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Rotor Dynamics of Turbocompressor Based on the Finite Element Analysis and Parameter Identification Approach

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Abstract. The article is devoted to improving methods for designing a finite element model of rotor dynamics. For this purpose, numerical implementation of the authors' computer program "Critical frequencies of the rotor" was developed based on the computer algebra system MathCAD. As a result of the scientific work, a refined mathematical model of rotor dynamics using finite beam elements was created. This model considers the dependence of the radial stiffness characteristics of the bearing supports on the values of the critical frequencies. The reliability of the mathematical model was justified by the permissible differences of the obtained results within 2% compared with the results of finite element analysis using the ANSYS software. The theorem was also proven by the mutual location of the spectra of the natural and critical frequencies. Overall, the proposed scientific approach reduces preparation and machine time compared to numerical modeling using the ANSYS software without losing the accuracy of the calculations.

Keywords: centrifugal machine, process innovation, critical frequency, finite element analysis, Campbell diagram.

1 Introduction

Rotary machines belong to the largest class of power machines [1]. During the operational process, polyharmonic disturbances act on the rotor in the form of forces and moments of inertia of unbalanced masses. They cause forced oscillations of the rotor.

In most cases, the technical level of such machines is assessed by their vibroacoustic characteristics determined by the vibration state of the rotor [2]. Therefore, the rotordynamic problems are of significant practical importance, and the range of these tasks is predetermined by the number of constructive types of rotor machines, their features, and operating conditions.

Among rotary machines, high-pressure multistage centrifugal pumps and compressors are characterized by permanently increasing operating parameters (e.g., speed, pressure) that lead to a single unit's capacity.

The pressure in centrifugal machines is proportional to the square of the rotor speed. Therefore, an increase in the rotation speed turns out to be the most rational way of achieving high pressures. As a result, high-pressure centrifugal machines are usually high-speed. And for such machines, the rotordynamic problems are particularly relevant [3].

The reasons for the unsatisfactory vibration state of the rotor are imbalance and misalignment, force and temperature deformations of the housing and its individual elements, loss of dynamic stability, structural and technological defects, wear of gap seals, supports, drive couplings [4, 5].

Increased vibrations are accompanied by operations close to critical frequencies. The calculation of the latter cannot guarantee a reliable detuning from resonance due to the lack of reliable data on the stiffness of the supports and the impact of randomly variable factors [6].

Up-to-date trends in studying the dynamics of rotary systems are mainly based on computer programs that implement finite element analysis [7].

To solve such problems, 2D and 3D finite elements are used [8, 9].

Calculating rotor dynamics, e.g., using numerical modeling using the ANSYS software, is time-consuming. This is mainly due to the time for the preparation of initial data. It also requires a lot of machine time.

According to those mentioned above, the work aims to improve the vibration state of the rotor system by creating a reliable model for evaluating the dynamic state.

The main research objectives are as follows:

– development of a refined mathematical model of free and forced oscillations of the rotor considering the predetermined dependence of the stiffness of the supports on the frequency of rotation of the rotor;

– parameter identification of the coefficients for the dependence of the stiffness of the bearing supports;

– simulation of rotor dynamics for a multistage centrifugal compressor 225GC2-135/12-50M1245 using the program “Critical frequencies of the rotor” realized in the computer algebra system MathCAD;

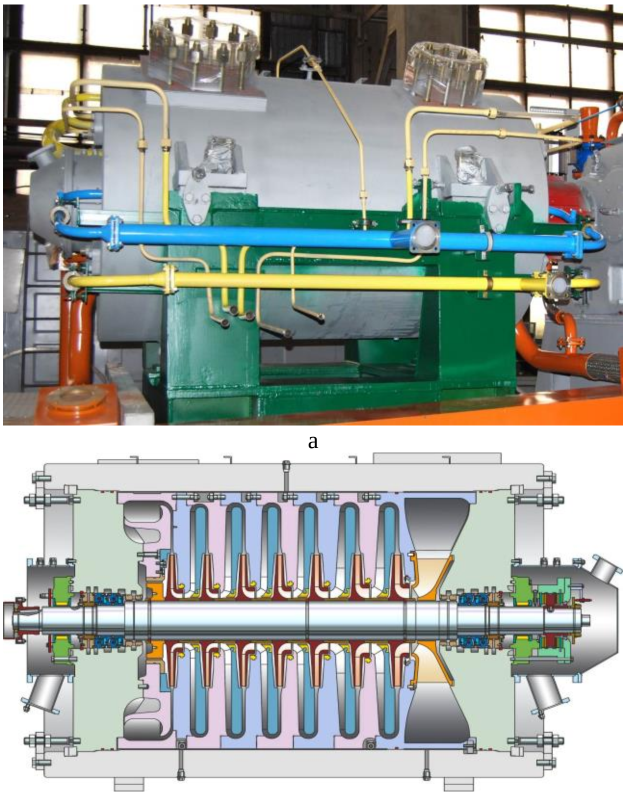
– verification of the reliability of the finite element model in the ANSYS software;

Research methods include numerical simulation of free oscillations of the rotor using the finite element analysis and parameter identification approach comprehensively.

2 Research Methodology

2.1 Design scheme

The object of research is the free oscillations of a flexible rotor of a centrifugal compressor 225GC2-135/12-50M1245 (Figure 1). Its technical parameters are summarized in Table 1.



a

b

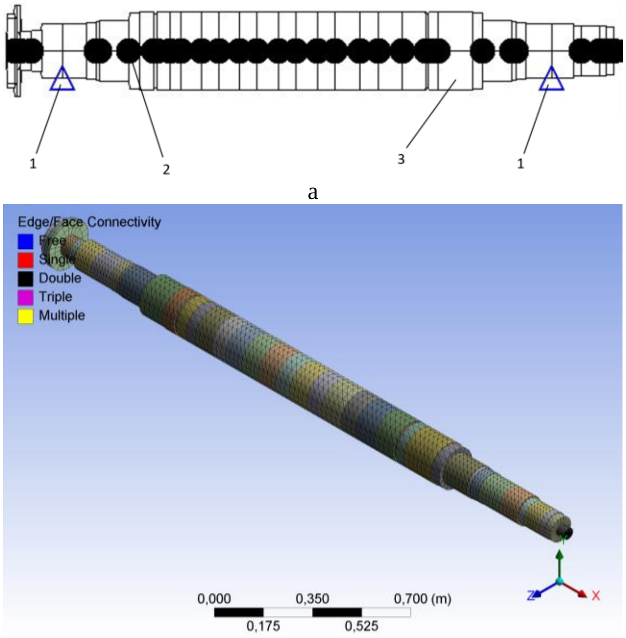
Figure 1 – General view (a) and cross-section (b) of the turbocompressor

Table 1 – Technical parameters

Parameter	Value
Normal performance, m^3/s	25.0
Inlet pressure, MPa	1.20
Outlet pressure, MPa	4.95
Polytropic efficiency, %	70.0
Rotor speed, rpm	8920
Nominal power, MW	7.33
Inlet gas temperature, °C	28.1

To further study rotor dynamics, it is necessary to create its calculation model. A beam finite element model was compiled using the authors’ program “Critical frequencies of the rotor” [10] (Figure 2a).

Figure 2b presents 3D finite element model created in the ANSYS software.



a

b

Figure 2 – Design models in MathCAD (a) and ANSYS (b): 1 – radial supports; 2 – local masses; 3 – finite elements

2.2 Program description

The program “Critical frequencies of the rotor” is based on the finite element method. It is intended to evaluate the critical and eigenfrequencies of radial oscillations of rotors. The calculation results in the value of critical frequencies and forms of oscillations.

The program was created using the computer algebra system MathCAD.

The input data required are density and Young’s modulus of the material, lengths of sections, inner and outer diameters, added masses of mounting parts, and rigidity of bearing supports and gap seals.

The matrix equations of free oscillations of a rotating rotor can be presented in the following form:

$$\bar{M}\ddot{\bar{U}} + \bar{K}\bar{U} = 0, \quad (1)$$

where $\bar{U} = (x_1, \vartheta_1, x_2, \vartheta_2, \ldots, x_{n+1}, \vartheta_{n+1})$ – column-vector of deflections and angles of rotation for sections at the joints of finite elements.

Local matrices $\bar{K}_i$ and $\bar{M}_i$ by the dimension of 4×4 have a standard form according to the fundamentals of beam's oscillations [11, 12].

The formation of global matrices $\bar{K}$ and $\bar{M}$ by local matrices $\bar{K}_i$, $\bar{M}_i$ for $i = (1, n + 1)$ is based on the algorithm, which uses the conditions of the junctions for all the sections [13].

The calculation of critical rotor frequencies is based on the numerical determination of the eigenvalues λ of the following equation:

$$det[\bar{K} - \lambda\bar{M}] = 0. \quad (2)$$

The program for calculating free oscillations consists of the following stages:

1) calculation of coefficients for stiffness and inertia matrices;

2) solving the frequency equation (2) and finding a given number of eigenfrequencies;

3) calculation of the mode shapes of free oscillations corresponding to their frequencies.

2.3 Mathematical model of bearing stiffness

In this work, it is proposed to approximate the dependence of the stiffness of the supports on the frequency of rotation of the rotor by the following analytical formula:

$$c(\omega) = c_0 + (c_{max} - c_0)(1 - e^{-k\omega}), \quad (3)$$

where c_0 – initial stiffness, N/m; c_{max} – maximum stiffness, N/m; ω – operating speed, rad/s; k – time parameter, s.

Such a model has the following advantages compared with the polynomial approximation. Firstly, instead of an infinite increase in stiffness, it asymptotically approaches its maximum value.

Secondly, when approaching this value, the stiffness gradient asymptotically approaches zero.

These facts are because parameter k has the following physical meaning:

$$\frac{dc(\omega)}{d\omega} = k[c_{max} - c(\omega)]. \quad (4)$$

According to the quasilinear parameter identification approach, the unknown parameter k in expression (1) can be evaluated by minimizing of the following least square error:

$$R(k) = \sum_{n=1}^{N}\left[k\omega_n - ln\left(\frac{c_{max} - c_0}{c_{max} - c_n}\right)\right]^2 \rightarrow min, \quad (5)$$

where n – a number of the experimental point; N – the total number of the experimental dataset; ω_n – n-th operating speed, rad/s; c_n – experimentally obtained stiffness, N/m.

The last condition is satisfied for values of k, at which the following derivative equals a zero value:

$$\frac{dR(k)}{dk} = 2\sum_{n=1}^{N}\left[k\omega_n - ln\left(\frac{c_{max} - c_0}{c_{max} - c_n}\right)\right]\omega_n = 0. \quad (6)$$

Finally, the following regression dependence can be obtained:

$$k = \frac{\sum_{n=1}^{N}\omega_n ln\left(\frac{c_{max} - c_0}{c_{max} - c_n}\right)}{\sum_{n=1}^{N}\omega_n^2}. \quad (7)$$

Thus, the determination of the unknown parameter k requires preliminary experimental determination of stiffnesses c_0 and c_{max}.

3 Results

3.1 MathCAD simulations

The calculation of eigenfrequencies involves calculating dangerous speeds during the rotor's constant rotation at its operating frequency. For the considered case study, such a frequency equals 934 rad/s.

The stiffness of the bearings was also assumed to be constant and equal to the stiffness at the operating speed ($c_p = 2.94 \cdot 10^8$ N/m).

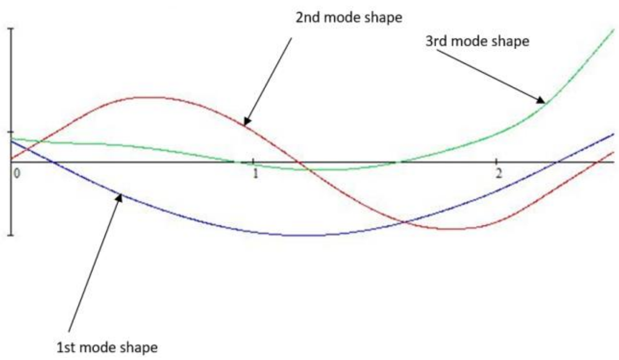
Remarkably, critical frequencies are dangerous for the rotor during its acceleration to the operating mode. For their calculation, it is necessary to consider that the bearings have different stiffness at different rotation frequencies, which depends on the values of the natural frequencies of the rotor.

The program "Critical frequencies of the rotor" allows setting directly the dependence of the rigidity of the rotor supports on the frequency of its rotation.

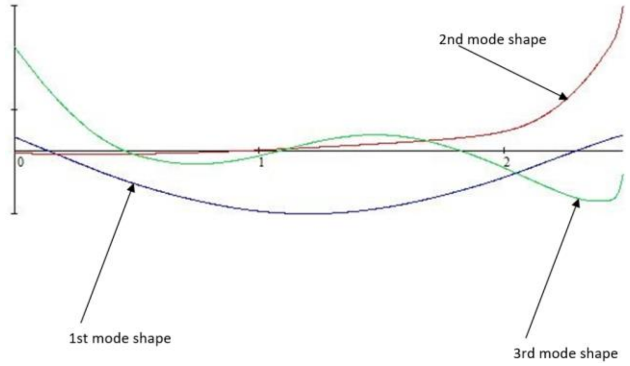
Using the dataset of JSC "Sumy NPO" (Sumy, Ukraine), the following values were evaluated: $c_0 = 1.77 \cdot 10^8$ N/m and $c_{max} = 1.51 \cdot 10^9$ N/m.

Therefore, according to formula (7), the evaluated time parameter $k = 1.0 \cdot 10^{-4}$ s.

The program "Critical frequencies of the rotor" outputs the result in the form of natural frequency values and their corresponding forms. Figure 3 presents screenshots from the program with calculation results for the investigated rotor.



a



b

Figure 3 – Natural (a) and critical (b) frequencies of the rotor

3.2 ANSYS simulations

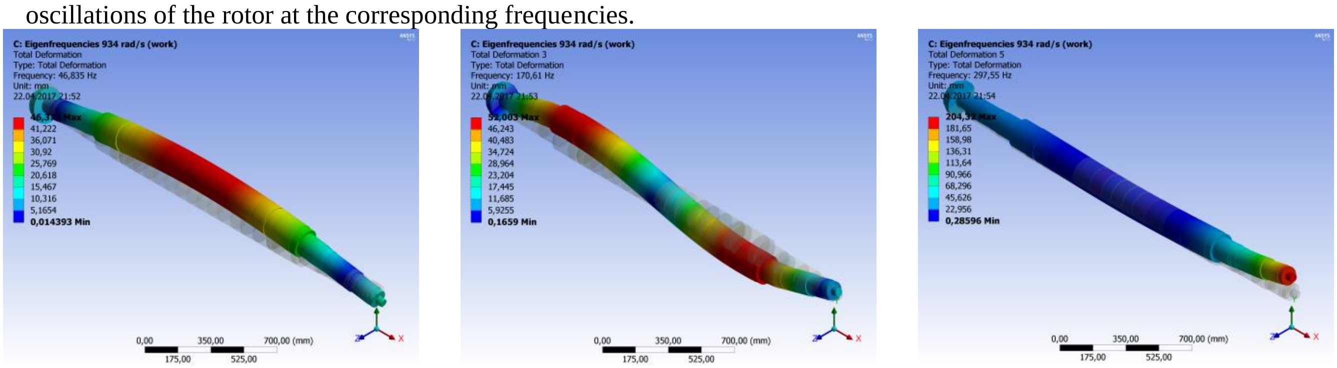
As a result of numerical simulations using the ANSYS software, Figure 4 presents mode shapes of free oscillations of the rotor at the corresponding frequencies.

Unfortunately, ANSYS does not allow directly evaluating the critical frequencies as is possible in the program "Critical frequencies of the rotor".

In general, the calculation of critical frequencies using the ANSYS software is like the calculation of eigenfrequencies. The only difference is that it is necessary to consider the property of bearings to change their stiffness depending on the rotation frequency non-linearly. However, the modal analysis ignores the initial model's nonlinearities [14].

During the simulations, the stiffness of the bearings was calculated in the range of rotation frequencies from 0 to 3000 rad/s with a step of 250 rad/s.

Then, changing the stiffness of the bearings in ANSYS, the eigenfrequencies of the rotor at the corresponding frequencies of its rotation were found as a Campbell diagram(Figure 5).



a b c

Figure 4 – 1st (a), 2nd (b), and 3rd (c) mode shapes of free oscillations of the rotor

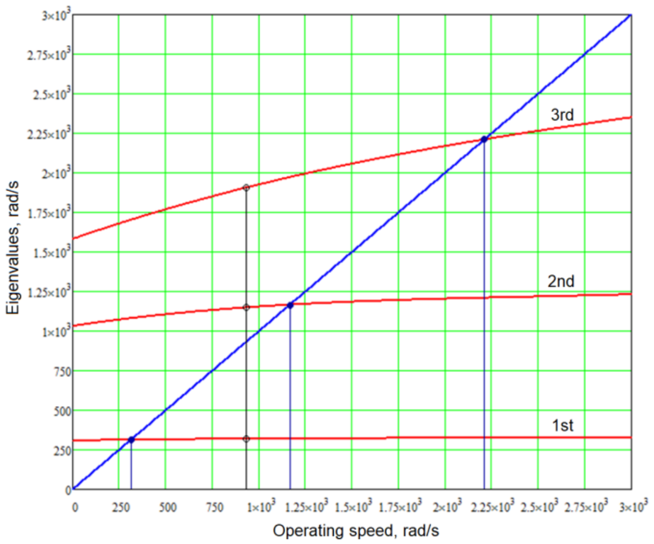


Figure 5 – Campbell diagram

The essence of Campbell's method for calculating the critical frequencies of the rotor is that the ordinates of the points of intersection of the bisector on the diagram drawn from the origin of the coordinates with the eigenfrequency curves are equal to the critical frequencies of the rotor.

Remarkably, the Campbell diagram serves as visual proof of the theorem on the mutual location of the spectra of critical and eigenfrequencies [15]. Particularly, critical frequencies below the operating speed are lower than the corresponding eigenfrequencies. Vice versa, critical frequencies above the operating speed are higher than the corresponding eigenfrequencies.

3.3 Comparison of the results

To confirm the reliability of the mathematical model of the rotor dynamics created in the program "Critical frequencies of the rotor", it is necessary to compare the results obtained in it with the data of numerical simulation in the ANSYS software. Such a comparison is presented in Table 3. It shows values of critical and eigenfrequencies evaluated by both the MathCAD and ANSYS software, as well as the percentage difference between the obtained results.

Table 2 – Comparison of the calculation results

No.	Natural frequencies, rad/s		Relative deviation, %	Критичні частоти, рад/с		Relative deviation, %
	ANSYS	MathCAD		ANSYS	MathCAD	
1	318.8	318	0.25	312.9	312	0.28
2	1125.0	1148	1.97	1140.0	1164	2.06
3	1867.0	1906	2.04	2150.0	2210	2.72

4 Conclusions

As a result of the work, a refined mathematical model of rotor dynamics using finite beam elements was created. This model considers the dependence of the rigidity of the radial supports of the rotor on the frequency of its rotation. The reliability of the mathematical model is confirmed by the permissible differences of the obtained results within 3%, as well as by the observance of the theorem on the mutual location of the spectra of natural and critical frequencies.

This indicates that using the working file "Critical frequencies of the rotor" of the computer algebra system MathCAD is more appropriate for solving similar problems. This is due to the possibility of directly considering (in an arbitrary analytical form) the dependence of bearing stiffnesses on the frequency of rotation of the rotor.

Additionally, the calculation using the proposed method reduces the preparation and machine time by order of magnitude compared to the numerical simulation in the ANSYS software without losing the accuracy of the calculations.

The obtained results can be further applied to study centrifugal machines' rotordynamic stability under fractional-order internal friction.

5 Acknowledgment

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