P, D, Q, S, X_p and X_e. Naming of different classes is provided by acronyms explained in the text. Numbers 1-16 refer to cases shown in Figure 4. Classes and cases within them that are common with electron *L*-lexicons are bolded. Lower dot denotes a missing category.

Classes QFX (A) and QSX (B) are the most complete classes. They contain at least seven shared properties, which include properties referring to at least one of the experiments (X_e or X_p), field- and particle-properties (F and P), properties referring to the dualistic nature of entities or dualistic models of their description (D), properties referring to quanta and quantization (Q) and to stochastic or probabilistic nature of events or measurements (S). Lexicons in these classes are fully quantum physical. They contain the terms essential to writing and speaking about electrons and photons as quantum entities, their wave-particle dualism (or alternatively, wave-particle dualism of observation events), about quanta and quantization, the probabilistic or stochastic nature of events, and the epistemic basis of these aspects as they appear in double-slit experiments.

Class QS contains terms that are needed to discuss and communicate the quantization and quantum-nature of entities and events as they are related to either entities themselves or measurement events. As expected, the terms referring to stochasticity and probability are also essential in this case. Terms related to particle- and field-nature of electrons and photons are also contained in this class, but it lacks the vocabulary to discuss dualism. Consequently, dualistic nature remains a topic that cannot be approached properly if QS remains the only shared lexicon.

Class FDQ is interesting in focusing primarily on field aspects but also containing vocabulary for discussion about wave-particle dualism, augmented with terms relating to quanta and quantization. In many ways, it is a scaled-down version of QFX. As such, field-based views, like Hobson's [24], can be discussed adequately using this shared lexicon.

Class SC (G) is semiclassical, containing mostly terms referring to particles and fields and requiring only three ($L = 3$) or four ($L = 4$) common properties. These properties are also found in many other L-lexicons, demonstrating that semiclassical lexicons underline many of the L-lexicons and nearly all pre-service teacher pairs can discuss wave-particle dualism using that level of lexicon.

The L-lexicons and their connections for photons (shown in Figure 5) and the most important L-lexicons (summarized in Table 2) are in many ways like the L-lexicons of electrons. As with electrons we can distinguish certain interesting lexicons shared by many pre-service teachers. The characterizations for QFX (A) at level $L7$ and higher and QSX (B) at levels $L = 6$ and $L = 5$ remain as for electrons. However, only for cases 1-3, 5 and 6 are both electron and photon L-lexicons in class QFX. This means that cases 1-3, 5 and 6 are the only ones with which knowledge about wave-particle dualism of electrons and photons can be discussed and communicated with a rich and adequate vocabulary, which is symmetric in that the properties of both entities can be communicated using very similar lexicons. This, of course, is desirable if the target is to understand at a deeper level the similarity of description of quantum entities.

Classes FPDQ (E) and PDQ (F) are not found for electrons, at levels $L = 6$ and $L = 5$. In these lexicons, the particle-nature of photons is dominant. The particle-nature, however, is strongly connected to quantization and photons are understood as particle-like quanta. In the case of FPDQ, the field/wave-nature and dualism are central to making this connection. The semiclassical lexicons SC (G) that require only three ($L = 3$) or four ($L = 4$) common properties are again found in many cases, as for electrons, demonstrating that such semiclassical lexicons underline nearly all L-lexicons and are shared by nearly all pre-service teacher pairs.

The lexicons as they are summarized in Tables 2 and 3 contain terms that are connected in many ways to the mental models reported in studies based on traditional, interpretative methods. The mental models reported in research literature and our interpretation of how their expression by speaking or writing would parallel our L-lexicons are summarized in Appendix C. It is not surprising to find that our pre-service teachers' lexicons resemble the mental models as reported in research literature because traditional phenomenographic and interpretative methods are also based on analysis of expressions, which are either in written or spoken form.

5.3. Relevance and Reliability

The relevance of the study hinges on two key issues. The first is a fundamental question of theoretical underpinnings: to what extent lexicons are relevant to research that attempts to discover the structure of pre-service teachers' knowledge or shed light on pre-service teachers' conceptual models. The answer to this question is lexicons, as sets of terms and words (i.e., vocabulary), are how knowledge is expressed in speaking and writing, and thus they indirectly provide information on the conceptual models (see Section 2). The lexicons are directly accessible to investigation by analyzing the text, its vocabulary and syntax, while the conceptual models and mental models, understood as cognitive structures, are inherent, and not directly accessible by any method based on analysis of spoken or written expressions of declarative knowledge. However, such structures are externalized in verbal and symbolic form, in lexical or symbolic structures. From this perspective, lexicons are not only relevant but also the most obvious targets of study.

The second issue of relevance is related to methodological issues. The terms and words as they appear in the written and spoken text are parts of complex syntactical structures. Here, we have simplified such structures in the form of simple main clauses and constructed the network of relations through these main clauses. This procedure loses information about the more delicate syntactical constructs contained in the text. However, we believe that enough of the intended meaning is retained and our analysis retains its relevance. Finally, a decision must be made regarding, which the key terms are relevant. This issue is largely a normative decision about the relevance of key terms regarding the goals of teaching, not an issue that can be empirically settled by examining the students' reports to find out which terms are relevant for them for other reasons. Our decision is to take the terms summarized in Table A1 in Appendix A.

The third issue of relevance is also methodological and related to how the connections between the terms and words are modelled as a network. To define the connectivity between terms, as they are established through context-related sentences, one must decide the connectivity property of interest. Here we have chosen path-length counting, through counting the number of walks by which a given node is available from any other node in the network. Such path counting is global by nature and sensitive to global as well as local connections. Such a measure, accounting equally for local and global connections, is well justified and provides the relevant information.

Reliability, as a criterion for reproducibility of the analysis method, is in many ways an easier issue to settle than relevance. The first issue related to reliability is the part of the analysis where interpretation is needed in the pruning of text in the form of simple main clauses. The analysis of sentence structure is not interpretative but follows grammar (finding verbs, nouns and objects in clauses). Everything that follows is carried out by exact enumerative and combinatorial methods of analysis and is not at all dependent on human interpretation or agreements between interpreters about the application of interpretation protocols. The second issue is related to division into sub-contexts and contexts, which is not always clear from the students' reports. However, this division is related more to the structural stratification of the text as shown in Figure 1. Its importance is that the level of sub-context is present and it is sub-ordinated to context. How exactly different clauses fall into different sub-contexts is not so crucial for the lexicons although it affects the weight of different dimensions somewhat. Such changes are always only 10% to 15% of the relative weights of the dimensions shown as the octagons in Figures 3 and 4 and have no effect on the *L*-lexicons. The third issue is the categorization of key words into eight property classes as summarized in Table 2. For this categorization, very little if any interpretation is needed, since the categories are rather clearly different ones and sometimes even exclusive (e.g., particle-like notions cannot go into any other of the categories). In that classification, no disagreement emerged during the analysis. It is important to note that all other parts of the analysis are exact and thus fully reproducible.

6. Discussion and Conclusions

We have examined pre-service teachers' written reports on how they use the terms "electron" and "photon" and other terms in two contexts: double-slit interference of dim light (single photons) and electron beams. On this basis, we constructed the lexicons and shared lexicons that the pre-service teachers used in these contexts. Such lexicons are central to learning scientific knowledge [16–18] and they also provide important access points to the underlying conceptual structures [14].

The double-slit experiments are standard topics used in science education research attempting to reveal pre-service teachers' mental models of electrons and photons [1,2,5], thus allowing us to deduce what lexicons are needed to express and discuss certain mental models reported in research literature, as summarized in Appendix C. The lexicons, which consist of eight different dimensions, each of them containing seven related features, reveal remarkable variation and differences and are strongly context-dependent. The pre-service teachers express themselves in a multidimensional manner, showing some expected coherencies but also many contingent ways of speaking about electrons and photons. However, we find a multitude of different lexicons, only some of which are connected to the mental models reported in research literature (see Appendix C). Now, when it is evident that the more traditional, interpretative methods are based on written reports and their interpretation, a justified question is: to what degree do such categorizations reflect the assumed underlying conceptual models? Furthermore, to what degree are the robustness and coherency of the categories outcomes of the limited ability of human interpreters to pay attention to significant details, thus leading to oversimplifications when mental or conceptual models are categorized?

The analysis of lexical networks and lexicons reveals how pre-service teachers use words and terms in relation to each other, but it does not necessarily contain information about the semantic meanings of the words and terms (compare with e.g., [19]). As has been pointed out, lexical and semantic structure are related. However, revealing such deeper connections reliably would require paying closer attention to how semantic connections build up, which is beyond the scope of this study.

Regarding practical issues of teaching and learning, the major message of the study is that only some pre-service teachers use sufficiently rich vocabulary to allow them to participate in many-faceted discussions and communication of their views about quantum entities. This study is not able to reveal the reasons behind these shortcomings: Whether it is because of inadequate content knowledge, inadvertent careless use of language or focus of attention on only part of the phenomenon. Nevertheless, the task would require a broader vocabulary than many pre-service teachers use. According to the results found in this study, in most cases, the pre-service teachers' lexicons are limited so that they share too few features for such discussion, and lack, for example, important terms that refer to the stochastic and probabilistic features of quantum events. In some cases, even terms referring to quantization are not very well represented. Such deficiencies in lexicons seriously limit teachers' ability to convey the rich picture of quantum phenomena and quantum entities in their teaching. It is clear that without a rich lexicon it is impossible to express deeper ideas about complex phenomena such as wave-particle dualism. A rich lexicon can be seen as a necessary requirement for good teaching, not of course sufficient in itself, but with shallow lexicons, good teaching is outright impossible.

A question of practical importance is how extensive a lexicon a pre-service teacher needs to possess to teach introductory quantum mechanics properly. In the case of teaching wave-particle dualism, it is clear that one needs the vocabulary to describe the stochastic aspect, dualistic aspect and quanta and quantization. When the proper vocabulary is lacking, it is not possible to cover these topics. In addition, when vocabularies are restricted, the exchange of ideas and views, and also perhaps the ability to acquire more in-depth ideas and views from written text, remains limited. Therefore, it is important to ensure that pre-service teachers have rich enough vocabulary and that they use it in the contexts where it is demanded. Based on the results presented here, L-lexicons containing six or seven key features (dimensions) appear as minimal requirements. Unfortunately, only a fraction of pre-service teachers possesses lexicons that are that extensive and which can be shared. This means that pre-service teachers are not well equipped to communicate and share their ideas about these multifaceted quantum phenomena (cf. [16–18]). Improving the situation in teacher education remains a challenge. However, the lexical analysis helps teacher educators to better identify the extent of pre-service teachers' lexicons and thereby guide them to pay more attention in teaching multifaceted vocabulary and terms.

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Appendix A

In the text analysis, the written text is first pruned in the form of main clauses, and closely synonymous terms (nouns) are replaced by key words as shown in Table A1.

Table A1. Examples of the text analysis.

Excerpt	Pruned	Key Words and Terms
Although photons do not arrive at the same time, we observe an interference pattern forming of single spots.	Photon double-slit experiment form single hit interference pattern of photons	- photon double-slit experiment - photon - single hit - interference pattern
This indicates the particle nature of light.	[Photon double-slit experiment and single hits] indicate that light has particle nature.	- photon double-slit experiment - single hit - light - particle nature

Each of these properties *P* are then connected to the use of key words, some of them synonymous, some of them closely related. The terms and words we have here taken as indicative of a certain property *P* are listed in Table A2.

Table A2. Properties P1-P8 relevant to lexicons and their indicator terms. Double-slit experiment (DSE) is shown separately for electrons and photons. Note that in both cases Young's experiment is included since in most cases it was difficult to decide which experiment it was connected to in written text. *P* in column at left refers to basic tag word while P_x is for synonymous tag words. These symbols are used in Figure 2.

	Property							
	$\mathbf{P_1}$	$\mathbf{P_2}$	$\mathbf{P_3}$	$\mathbf{P_4}$	$\mathbf{P_5}$	$\mathbf{P_6}$	P_7	P_8
	W	P	D	E	Q	S	Xp	Xe
	Wave	Particle	Dualism	Energy	Quantization	Stochastic	DSE photon	DSE electr
P	wave	particle	dual model	energy	quantization	probability	DSE photon	DSE electron
P_a	wave motion	particle nature	single hit	intensity	quantum	Probability distribution	young-experiment	young-experiment
P_b	wave property	mass		radiation energy	field quantum	Probability wave		
P_c	wave nature	charge		kinetic energy	light quantum	wave-function		
P_d	interference	light particle		work function	radiation quantum	born-rule		
P_e	EM wave	electron flow		light energy	quantum hypothesis	hit distribution		
P_f	EM field	elastic collision		linear momentum	uncertainty relation			

Table A3. One example of an explanation produced by a pre-service teacher (translated into English). The context here is double-slit experiment with electrons. The numbering indicates the sub-contexts as shown in Figure 1. The expressions used as key words are underlined.

K1	In the Young double-slit experiment the wave entering the slit changes so that both slits act as the source of a wave. The resulting new wave interferes. The wave with same phase strengthen and with the different phases weaken each other. This results in a minima and maxim in the interference pattern on the detection screen. The interference pattern found in the double-slit experiment is still explained by the wave model of light.
K2	The principle of the double-slit experiment can be applied in a new experiment with the low intensity light. In the experiment, the intensity of the *light* is reduced to such an extent that it is possible to think of the photon as opposed to the Young's experiment.
K3	Finally, in the double-slit experiment the interference pattern is formed but the pattern is found to be based on the individual hits. The classical wave model is not able to explain and the experiment is a clear proof of the particle nature of the light. The probability of photon detection locations is described by the wave function.
K4	Light is not really the wave nor the particle but both models describe light. Depending on the situation, light can be interpreted as electromagnetic wave motion or as photons which are massless particles.
K5	The wave function combines the particle model and the wave model. In the wave-particle dualism particles are reduced as momentarily and locally existing quanta. The wave motion becomes the wave function that regulates the probability of the photon observation. The wave function itself has no detectable physical properties.
K6	In the previous didactic reconstruction of the photon phenomenon, the double-slit experiment resulted in the wave-particle dualism type of the quantum model of the photon.
K7	Louis de Broglie put forward a hypothesis that the wave-particle dualism is the nature of all the fundamental entities of nature. The idea of dividing energy quantum into particles and waves should be discarded. The wave-particle dualism does not only concern photons.
K8	According to the classical model, the electron is small particle that is identifiable and localised. The movement of the particle follows basic laws of the classical mechanics and their behavior in interaction situations can be predicted by the classical physics. The mass and the charge and the linear momentum and energy are localized on the location indicated by the particle location vector.
K9	The interference pattern is observed in the double-slit experiment for the weak electron beam. In this case the interference pattern consists of individual hits. From this, it can be concluded that the electron must have both particle nature and wave nature. In the double-slit experiment the geometry of the slit and the de Broglie law make the probability distribution of the individual hit of the electron predictable
K10	The wave nature of the electron can only be detected as interference phenomena such as for example in the double-slit experiment. The wave related to the electron is the probability wave represented by the wave function. The square of the wave function indicates the probability distribution of the presence of the electron. It is also noteworthy that the wave function is not the same as in the case of photon.
K11	Based on the double-slit experiment the electron has both the particle nature and wave nature. The combination of the particle nature and the wave nature results in the wave-particle dualism that applies to the electron. The particle is the field quantum which is detected only in interactions and does not exist as individuals. The probability distribution expressed by the wave function also follows the uncertainty principle.

Appendix B

The essential information of connectivity of terms in the lexical network is coded in the links connecting nodes p and q, which stand for terms and words. The basic idea is to find a set of key terms (nodes) that are connected to the term of interest by contiguous (connected) paths in the lexical network. The counting of such paths of various length, weighted by their lengths, is then used as a measure of lexical proximity. Such a measure is related to how a node can pass information through the network to nodes to which it is connected.

The lexical network consists of N nodes and (at maximum) $N \times (N-1)$ possible links between the nodes. The network can be described by a $N \times N$ adjacency matrix $\mathbf{A}$ with elements $[\mathbf{A}]_{pq} = a_{pq}$, where $a_{pq} = 1$ when nodes are connected and $a_{pq} = 0$ when they are not connected. The adjacency matrix $\mathbf{A}$ can be now used to obtain the number of walks in the lexical network needed to connect the two nodes within it. This is based on the notion that there is a walk from p to q if $a_{pq} \neq 0$, walk $p \rightarrow j \rightarrow q$ if $a_{pj}, a_{jq} \neq 0$, walk $p \rightarrow k \rightarrow k' \rightarrow q$ if $a_{pj}, a_{jk}, a_{jk'} \neq 0$ etc. Thus, for a walk involving two nodes $\left[\mathbf{A}^2\right]_{pq} \neq 0$, for three nodes $\left[\mathbf{A}^3\right]_{pq} \neq 0$ and N nodes $\left[\mathbf{A}^N\right]_{pq} \neq 0$, respectively. Now, in a connected network, the number of long walks increases rapidly, nearly factorially with the length of the walk, because different combinatorial possibilities emerge, and one is interested in the relative weight of such walks. Therefore, the number of walks is usually divided by the factorial, to obtain the communicability [9,25]

$$G_{pq}(\beta) = \frac{1}{1!}\beta[\mathbf{A}]_{pq} + \frac{1}{2!}\beta^2\left[\mathbf{A}^2\right]_{pq} + \frac{1}{3!}\beta^3\left[\mathbf{A}^3\right]_{pq} + \cdots = \left[e^{\beta\mathbf{A}}\right]_{pq} - 1 \tag{A1}$$

where $e[\ \ldots\ ]$ is matrix exponential and $[\ldots]_{pq}$ is its element at row p and column q. The communicability G_{pq}, as operationalized through walks, has well-defined mathematical properties [25].

The definition in Equation (A1) includes a free parameter $\beta \geq 1$ which is used to tune how extensively parts of the network are included in counting the walks. An optimum value of parameter β is such that all paths that increase the diversity of key terms and words that contribute to the total communicability are included with the lowest possible value of β. In practice, values $1.5 < \beta < 2.0$ provide the optimal estimates.

The lexicons $\boldsymbol{\pi}(T) = \{\pi_T(P1),\ \pi_T(P2), \ldots, \pi_T(P8)\}$ of term T is an eight-dimensional vector consisting of its property components $\pi_T(P)$ which are defined as a sum of logarithms of communicabilities, in the form

$$\pi_T(P) = \sum_p \log\ G_{pT} \text{ where } p\ \in\ Y \text{ and } p \neq T \tag{A2}$$

where Y is the set of key terms and words relevant for property P. The logarithm is used to make the communicabilities G_{pq} comparable because the definition of G_{pq} is exponential.

Finally, we will refer to the shared lexicons of L-shared components as L-lexicons Π_L. The shared lexicons are found by comparing the similarities between lexicons π and π' as based on a threshold similar in form

$$S = \sum_P \text{STEP}\left[\sqrt{\pi(P)\pi'(P)} - \pi^*\right] \tag{A3}$$

where STEP is a step-function that has a value of 1 when the geometric mean of total communicabilities exceeds the threshold value π^*. L-lexicons are then constructed as L-dimensional vectors consisting of those components of the pairs of lexicons π and π' whose geometric mean exceeds the chosen threshold value. Note that for L-lexicons $S = L$.

Appendix C

The L-lexicons can be compared indirectly with mental models reported in research literature. The comparison should be understood such that the lexicon in question is the minimal lexicon required for discussing the given mental model or communicating knowledge about it and, therefore, accessible in interpretation of written text or analysis of interviews. Ayene et al. [1] report, in the context of the double-slit interference phenomenon, three categories (cat I-cat III) of mental models for each context. According to our interpretation, expressing views related to these categories requires that pre-service teachers should master and be able to share at least certain L-lexicons as found in our analysis. These minimal requirements of *L*-lexicons are summarized in Table A4. Similarly, categories reported in [5] can be paralleled with certain minimal *L*-lexicons. In this sense, many of the *L*-lexicons closely resemble mental models as they are resolved in traditional interpretative analysis.

Table A4 shows how the nine categories as reported by Ayene et al. [1] and four categories by Mannila, Koponen & Niskanen [5] can be paralleled with the L-lexicons we have reported in Tables 1 and 2. In the comparison, it should be noted that serious ambiguity for the comparison arises from the fact that the contexts used in this study and by Ayene et al. [1] and Mannila et al. [5] are not the same, although they overlap.

Cheong and Song [6] have discussed wave-particle dualism from the perspective of what kinds of wave-particle dualism interpretations could be possible in teaching and recommendable at different levels of instruction. They distinguish three levels: introductory, undergraduate and graduate levels of instruction, the ascending levels requiring more sophisticated conceptual models. In Table A4 we have summarized the lexicons that we think are required to meet the recommendations by Cheong and Song [6] at the three levels they introduce. It should be noted that the summary in Table A4 is only a tentative view of which lexicons are required to describe and communicate the ideas behind the given conceptual models.

Table A4. Comparison of L-lexicons with categories (cat) reported in (A) Ayene et al. [1] in their contexts III of double-slit experiment (CX III), with four models reported in (M) Mannila et al. [5] and the three levels of interpretation recommended by Cheong & Song [6] for introductory (I), undergraduate (U) and graduate (G) courses.

Category		**Description in Original Reference**	**Lexicons (min.)**
Ayene et al. [1]	cat I	Classical wave and intuitive model	SC
	cat II	Mixed model	QS,FDQ,FPDQ
	cat III	Incipient Quantum model	QSX, QS
Mannila et al. [5]	Clas-M	Classical (hybrid) model	SC
	Traj-M	Trajectory-based model	QS,PDQ,FDQ
	Stat-M	Statistical/ensemble model	QS,QSX,FPDQ
	Quant-M	Fully quantum model	QFX, QSX
Cheong & Song [6]	I	Introductory physics course	QS,FDQ,FPDQ,PDQ
	U	Undergraduate quantum mechanics	QSX,QS,FPDQ
	G	Graduate quantum mechanics	QFX

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