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The association between executive functions and early numeracy in 4-year-old preschoolers

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Abstract

This study aims to reveal how executive functions are related to early numeracy skills. Several articles have been published in this respect. The present one focuses on just two executive functions, inhibition and switching, and two early numeracy skills, counting and numerical relational skills. The study wants to determine how the accuracy and reaction time in inhibition and switching tasks correlate with the counting and numerical relational skills in four-year-old preschoolers, and if there is any general latent condition under which these relations are modified.

The participants of this study are 4-year-old preschoolers (N=189) from preschools in the Helsinki Area (N=21), Finland. They have done two different tests that have been used to gather the data. A digital version of the Flanker tasks (modified from Fan, et al. (2002)) has measured inhibition and switching accuracy and reaction time. The Early Numeracy Test (Aunio, Hautamäki, Heiskari, & Van Luit, 2006) has measured the preschoolers' performance in counting and relational skills.

The data has been quantitatively analysed with SPSS and R. A correlation analysis has been performed to understand how the variables are related (calculation of Spearman's rank correlation). A latent profile analysis has been run using the mclust package, to see if there could be extracted any latent variable that could drive the correlation in different directions.

The main results reveal that accuracy in inhibition and switching tasks have a weak to moderate positive correlation with the successful use of counting and relational skills in 4-year-old preschoolers. Reaction time seems to be a variable whose implications change depending on the participants' EN-performance, as visible in the latent profile analysis. However, there have not been yielded any robust conclusions about the existence of latent variables.

Keywords

inhibition, switching, counting skills, numerical relational skills, accuracy, reaction time, executive functions, early numeracy skills, 4-year-old

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1. Introduction

To effectively teach, it is necessary to know how learning takes place. Learning is the development and adaptation of mental processes to different situations and scenarios. Therefore, understanding mental processes and their relation to learning helps to know how learning occurs. This general narrative frames the present study focused on the development of mathematical skills, disentangling nuanced aspects of the relationship between Executive Functions (EF) and Early Numeracy skills (EN)

EF are the most basic neurological functions needed to learn or reason (Clements, Sarama, & Germeroth, 2016). There have been described four basic EF (Bull & Lee, 2014): inhibition, switching, working memory, and updating. Inhibition refers to the omission of unimportant information. Switching refers to changing from using one information or idea to using another one. Working Memory is related to retaining and using the critical information we need to perform concrete mental tasks. Updating refers to the ability to add new relevant information to mental tasks. Several studies have proven that EF are positively correlated with the performance of EN (Bull & Lee, 2014; Clark, Pritchard, & Woodward, 2010; Espy, et al., 2004; Lan, et al., 2011; Yang, Chung, & McBride, 2019).

EN underpin our mathematical reasoning and allow us to operate and relate concepts that involve numbers and quantities (Aunio & Räsänen, 2015). There have been described four EN: numerical relational skills, counting skills, basic arithmetic skills, and symbolic and non-symbolic number sense (Aunio & Räsänen, 2015). Numerical relational skills are the skills that allow us to analyse and relate elements that include numbers or quantities. The counting skills are the ones that allow us to determine exact quantities according to the rules of a concrete numerical system. Some studies have shown that EN are also related to the future academic performance in mathematics (Aunio & Niemivirta, 2010; Braak, et all. 2022)

As already indicated, some studies have already analysed the relationships between EF and EN (Bull & Lee, 2014; Clark, Pritchard, &

Woodward, 2010; Espy, et al., 2004; Lan, et al., 2011; Yang, Chung, & McBride, 2019). They draw conclusions that should help develop more effective teaching methodologies. Considering that by understanding the learning process, teaching can be improved, the present study wants to give some hints to improve the teaching methodologies of early mathematics. Therefore, it makes a deeper analysis of the relationships between EN and EF, wanting to reveal which aspects of inhibition and switching have a more significant relationship with counting and numerical relational skills. The two considered aspects of inhibition and switching are accuracy, and reaction time (RT). Accuracy is related to the precision in which the EF is performed, and RT is related to the invested time in the performance of the EF. Camerota et al. (2020) already studied these two aspects as different metrics of EF. They concluded that their relevance for describing an EF depends on the learner's overall performance. Thus, this paper will be answering questions such as, what is more related to having good counting skills, being accurate in inhibitory tasks, or having a short RT? Is there any latent variable that could affect this correlation?

To better understand the whole study, the introduction is organized in different subsections to introduce the definition of the main concepts discussed in the study and how previous literature connects them to the research questions. The main topics covered in this chapter are EF (focusing on inhibition and switching, and accuracy and RT), EN (focusing on counting and numerical relational skills), and how these are related to each other. Finally, based on the above theoretical framework, the research questions of this study are presented and justified.

1.1. Executive Functions in four-year-old preschoolers

EF (Executive Functions) are the first general variable considered in this study. As previously said, it is agreed by the scientific community that they are the neurological functions that act as prerequisites or cognitive support for learning and reasoning (Clements, Sarama, & Germeroth, 2016). However, when defining their different components, there is a controversial debate (Karr, et al., 2018). Several models have been presented, but they can be grouped into two: the working memory model (Clements, Sarama, & Germeroth, 2016; Diamond, 2013), which locates working memory in the center of the model, as the most important EF; and the inhibition-switching-updating model (Karr, et al., 2018) that recognizes a more balanced contribution of the different EF. Besides the two descriptive models, there are two perspectives from which EF can be analysed (Clements, Sarama, & Germeroth, 2016): first, as prerequisites or cognitive support for learning and reasoning, and second, as tools for accepting and following social rules, benefiting the subject to be able to learn in various social contexts.

The present study considers the inhibition-switching-updating model since it focuses on two of these EF; and the first perspective, as it focuses on understanding the mental processes underpinning learning early mathematics.

Following the model presented by Karr et al. (2018), Yang, Chung and McBride (2019) published an extensive study on how EF affect the development of early mathematics, closely related to the aim of the present study. The definitions Yang, Chung and McBride (2019) used for each of the EF are the concrete ones being followed in this study. Thus, working memory consists of the retention of operating information in mind to utilize it; it has been proven to improve calculation and applied problems. Then, updating refers to the ability to monitor our mental representations and renew (update) them according to new and relevant information we gather. Many studies have proven its influence on mathematical skills, although they remain controversial due to its close similarities with working memory (Bull & Lee, 2014). Next, inhibition refers to the suppression of a dominant response in favour of another response or non-response that is more useful at the time. Finally, switching (also known as shifting or mental flexibility) is the ability to change (switch) between tasks, sets, or strategies. At this point, it is essential to specify that, while described as independent entities, EFs collaborate with each other to accomplish different mental tasks. For this reason, studying them in a practical setting as isolated entities is quite tricky.

In the present study, in addition to the difficulty of wanting to analyse two executive functions separately (inhibition and switching), it has been added difficulty in assessing switching in 4-year-old preschoolers. Switching performance emerges during the school-age (starting at 6-years-old), and no model has identified it as a separate factor among preschoolers (Karr, et al., 2018).

Bull and Lee (2014) explain how EF gradually differentiate with age. For example, one study (Lee, Bull, & Ho, 2013) reported the existence of only two domains of EF in 5–13-year-olds (updating and an amalgamated inhibition-change factor) but three domains in 15-year-olds. Moreover, the reality is that "no preschool sample identified shifting as a separate factor, while all three factors were represented in all groups above six years of age" (Karr, et al., 2018, p. 1162).

Nevertheless, Lee and Bull (2014) claim that a new test needs to be developed to measure inhibition and switching as independent EF in young children. They are confident of their influence as isolated components of EF in young children, which has also been demonstrated by Moriguchi & Hiraki (2011). They concluded that children from age 3 to 4 improve their performance on conceptual switching tasks. This study also supports this idea and engrosses the data and empirical knowledge around this particular EF among 4-year-old preschoolers. It uses flanker tasks to assess both inhibition and switching, classifying their items into "switching items" or "inhibition items" depending on their characteristics (explained in depth in the chapter of Research work / Materials in this document).

Consequently, the relevance of the executive functions in the present study lies in the idea that they conform the prerequisites and cognitive support for learning and reasoning. This suggests that EF might allow those students with limited initial math skills who have high levels of EF to "catch up" (Braak, et al., 2022). Nevertheless, an exciting idea is the possibility of training EF as a tool to improve the learning of mathematics. However, Thorell et al. (2009) published some results saying that not all EF can be trained, stressing that out over inhibition. The present study aims to shed some light on this issue by understanding the role that accuracy and RT play in the relationship that inhibition has with EN.

1.1.1. Inhibition and Switching

Inhibition and switching have already been introduced in this study report. However, since they are the two EF assessed in this study, this subsection wants to present them in a more concrete manner related to the current literature background.

In the first place, Yang, Chung, and McBride (2019) consider **inhibition** as the capacity to suppress a dominant or pre-ponent response in favour of another response, which also can be the absence of any response. This means that inhibition is the EF that helps the suppression of irrelevant-information-processing or the use of inappropriate strategies during mathematics problem-solving. For example, when solving a problem that relates two numbers in a number retrieval (e.g. '2×3'), inhibition is the process that helps to omit other number retrievals that relate those same two numbers but are not the posed problem (e.g. '2+3') and to process the solution of the given problem ('2×3').

Many scholars have already studied to understand the relationship between inhibition and mathematics performance, yielding different results. Espy et al. (2004) found that an inhibition composite variable accounted for unique variance in simple applied problems. Lan et al. (2011), in a study with preschoolers, found that inhibition could just explain basic counting skills but not any other more cognitively demanding tasks. However, concerning the assessment of inhibition, Lee and Bull (2014) explain that the measurement of inhibition is also complicated. They point out that the used tasks must have a minimal memory load and not require rule switching.[1]

Switching (or shifting) is described as the ability to switch between two or more tasks, solution strategies, steps, or notions (Bull & Lee, 2014; Yang, Chung, & McBride, 2019). As it has been established, switching involves changing a "'mental set' from one aspect of a situation to another as the situation requires" (Clements, Sarama, & Germeroth, 2016, p. 80), the reason why switching is also

[1] Bull and Lee (2014) have studied how and why updating normally reminds as the unique predictor of maths achievement, while the obtained results around inhibition and switching resemble less conclusive. They claim for better tests to measure inhibition and switching as independent EF. However, integrated models are still being used and also working as predictors of mathematical academic performance (Braak, Lenes, Purpura, Schmitt, & Storksen, 2022).

known as *cognitive flexibility*. Therefore, in the subject of mathematics, switching contributes to changing from one step to the next in complex multistep problems, as well as to making the decision between different strategies to solve the problem in its optimal way. An example of good switching abilities is the capability to count by twos, by threes, or so on; and also changing the solving strategy in a problem when seeing that the last one failed.

When it comes to studying its direct association with mathematical skills, Bull and Scerif (2001) found out that those children who are poor in inhibition and working memory may also show a poor performance in switching; probably due to the previously mentioned idea of switching interacting together with working memory. In this concern, Moriguchi and Hiraki (2011) showed how, from the age of 3 to 4, children improve their performance on cognitive switching tasks (such as the Wisconsin Card Sorting Test[2]), which are those tasks that require a process of conscious redirection of one's attention from one fixed 'mental set' to another[3]. This is probably the reason why Bull and Lee (2014) hypothesise that switching could help in the shifting between operations, solution strategies, notations, and the steps of a multistep problem. However, even nowadays, most studies keep on using a testing model that integrates working memory, inhibition and cognitive flexibility (Braak, et al., 2022).

In general, studies have drawn a direct positive correlation between EF and EN (Bull & Lee, 2014; Clements, Sarama, & Germeroth, 2016). Not many results have been published around inhibition and switching as independent EF. Nevertheless, Clark, Pritchard, & Woodward (2010) measured inhibition and switching in 4-year-old children, finding out a correlation with later academic achievement in mathematics; implying that they can be used as predictors of academic achievement or difficulties in mathematics. The used task to assess these EF was Shape School (Espy K. , 1997) which requires good use of

[2] The Wisconsin card sorting test (or WCST) requires the participant to short out cards according to one of their characteristics, for example, shape. Then, after the correct classification of ten cards, the participant must change and start sorting them out according to another characteristic, for example, colour. However, the WCST has been described as a test that requires the engagement "of different EF and auxiliary processes at different stages of the test" (Lee, Bull, & Ho, 2013, p. 1936).

[3] If this process occurs unconsciously, then it is related to task switching. Both unconscious and conscious switching processing are forms of cognitive flexibility.

language and vocabulary. Therefore, to make sure that the use of language and vocabulary was not an impairment, in this study, there have been used digital adapted flanker tasks (modified from Fan, et al. (2002)).

1.1.2. Accuracy and reaction time

The present subsection focuses on the concepts of accuracy and RT (reaction time) as the two different metrics that are utilised when measuring inhibition and switching. In contrast to the other concepts presented so far, accuracy and reaction time as metrics of executive functions have been studied in less detail. However, there can be found some interesting studies that give us some valuable information in this regard.

In the first place, Camerota et al. (2020) present valuable definitions and conclusions in their study. They defined accuracy and RT as follows: accuracy is the proportion or percentage of correct answers, and RT is the average time used to answer the items.

To this concern, interesting results were presented by Diamond et al. (2007)and Clark et al. (2014), who accomplished that the measuring power of accuracy is more relevant in younger children and, afterwards, RT starts to play a more important role. In this line, Camerota et al. (2020) ran a study with 1015 6-year-old participants to see if the relevance of accuracy and RT could change among participants of the same age. They concluded that when accuracy reaches a score over 80%, RT starts to have greater importance as a metric of EF. This means that there can be two profiles of children with high EF accuracy. The ones that find the task easy, so they may be able to keep a high accuracy and give a quick answer. Whereas the ones that find the task more challenging may use more cognitive resources to respond accurately, but they will have a higher RT than the children in the previous group.

In light of these results, this study makes a latent profile analysis to find out if such behaviour also happens among younger children, 4-year-old preschoolers that have not yet reached the best of their cognitive abilities.

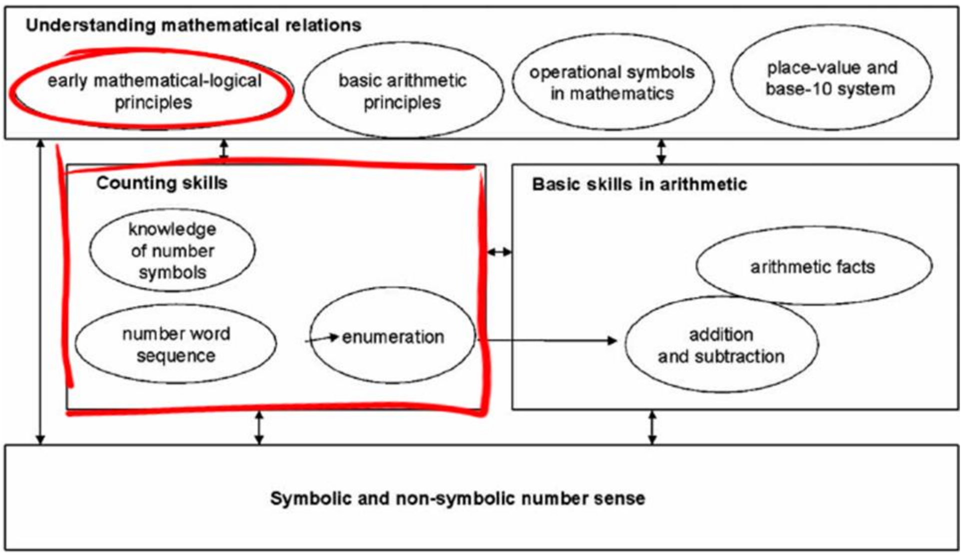
1.2. Early Numeracy skills in four-year-old preschoolers

EN (Early Numeracy skills) are the second variable that is considered in this study. EN are a set of skills that has also been described under the name of 'number sense' and is settled in the preschool years. EN are the precondition for mathematical competency, which is developed from the stabilized link between counting and the ability to discriminate and relate quantities (Krajewski & Schneider, 2009). Another definition of EN is given by Aunio and Niemivirta (2010, p. 427), who describe them as "the set of skills to operate with the number–word sequence and to make relational statements in a numerical context". EN have been proven to correlate with a better academic mathematics performance in primary school when being worked and correctly developed in preschool, before the age of six (Aunio & Niemivirta, 2010). Therefore, understanding EN helps the understanding of learning mathematics, so its teaching methods can be improved.

This study follows the definitions established by Aunio and Räsänen (2015) to narrow it down. They looked at early numeracy from different views and found diverse mathematical skills with a prediction power for later mathematics learning. They categorize these skills into four groups (summarised in Figure 1).

Figure 1

Early numerical skills for learning mathematics (modified from Aunio and Räsänen (2015, p. 16))



Note. The red lines are framing the core early numeracy skills the present study is focusing on.

The first group is called 'symbolic and non-symbolic number sense', and it is used in "processes where an approximate evaluation of magnitudes or symbols representing magnitudes is used" (Aunio & Räsänen, 2015, p. 10). The second set of skills, 'understanding mathematical relations' refers to the capacity to understand quantitative and non-quantitative relationships between elements in a task. Ordinal numbers, mathematical symbols, place value, and base-10 operations are included in this group, which explains that early mathematical-logical principles and basic arithmetic principles are also embedded in this group. This group is at the core of the present study, so it will be explained in more detail in the next section of this chapter. The third group gathers the 'counting skills' divided into three different aspects: the knowledge of number words and symbols, the skills to move through the number words sequence, and enumeration. As well as the previous group, counting skills are also at the core of the present study, so they will be explained in more detail in the next subsection of this chapter. Finally, the fourth group refers to the 'basic skills in arithmetic'. It calls to the master of addition and subtraction tasks with symbols. This last group has been excluded from the current study because arithmetic is not introduced in the Finnish curriculum until the age of seven (Finnish National Agency for Education, 2019).

The present study focuses on the second and fourth groups: understanding mathematical relations and counting skills in 4-year-old preschoolers. Therefore, the following subsection is devoted to further elaborating on the theoretical framework that describes these two sets of skills.

1.2.1. Numerical Relational skills and Counting skills

This subsection presents the two EN that are assessed in this study; early numeracy skills and counting skills. It focuses on their definitions and some other studies that have been done in the same or similar field.

The **numerical relational skills** are part of the skills to understand mathematical relations. Aunio and Räsänen (2015) described them as the “early mathematical-logical principles”, which are necessary to understand mathematical relations. They include skills such as seriation (that allows the understanding of the number-word sequence and its ordinal and cardinal properties), classification (fundamental for mathematical reasoning in general), and one-to-one correspondence (which is needed to develop accurate counting skills). They emerge earlier than the ability to operate with the number-word sequence (Aunio, Hautamäki, Heiskari, & Van Luit, 2006) and are basic for the articulation of thoughts based on comparative thinking, which is an essential prerequisite for developing the logical thinking that enables us to solve basic arithmetic mathematical problems (Aunio & Niemivirta, 2010; Krajewski & Schneider, 2009).

The quantitative relations start to be expressed when children learn vague non-numerical terms that refer to quantities; they are called 'proto-quantitative terms' and allow the development of three 'protoquantitative schemas' (see the work of Resnick for a deeper explanation, referenced by Krajewski and Schneider (2009, p. 513)) which are closely related to the different schemas of the additive structure mathematical problems.

Counting skills refer to the use of number words and the general understanding of numbers. Around the counting skills, Aunio and Räsänen (2015) enumerate three important aspects to be considered: the knowledge of number symbols, the skills to move in the number-word sequence, and the skills to enumerate. The knowledge of number symbols is essential for understanding the cultural number system and how it works. The skills to move in the number-word-sequence are the predecessors of self-assessment in enumerating skills but also the key to introducing arithmetic problem-solving using counting strategies and the development of numerical flexibility, moving through the sequence skipping by twos, tens, or fives. Finally, enumeration is related to numerosity; it is frequently described as the skill for calculating the numerosity of a set. Enumeration requires good skills moving through the number-word sequence, but also the understanding of how a number indicates the addition of a new object in the enumeration process and how the last said number corresponds to the numerosity of the set.

The development of counting skills is divided into six stages (see the work of Fuson for more details, referenced by Aunio and Räsänen (2015, p. 13)). The first is related to the understanding of amounts and emerges around the age of two; children show how different quantities are expressed with different number-words. The second one is the acoustic counting stage, around the age of three, when they start to learn that there is a number-word sequence. However, they do not know it yet. They just say numbers, ones after the others. The third one is normally reached at the age of four (the age of the participants in this study). It is the a-synchronic stage, and, in it, preschoolers become able to say number words while pointing to objects, although this pointing is not coherent to the number-words said. Then, in the synchronic stage, achieved at the age of four years and six months, the pointing and counting are done in a coherent way. The resultative counting stage emerges around the age of five, and then, around six months after, they are able to recognise figures of numbers and continue counting upwards from that (Aunio & Räsänen, 2015).

The participants of the study are just in the middle of the counting skills development. From a theoretical viewpoint, they are between the age of reaching the asynchronous stage (being able to say number words in the correct order and point to objects, but the words and pointing are not coherent) and the synchronic stage (being able to recite numbers while pointing at the counted objects). Accordingly, the resultative counting skills are not yet expected among the participants of the present study, which explains the omission of seven items from the counting skills in the Early Numeracy Test because of having a response probability lower than ,10.

1.3. Relations between executive functions and early numeracy skills

Once the definitions of the elements of the study are clear, this section is to give information about how the existing literature describes the relationship between EF and EN.

The existing relations between EF and EN have been vastly studied since the decade of the 90s. In this light, Clements, Sarama and Germeroth (2016) made a review gathering almost all the findings of three decades. They conclude that it has been proven a positive correlation between EF and EN since it is also the area of mathematics the most related to EF (over others such as literacy or sciences). Moreover, it is mentioned how several studies came up trying to understand both if EF correlate the same way with the different areas of mathematics, or if there is any particular EF that has a higher correlation with the mathematical skills. In this light, they do not present any agreement among the scholars but some interesting studies.

In relation to how the different EF are related to EN, Lan et al. (2011) found out that inhibition works as a powerful predictor of counting but not of further calculations, where working memory plays a more important role. In this line, Yang, Chung and McBride (2019) published an extensive study on how EF affect the development of early mathematics, concluding that working memory is the most powerful predictor out of the four EF. Therefore, updating, inhibition, and switching complement the power of working memory in more complex mathematical processes. Finally, Espy et al. (2004) also found a bigger correlation from working memory, but also interesting information from inhibition in relation to the counting skills. Thus, there is a general agreement about the existing correlation between EF and EN, but not a common agreement that describes how each EF affects the mathematics' performance (for concrete examples, see the work of Bull and Scerif (2001), Diamond (2013) or Espy, et al. (2004)).

Looking at how accuracy or RT in each of the EF are related to EN, Camerota et al. (2020) show that the measuring power of accuracy and RT are different depending on the students' EF-performance. As previously mentioned, they found out that when the students have a good EF-performance (when reaching an accuracy over the 80%), there is a more powerful correlation (negative correlation) between the RT and the performance of the student, whereas when the student has a low EN-performance, the most powerful correlation lays on accuracy. Meaning that when a student has better performance, having a lower RT is more important than when the performance is lower, moment in which accuracy is more important. That could be explained because, in a low-performance student, a high RT can mean a lack of thinking and reflection whereas, in a good performance student, a high RT can reveal a better agility, since less time is used to process the information.

Finally, Clark et al. (2014) conclude that, although EF have a predictive power of later mathematics for all ages; it starts to lose its power with age, once processing speed starts to be taken into account as well as bigger language competency starts to be demanded (from 4-years-old onwards).

1.4. The present study

The present article wants to answer the question about how are switching and inhibition correlated with counting and numerical relational skills in 4-year-old preschoolers. The interest underlying this question comes from the findings from different studies that correlate the correct development of executive functions with a better academic performance in mathematics (Aunio & Niemivirta, 2010; Clements, Sarama & Germeroth, 2016). Therefore, understanding how executive functions affect the performance of early numeracy skills will allow us to intervene in the development of mathematical thinking as soon as possible.

To better understand this relationship, this study differentiates between accuracy and the RT when solving executive function tasks, which have been proven to be different indicators of EF ability (Camerota, et al., 2020). Hence, these are subsequent research questions (see Figure 2 for a better understanding of the relationships mentioned in the research questions):

1. How does accuracy in inhibition and/or switching affect counting and numerical relational skills?
2. How does RT in inhibition and/or switching affect counting and numerical relational skills?
3. Is there any latent pattern affecting the relationship between accuracy and RT, and counting and numerical relational skills?

To answer these questions this study makes a statistical analysis of the data gathered by *Active Early Numeracy* (the research group headed by Dr. Pirjo Aunio, at Helsinki University), which has been collecting information in 21 Early Education Schools in the Helsinki area, with 186 4-year-old preschoolers.

The decision to work with 4-year-old preschoolers has been made because it is claimed that EF are developed in the preschool years, but its components are not divided until the age of three (Yang, Chung, & McBride, 2019); however, not much data about inhibition and switching has been measured among 4-year-old children. Moreover, this study comes to engross the list of studies about inhibition and switching as independent EF in 4-year-old preschoolers. Then, the age component of the participants is the main reason why this study only measures numerical relational skills and counting skills. This study is just measuring numerical relational and counting skills because, accordingly to the Finnish curriculum of Early Education, 4-year-old preschoolers have not started to learn arithmetic yet (Finnish National Agency for Education, 2019).

The data has been gathered using a digital version of the Flanker tasks (modified from Fan et al. (2002)) to measure inhibition and switching accuracy and RT, and the Early Numeracy Test (Aunio, Hautamäki, Heiskari, & Van Luit, 2006) to measure the preschoolers' performance in the use of counting and numerical relational skills. Even if the description of the tests' particularities is not developed in this section, this information is valuable in explaining how EF and EN are related in the present study. That is very relevant question that must be approached, giving sense to the whole study.

Therefore, how are the participants needing inhibition and switching when making counting and numerical relational skills tasks in the Early Numeracy Test?

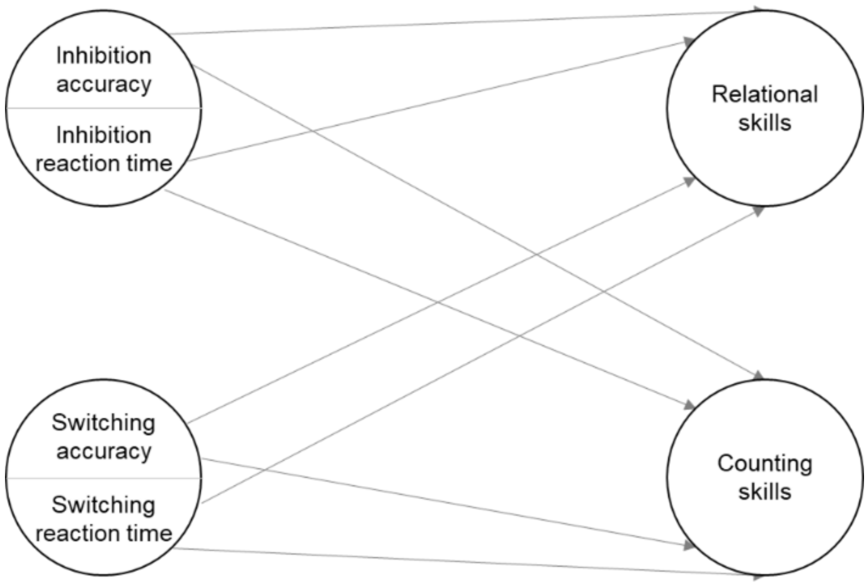
The performance of numerical relational skills requires the preschooler to inhibit all the information that is given to them (the different features of the elements that are being shown to them) and to only focus on the characteristics (information) that the administrator of the test is pointing out. Moreover, switching is involved in any transition between items but also when the preschooler is asked to follow instructions, where they have to switch from their way of acting to the one that the administrator is suggesting.

To solve the tasks that require counting skills, the preschooler must inhibit natural tendencies such as counting forward or pointing to the objects that are being asked to be counted. Here, switching is also involved in any transition between items but also when the administrator is asking to solve the task in a concrete and unusual way, so the preschooler must change from the established way of proceeding to the one that now is being demanded, i.e., counting by two instead of one by one.

Moreover, after this analysis, it can be seen how switching and inhibition are certainly very close to each other, so let us point out their differences so they can be better understood. As it is seen, they both refer to the analysis and discrimination of information. However, inhibition refers to the control and analysis of chunks of external information, while switching raises to the control of mental processes that will guide us to perform in one way or another. As Lee, Bull, and Ho (2013, p. 1933) describe: “Switching or shifting refers to the ability to move between alternative sets of mental operations. Inhibition refers to the ability to resist interference from competing or prepotent responses or processes.”

Figure 2

Scheme of the analysed relations between variables.



2. Research work

This chapter presents the research method implemented to learn how switching and inhibition are related to counting and numerical relational skills in 4-year-old preschoolers, digging into how accuracy and RT in these two EF can affect the two studied EN. This chapter first focuses on how data has been gathered, talking about participants and materials, and how this data has been statistically analysed lately.

2.1. Participants

A total of 189 4-year-old[4] preschoolers (89 boys and 100 girls) took part in the study. Of these, 107 (51 boys and 56 girls) participated and finished the two required tests. Therefore, the data used in the present study come from these 107 participants. All the participants come from volunteer families eager to participate in the study. The *Active Early Numeracy* research group (at Helsinki University, led by Dr. Pirjo Aunio) reached several schools in Helsinki Area. Twenty-one of them accepted to participate and reached their parents or legal tutors, who were the ones permitting the participation of the preschoolers.

2.2. Materials

The participants performed one test to evaluate the EN and another one to evaluate the EF. The Early Numeracy Test (ENT) (Aunio, Hautamäki, Heiskari, & Van Luit, 2006); was used to assess counting and numerical relational skills (as EN). To assess inhibition and switching (as EF) there was used a computer-based flanker set of tasks (modified from Fan et al. (2002)). Both of the tests were provided in Finnish to Finnish speaker preschoolers by a Finnish speaker research assistant from the *Active Early Numeracy* research group (at Helsinki University, led by Dr. Pirjo Aunio), and, in any case, the preschoolers were being informed about any possible mistake they could make.

This data gathering process lasted one year and a half, from the 12th of March 2019 to the 20th of November 2020, being paused from April to August 2020 due to the rise of the Covid-19 pandemic.

[4] The exact age of the participants is unknown.

2.2.1. The Early Numeracy Test

The two EN (numerical relational and counting skills) are assessed in this study with the ENT (Early Numeracy Test). The test takes a developmental perspective on children's EN and aims to describe eight aspects of numerical knowledge. The first four (concepts of comparison, classification, one-to-one correspondence, and seriation) focus on the logical principles that underpin the described numerical relational skills related to the understanding of quantities and its relations. The other four scales (the use of number words, structured counting, resultative counting, and general understanding of numbers) refer to the use and understanding of numbers, which is named in this study as counting skills.

The target group of the test is four- to seven-year-old children, so the participants of this study are at its lower edge. The test is given individually and takes about 30 minutes for a child to complete. The 40 items are scored by giving one point for a correct answer and zero for a wrong answer, with a maximum score of 40. The children are not given feedback as to whether their response is correct or incorrect, and the time spent on the test is not registered. The ENT is assumed to yield a unidimensional measure of children's early numeracy, but previous studies have shown that the ENT differentiates two closely related factors reflecting slightly different aspects of children's early numeracy. The first four scales of the test (items 1 to 20) concern numerical relational skills, while the other four scales (items 21 to 40) focus more explicitly on the use and understanding of the number–word sequence, counting skills (Aunio & Niemivirta, 2010).

As just mentioned, the numerical relational skills are measured by items 1 to 20, divided into four groups of five tasks that focus on (Aunio, Hautamäki, Heiskari, & Van Luit, 2006). (1) Comparison, which refers to quantitative and non-quantitative relationships and helps the development of numerical reasoning (Aunio & Räsänen, 2015). (2) Classification is considered the pillar of mathematical reasoning (Aunio & Räsänen, 2015), asks the preschoolers to group objects according to one or more characteristics. (3) Making correspondence assesses the participant’s understanding of one-to-one correspondence. One-to-one correspondence leads to counting accuracy and makes reference to the ability to connect entities in two different sets. It is necessary to make sense of counting tasks that require the connection between the number-word sequence and the set of elements that are being counted (one number word, one object) (Aunio & Räsänen, 2015). Finally, (4) the seriation set includes items that invite the participant to deal with discrete and ordered entities. This, as Aunio and Räsänen (2015) explain, is important to understand the number-word sequence that, together with cardinality, helps to understand the number-word sequence, which is built in order of growing cardinality value.

Counting skills are measured by items 21 to 40, divided into four groups of five tasks that assess (Aunio, Hautamäki, Heiskari, & Van Luit, 2006). The first five tasks assess the use of number words in the number-word sequences up to 20, breaking the number line and counting both forwards and backwards. Knowledge of number words has been shown to be crucial for performing simple one-to-one counting tasks, and for understanding how the number system works (Aunio & Räsänen, 2015).[5] The second group demands the use of synchronous and shortened counting of objects in an organized and unorganized setting with the help of pointing. In these tasks, the participants are showing their ability to use their number-word sequence skills to solve an enumeration problem (Aunio & Räsänen, 2015). The third group is a set of resultative counting in tasks in which pointing is not allowed, and the participants are asked to say how many elements are presented in a concrete set. So, these items go a step further, expecting the participant to understand that "the last named number word states the numerosity of the set" (Aunio & Räsänen, 2015, p:13). Finally, the general knowledge of the numbers is assessed, which refers to the daily application of EN with different verbal mathematical problems, which is the final stage in the development of counting skills (Aunio & Räsänen, 2015).

Finally, to optimise the psychometric properties of the measures, all items that were too easy or too difficult (response probability greater than .90, or less than .10, respectively) were removed. Therefore, items 23, 24, 25, 30, 31, 34 and

[5] This statement shows how EN are developed in a continuous way, where the correct development of one affects the development of the other.

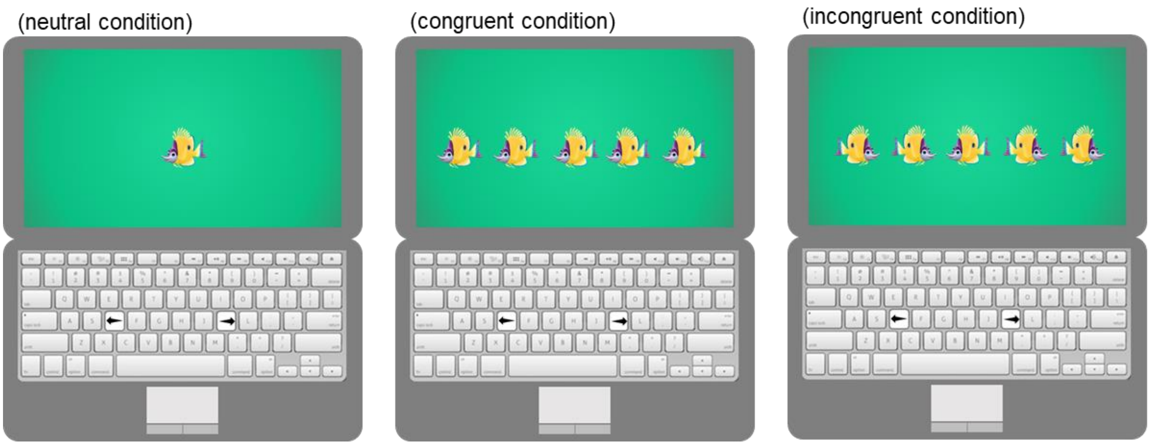
35 were removed. This makes sense, as they are all items of the counting ability, which is fully developed by the age of five.

2.2.2. The computer-based flanker task

The computer-based flanker task (modified from Fan et al. (2002)) presented a row of five fish on the screen; they will be facing either left or right. The target fish is the one in the centre of the computer screen. There are three possibilities to present the target fish (as visible in Figure 3). The target fish can appear on its own (this is called 'neutral condition'), or flanked by two fish on each side. If the fish are facing the same direction, it is called a 'congruent condition'; if they are facing the opposite direction, it is called an 'incongruent condition'. In each trial, preschoolers are asked to identify, by key pressing, the direction that the target fish is facing.

Figure 3

Conditions in which the target fish can be presented on the screen.



The test consists of six blocks of items. In the first block, 20 items are presented in a neutral condition. Then, the following two blocks will present 20 pure congruent trials, and 20 pure incongruent trials (or vice versa). Afterwards, the following three blocks consist of 28 items in which preschoolers will have to switch between congruent and incongruent conditions in approximately one-third of the trials. Therefore, preschoolers must inhibit the information given by the fishes in the flankers (sides) of the target fish. One point was given for each correct answer, and the sum-scores were used in the analysis. However, in the used dataset, there is a maximum of 48 points for inhibition accuracy and 24 for switching accuracy.

The points presented in the dataset come from the following analysis. First, there were excluded each of the 4 first items of each of the last three blocks, which makes a total of 72 items. These 72 items have been divided into two groups: (1) 48 no-switching tasks, used to assess inhibition with sequences of congruent conditions; and (2) 24 switching tasks, used to assess switching with sequences of congruent-incongruent conditions. At this point, the advice from Lee and Bull (2014) was taken into account since the test requires a minimal memory load and does not require rule switching; the same rule is applied during the whole test; ‘indicate the direction the middle-fish is looking at’.

Lastly, in this test, RT was directly registered as the time (measured in ms) that elapsed between the exposure of the item and the response of the participant.

2.3. Statistical analysis

After all the data was gathered, it has been statistically analysed with SPSS (version 27) and R (version 4.2.0). The analysis was done with the aim of answering the research question and finding out how RT and accuracy in inhibition and switching are related to counting and numerical relational skills performance in 4-year-old preschoolers. Therefore, the main performed analysis were a correlation analysis (with SPSS v.27) and a latent profile analysis (LPA) (with R v.4.2.0).

The utilized variables in this analysis were the following calculated ones:

1. NUM_REL_SUM: the average punctuation in numerical relational skills, corresponding to the 20 first items of the ENT.
2. COUNT_EN_SUM: the average punctuation in counting skills, corresponding to the 13 items from the second half of the ENT. They result from subtracting items 23, 24, 25, 30, 31, 34, and 35, which have their response probability below .10.
3. INH_SUM: the average punctuation in inhibition, with 48 items from sequences of congruent conditions in the flanker tasks.
4. INH_reac_time: the average time response in the inhibition items, measured in ms.

5. SWI_SUM: the average punctuation in switching, with 24 items from sequences of incongruent conditions in the flanker tasks.
6. SWIT_reac_time: the average time response in the inhibition items, measured in ms.

These all were independent variables, and the data in each of them was not normally distributed, which led to the computation of a Spearman's rank correlation between each of the variables from EF to EN (as visible in Figure 2), a Spearman's rank correlation between 6 different variables.

With the aim of knowing if there was any latent variable that could drive these correlations differently in different situations, a LPA was run in R, using packages mclust (v5.4.9; Scrucca et al., 2016) and reshape2 (v1.4.4; Wickham, 2007). LPA is a categorical latent variable approach that focuses on identifying different subpopulations within the main observed population based on the metrics of a different set of variables (Spurk, et al., 2020). The LPA assumes that people can be classified, with different degrees of probability, into different groups/classes/profiles that have different configuration metrics of the input variables. In the case of this study, the LPA classifies the participants according to their numerical and counting performance, their inhibition and switching accuracy, and their average RT in the inhibition and switching tasks. Giving each of the participants a concrete probability of belonging to one profile or the other.

When it comes to performing a LPA, one of the most complicated steps is the decision on the model to be used and the number of profiles into which the population is to be divided. The different models make different assumptions about the population and the way in which the data is organized in relation to the variables (to know more about the different models, see the work from Scrucca et al. (2016)). To assist in this decision, different indicators have been used to describe how well the data fit each of the models with a different number of profiles. The indicators that have been used in this study are the Bayesian Information Criterion (BIC), the Integrated Complete-data Likelihood (ICL), and the p-value of the bootstrapped likelihood ratio test (BLRT); even if the theoretical value of the models has also been taken into account (Scrucca, et al., 2016), for which it is necessary to have a good knowledge of how the data are distributed among the different variables.

3. Research results and their interpretation

This chapter presents the obtained results and interprets them considering the research questions and the theoretical framework presented in the introduction.

The results obtained from the Spearman's rank correlation are visible in Table 1, and they show that accuracy in EF has a higher positive correlation with EN than RT. The correlation between the accuracy punctuation and the EN-performance punctuations was weak to moderate positive correlation ($p < .001$), while the correlation between the average RT in the EF and the EN-performance punctuations was weak or close to zero, with lower statistical significance ($p < .05$)[6].

Table 1

Correlations among inhibition and switching accuracy and RT, and counting and numerical relational skills.

	Numerical relational skills	Counting skills	Inhibition Accuracy	Inhibition RT	Switching Accuracy	Switching RT
Inhibition Accuracy	,376**	,383**	-			
Inhibition RT	,220*	-,029	-,172	-		
Switching Accuracy	,322**	,395**	,796**	-0,168	-	
Switching RT	,192*	-,073	-,283**	,861**	-,398**	-

** $p < .01$
* $p < .05$

The results of the LPA show two statistically significant models, which are interesting (see Table 2). One of them a 3-profile model (BLRT(p) = ,003); the other one a 2-profile model (BLRT(p) = ,001). The p-value calculated in the BLRT indicates the likelihood of being wrong in the classification of the participants represented in the dataset (Nylund, Asparouhov, & Muthén, 2007). That means,

[6] The classification of the obtained correlation factor as weak, moderate, or strong has been obtained from the review article from Akoglu (2018), presenting a user's guide to correlation coefficients.

in a one-profile model, the likelihood of being wrong is 0%; then, according to the data exposed in Table 2, the likelihood of being wrong in the classification of any participant into their profile is up to 0,1% (BLRT(p) = ,001), and, consequently, in a three-profile model, the error will be up to a 0,3% (BLRT(p) = ,003).

According to the BIC and the ICL, a VEE model has been chosen, which considers profiles with ellipsoidal figures with the same shape and orientation. The VEE model is the most represented among the six best ones with the best BIC and or ICL values (see Table 3) and has a good p-value from the BLRT (see Table 2). Both the BIC and the ICL are IC (information coefficient) that are based on the log-likelihood of a fit model. Each of the ICs applies different penalties regarding the model, the sample size, or both. That is the reason why their results can be different (Nylund, Asparouhov, & Muthén, 2007). Nevertheless, they are always interpreted as "the lower, the better" (to see examples, the reader can check the work of Spurk et al. (2020) or Borden et al. (2014)).

Table 2

BLRT for the number of mixture components in model VEE (999 replications)

	BLRTs	BLRT(p)
1 vs 2 profiles	52,299	,001
2 vs 3 profiles	34,104	,003
3 vs 4 profiles	23,841	,065

Note. BLRT(p) = p-value for the bootstrapped likelihood ratio test.

Table 3

Models with the best BIC and ICL values

	VVE,2	VEE,2	VEE,3
BIC	15072,78	15248,26	15281,04
BIC diff	0,00	17,55	20,83
	VVE,2	VEE,2	EVE,3
ICL	1518,58	1530,84	1537,02
ICL diff	0,00	12,26	18,45

Note. The set of three letters on the headers of the colons refer to the model (VVE, VEE, and EVE). The number after the coma refers to the number of profiles (2 or 3).

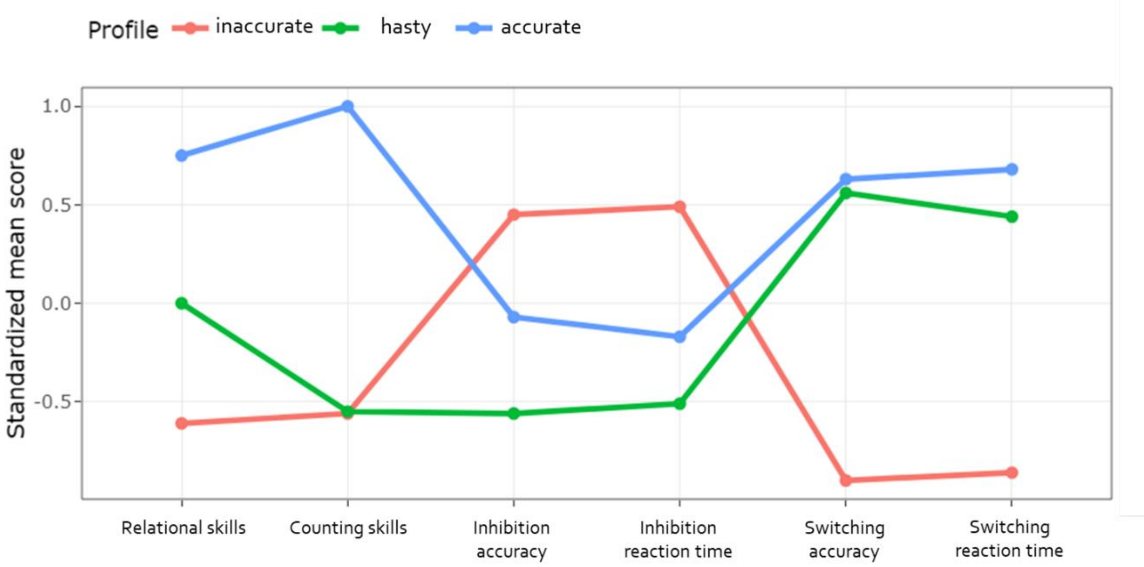
The 3-profile model shows two apparent patterns: one with inaccurate responses and high RT (in red in Figure 4) and the other with really accurate responses and average RT (in blue in Figure 4). The third group (in green in Figure 4) shows the lowest RT. It brings together the fastest participants with high EF accuracy but average EN-performance. This last group is quite interesting because it does not follow an expected pattern.

The three profiles where calculated with the standardized mean score of each of the variables (see Figure 4). They have been named in the following way:

- Profile one (N=42): describes the results of the participants that have shown higher RT concerning the mean and worse performances than the mean. Therefore, this group is named "Inaccurate".
- Profile two (N=29): describes the results of the participants that have shown the lowest RT in relation to the mean and EN-performance not better than the mean. Therefore, it is named "Hasty".
- Profile three (N=36): describes the results of the participants that have shown average RT in relation to the mean and better performance than the mean. Therefore, it is named as "Accurate".

Figure 4

Standardized means in the indicators of numerical relational skills, counting skills, inhibition RT, switching RT, inhibition accuracy, and switching accuracy for the three-profile solution.
Note. 99% CI, N = 107, p<0,01

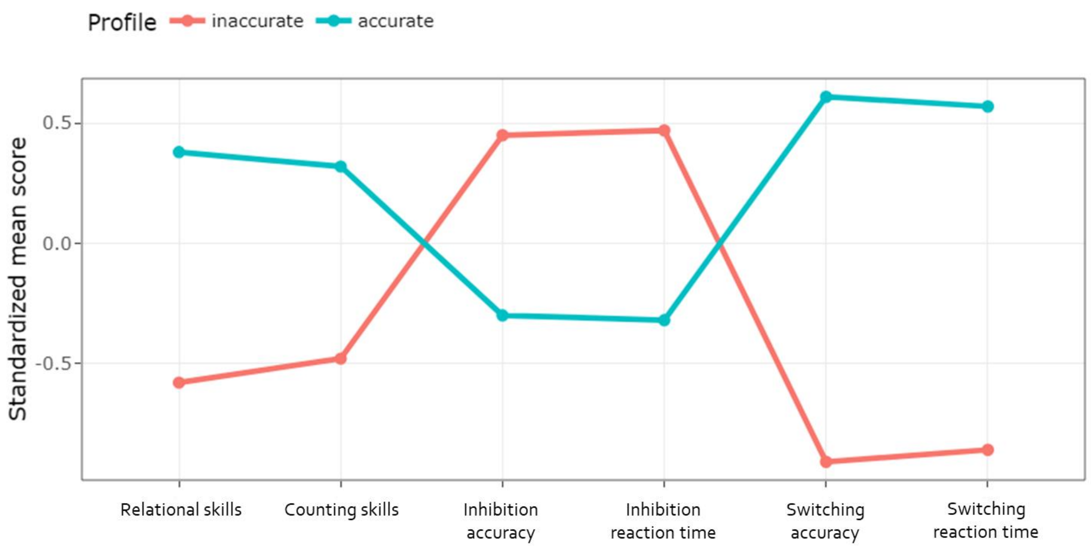


The 2-profile model shows two clear patterns: one with inaccurate responses and high RT (in red in Figure 5) and the other with really accurate responses and average RT (in blue in Figure 5). The two profiles were calculated with the standardized mean score of each of the variables (see Figure 5), and they have been named in the following way:

- Profile one (N=42): describes the results of the participants that have shown higher RT in relation to the mean and worse responses than the mean. Therefore, it is named as "Inaccurate".
- Profile two (N=65): describes the results of the participants that have shown lower RT in relation to the mean and better responses than the mean. Therefore, it is named as "Accurate".

Figure 5

Standardized means in the indicators of numerical relational skills, counting skills, inhibition RT, switching RT, inhibition accuracy, and switching accuracy for the three-profile solution.
Note. 99% CI, N = 107, *p*<0,01



Concluding, the results show that even if the Spearman's rank correlation factor shows a better correlation factor between accuracy than RT with EN, each correlation must be understood according to the value it has, which is weak to moderate. That is probably why the 3-profile LPA shows some preschoolers that, even if having good inhibition and switching accuracy, have not obtained as good results in their EN-performance (profile "hasty", in green in Figure 4). This could also be helped by the power that RT still has in correlation to numerical relational skills performance: $r_{s(105)}$=,220 between inhibition RT and numerical relational skills, and $r_{s(105)}$=,192 between switching RT and numerical relational skills. Therefore, since participants' RT in the "hasty" profile is the lowest, their numerical relational skills performance might be lower. However, the counting skills performance is the one that appears more decremented in this profile, which is not expected from the results obtained from the correlation analysis.

Moreover, the results obtained by Camerota et al. (2020) claimed an opposite behaviour to the one obtained in the present study. According to their results, participants with high accuracy and low RT may have a better overall EF-performance than the ones with high accuracy but higher RT. Due to the proven correlation between EF and EN (in the present study and others; see Clements, Sarama and Germeroth (2016)), the overall EF-performance is considered similar to the EN-performance which, in participants with high EF-accuracy and RT (blue in Figure 4) is higher than in participants with high EF-accuracy and low EF-RT.

Finally, the research questions have been answered in the following way. Concerning the first question, 'How does accuracy in inhibition and switching affect counting and numerical relational skills?', it can be concluded that accuracy in inhibition and switching has a weak to moderate positive correlation with counting and numerical relational skills. In relation to the second question, 'How does RT in inhibition and switching affect counting and numerical relational skills?', it can be concluded that RT in inhibition and switching has a weak and positive correlation with numerical relational skills, but this tends to zero with no significant values when is related to counting skills. Lastly, the last question, 'Is there any latent pattern affecting the relationship between accuracy and RT, and counting and numerical relational skills?' the answer is that it might be, even if the results are not conclusive. This hypothesis comes from the analysis of the obtained profiles, where the "hasty" profile (in Figure 4) presents unexpected behaviour, not described by the correlation between the variables. Moreover, it does not respond either accordingly to the results obtained in previous studies (Camerota, et al., 2020). That is why a study under the same conditions but with a bigger sample will be interesting. So, a more significant LPA could be performed in search of any latent variable that could affect the existing relations between inhibition and switching accuracy and RT, and counting and numerical relational skills in 4-year-old preschoolers.

4. Validity and ethical limitations

The following facts should be highlighted to ensure the validity of the obtained results and the development of an ethical study. They have helped the development of a valid and ethical study, but they have also shown other consequences.

To defend the privacy and freedom of the participants, all participant data were anonymised, and they participated as volunteers, having the possibility to leave the study at any time without giving any justification. However, it has meant a limitation in obtaining a more extensive dataset. Due to the size of the analyzed dataset (N=107), no general conclusions should be yielded, but hints to know what to expect from further similar studies with a more significant sample.

To ensure the validity of the results, the tests were always provided in the same way, following a previously determined plot and instructions and never giving any feedback to the participants. Moreover, different references have been used to ensure that the statistical analysis was optimal considering the characteristics of the data set. The Spearman's rank correlation factor has been performed and reported according to the work from Field (2009, pp. 166-196) and Akoglu (2018). For the calculation and interpretation of the LPA, the primarily used references have been the work from Spurk et al. (2020) and Samuelsen and Raczynski (2013); as well as the concrete references related to the used packages in R: mclust (Scrucca, et al., 2016) and reshape2 (Wickham, 2020).

Finally, to make sure that no violations of the responsible conduct of research were occurring, the Finnish National Board on Research Integrity TENK (2020) has been used as a model. Making sure that no fabrication or falsification has been done, reporting any analysis as it has been performed, and each result as it has been obtained, based on the data that was facilitated by the *Active Early Numeracy* research group, and always trying to give as much detail as possible; also with the aim of this study being reproduced in future occasions, to keep on learning from it. Moreover, plagiarism and misappropriation have been avoided by providing every reference that has been consulted in for the development of this study report, making sure that every idea guiding this study was correctly related to its original author. With that aim, all the references have been reported following the referencing rules from the APA (American Psychologist Association), so everyone can easily find the original works from which the ideas presented in this study report have been obtained.

During the whole working process in this study, I have learnt with admiration, respect, and a critical view from each of the researchers I have been reading articles from. I am very grateful for this, so I have tried to reflect this in the study report as well.

5. Discussion and conclusions

The obtained results show a weak to moderate positive statistically significant correlation between inhibition and switching-accuracy, and counting and numerical relational skills. Also, it yields a weak positive statistically significant correlation between inhibition and switching-RT, and numerical relational skills; and an almost zero insignificant correlation between RT and counting skills. These correlations could be perceived in the analysis of latent profiles, where two of the profiles respond to the existing correlations between accuracy and EN-performance. In addition, another profile ("hasty") was found, which could be a sub-profile of the so-called "accurate" profile. Then, the latter "hasty" profile highlights the importance of the described correlation between RT and numerical relational skills. However, it shows a decline in counting skills that cannot be explained.

Thus, it can be concluded that this study has succeeded in answering two of its research questions by finding the correlation values between accuracy and RT in EF, and EN-performance. However, the answer to the third question is not conclusive. Statistically significant latent profile models have been found, but it has not been possible to explain all the reasons for their behaviour. In order to give a reliable answer to this question, it will be interesting to replicate this study with a larger sample size. Camerota et al. (2020), who have inspired this part of the research, used a much larger sample size (N=1015).

Consequently, the present study yields some conclusions that should help develop more effective teaching methodologies. Considering what has been understood from the existing relations between inhibition and switching (EF) and relational and counting skills (EF), it can be concluded that: in the teaching and learning of early numeracy skills among 4-year-old preschoolers, the improvement of inhibition and switching accuracy will positively affect the EN-performance, while the effect of RT does not seem to be very relevant, even when the preschoolers show high accuracy. Nevertheless, the relevance of RT might be increased in the following years (Diamond, et al., 2007).

Therefore, these results also support the conclusions yielded by Clark et al. (2014), so processing speed has no major relation to EN at this age. However, it indicates that the power of RT over the EN variables should increase if adapting this study to older students. Meaning that it is better to focus on accurate responses in the early years of education. Whereas, in the following years of elementary education, time reaction starts being a variable considered by the teachers when wanting to impact the students' learning process through the training of EF. However, to confirm this hypothesis, it would be necessary to run a similar study to this one with participants of different higher ages (6, 8, or 10-year-old participants). It should also measure their inhibition and switching accuracy and RT, and their mathematic skills, including, this time, arithmetic and problem-solving.

In such a way, it also has to be considered how the accuracy of these EF (switching and inhibition) can be trained. Related to this question, Thorell et al. (2009) concluded no possible improvement of inhibition after going through a training programme. However, the levels of this training programme were established according to the RT of the student: "In the inhibition tasks, difficulty was manipulated through decreasing the time allowed for making a response." (pp. 108-109). The possibility of improving inhibition and switching must be tested with a training programme that establishes the levels according to the accuracy of the response, aiming to see if inhibition and switching accuracy can be improved. Therefore, the subsequent levels of the training programme could be organized, so the first level requires accuracy of X1 correct answers out of N, and the following ones an accuracy of Xn correct answers out of N.

Moreover, in contrast to the results drawn by Bull and Lee (2014) in a deep analysis of how EF are related to mathematics achievement, this study has concluded that switching has a significant impact on mathematics achievement (EN-performance). Which is something that they have not found in any of the analysed studies that they present in their report.

From the difference in the correlation factor between inhibition accuracy and switching accuracy (see Table 1), mainly concerning the numerical relational skills ($r_{s(105)}$=.376 and $r_{s(105)}$=.322 respectively), it can be said that switching has been effectively measured as an independent EF among 4-year-old preschoolers. This interesting result has probably been obtained thanks to the utilized test to measure inhibition and switching, with straightforward instructions.

This drastically reduces the amount of information that has to be retained by working memory, so switching and inhibition can be measured as isolated EF; also as Bull and Lee (2014) indicate.

To sum up, the present study contributes to the existing literature showing inhibition and switching-accuracy as a more robust metric than inhibition and switching RT in relation to EN-performance in 4-year-old preschoolers. It also adds empirical results favouring the definition of switching as a measurable isolated EF. However, to be sure of drawing definitive conclusions in this respect, it would be interesting to replicate this study with larger sample size.

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