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# Conceptual Modelling, Combinatorial Heuristics and *Ars Inveniendi*

## An Epistemological History in Four Acts: Plato, Llull, Leibniz & Zwicky

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SYNOPSIS

(**1**) An introduction to the principles of conceptual modelling, combinatorial heuristics and epistemological history; (**2**) the examination of a number of perennial epistemological-methodological schemata: conceptual spaces and blending theory; *ars inveniendi* and *ars demonstrandi*; the two modes of analysis and synthesis and their relationship to *ars inveniendi*; taxonomies and typologies as two fundamental epistemic structures; extended cognition, symbolic systems and model-based reasoning; (**3**) Plato's notions of conceptual spaces, conceptual blending and hypothetical-analogical models (*paradeigmata*); (**4**) Ramon Llull's concept analysis and combinatoric spaces; (**5**) Gottfried Leibniz's development of compositional analysis and synthesis as a general modelling method and a paradigm for *ars inveniendi*; (**6**) Fritz Zwicky's revival of the morphological method of analysis and construction, and its subsequent computerised applications.

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Contents (Chapters 1 & 2)

# 1 Introduction and Theoretical Background

## 1.1 Morphology

Nominally, *morphology* (from the Greek *morphé* = shape or form) is the *science of organic form*. This could be misconstrued to mean that it relates only to the study of "living organisms", but such is not the case. The term *morphology* is used in science in a number of different contexts and on a number of different conceptual levels.[1]

**I.** On an *Epistemological-Ontological* level, morphology refers to nothing less than a philosophy of science, or at least what Laudan calls a basic scientific *research tradition*.[2] That is, a set of theoretical (or ontological) assumptions and methodological ground rules upon which to approach scientific enquiry in general. At this level, morphology is ontologically committed to the reciprocal status of whole-part relationships, hierarchal causality and strong emergence. Note that although the *term* morphology was only coined at the end of the 1700's, the scientific tradition it represents dates back to antiquity.

**II.** On an *Epistemological-Methodological* level, we find morphology and *morphological analysis* denoting a general methodological approach used in a wide range of scientific disciplines concerned with the study of *structural inter-relationships within the context of a developing systemic whole*. For example, geomorphology is the study of landforms and the physical processes that shape them; urban morphology is the study of human settlements and the process of their formation. In linguistics, morphological analysis is the branch of grammar that studies the structure of word forms and their development over time. However, it is primarily in the field of biology – for which the term was originally coined by Goethe – that it first developed into a method for the study of "organic form", both in its structural and developmental (morphogenetic) sense.

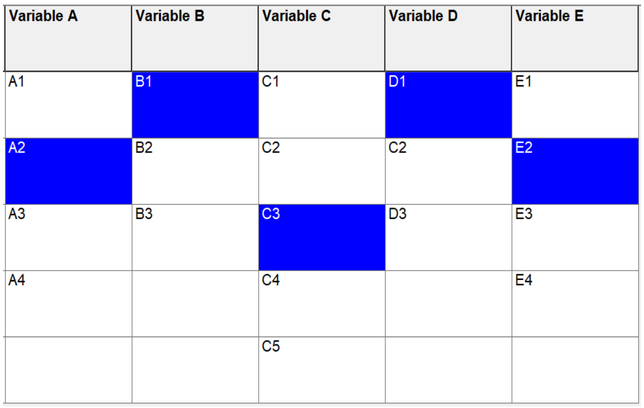
**III.** Finally, on a purely *Modelling Theoretical* level[3] what today is called *morphological analysis* (which should actually be termed *morphological analysis and synthesis*) refers to a conceptual modelling method based on the *analytic decomposition* of a complex concept and the reciprocal procedure of *synthesis by combinatorics*. It has a long history, being first (at least implicitly) conceptualised by Plato, prototyped by Ramon Llull (c. 1300), developed in its modern form by Gottfried Leibniz (1666) and then revived and popularized by Fritz Zwicky in the 1950s and 60s.

The ***Ars Morphologica*** series as a whole concerns all three of these levels. However, the present introductory study – an epistemological history of morphological modelling as a method of scientific discovery – concerns level **III** supported by concepts derived from level **I**. We begin by delineating three of the central concepts: "morphological modelling", "combinatorial heuristics" and "epistemological history".

## 1.2 What is a morphological model?

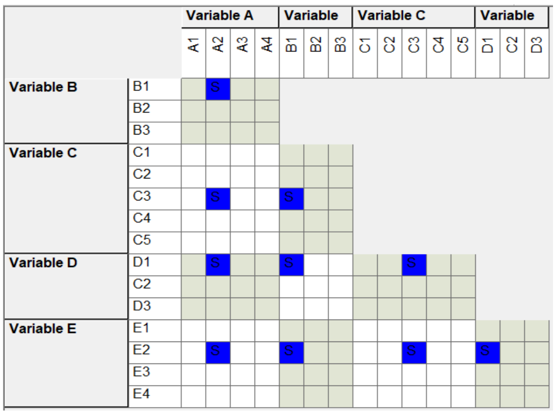
The epistemological principle underlying (discrete) *morphological modelling*[4] is that of *decomposing* a complex (multivariate) concept into a number of ("simple") one dimensional concepts (i.e. category variables), the domains of which can then be *recombined* and *recomposed* in order to discover all of the other possible (multidimensional) concepts which can be generated combinatorially. As such, it is both a generic modelling method and a *conceptual design* process. Note that this (analytic) *decomposition* and (synthetic) *recombination* process is exactly what we do – on a smaller scale and in a less complex format – when we create *typologies*, which are essentially low-dimensional (usually 2-D) morphological models.[5] (See § 2.6).

Technically, a morphological model consists of 1) a set of category variables forming a *configuration space* (a.k.a. *morphospace*, Figure 1a); and 2) a combinatorial matrix called a Cross-Consistency Assessment matrix (CCA-matrix – Figure 1b) which represents all of the dyadic (pair-wise) relationships between the domains of the category variables. If N = the number of category variables in the morphospace, then the CCA-matrix consists if ½ N(N-1) 2-dimensional typologies (or non-ordered Cartesian products). These pair-wise relationships are expressed by way of *modal* constraints concerning compossibility/compatibility.



| Variable A | Variable B | Variable C | Variable D | Variable E |
|---|---|---|---|---|
| A1 | B1 | C1 | D1 | E1 |
| A2 | B2 | C2 | C2 | E2 |
| A3 | B3 | C3 | D3 | E3 |
| A4 | | C4 | | E4 |
| | | C5 | | |

Figure 1.1a: 5-parameter morphospace showing a single configuration (dark cells).



| | | Variable A | | | | Variable | | | Variable C | | | | | Variable | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | A1 | A2 | A3 | A4 | B1 | B2 | B3 | C1 | C2 | C3 | C4 | C5 | D1 | C2 | D3 |
| Variable B | B1 | | S | | | | | | | | | | | | | |
| | B2 | | | | | | | | | | | | | | | |
| | B3 | | | | | | | | | | | | | | | |
| Variable C | C1 | | | | | | | | | | | | | | | |
| | C2 | | | | | | | | | | | | | | | |
| | C3 | | S | | | S | | | | | | | | | | |
| | C4 | | | | | | | | | | | | | | | |
| | C5 | | | | | | | | | | | | | | | |
| Variable D | D1 | | S | | | S | | | | | S | | | | | |
| | C2 | | | | | | | | | | | | | | | |
| | D3 | | | | | | | | | | | | | | | |
| Variable E | E1 | | | | | | | | | | | | | | | |
| | E2 | | S | | | S | | | | | S | | | S | | |
| | E3 | | | | | | | | | | | | | | | |
| | E4 | | | | | | | | | | | | | | | |

Figure 1.1b: CCA matrix showing the pair-wise relationships defining the configuration in Figure 1a.

The CCA-matrix functions as a *heuristic search space* involving two seemingly opposing tasks: on the one hand, by identifying those pair-wise relationships between category variables which are contradictory or otherwise incompatible, the total number of formal configurations in the morphospace can be reduced to a solution space of consistent (possible) configurations.[6] This is a form of *constraint-based modelling* and *inference by exclusion*. At the same time, one also needs to keep an open mind for the discovery of strange and novel combinations, which may initially seem impossible or implausible, but which represent "emergent attributes in concept conjunctions"[7] (a.k.a. "combinatorial creativity"[8] or "conceptual blending[9]). (See §2.1.) (The relationship between the *semantic* and *syntactic* aspects of these two "spaces" will be discussed in §6.)

For those who are not previously acquainted with discrete variable morphological modelling, methodological overviews are available in Ritchey (2006 & 2018), which can be downloaded from the designated URLs in the Reference List.

## 1.3 What is "combinatorial heuristics"

### 1.3.1 Combinatorics

***Combinatorics*** is a branch of *discrete mathematics* which studies the arrangements of countable sets of elements or objects by way of combinations, permutations, ordering, ranking, grouping, partitioning, etc. While arithmetic deals with integers and algebra with mathematical operations, combinatorics deals with discrete *configurations* of elements.

Combinatorics is usually broken down into three branches:[10]

- *Enumerative combinatorics*: This deals with the counting of combinatorial configurations, i.e. determining the number of possible formal arrangements of a set of objects.
- *Existential combinatorics* (a.k.a. constrained combinatorics): This deals with problems concerning the existence or nonexistence (possibility or impossibility) of different combinatorial arrangements, given specified conditions or constraints.
- *Constructive combinatorics*: This deals with methods for *finding specific configurations* – as opposed to merely demonstrating their possible existence – in order to solve a given problem. It also includes "the design and study of *algorithms* for creating arrangements with special properties."[11]

Morphological modelling involves all three of these combinatorial variants, although of special interest is existential (or *constrained*) combinatorics as a heuristic procedure for finding viable configurations in a search space – in this case a *morphospace*. Also of interest is the study of *combinatorial geometry*, which is concerned with "spatial relations and processes, such as those of inclusion, intersection, decomposition, and ... the combinatorial possibilities associated with these relations and processes."[12] This includes the study of taxonomies and typologies (see § 2.6), and generally problems concerning "places in conceptual spaces."[13]

In this context, we can speak of *combinatorial systems* consisting of a finite number of elements which can be combined and permuted to create larger structures with properties distinct from those of their elements. As will be discussed later, these can develop into *recursively generative combinatorial systems* which are essentially unbounded in their developmental potential and which act as mechanisms for discovery and invention. Human language and mathematical symbolism, as well as biological/ecological systems, are examples of such systems.[14]

In this context – and we will see this especially in the work of Leibniz – combinatorics is not simple a branch of mathematics, but has the status of a heuristic scientific method in itself: "...a universal method capable of revealing or representing the [whole-part] structure of objects ... the relationships between the objects of a particular science, or even of a science of being in general."[15]

### 1.3.2 Heuristics

> "If you will not let me treat the Art of Discovery as a kind of Logic, I must take a new name for it, *Heuristic*, for example" [first known use of the term in English, attributed to William Whewell in 1860][16]
>
> "The aim of heuristics is to study the methods and rules of discovery and invention." (George Polya)[17]

*Heuristic* comes from (classical Greek) *heurískō*: "to find out" or discover (*eureka*! being the first person indicative). The Latin translation is *inventio*, which literally means "to come to mind" (see § 2.2). Heuristic reasoning is distinguished both from (formal) deductive inference (i.e. inferring *necessary conclusions* from given *premises* or *axioms*), and from Bayesian (probabilistic) inference (i.e. inferring conclusions on the basis of *well grounded probability distributions*). It deals neither with *necessity* nor *probability*, but with *possibility* (and *plausibility* in the sense that Polya uses this term[18]). Generally, we can speak of *heuristic methods* and *heuristic principles*, the former being procedures and routines which can "provide promising search fields and directions for finding new connections, relationships and solutions."[19], the latter being general frameworks or "architectonics" which can guide and constrain hypothesis generation and theory building.[20]

By their very (open-ended) nature, *heuristic methods* represent an essentially unbounded domain. They consist of procedures such as "simplify", "scope", "use analogies and metaphors", "prioritize"; "partition" (e.g. decompose); "integrate" (e.g. recompose), "make a model". Indeed, Polya (1887-1985), one of the great researcher in, and proponents of, heuristic methods, admitted that he had no way to actually give a strict (positive) definition of heuristic reasoning.[21] For the purposes of the present work, however, four *general* heuristic methods of reasoning and knowledge generation are emphasized.

### *Hypothetical-analytic method*

Probably the most important – and potent – heuristic method for scientific discovery (also acknowledged by Polya[22]) is the method of "working backwards" (analytically) from conclusions to (hypothetical) premises, or from effects to (hypothetical) causes, a.k.a. Peirce's *abduction*. As will be discussed in more detail in §3, Plato prescribes iterating this procedure, i.e. by formulating hypothesis not only to account for specific phenomena or events, but also formulating higher-order hypotheses (meta-hypotheses) which can account for (or falsify) the initial hypothesis.[23] Historically this has been carried out in recursive phases, successively creating broader, more comprehensive theoretical frameworks.

One of the most important developments over the past few decades has been a renewed interest in, and increased recognition of, this Platonic/Neoplatonic stepwise ampliative procedure for theory generation and scientific discovery (only partially rediscovered by Peirce). It is now acknowledged by contemporary classical scholars, philosophers of science and modelling theorists.[24]

> "It is now time that "mainstream academia" recognize the supreme role of Abduction, and pay homage to Neoplatonism (predominantly through Proclus) and to Peirce for carefully preserving for us this great technique of discovery that had its birthplace and refinement in the hallowed halls of the first Academy."[25]

### *Combinatorial-synthetic method*

*Combinatorial heuristics* consists of non-deductive, non-determinate methods that can guide discovery by generating a combinatorial search space that can be mined for "*conjunctions of epistemic possibilities*"[26]. Indeed, one of the most general and fundamental forms of heuristic procedures that Polya takes up in his celebrated book "How to solve it" is just that of the morphological "*decomposition and recombination*" of combinatoric spaces.[27] How such "spaces" are generated, how they are constrained, and the notion of "creative conjunction" and "productive ambiguity" is one of the main subjects of the present opus, and will be taken up in more detail below.

## *Analogy and Teleology*

Two of the most powerful general heuristic devices for facilitating hypothesis generation and scientific discovery are *analogy* and *teleology*. Whereas analogy is concerned with *structural similarity* between domains (in the broad sense of the term "structure"), teleology is concerned with *evaluative principles* which guide and constrain hypothesis generation.

### Analogy

An *analogy* is "a *mapping of structure* from one domain (*source*) to another (*target*)". [28] It compares a *source* and a *target* on the basis of certain similar *formal* structures (functions or properties), and then, on the basis of such similarities, *further extends this comparison* by projecting other properties of the source onto the target, as *hypotheses which can be tested for* (on the target).[29] Here the concepts of partial isomorphism and homomorphism are often used. In this sense, analogies are expressions of *model-based reasoning* (§ 2.5.4) where the source acts as a hypothetical-analogical *model* of the target. Note however, that "[Analogies] rest on the principle that certain more or less deep insights (schemata, structures) are transferred from a source to a target domain"[30] ... that they are "…based on deep connections between things rather than some simple surface similarities such as physical resemblance" ... they are "... mappings or alignments of hierarchically structured, causal relationships shared between source and target analogues".[31]

One of the textbook examples of this is the analogy between (ideal) fluid dynamics and electromagnetism – i.e. the *similarity* between the equations that govern these two domains. Maxwell exploited these formal similarities in what he called the method of *physical analogy*.[32]

> "By a physical analogy I mean that partial similarity between the laws of one science and those of another which makes each of them *illustrate the other* . . . we find the same resemblance in *mathematical form* between two different phenomena."[33]

A fascinating twist on this illustrates the power of analogy: Hermann von Helmholtz later made the process recursive by using Maxwell's laws of electromagnetism – which were derived from an analogy with the laws of fluid dynamics – to further develop the laws of fluid dynamics.

Thus analogy is well established in the history of science as a creative (heuristic) tool for hypothesis generation and theory construction. Bernhard Riemann called it "das Dichten von Hypothesen" ("the poetry of hypothesis") and maintained that it is the very motor of scientific discovery.[34] Although we are sometimes warned about the "dangers" of making comparisons by analogy – that we can easily go too far and endow a target with properties that it simply does not have –, as Riemann pointed out, there is no danger here if analogies are consciously treated a *hypotheses to be tested*.

### Teleology

The use of *teleology* – or final causes – as a heuristic device is a different and more controversial matter. It was not long ago that the promotion of teleology explanation – even if regarded in terms of "function" or a purely heuristic devise – was an invitation to professional suicide. 20th century analytic philosophers and logical empiricists treated the concept "... as an insidious metaphysical notion that was to be tossed out with the rest of metaphysics".[35] However, the concept has been resurrected (again!) not only in modern biology, but also in modern physics.

There are two principle types of teleological explanations. The first concerns goal-directed behaviour in human *decision-making* and *problem solving*. This is called *intentional teleology* (or the "Human Design Model"[36]) and is relatively uncontroversial (unless you are a devout epiphenomenologist).[37] The second (more controversial) type concerns what is usually called *natural teleology* which is expressed as an evaluative (architectonic) *principle* which can give rise to higher-order explanations, e.g. principles of harmony, simplicity, optimality and (more generally) the notion of *axiological systematic fitness*.[38]

> "[Final causes] serve a variety of necessary functions: they mediate the unification of empirical laws, guide choice between competing hypotheses, and inform the classification of natural kinds. In these roles, teleological principles are *not merely heuristical*. They do not simply provide an easier method for deducing physical laws, or a convenient scheme for organizing experimental data, but are indispensable for *interpreting empirical results as expressive of nature*. In other words, on the reasonable supposition that the working scientist takes herself to be investigating the truth about nature, teleological principles inescapably figure as constraints on the semantics of any theory.[39]

Of special interest here are principles of *optimality* (or optimal form) which is "one of science's most pervasive and flexible meta-principles", used e.g. in physics, chemistry, evolutionary biology and economics.[40]

> "Optimality principles have been used to formulate *laws of nature* since the very beginning of science, be it that such principles suit scientists aiming to unification and simplification of knowledge..." For all the resistance against the use of optimality "... it remains the fact that many if not all laws of nature can be given the form of an *extremal principle* and many of the mathematical structures have their sources and their underlying texture in extremal principles."[41]

In physics, for instance, the optimality Principle of Least Action (PLA) is ubiquitous:

> "We have found that the Principle of Least Action applies across all scales, from realm of the microscopic to the everyday (classical mechanics, engineering, optics, the transmission of radiation, physical chemistry, statistical mechanics, continuum mechanics), and on to the whole cosmos (gravitation due to stars, planets, black holes, and gravity waves)."[42]

Gottfried Leibniz, Bernhard Riemann, Max Plank and Albert Einstein, among others, successfully employed *principles of optimality* in order to discover new relationships and develop higher-order explanatory frameworks. As Leibniz pointed out more than 300 years ago:

> "Final causes may be introduced with great fruitfulness into the special problems of physics ... to help us make predictions by means of them which would not be as apparent through the use of efficient causes."[43] ... "[They] present principles of discovery … for finding those properties of matters whose inner nature is not yet known to us so clearly such that we could use the closest efficient causes and explain the mechanisms."[44]

Although even empiricists nominally agree that one must chose those explanatory laws and principles that are *most general* and which successfully *cover the greatest range of phenomena*, PLA and natural teleology in general (even in its softer *functional* version), remain one of the last great empiricist taboos.

## 1.4 Epistemological History

> "It is the obligation to throw light on the historical nature of science by reference to the modernity of science which makes the history of science a perpetually youthful discipline, one of the most lively and most educationally valuable scientific disciplines." - Gaston Bachelard[45]

This work is a contribution to the field of study called *Epistemological History*. Since this is a relatively new, diverse and still contested area, it needs some clarification.[46]

The so-called "historical turn" was the post-World War II Anglo-American revival of a *historical perspective* concerning the philosophy of science. It was, in some ways, a reaction to the earlier dominance of positivism, logical empiricism and behaviourism in the first half of the 20th century. Among the central figures involved in this Anglophone "turn" were Stephen Toulmin, Thomas Kuhn, Russell Hanson, Imre Lakatos, Larry Laudan and Alistair Crombie.

The rationale behind the "turn" was the need to integrate *history of science* and *philosophy of science* – the former focusing mainly on a historical account of particulars rather than general structures; the latter emphasizing such general methodological and metaphysical structures, but mainly ahistorically.[47] What was sought was both a broad and deep historical account of the evolution of general epistemological-methodological structures and principles in order to better understand the long-term, discontinuous process of scientific development.

In 1968, Laudan endorsed it as follows:

> "... both the history and philosophy of science have a common concern about, and vested interest in, one particular subject – the *history of theories of scientific method*. Indeed, it is difficult to understand either the history or the philosophy of science without dealing with the evolution of *theories of method*."[48]

At around the same time, the Australian historian of science Alastair Crombie (1915-1996) was pondering what he called different historical-methodological "styles of thinking" in Western science (which Ian Hacking reworded to "styles of reasoning"). It culminated in a comprehensive three volume work.[49] In it, he made

> "…a detailed comparative analysis of the forms of scientific reasoning that were developed within European intellectual culture, beginning with the Greek search for the principles of nature and argument itself, and applied to an ever wider variety of subject-matters."[50]

Crombie put forward six "methodological styles" which had been studied by earlier philosophers of science, but which hitherto (at least in the Anglophone environment) had not been given a comprehensive *historical treatment* [see §2.2]. As will be shown, two of his "styles" – historically corresponding to *ars demonstrandi* and *ars inveniendi* respectively – emerged with the very genesis of rational science in 5th Century BC Greece and run like golden threads thought its entire history. Indeed, the historical tension between these two methods of reasoning is one of the basic features of Western science and philosophy – and one of the central themes of this essay.

Note that the general tendency in the Anglophone history and philosophy of science was that of an *empirical-descriptive* and *naturalistic* (i.e. non-evaluative) approach.[51] Furthermore, it almost totally ignored what had been taking place on the "Continent" decades earlier.

> "[In the 1920's], Gaston Bachelard had already begun to elaborate a theory of 'epistemological blocks' and ensuing 'ruptures' [and] ... Georges Canguilhem systematically elaborated the details of scientific revolutions over the whole panoply of science. So Kuhn was a sensation for us, but rather old hat in France."[52]

> "[Bachelard and Canguilhem] were already doing something very similar to Kuhn, namely trying to understand physics or biology by examining their specific historical developments. ... Bachelard, in fact, proposed a philosophy which *combines discontinuity and rationality*, and thus evades certain forms of [Kuhnian] relativism."[53]

In contrast to the Anglo-American tradition, the French tradition in the *history of science* had always had closer institutional links with *philosophy of science*.[54] It had also been more sceptical of "positivism" and regarded *empiricism* and *rationalism* not as in opposition, but as necessarily complementary and interactive epistemological approaches. These circumstances naturally fostered a more integrative approach to the history and philosophy of science. Furthermore, the French notion of *epistemology* is broader and more dynamic than the Anglophone version of "theory of knowledge". It goes beyond inquiring into the nature of knowledge and what it is that makes knowledge" scientific" or "rational", but also reflects upon

> "...the historical conditions *under* which, and the *means* with which, things are made into objects of knowledge. It focuses thus on the *process of generating knowledge* and the ways in which it is initiated and maintained."[55]

There is another interesting aspect of this matter which is not always considered but which is significant nonetheless: the French scientific community of the late 19th century was far more familiar with the works of Gottfried Leibniz than its Anglo-American counterpart.[56] Indeed, some of Leibniz most important works on metaphysics and epistemology were originally written – and published – in French.[57] Of special importance here is Leibniz's version of a progressive *integrative-rationalism*, i.e. the notion of the continual qualitative development in science through the *reciprocal interaction* of empirical research and principles of rationality[58] [see §5]. Thus we find a number of prominent French scientists and philosophers of science involved in the early 20th Century "Leibnizian revival". These included Henri Poincaré (1854-1912), Louis Couturat (1868-1914), Lucian Levy-Bruhl (1857-1939) and Leon Brunschvicg (1869-1944).[59]
It was in this environment that Gaston Bachelard (1884-1962) and Georges Canguilhem (1904-1995) developed an integrated research program – a generation earlier than their Anglophone counterparts – that became known as *Épistémologie historique*.[60] Note that the original French concept has been variously translated into English as "Historical Epistemology", "History of Epistemology" and "Epistemological History". There has been some discussion as to possible differences in the meanings of these terms, which will not be addressed here. Following Canguilhem[61], we choose "Epistemological History" (EH), which is clear enough and sits better in English than "Historical Epistemology".

In a nutshell, French EH promoted "the establishment of a dynamic relationship between the past and the present"[62] by which to study "the history of the *production of scientific concepts*" where "new theories integrated old theories in new paradigms, *changing the sense of concepts*."[63] This nutshell contains three central aspects:

1. *The reciprocal relationship between empirical research and rational order*. The long-term his-

tory of the development of scientific knowledge is not based solely on advances in empirical research within a *fixed epistemological framework* (i.e. of immutable Kantian "categories of thought"), but also on the basis of new "epistemic requirements that ... change in the historical process of doing research".[64] Following Leibniz and Brunschvicg concerning the "happy marriage between the theoretical and the empirical"

> "... Bachelard's ambition as a philosopher was to draw lessons from the fact that rationalism and empiricism are actually combined in scientific activities, as two complementary professional attitudes. ... [leading to a] dialogue through which theory and experiment *conjointly evolve and shape each other*, as theoretical suggestions meet experimental objections, or unexpected experimental results look for their theoretical interpretation."[65]

2. *Epistemic concepts and structures are the principle historical objects of study*. Brunschvicg protégé Jean Cavaillès first put forward the notion of a "philosophy of the concept" (later promoted by Foucault). Such an approach works to "understand the sciences by examining the history of [epistemological-methodological] concepts and *the norms that govern their use*."[66]

> "It is not a philosophy of consciousness but a philosophy of the concept that can yield a doctrine of science. The generative necessity is not that of an activity, but of a dialectic."[67]

3. *A normative-comparative approach to an engagement with the past.*

> "... the methodological choice of the use of history does not consist in a simple work of *reconstruction of the past* ... but it is rather open to future perspectives, possible innovations that are going to enrich different scientific theories. Historical analysis looks backwards, but it is completely projected in the forwarding movement of science. This projection in the future corresponds to a need that comes from the nature of scientific work itself."[68]

Such a normative (or evaluative) engagement with the past is based on what has been termed the "controlled" or "regulated" use of anachronism[69], where historians "acknowledge anachronism as part of their awareness of the relationship between their own present and the past that they are studying."[70] It strives both to "uncover the contextual origin of supposedly obvious concepts, bringing to light the assumptions on which they rest" as well as furnishing "innovative perspectives on contemporary issues in Philosophy of Science."[71]

The spirit of this is expressed admirably by Erwin Schrödinger in "Nature and the Greeks" (1954). In studying ancient Greek science and philosophy:

> "There is not only ... the hope of unearthing obliterated wisdom, but also of discovering inveterate error at the source, where it is easier to recognize. By the serious attempt to put ourselves back into the intellectual situation of the ancient thinkers, far less experienced as regards the actual behaviour of nature, but also very often much less biased, we may regain from them their freedom of thought."[72]

As is evident from the two approaches outlined above, the epistemological history of "Epistemological History" itself reveals the same old lamentable methodological divergence that we see running through the entire history of western science and philosophy – in this case expressed as the divergence between an empiricist-descriptive and naturalistic (Anglophone) approach, and an analogical, comparative-evaluative ("Continental") approach to epistemology history.

The present article represents a particular variant of this latter comparative-evaluative approach. It concerns the following:

If one of the primary aims of science is to create new, useful knowledge, then one of the legitimate aims of an epistemological history must be that of attempting to identify such concepts, heuristic methods and principles that transcend any specific scientific "paradigm" or epoch, but which are found to be involved, generally, in successive historical instances of scientific discovery, both in the case of small, "local" advances, and in the case of major structural transformation.[73] By comparing these concepts in their long-term development, we attempt to identify *to what extent* and *in what manner* modern versions of these concepts are expressed in earlier periods ("controlled anachronism"); to what extent those earlier versions contain the seeds of later versions – as stepping stones in the history of the development of our modern concepts. [74] This represents both continuity and discontinuity in the history of science and scientific concepts. Writes Canguilhem:

> "This history [of science] could no longer be a collection of biographies, nor a table of doctrines, in the manner of a natural history. It must be a history of conceptual filiations. But this filiation has a status of discontinuity, just like Mendelian heredity."[75]

[In this endeavour one might – as an entertaining, and not altogether unreasonable thought-experiment – employ the *time-machine image* to ask if earlier proponents of a concept would understand current versions. For instance, if we could send Charles Peirce back in time to meet with Plato at the Academy to explain his (Peirce's) concept of abduction-deduction cycles as hypothesis generation and evaluation, would Plato understand it? Or send Kurt Gödel back to early 18th century Hanover in order to explain mathematical model theory to Leibniz. (Both would have given their right arms to be able to do this!) On reading the relevant texts on Plato's *method of hypothesis* (e.g. *Phaedo* 101d3-e1; *Republic* 509d-511e) [see §3], and Leibniz's texts on the differentiation of form and content in symbolic systems (*ars characteristica* and *calculus ratiocinator*) [see §5], I am inclined to believe that they certainly would understand. Leibniz would be delighted. What Plato would think is another matter. On the one hand, it would be gratifying to know that one's "method" was being practiced by scientists 2300 years in the future. On the other hand, he might wonder why we haven't gotten further than this after more than two millennia.]

Finally, in tracing the epistemological history of the notions of *ars inveniendi*, model-based reasoning and related concepts, we quite naturally highlight – and *evaluate* – that scientific "research tradition" which has most clearly – and self-consciously – promoted such concepts: the Neoplatonic tradition in the philosophy of science.

## 2 Methodological concepts for the history of *ars inveniendi*

Here we review a number of perennial concepts in the history of Western science and philosophy which are associated with methods of scientific discovery. These are (1) the notions of *conceptual spaces* and *concept blending* as a basis for human reasoning and creativity; (2) the relationship between *ars inveniendi* and *ars demonstrandi* as two pillars of rational science; (3) the two main forms of *analysis* and *synthesis* and their relationships to the art of discovery; (4) the distinction between *taxonomic* and *typological* epistemic structures; and (5) the notions of *extended cognition*, *operative symbolism* and *model-based reasoning* (although these are, for the most part, more recent "Enlightenment" developments).

## 2.1 Conceptual spaces and blending theory

During the much of the 20th Century, under the dominance of positivism, logical empiricism and behaviourism, it was generally thought that human reasoning is *principally* based on some form of tacit "logic" involving formal-propositional rules and deductive inference. Essentially, this perspective maintained that in dealing with a specific "reasoning problem" the human mind 1) recognises and recovers the logical form of the premises involved in the problem and then 2) employs formal rules of inference to draw proper (valid) conclusions.[76]

However, this has not been the predominant approach in the history of science (see below). Even during the empiricist onslaught in the 1930's the German Gestalt psychologists opposed this approach[77], as did the (usually cited) Scottish psychologist Kenneth Craik in the 1940s.[78] They argued that basic human reasoning is more accurately depicted as a (mental) *modelling process* than a process of *rule-based deductive logic*. Certainly we *use* deductive logic as one among other tools for reasoning, but this is not necessarily the way that the human mind fundamentally works. The notional basis for this standpoint is that the human mind has evolved to navigate within a dynamic spatio-temporal world – both physical and social – which it represents in the form of conceptual spaces and topological relationships.[79] As will be demonstrated, this mode of reasoning – mental models and conceptual spaces – serves as a framework for both *formal deductive* reasoning and *heuristic non-deductive* reasoning, the latter being broader and more generative than the former for fostering new concepts and scientific discovery. (Note the difference between *mental models* and *conceptual models*: "A mental model coordinated with symbolic representation is called a *conceptual model*. Conceptual models provide symbolic expressions with meaning."[80]) (See §2.5.)

Since the 1960's, theories of mind have been proposed – and experimentally tested – giving strong evidence that human reasoning fundamentally takes the form of non-deductive (non-propositional) mental models based on conceptual spaces and typological combinatorics[81], and that a basic way that people solve problems is by way of a heuristic search through a combinatoric space.[82] These are no longer considered to be simply spatial *metaphors*, but are associated with the same basic properties that we ascribe to articulated scientific models: e.g. the formation of linked domains and combinatorial variation:

> "Mental spaces, on the view of cognitive linguists, behave in two important ways: they link together to form networks which are salient or active for a period of time (as when one mentally co-ordinates *variables in a mathematical problem*, or persons in a narrative) and they also at times *integrate or blend*, as when a relationship of identity – whether exact and literal, or approximate and figurative – is found between them."[83]

The more recent field of "Conceptual Integration Theory" and the notions of *conceptual blending* and *cross-space mapping* (see §2.6) developed by Fauconnier & Turner (1998, 2002) represents human reasoning as based on conceptual spaces that can combine or blend in order to create new ("emergent") concepts.[84] (In experimental psychology and in the area of Artificial Intelligence this is called *conjunctive concept formation*[85] and *conjunctive concept combination*[86] respectively.)

> “Conceptual integration, which we also call *conceptual blending*, is a basic mental operation, highly imaginative but crucial to even the simplest kinds of thought “ [87]… [It involves, inter alia] “setting up mental spaces”, “matching across spaces”, “locating shared structures” and “running various operation in the blend”.[88] … “The essence of the operation is to construct a partial match between two inputs, to project selectively from those inputs into a novel 'blended' mental space, which then dynamically develops emergent structure. It has been suggested that the capacity for complex conceptual blending ("double-scope" integration) is the crucial capacity needed for *thought and language*.”[89]

The last sentence of these quotations is intriguing. The authors express it in another way in an earlier work: “Mental spaces are small conceptual packets constructed *as we think and talk...*”.[90] Remember these words; they turn up later in a most fascinating place (§3).

We can take a (very) simple example – called the “boathouse-blend” – by which to display different blending formats which emphasise different approaches to concept formation (Figures 2.1a-c).

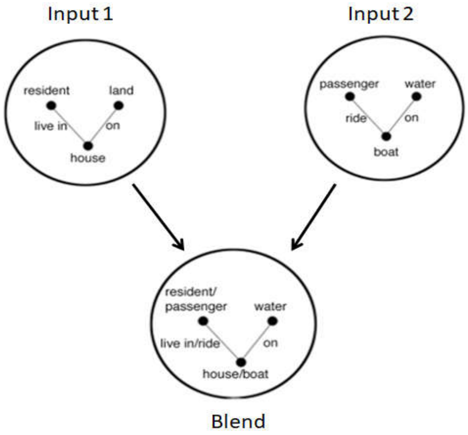


Figure 2.1a: Blending graph structure[91]

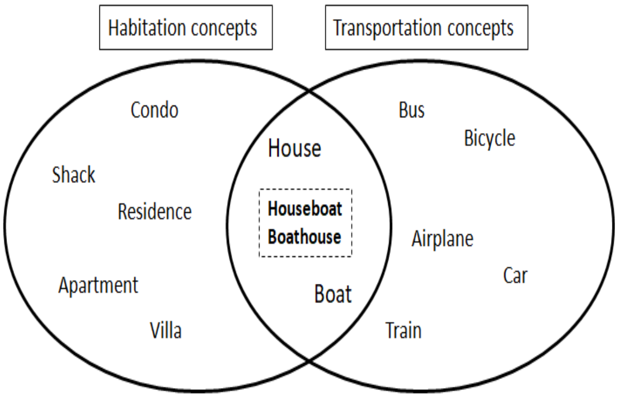
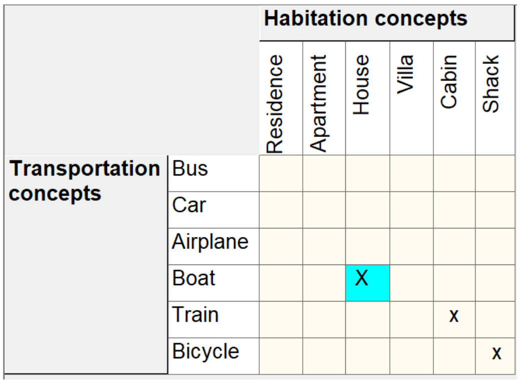


Figure 2.1b : Conjunctive Euler spaces



| | | **Habitation concepts** | | | | | |
|---|---|---|---|---|---|---|---|
| | | Residence | Apartment | House | Villa | Cabin | Shack |
| **Transportation concepts** | Bus | | | | | | |
| | Car | | | | | | |
| | Airplane | | | | | | |
| | Boat | | | X | | | |
| | Train | | | | | x | |
| | Bicycle | | | | | | x |

Figure 2.1c: Typological “cross-order” structure

In contrast to 2a and 2b, the typological or “cross-order” format deliberately displays *all* (formal) combinatorial blends among concepts in a matrix. Here we can detect other semantically compatible pairs of concepts: e.g. “Train-cabin” and “Bicycle-shack”. Are there others? (*Boathouse* is special here in that it is fully reversible: i.e. *Houseboat*.)

The basic ideas of *conceptual spaces* and *concept blending* are, of course, nothing new; they go back to antiquity where they were promoted at the Platonic academy. Plato's "theory of ideas" and classification of "kinds" are prototypical examples of the former; his notion of "blending" and "combining" concepts in the Sophist is an example of the latter. (Plato's spatial metaphors are best represented by Euler spaces. Llull – as we shall see – mainly adopted the typological format, as he was more into combinatorics (§4)).

The same can be said for the idea of a *semantic search space*. The classical notion of *topoi* (Latin: *loci* – see §2.2) "functions rhetorically as a conceptual *place* to which an arguer may mentally go to find arguments", which gives both "the possibility of novelty hidden in the commonplace", and serves as "a space for combination and recombination" ... "viewing a problem from the vantage of a *topos* can reveal or make possible new combinations, patterns, relationships that could not be seen before".[92]

This spatial metaphor is still with us today. In English, for example, when we want to enumerate arguments, we say "in the first place; in the second place" etc.; we talk about ideas and facts "falling into place"; and "commonplace" refers to shared ideas and attitudes. This metaphor is also associated with the long history of the "art of memory" (*ars memorativa*) which is concerned not only with "finding" concepts, but with the systematic ordering of concepts and the structure of knowledge in general.[93]

## 2.2 Ars inveniendi and ars demonstrandi

*Ars inveniendi* and *ars demonstrandi* are two of the most important methodological concepts in the history of western science. The epistemological status of these concepts – and the relationship between them – has been a subject of contestation starting with Plato and Aristotle and continuing right up to the present moment. We begin by defining these concepts and then – in keeping with the spirit of Epistemological History – trace out their origins and development.

- *Ars demonstrandi* (***ars dem***, a.k.a. *ars iudicandi*) is concerned with formal (deductive) methods of demonstration, justification or "proof".[94] It starts from given principles (axioms, premises, hypotheses) and by rules of valid inference deduces *necessary conclusions*. Today this is also known variously as the *synthetic-deductive* or *axiomatic-synthetic* the method.
- *Ars inveniendi* (***ars inv***) is concerned with non-deductive, *heuristic* methods used to search for and discover new knowledge or principles, which can serve as starting points (premises, axioms, hypotheses) for ***ars dem***. Thus it starts from conclusions or a given state of affairs, in order to find possible principles or premises by which such conclusions can be drawn or by which such a state of affairs can be explained. (It can also start from a hypothesis and proceed towards a *higher-order hypothesis*, from which the initial hypothesis can be *demonstrated* or *justified* – see §3.) Today, *ars inveniendi* is also known as the Abductive or Hypothetical-Analytic method.

Some general observations:

First of all, it is obvious that these are reciprocal procedures: ***ars dem*** requires a prior ***ars inv*** to furnish "true premises", or at least plausible hypotheses; and ***ars inv*** requires a subsequent ***ars dem*** in order to test the implications of such premises and to draw from them valid conclusions.

Secondly, based on these definitions, we see that they inquire into two fundamentally different things: ***Ars dem*** inquires into the "internal" *formal* principles of valid inference (*in intellectu* or so-called *second intensions*), whereas ***ars inv*** inquires into "external" *ontological* principles and relationships (*in re* or *first intensions*).[95] (See §2.3 & §4).

Finally, ***ars dem*** and ***ars inv*** look suspiciously like the so-called "geometric method" as practiced by the classical Greek geometers: *synthesis* starts from given geometric premises (axioms) and draws necessary (deductive) conclusions[96]; whereas *analysis* starts from a given or desired conclusion in order to "find" (or hypothesize) premises from which such a conclusion can be drawn [see §2.3.].

However, as usual, things are not that simple. We need to look at the historical epistemological development of these methodological concepts in order to understand their contemporary status. It starts with Plato, not necessarily because Plato was the first to ever think about this issue, but because he was the first to systematically report and elaborate on it. In *Republic* (510 b4-9) he has Socrates explaining that there are two ways for the mind to investigate things: the first by proceeding from principles to inferred conclusions; the other by "finding" principles which are *sufficient to explain* a given situation or conclusion. Indeed, Aristotle attests to the fact of Plato routinely bringing up this issue in discussions at the Academy:

> "Let us not fail to notice, however, that there is a difference between arguments *from* and those *to* first principles. For Plato was right in raising this question and asking, as he used to do, "Are we on the way from or to first principles?" (Aristotle, NE. 1.4, 1095a.)

Of course, Plato and Aristotle did not use the terms *ars demonstrandi* and *ars inveniendi*. These are Latin translations. The Classical Greek concepts are *apódeixis* (literally "to show-off", i.e. demonstrate or prove) and *heuresis* (literally "to find-out", discover). The origins of the Latin terms are usually traced back to Cicero (see below). Here we will use the more familiar Latin terms, even when considering their Greeks origins.

Thus, ever since Plato and Aristotle, ***ars dem*** has been associated with formal deductive inference from principles (e.g. Aristotelian *syllogistic* or "logic"), and ***ars inv*** with the critical analysis of concepts and hypothesis generation which would formulate principles (i.e. Plato's "method of hypothesis" or what Peirce would call *abduction* or *retroduction*). Note that the former is thought of as a *descent* from principles to conclusions; the latter as an *ascent* from conclusions (or a given state of affairs) to subsuming principles.[97]

The main issue here is that Plato and Aristotle were not in agreement about the *epistemological status* of these two procedures. They used them to define "science" or "scientific knowledge" (*episteme*) in two different ways. For Plato (and the subsequent Neoplatonic tradition), "science" is based on the *reciprocal process* of ***ars dem*** and ***ars inv*** – i.e. "science" should include methods of development and discovery of new knowledge as well as the justification of already acquired knowledge. This reciprocal relationship is one of the expressions of Plato's original notion of "dialectic" [see §3]. For Plato, the main point of "science" is the critical analysis of hypotheses through the formulation of higher-order hypotheses (principles) by which to frame and create new knowledge. *Demonstration* or proof by deductive inference is necessary for the formal evaluation of such discoveries, but is not, *in itself*, generative or ampliative.[98]

Aristotle, on the other hand, does not consider ***ars inv*** as belonging to *science proper*, since it does not produce "certain knowledge". For him, scientific knowledge is only gained through demonstrative proof, based on a relatively small number of *already accepted* first principles (akin to the "axiomatic-synthetic" method of the Greek geometers[99]). This is expressed in his oft quoted text in the Nicomachean Ethics (NE - Book 6:3:4):

> "Scientific Knowledge, therefore, is the quality whereby we *demonstrate* [i.e. *apodeikný* = proof by deduction from first principles] … Let this stand as our definition of scientific knowledge [*episteme*]."

Now, among the diverse texts attributed to Aristotle there are more subtle expressions of this "definition", and even seemingly contradictory expressions of it.[100] The point, however, is that the "theory of science" expressed in NE and the *Posterior Analytics*, "... established itself at such a level of perfection that for centuries its history coincided largely with the history of the *interpretation of that text*."[101] And although it would be going too far to attribute to Aristotle the Enlightenment empiricism of Locke and Hume – let alone that of modern logical empiricism – nonetheless "... the theory of explanation that the present interpretation ascribes to the Posterior Analytics remains consistent with an *empirical foundationalism* – with the idea that all knowledge is justified ultimately on the basis of sense perception"[102]. The historically received "interpretation of that text" became the foundation for the neo-Aristotelian "classical model of science".[103]

So where does ***ars inv*** end up in Aristotle's version of science? He separates demonstration from hypothesis generation by regulating the latter to the realm of "general opinion" (*endoxa*).[104] In this way, ***ars inv*** is moved out of science proper, and Plato's reciprocal ***ars inv-ars dem*** "dialectic" becomes – over time – mainly associated with *rhetoric*.

> "Aristotle widens the scope of application of dialectic and, at the same time, downgrades its epistemic rank. His dialectic is concerned with a certain kind of intellectual debate. Concerned neither with truth of the highest order nor "ordinary" philosophy, but being the art of attacking an adversary's thesis or defending one's own, dialectic continues the *agôn* (contest), characteristic of Greek culture, by intellectual means, including obfuscation (*krypsis*), which leaves the opponent in the dark concerning the aim of the proof (*Top*. VIII 1). ... Given that [Aristotle's] dialectic does not question its premises – *the endoxa* – any further, it cannot in the end achieve more than a demonstration of the internal consistency and coherence of a multitude of theses."[105]

What is historically important about this separation is not only that it is a *tour de force* of misrepresentation of Plato's notion of *episteme* and *dialektikḗ* [see §3], but that it is the opening shot of 2400 years of epistemological warfare. The expulsion of the *discovery process* from science proper will repeat itself (*mutatis mutandis*) several times during the next two and a half millennia, viz. the 13th Century introduction of Aristotelianism into Medieval Scholasticism; with European Enlightenment "empiricism"; and 19th - 20th Century positivism and logical empiricism.

Thus several centuries after the time of Plato and Aristotle, the philosophers of the "early" (Neo-) Platonic revival (2nd Century AD) woke up to find themselves confronted with two inherited versions of "science": one based on ***ars dem***, going under the name of "logic"; the other based on the reciprocal procedures of ***ars inv*** & ***ars dem***, going under the name of "dialectic". Indeed, some

historians of science have regarded the tension between these two concepts of "science" as one of the central features of the history of Western science.[106]
Prior to the 2nd Century Neoplatonic revival, however, Cicero (106-43 BC), using what were possibly his own Latin translations, employed the notion of ***ars inv*** in the Aristotelian sense, i.e. not primarily as a generative discovery process in the context of *scientific inquiry*, but in the context of *rhetoric*, i.e. the art of oratory, disputation and persuasion. In this context, the role of ***ars inv*** was to "find" the right terms, expressions or concepts to be used as arguments in a given speech, debate or judicial line of reasoning. This "finding" process takes "place" within a conceptual space – *topoi* (Greek) or *loci* (Latin) – aided by memory and some form of (combinatoric) heuristic procedure. The role of ***ars dem***, on the other hand, is concerned with evaluating and structuring "arguments" into a proper and effective "disposition" or "arrangement" which binds them together in good *demonstrative order*. This distinction between discovery and demonstration remained in the medieval European curriculum of liberal arts as essentially rhetorical concepts (with the exception of notable Neoplatonic rebels such as Robert Grosseteste (1168-1253), Roger Bacon (1220-1292) and Ramon Llull (1236-1316)).[107]

The Renaissance revival of the notion of ***ars inv*** as a rational method of scientific discovery [§5] again took backseat to ***ars dem*** both in the 18th Century British and French Enlightenment period, and in the 19th and early 20th century with the rise of logical empiricism and an "analytic philosophy of science". In "Why was the Logic of Discovery was Abandoned", Laudan maps out how 19th century establishment science regulated ***ars inv*** "... to the domain of the irrational".[108]

> "An event of major significance occurred in the course of 19th- century philosophy of science. The task of articulating a logic of *scientific discovery* and *concept formation* ... was abandoned. In its place was put the very different job of formulating a logic of *post hoc* theory evaluation, a logic which did not concern itself with how concepts were generated or how theories were first formulated."[109]

This latest separation of ***ars dem*** and ***ars inv*** was subsequently reformulated by the Vienna Circle of logical empiricism. It now became the *context of justification* and the *context of discovery*, expressions which are usually attributed to Hans Reichenbach.[110] Since no formal-logical rules could be formulated for the *context of discovery*, it was regarded as a psychological and sociological issue, with no place in the (logical empiricist) philosophy of science – strikingly analogous to Aristotle expelling Plato's *heuristic method of hypothesis* out of *episteme* into the realm of *endoxa*.[111] As Reichenbach put it (in this oft-cited statement):

> "The philosopher of science is not much interested in the thought processes which lead to scientific discoveries . . . that is, he is interested not in the context of discovery, but in the context of justification."[112]

Warkosky (1980) calls the whole situation the "scandal of [analytic] philosophy":

> "Although creativity in science, mathematics, and technology is crucial to the fundamental processes of discovery and invention, it has largely been ignored by the [analytic] philosophy of science, or it has been regarded as a question which lies outside the domain of philosophy of science proper. This has been the scandal of contemporary philosophy of science. But it has not been a hidden scandal, tacitly acknowledged and whispered about behind closed minds." [113]

Although the Neoplatonic tradition in science was certainly alive and thriving in the early 20th century in some of the most creative "working scientists" of the time – for instance Henri Poincaré, Albert Einstein, Kurt Gödel, Ervin Schrödinger, Louis de Broglie, Hermann Weyl (Fritz Zwicky's thesis advisor [§6]), David Bohm and Abraham Robinson (all of whom were philosophically inclined) – the notion of a *rational art of scientific discovery* only began to gain attention again in Anglophone philosophy of science with the publication of a number of anthologies based on work during the 1960's and 70's.[114] As Thomas Nickles expressed it some 40 years after the publication of his two anthologies:

> "Notice that as originally conceived, scientific method was a method of discovery, not merely a method of justification. In fact, discovery and justification were one."[115]

It is indeed intriguing – as well as disturbing – to find these divergent attitudes to the nature of scientific discovery running through the entire history of Western science and philosophy. It would seem that every historical epoch, maybe every generation, has to *rediscover* and *relive* it *(pace* Santayana). Note also, that even present-day discourse on the *discovery-justification* distinction is hardly settled, has become all the more involuted, and remains a subject that seems to engender "almost deliberate misunderstanding".[116]

## 2.3 Analysis and synthesis: Sequential and Compositional

As mentioned above, the distinction between ***ars inv*** and ***ars dem*** looks suspiciously like the methods of *analysis* and *synthesis* used in classical Greek geometry. Here is how Pappus (290 - c. 350 AD), one of the great Greek mathematicians, describes the matter:

> "Analysis, then, takes that which is sought as if it were admitted and passes from it through its successive consequences to something which is admitted as the result of synthesis: for in analysis we admit that which is sought as if it were already done and we inquire what it is from which this results, and again what is the antecedent cause of the latter, and so on, until by so retracing our steps we come upon something already known or belonging to the class of first principles, and such a method we call analysis as being *solution backwards*. ... "But in synthesis, *reversing the process*, we take as already done that which was last arrived at in the analysis and, by arranging in their natural order as consequences what before were antecedents, and successively connecting them one with another, we arrive finally at the construction of what was sought; and this we call synthesis."[117]

Thus:

- Synthesis: ***progressing*** *from* given axioms or premises *to* (inferring/proving) theorems or necessary conclusions. Today we call this the "axiomatic-synthetic" method and, because of its use by the early Greek geometricians, it is often referred to as the "geometric method".
- Analysis: ***retrogressing*** *from* given (or sought after) theorems or conclusions *to* axioms or premises *from which they* (the theorems or conclusions) *can be deduced.*

This form of analysis and synthesis we shall call *sequential* or *gressive* (pro*gressive,* retro*gressive*). In subject-predicate form it is called *propositional* or *sentential*. However, it can also be expressed in *causal* terms, both as "formal causes" and for "efficient" causal explanations:

- Synthesis: progressing from causes to (*explain*) effects
- Analysis: retrogressing from effects to (*find/discover*) possible causes which can explain

such effects

So here we see the correspondence between the concepts of *sequential* (propositional or causal) analysis and synthesis and of ***ars inv*** and ***ars dem*** (a.k.a. *ars judicandi,* the art of justification).

> "Analysis concerns heuristic methods employed to solve problems, whereas synthesis has to do with proofs; that is, with methods employed to deduce theorems from the axioms. *In this respect*, analysis corresponds to what the tradition called an art of discovery (*ars inveniendi*) and synthesis to what was called an art of judgement (*ars judicandi*)."[118]

However, this is not the whole story. There is another "mode" of analysis and synthesis that complements the *sequential* or *propositional* mode: that is, *compositional* analysis and synthesis, which is not about *subject-predicate* or *cause-effect* relationships, but about *whole-part* relationships.[119]

- Analysis: decomposing a conceptual or substantial *whole* into constituent *parts*
- Synthesis: combining *parts* in order to compose or configure a *whole*.

This "mode" developed in parallel with *sequential* analysis and synthesis and was initially prototyped by Plato in his method of *divisions* and *collections* (see §3). The relationship between *sequential* and *compositional* modes of analysis and synthesis is complex and has led to a good deal of confusion in the history of science. When we get to Leibniz (§5), who re-instated *ars inveniendi* at the center modern science, we will see how these two modes combine: how ***ars inv*** and ***ars dem*** cut across both versions of analysis and synthesis, and how both compositional analysis and compositional synthesis can themselves be regarded as methods of scientific discovery.

Figure 2.2 visualises the relationships between these two modes. (Note that, especially the concept of *analysis* has been called a polycephalous monster,[120] and that this heuristic diagram does not cover all of the aspects that have been associated with it.) We designate Causal and Morphological ana-syn as *first intensions* operating *in re* (on the *territory*), with Propositional and Mereological ana-syn as second intensions operating *in intellectu* on first intensions (the *map*). (See footnote 95.)

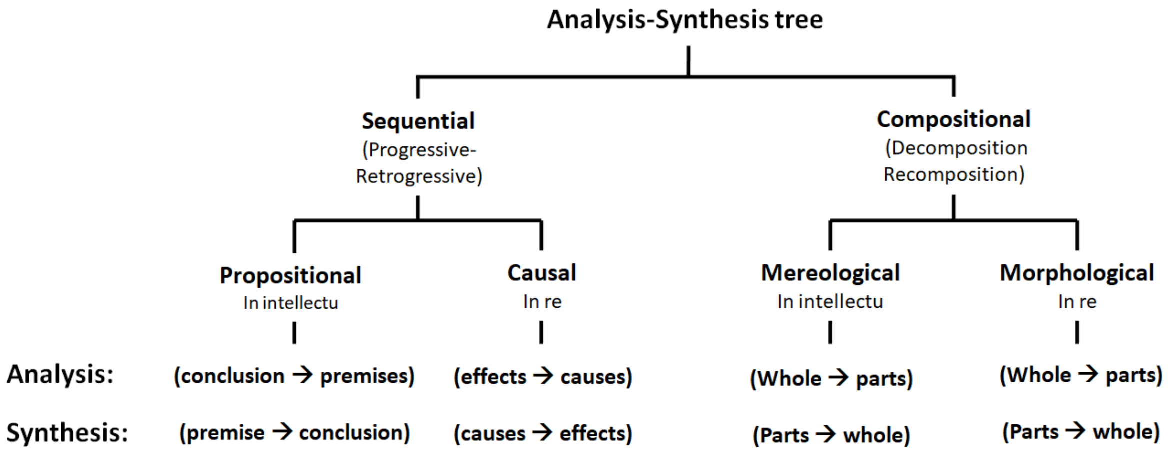


Figure 2.2: Analysis-Synthesis taxonomy

### 2.3.1 Analytic and synthetic forms of *ars inveniendi*

Here we consider the relationship between analysis, synthesis and *ars inveniendi*. Since there is currently no generally agreed upon *standard terminology* for all of these concepts – and there is still an ongoing debate about them in general[121] – we include some of their different terminological variations.

1. *Analytic-generative* discovery, based on "hypothesis generation" or *abduction*. Magnani calls it *creative abduction*.[122] This is, so to speak, "out-of-the-box" generative discovery.
2. *Synthetic-combinatoric* discovery, based on Leibniz's compositional "synthesis by combinatorics" and "conceptual blending".[123]

#### 1. Analytic inveniendi as hypothesis generation (abduction)

The analytic mode of ***ars inv*** derives from Plato's method of hypothesis and meta-hypotheses presented in the famous divided line section (510b5-511c2) in of the *Republic*. This is what Peirce spent a good deal of his life trying to formulate (or rediscover) with his notion of *abduction*.[124]

This mode of inference, as "out-of-the-box" creativity has two main phases or aspects:[125]

- Hypothesis Generation (Abduction): The process of formulating (discovering, creating) a hypothesis which is/or may be sufficient to explain a situation, an event or other object of scientific enquiry. This can be done through e.g. analogical extension and by extending or generalising heuristic principles.[126]
- Epistemological Evaluation (Deduction): The process of evaluating (justifying) a hypothesis by drawing out its consequences and testing these for consistency with other concurrently accepted assumptions, premises and theorems, and by comparing it with other rival hypotheses and their conclusions.

In the Neoplatonic tradition, from Plato to Leibniz to Riemann to Peirce, analytic or abductive hypothesis generation is not only considered as something rational, but seen as the very motor of science: "the mode of reasoning that sets the scientific mind in motion"[127]; "... an entry point into a new space of entities and relationships".[128]

Michel Serres beautifully expresses this analytic-retrogressive process of re-axiomatization in mathematics:

> "Questioning backwards, questioning the foundations, and the refined analysis of original elements perceived retroactively as layered, stratified ideas, as complex particular cases of elements that are even more original still ... We would never get to the end of repeating how many times questions reconstructed deeper foundations, not only for the starting axioms, but in the very constitution of the idealities in question."[129]

### 2. Synthetic inveniendi as combinatorial blending

Synthetic-combinatoric discovery is based on having an appropriate *heuristic search space* in order to "find" new combinations or blends of concepts. Here the use of tables, grids, matrices, typologies and morphologies works to superimpose or "blend" different concepts as *conjunctions of epistemic possibilities*.[130]

Emily Grosholz expresses this ampliative process in the context of mathematical discourse:

> "When distinct representations are juxtaposed and superimposed, the result is often a ***productive ambiguity*** that expresses and generates new knowledge. Mathematical experience emerges from traditions of representation and problem-solving, as it explores 'combinatorial spaces' ... produced in polyvalent mathematical discourse.[131]

Some would argue that the heuristic procedure of existential and constructive combinatorics is not a true ***ars inv***, since once the "space" is formulated, then all the possibilities are already – at least potentially – in "place" and need only be identified (Francis Bacon's "*a chase of deer in an enclosed park*"[132]). However, if such a combinatoric space is created in the act of *concept definition by analysis*, then such definitions "are not arbitrary stipulations but rather hypotheses and, as all hypotheses, are means of discovery"[133]. Thus if the analysis phase is itself *generative*, then the combinatoric-synthetic phase can use these new concepts to facilitate emergent blending. However, one can also regard the combinatoric *blending process* itself as a form of *inveniendi*.

It is interesting to note that Henri Poincaré – who was highly interested in such discovery processes – described it in terms of an "unconscious combinatory activity".[134] Poincaré was well acquainted with the work of Louis Couturat on *La Logique de Leibniz*, which included a detailed account of Leibniz's doctorial thesis extension "De arte combinatoria" (see §5), with its description of compositional analysis and combinatorial synthesis.

> "Poincaré ... draws on the idea of expressing and formalizing the unconscious processes that underlie the search for a solution to a problem. He argues that in order to solve problems our unconscious first combines and then selects ideas: when we face a problem and our conscious attempt to solve it does not succeed and stalls, the unconscious processes step in and work by creating all the possible combinations starting from certain ideas."[135]

Since combinatorial synthetic "discovery" is one of the primary objects of interest in this article, we shall take it up in more detail in the examples presented in §3-6.

## 2.4 Two perennial epistemic structures: Taxonomy and Typology

> "Trees, wheels, columnar tables … are all structures that can be visually compartmentalized … As diagrammatic structures these compartments become enclosed areas for the accommodation of verbal material, serving to illustrate the *conceptual relationships between the various components and the relations between each part and the whole*."[136]

In the 1930's, William Chase Greene and colleagues at the *Societas Philologica Americana* carried out a comprehensive study of logical and geometric (diagrammatic) symbolism in the Platonic *Scholia* – i.e. the 2000+ year old scholarly tradition of commentary, annotation and interpretation of the Platonic corpus.[137] Two of the most persistent and important of these diagrammatic structures fall into two natural categories: (genealogical) *trees* and (combinatorial) *matrices*. Figures 2.3a & 2.3b are examples of these two structures from the late medieval *scholia*.[138]

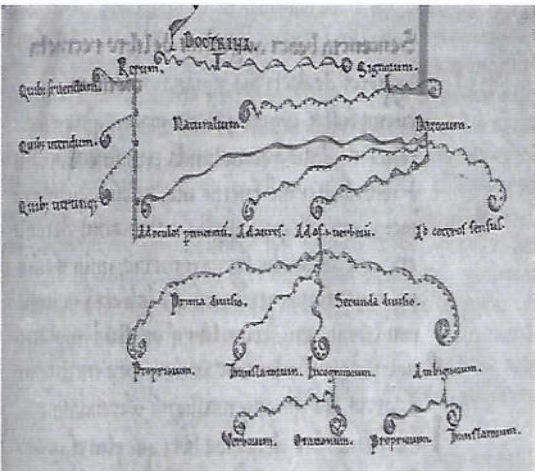


Figure 2.3a: Multi-level medieval tree diagram

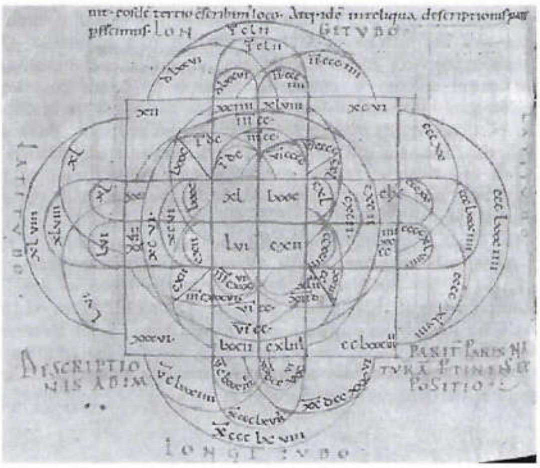


Figure. 2.3b: Multiple crossed orders

Such diagrammatic classification schemes can embody epistemic structure and mapping devises, both for knowledge representation and knowledge generation.

> "By manipulating and inspecting a diagram we can, due to the underlying conceptual mapping, draw inferences about the objects represented by the diagram (or rather: about the objects as they are conceptualised under the given metaphor or blend). ... To give full cognitive characterisation, diagrams are material anchors for conceptual mappings."[139]

As far back as Plato and his "Theory of Ideas", objects of knowledge were thought of as being defined by way of "containment" – i.e. inclusion within or conjunction with – more general concepts or properties. Indeed, the idea of being properly "defined" was synonymous with being properly "classified" within a conceptual (hierarchal and/or combinatory) framework. Furthermore, properly defined concepts are a prerequisite for discovering new types of relationships and new forms of knowledge.

> "Classification schemes not only *reflect* knowledge by being based on theory and displaying it in a useful way, ... but also classifications *in themselves* function as theories do, and serve a similar role in inquiry: that is, the role of explanation, parsimonious and elegant description, and the generation of new knowledge."[140]

Today, these two fundamental forms of classificatory structures – *trees* and *matrices* – go under the names of *taxonomies* and *typologies*. We will define these two structures and then take a look at their wider epistemological import.

Before continuing, however, a brief warning is in order. For historical reasons, classification theory labours under an unfortunate ambiguity in terminology that has, on occasion, led to great confusion. The classical Greek word *taxis* (τάξις) means *order* or *arrangement*. On this basis, the term "taxonomy" has/is sometimes used *generally* to denote *any and all types* of classification schemes or structures. On the other hand, *taxonomy* is also used in a more specific sense in biological classification schemes and evolutionary theory to denote a *hierarchal structure of linear inheritance* (*genealogy*), represented by a *tree structure*. In this context, a pure (or "proper") *taxonomy* has the property of *strict hierarchal class inclusion* with no overlapping categories (see below). This distinguishes it from a "proper" *typology*, which is non-hierarchal and consists *exclusively* of overlapping categories. This ambiguity between the two meanings of *taxonomy* has led to a number of futile controversies, one of which concerns the relationship between taxonomies and deductive reasoning.

However, in contemporary *classification theory and research*, the difference between taxonomic and typological structure is important, even though the specific terms themselves are not always applied consistently. And although taxonomic and typological structure can certainly be combined, for instance in so-called *faceted* information retrieval systems[141] (see below), we nonetheless want to clearly distinguish between the "proper" notions of these two classificatory systems, since their formal structures represent two quite different, albeit complementary, forms of reasoning – which is exactly what we wish to highlight here.

### 2.4.1 Taxonomies

A taxonomy is a system of classification that categorizes phenomena into mutually exclusive and exhaustive sets with a series of discrete, hierarchically nested decision rules. Proper taxonomies are strict hierarchal tree-structures based on genus-species and/or whole-part relationships, where only *logical inclusion and disjunction* are allowed.

Put another way, a proper taxonomy is an ordering structure which classifies concepts in descending hierarchal order of "class inclusion", from the general to the more specific (i.e. a "nested" genera-species hierarchy). Except for the "highest" category (*summum genus*), each category is entirely included within at least one higher-order category. Such a *strict* species/differentia hierarchy allows for no overlapping (intersecting) categories. Figure 2.4 shows three ways to visually represent a proper taxonomic structure.

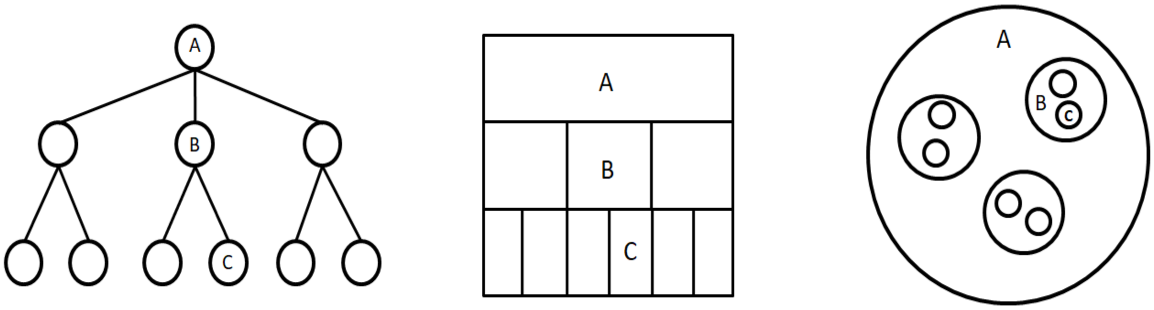


Fig. 2.4a
Tree format

Fig. 2.4b
Block format

Fig. 2.4c
"Euler" diagram

Figure 2.4. Three representations of a proper taxonomic structure

As can be seen most clearly in the spatial logic of the so-called Euler diagram[142] (Figure 2.4c), the only types of logical relations allowed are *class inclusion* and *class disjunction*.[143] *Class inclusion* (or subsumption) is a relation expressing *logical entailment*:

> "Rational taxonomy is based on the assumption that the key characteristic of rationality is deductive inference of certain partial judgments about reality under study from other judgments taken as more general and *a priori* true." [144]

The textbook example is the deductive structure of the syllogistic form BARBARA (as is most clearly seen in Figure 2.4c) :

All B is A
All C is B
∴ All C is A

Thus, proper taxonomies are "significant conceptual structures" which express – and visualise – deductive inference.

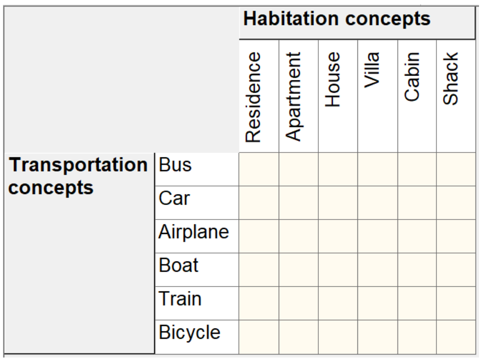
Summing up the following properties of proper taxonomies:

- **Structure**: Strict hierarchal (tree structure)
- **Logical relationship.** Only *species/differentia*, i.e. only strict class *inclusion* vertically (*genera/species*), and only disjunctive relationship between horizontal sibling classes (*differentia,* i.e. classes sharing the same superclass).
- **Mutual exclusivity**: A given entity can only belong to one class (no overlapping entities).
- **Inheritance**: Attributes are inherited by a subclass from its superclass.
- **Asymmetric inheritance** (if all B is A, then all A is not B).
- **Transitivity**: Inheritance is *transferred* to subclass not only from its immediate superclass, but also from every other higher superclass (i.e. *full transitive inheritance*).

### 2.4.2 Typologies

> "... perceiving the world and structuring it by means of types and typologies, is an essential and intrinsic aspect of the basic orientation of actors to their situations. ... typologies are ubiquitous, both in everyday social life and in the language of the social sciences." [145]

A typology (a.k.a. multiple crossed-orders)[146] is a *combinatorial classification* system based on multiple independent criteria. It consists of a set of category variables or attribute lists which are cross-referenced or *co-ordinated* (as in a Cartesian product), thus superimposing their domains upon one other (Figure 2.5). It identifies and inter-relates "simple" (one dimensional) concepts in order to create and explore the more complex multidimensional (multivariate) domain of concepts which can compounded out of these simple concepts. This gives rise to a combinatoric space called a *typological field* or *type-space*. Each specific formal type in the type-space consists of a unique (non-ordered) combination of the defining attributes.



| | | Habitation concepts | | | | | |
|---|---|---|---|---|---|---|---|
| | | Residence | Apartment | House | Villa | Cabin | Shack |
| **Transportation concepts** | Bus | | | | | | |
| | Car | | | | | | |
| | Airplane | | | | | | |
| | Boat | | | | | | |
| | Train | | | | | | |
| | Bicycle | | | | | | |

Figure 2.5: 2-D typological format involving the "boathouse blend" in §2.1.

Note that, in contrast to taxonomies, no decision-rules are used in creating the *initial* type-space. However, it acts as a heuristic *search space* that can be constrained by way of rule-based (existential) combinatoric constraints. Constrained typologies provide *inference by exclusion*: i.e. an independent variable can be selected and manipulated to produce variance in dependent variables.

Thus a (proper) typology – in contrast to a (proper) taxonomy – contains *only* overlapping (or interesting) categories. A typology is called "complete" or "full" when all of the possible (formal) combinations of attributes are designated in the type-space. Figures 6-8 show 2-, 3- and 4-dimensional "dyadic variable" typologies. They are shown in three different formats: Euler diagrams; typological "coordinate" (or "cross-order") format; and morphospace format. Note that the simplest form of a typology (Figure 6b, below) is called a four-fold (or 2x2) table.

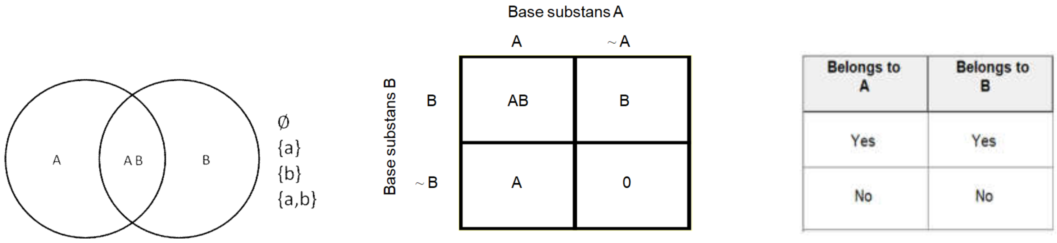


Fig. 2.6a
Euler diagram

Fig. 2.6b
Typological format

Fig 2.6c
2-D Morphospace

Figure 6. 2-species typology giving 4 types (n=2, k=2)

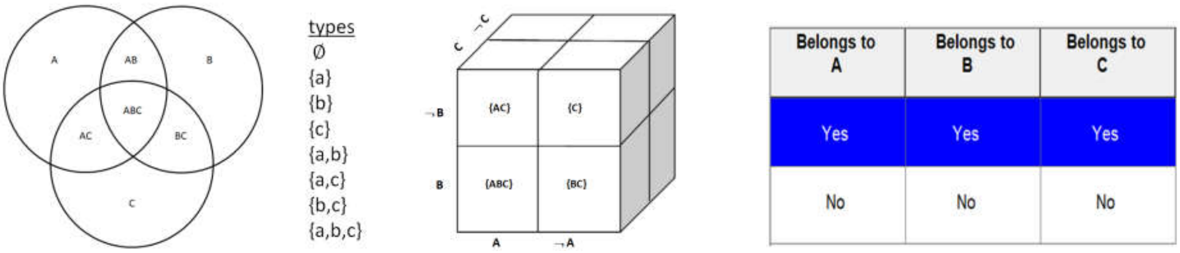


Fig. 2.7a
Euler diagram

Fig. 2.7b
3-D Typological format

Fig 2.7c
3-D Morphospace (showing {a,b,c})

Figure 2.7 3-specis typology giving 8 types (n=3, k=2)

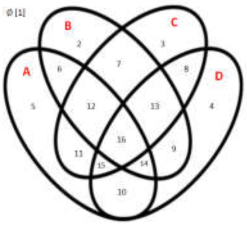
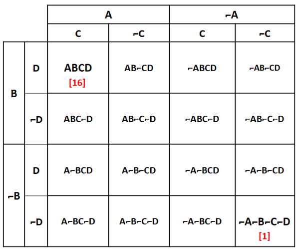
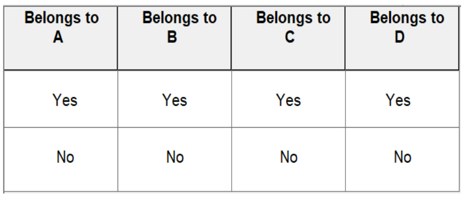


Fig. 2.8a
Euler diagram



| | | A | | ¬A | |
|---|---|---|---|---|---|
| | | C | ¬C | C | ¬C |
| B | D | ABCD<br>[16] | AB¬CD | ¬ABCD | ¬AB¬CD |
| | ¬D | ABC¬D | AB¬C¬D | ¬ABC¬D | ¬AB¬C¬D |
| ¬B | D | A¬BCD | A¬B¬CD | ¬A¬BCD | ¬A¬B¬CD |
| | ¬D | A¬BC¬D | A¬B¬C¬D | ¬A¬BC¬D | ¬A¬B¬C¬D<br>[1] |

Fig. 2.8b
4-D Embedded typological format



| Belongs to A | Belongs to B | Belongs to C | Belongs to D |
|---|---|---|---|
| Yes | Yes | Yes | Yes |
| No | No | No | No |

Fig 2.8c
4-D Morphospace

Figure 2.8. 4-species typology giving 16 types (n=4, k=2)

Here are the formal properties of a proper typology

- **Structure**: Co-ordinate/orthogonal (matrix or faceted structure; non-hierarchal)
- **Logical relationship:** Only intersection/combination
- **Mutual inclusivity**: Entities can belong to more than one class (multivariate)
- **Symmetry/asymmetry**: Intersection is **symmetric** (if some B is A, then some A is B).
- **Compossibility**: All combinations are initially treated as formally possible

Typologies (and morphologies) are more than simply knowledge structures, but are true models in that they represent coordinate systems of linked variables, allow for inputs and outputs, and support hypothesis formulation.

> "Classifications have structural properties that lend themselves to representing knowledge in a given situation. The traditional trees and hierarchies [e.g. taxonomic structure] are powerful, but are difficult structures to construct in domains that are not fully understood, changing, complex, and *multivariate*. Facet analysis [typological/morphological structure based on compositional analysis and synthesis] proves to be a useful tool for building classifications in such circumstances and, if carefully and rigorously applied, allows for movement towards *theoretical understanding of the domain it classifies*."[147]

To sum up: taxonomies and typologies are two fundamental types of classificatory structures representing different types of topological relationships. When the relationships are solely inclusion and exclusion (strict hierarchal), such diagrams are inferential and correspond to a "natural deductive system".[148] When the relations are solely intersectional (multiple crossed orders) they represent a non-hierarchal combinatorial system of faceted attributes which facilitate discovery and theory building.[149]

## 2.5 Extended cognition, *cognitio symbolica* and model-based reasoning

> "... the introduction of symbolic writing in the seventeenth century was a true revolution in thought patterns that instituted a powerful tool for the creation of mathematical objects, without equivalence in natural language." - Michel Serfati[150]

### 2.5.1 Extended cognition

The notion of *extended cognition* is based on the idea that "mental modelling" can be extended into the external world in the form of iconic and symbolic systems, and utilised as a *functional part* of the human reasoning process.

> "Our thinking is not confined to what goes on inside the head; rather, it comes into being through interactions with an external set of *symbolic and technical tools*. What the physical tool is to corporal labour, the 'thinking-tool' is to mental work. Diagrams and related graphical artefacts – like scripts, lists, tables, graphs, and maps [and models in general] – lay the foundation for acquiring and evaluating knowledge, a foundation which is indispensible to scientific inquiry.[151] ... The surface of the inscription not only builds a representational space for thought, but it also constructs a space of operation: within this space we can – to formulate a paradox – undertake empirical investigation with non-empirical objects."[152]

Andy Clark & David Chalmers, who formulated the original notions of "extended mind theory" and "active externalization", justify it with what they call the "parity principle": i.e. "*were it done in the head*, we would have no hesitation in recognizing it as part of the cognitive process."[153]

### 2.5.2 Symbolic systems

Georg Heinrich Ferdinand Nesselmann (1811-1881), the German philologist and historian of mathematics, was especially interested in the history of algebra. In work from the 1840's he made the distinction between three *Entwicklungsstufen* (historical "stages of development") for algebra: the *rhetorical* stage (using only natural language), the *syncopated* stage (using abbreviations or indicative signs) and the *symbolic* stage (using operative symbols).[154] Although there were problems concerning Nesselmann's chronology concerning how these phases developed historically[155], its basic tenets remain applicable for identifying the crucial difference between *symbolic* and *non-symbolic* notational systems.

First we must emphasize the distinction between an *abbreviative sign* and an *operative symbol*. A *sign* is an arbitrary "mark" that is an abbreviation or index that denotes something, i.e. it signifies or indicates some specified thing or idea. A *symbol*, on the other hand, is a *sign* that is embedded within a non-arbitrary relational system, and which derives its operational meaning as a *constituent part of a unified operational whole*. That is, symbols are defined by the syntactical rules of the *calculized operational environment* in which they take part. Such a "... system of notation ... not only represents its objects but also makes them manipulable and thus *generates* and *constitutes* them, as the unity of objects arises from the unity of the operation."[156]

Such operative symbolic systems thus provide three functions:[157]

1. A medium for *representing* cognitive phenomena,
2. A tool for *operating* with this medium in order to solve problems or to prove theories,
3. A *formal (syntactic) structure* that has an *inherent generative potential* for the development of new entities and relationships.

It is this third property which is the "killer app", as the modern-day expression goes. It concerns a "symbolism that not only abbreviates words but represents the [deep structure] workings of the combinatory operations [involved in the target]".[158] This is what Leibniz called a *calculus*.

Thus symbols lead double lives: they serve as both *semantic* and *syntactic* objects. As semantic objects they carry content or meaning, given mostly by convention. As syntactic objects they make up purely formal systems of substitutions and transformations. It is this "dialectic between the use of symbols as semantic and syntactic objects" which facilitates the inventive process:

> "When symbols are treated as syntactic objects, the meaning of the symbols is suspended, and the problem at hand is solved by manipulating the symbols following purely formal rules ... There is no reference to the meaning or content of the symbol, only to the symbolic forms and the transformations we make on them; ... The meaning of the symbols is only restored when the solution is found." [159]

Krämer calls this a process of *desemantification*, followed by defining new relationships by substitutions and transformation, and then *re-semantifying* the results.[160]

> "With the introduction of this notion of symbolic knowledge an important methodological innovation was achieved: the knowledge obtained through *symbolic manipulation*, produced in the form of calculi, has a prominent position in the whole structure of human knowledge. In this manipulation, symbols are seen as objects independently of their meaning. Moreover, symbolic systems provide proof procedures and decision methods. Proof is then understood as calculation in a symbolic system…".[161]

Serfati sums up: "After the introduction of the symbolic writing system, nothing in mathematics was anymore like before. The outcome was, strictly speaking, a (symbolic) revolution, one of the major components of the scientific revolution."[162]

### 2.5.3 *Cognitio symbolica* and three methods of *ars inveniendi*

In general, there are three ways to generate new knowledge through operative symbolic systems:

1. As above, by the *manipulation* of its symbols according to syntactic rules of substitution and transformation (the basis of a formal calculus)[163]
2. By *concept expansion*, including the introduction of new mathematical or symbolic entities into the system's formal syntax, and *discovering* how the syntactic rules apply to them (e.g. Leibniz introduction of infinitesimals into algebraic syntax)[164]
3. By *domain extension and analogy*, i.e. applying the symbolic system or its structure to new domains (other than the one it was originally intended for) thus revealing new properties of said domains (as with Maxwell's *physical analogy*)[165]

Thus symbolic systems – and scientific models supporting them (see below) – can contain hidden (sub-) structure and "latent information" about the deep structure of their targets[166]; they can "embody unseen hypothetical entities that interact to produce emergent behaviour".[167] Especially the introduction of new types of entities or operations into such formal structures can generate totally new systems of knowledge.[168] This is something that *lexicographical* (or *rhetorical*) and purely *abbreviative* (or *indexical*) sign systems cannot facilitate.

> "As a combinatorial system, mathematical symbolism permits the generation of formulas "blindly", i.e. independently of their meanings. ... which frees the mathematician from the traditional enslavement to a pre-ordained semantics... It also provides a new method of dis-

> covery, whose results are not submitted beforehand to semantic or logical "rational" constraints, and can therefore lead to innovative breakthroughs."[169]

As will be discussed in more detail in §5, Gottfried Leibniz seems to have been the first to fully appreciate the significance of this property of operative symbolic systems for extending the powers of human reasoning. He noted its facility to reveal hidden ("deep") structure:[170]

> "[It] is the highest rule of the characteristic art [i.e. the art of devising symbolic systems] that the characters [symbols] express everything which *is hidden in the designated thing*."[171] ... "No one should fear that the contemplation of characters will lead us away from the things themselves; on the contrary, it leads us into *the interior of things*."[172]

The maxim "A beloved child has many names" is certainly applicable here: Besides Leibniz's *symbolic cognition*, modern terms include *operative writing*[173], *symbolic realism*[174], *generative symbolism*[175], *surrogative reasoning*[176], *extended mind, active externalization*[177]*,* and *vehicle externalism*[178] with *non-derived content*[179] – all of which can go under the general rubric of *epistemic representation*[180]. Note that C.S. Peirce was aiming at the same general principle with what he called *theorematic reasoning*, i.e. manipulation/experimentation with purely symbolic forms in order "to discover unnoticed and hidden relations among the parts".[181] Ditto Bachelard's concept of *noumenology*, i.e. symbolic (mathematical) systems which "possess their own dynamic and may legitimately explore paths that do not seem to have [as yet] any empirical correlate."[182]

In the 1600's, the notion of epistemic representation and symbolic systems as generators of new structures, properties and knowledge was controversial, to say the least, and Leibniz's epistemological enthusiasm was initially resisted (by *both* British empiricists *and* Continental rationalists!). Today, we take such systems so thoroughly for granted that we have pretty much lost our wonder of them. For they are one of the riddles concerning "The Unreasonable Effectiveness of Mathematics in the Natural Sciences".[183]

### 2.5.4 *Cognitio symbolica* and Model-Based Reasoning

Beginning in the late 1990's, a number of monographs and anthologies were published concerning how scientific models – as symbolic structural analogues to real-world systems – are seen as epistemological tools for scientific discovery.[184] This was referred to as Model-Based Reasoning (and more recently "Model-Based Science").

Model-Based Reasoning (MBR) is basically a generalisation of the principles stated above concerning *generative symbolic systems*. Or put the other way around, generative symbolic systems (Leibniz's *cognitio symbolica*) are stellar examples of *model-based reasoning*. What the MBR initiative did was to re-emphasise and generalise this epistemological principle as an essential aspect of scientific modelling in general. Such epistemic models are "…vehicles for surrogate reasoning – reasoning from premises *in a vehicle* [the model] to conclusions about the [real world] target system."[185] I.e. the model *itself* can be treated as an *empirical object of enquiry and manipulation*, in order to draw conclusions about the real-world target. It not only renders a description of surface phenomena, but penetrates into the target's deep structure – "*leading us into the interior of things*". Note that this also includes certain types of scientific diagrams employed as "representational models".[186]

The connection between MBR and abduction is crucial. If we adhere to Peirce's general concept of abduction – "... the process of forming an explanatory hypothesis"[187], then hypothetical-analogical modelling mediates the discovery of new knowledge and new theory formation. It is a true dialectic – a *filioque* : abductive hypotheses – involving analogy and teleology – lead to new modelling concepts, and the manipulation and development of the model mediates the development of new hypotheses.

> "[Abductive] reasoning addresses modeling. Hence, the *concept about the phenomenon* is a model that develops while seeking for explanations. Thus, scientific reasoning, in terms of searching for explanations to obtain insight into a phenomenon, is related to the construction of models. The derivation of hypotheses from these models and their application in empirical investigations allows the evaluation of the phenomenon. As such, modeling is a prominent style of scientific reasoning that also is understood as a skill that needs to be practiced."[188]

* * *

On the basis of the methodological concepts outlined above, we can now trace out four historical phases of the development of conceptual modelling and combinatorial heuristics, as this has evolved in the works of Plato, Llull, Leibniz and Zwicky. We start by demonstrating how Plato's basic approach to the theory of knowledge and conceptual models supports both formal deductive and informal-heuristic reasoning; both compositional and sequential analysis and synthesis, and both taxonomic and typological knowledge structures.

* * *

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# Notes

[1] The first four paragraphs appearing here, originally intended for the present article, were temporarily hijacked (by the author) for the opening paragraphs of the 2018 Special Issue of Technological Forecasting and Social Change on "General Morphological Analysis: Modelling, Forecasting, Innovation". Reusing one's own earlier texts without informing the reader is called "self-plagiarism" – a silly concept. As G. F. Händel replied to someone who accused him of using the same musical theme in several works: "How many good ideas do you think that you get in a lifetime, sonny?"

[2] Laudan (1977). Lakatos (1968) called it a basic scientific research program. I prefer Laudan's designation, as it includes a historical-developmental dimension as well as treating the issue of progressive conceptual change as a driver of scientific development, and not only developments in empirical knowledge. Laudan's definition of a research tradition is: "… a set of general assumptions about the entities and processes in a domain of study, and about the appropriate methods to be used for investigating the problems and constructing the theories in that domain." (p.374)

[3] The expressions "modelling theory" and "modelling theoretical" used here are not synonymous with (mathematical) "model theory", although there is, of course, *some* common ground. See e.g. Ritchey (2018).

[4] There is also what is called "morphodynamic modelling" which employs continuous variables to model two or more dynamic systems which co-determine one another, e.g. concerning the dynamics of riverbeds or coastal environments. This will be treated in a later article.

[5] Lazarsfeld (1937) called this *substruction* and *recombination*.

[6] For a detailed description of this CCA process see Ritchey (2015).

[7] See Hampton (1997); Cf. Thagard (1997)

[8] *Combinatorial creativity* is "...an unusual combination of, or association between, familiar ideas. Poetic imagery, metaphor and analogy fall into this class." (Boden (1999), p, 352). Boden also identifies two other types of creativity termed *exploratory* and *transformational* creativity (ET-creativity) based on *conceptual*

*spaces* or *a matrix.* [Ibid.] However, as Pereira (2007) has pointed out,

> "If one sees the problem of combinatorial creativity as the generation of a new concept from the association of previous ones (as the definition says), one can also accept a conceptual space containing all the possible combinations. By doing so, there is no difference between the act of exploring the conceptual space of possible combinations, and the act of generating a combination (which would exist in that conceptual space)." Pereira (2007), p. 25.

[9] Fauconnier & Turner (2002),

[10] Cf. Polya *et. al*. (1990); Berge (1971); Mormann, (1997).

[11] Harris, *et. al*. (2008), p. 128.

[12] Hadwiger & Debrunner (1964), Introduction. (Emphasis added)

[13] Cf. Cipra (2006); Gärdenfors, (2004).

[14] A.k.a. *recursive hierarchical, semantically open systems*. Cf. Newman (2005).

[15] Doucet-Rosenstein (2018), p. 67.

[16] The Oxford English Dictionary, 1971, p. 259.

[17] Polya (1957), pp. 112.

[18] Polya (1968). A hypothesis is considered *plausible* if the arguments *for* it are stronger than the arguments *against* it, on the basis of experience. Strangely, while the logic of *deductive reasoning* has its roots in antiquity, and the logic of *probability* was developed out of work in the 17th and 18th century (e.g. Pascal, Leibniz, Bernoulli, Bayes), we still await a comprehensive, "generalised" logic of *possibility.* For a discussion on a possible Generalized Possibility Theory, see e.g. Dubois & Prade (2011, 2015).

[19] Deckert, (2017), Abstract.

[20] Rescher (2010), p. 121. "... fundamental generalities governing our understanding of the *modus operandi* of some knowledge-assessable domain" and "guidelines to be followed if error is to be avoided".

[21] Polya (1971), p. 124: "In brief, heuristics is a kind of tactics of problem solving. Is this enough as a first orientation? At any rate, a stricter definition is difficult, because heuristics is an interdisciplinary no man's land which could be claimed by scientists and philosophers, logicians and psychologists, educationalists and computer experts."

[22] Idid., p 172.

[23] In Phaedo (101) and Republic (510b5-511c2).

[24] For those who would like to delve more deeply into this issue I would especially recommend – among classical scholars – Sayre (1969) (Plato's Analytic Method, § 1.4) and Olsen (2002) (Plato, Proclus and Peirce: Abduction and the Foundations of the Logic of Discovery). Other relevant sources include Scolnicov (1974/2018); Menn (2002); Benson (2005); Patterson (2007) and Bonicalzi (2009) to name a few.

For modelling theorists and philosophers of science I would recommend Jetli (2016) (Abduction and Model-Based Reasoning in Plato's Republic) and Cellucci (2017) (Reconnecting Logic with Discovery). Other relevant sources include Grosholz (2007); Cellucci (2013) and Ippoliti (2018).

[25] Olsen (2002), p. 99.

[26] Khemlani et. al. (2018).

[27] Polya (1957), pp. 75-84 & pp. 225-233.

[28] Hestenes (2015), p.7.

[29] One way to define the notion of *analogy* is through the following formalism:

$a \approx b: X(a) \rightarrow X(b)$ which reads:

"**a** *is similar to* **b**, therefore if **a** has the property of X, then **b** (hypothetically) has the property of X."

Cellucci treats this as a generalization of *substitutivity of equality*, which has the same formal expression using "=" instead of a "≈". It is noted, however, that *substitutivity of equality* is a *deductive* procedure, whereas analogy – as *substitutivity of* ***similarity*** – is non-deductive hypothetical *heuristic*. See Cellucci (2013), p. 336f.

[30] Minnameier (2010), p. 107.

[31] Shelley (2003), p. 2&7. "The constraint of structural consistency … concerns the syntax of the predicates in the analogical mappings. It means that, ideally, (i) each predicate in the source is mapped to a unique predicate in the target and vice versa, and that (ii) when two predicates are mapped, their respective arguments, if any, are also mapped. These constraints are treated as *soft* constraints – that is, their satisfaction is encouraged but not absolutely required. When both criteria are completely satisfied, the analogy is a structured isomorphism." (P. 44.)

Cf. Vitti-Rodrigues & Emmeche (2021), p. 1411: "According to Minnameier, analogical reasoning involves the relationship between abduction and induction in the generation of a hypothesis by connecting the target domain – "the domain where a problem has to be solved" – and the source domain—"the domain from which the analogy is drawn". As common in abduction, the reasoning starts with a problem to be solved or an explanation or action seeking clarification. The abductive step within the analogical style of thinking lies in finding the deep structured source domain to be compared with the target structure. In this step, scientists discover a similar situation to the one seeking for explanation that can potentially solve their problem or answer their question. The inductive step draws the deep similarities from the discovered source domain to the target domain, completing the generation of an explanatory hypothesis. Once a hypothesis is generated, the researcher deductively draws the possible consequences of this hypothesis to be subject to experimental test. Finally, s/he proceeds with the inductive phase in order to confirm, deny, or see how far the hypothesis corresponds to the solution of the given problem."

[32] For a more detailed discussion, see Bokulich (2015), p. 28f.

[33] Maxwell (1890), p. 156.

[34] Riemann (1953) [*Mechanik des Ohres*], p. 341. Cf. Ritchey (1991/1996) for an extended discussion.

[35] Perlman (2004), p. 4.

[36] See Nickles (2009).

[37] See e.g. Canfield (Ed.) (1966) for good and varied introduction to the teleological issue. Cf. Simon (1996); Young (2020).

[38] An axiological term often used by Nicholas Rescher (2006).

"From its earliest days, metaphysics has been understood also to include "axiology," the evaluative and normative assessment of the things that exist. Here lies the doorway to another mode of explanation – explanation of facts in terms of values and of reality in terms of optimality. ... The approach rests on adopting what might be called an axiogenetic optimality principle to the effect that value represents a decisive advantage in regard to realization..." Rescher (2006), p. 503f.

[39] Hamid (2019), p. 275. (Emphasis added.)

[40] Schoemaker (1991), p. 205. For a fascinating historical overview, see. Hildebrandt & Tromba. (1985).

[41] Freguglia & Giaquinta (2016), p.1-2.

[42] Coppersmith (2017), p. 195.

[43] Leibniz, G. (1695/1959), p. 723.

[44] Leibniz (1682).

[45] Bachelard (1971), note 12, p. 203. (Cited in Tiles (1987), p. 156.) Wilhelm Ostwald concurred: "Farsighted educators ... have repeatedly been forced to point out a deficiency which too often attaches to the present scientific education of our younger talent. It is the absence of the historical sense and the want of familiarity with the great researches upon which the edifice of science rests." Cited in Cajori (1899), p. 278.

[46] For a more detailed review see e.g. Feest & Sturm (2011); Sturm (2011); Nickles (2017) and Vagelli (2019).

[47] Cf. Renn, (1995).

[48] Laudan (1968), p.1. (Emphasis added)

[49] Crombie (1994): *Styles of Scientific Thinking in the European Tradition*.

[50] The British Academy (1998), p. 265.

[51] Cf. "Evolutionary Epistemology" (Bradie & Harms (2020)). Laudan (1990) coined the term "Normative Naturalism", but it was more natural than normative.

[52] Hacking, (2002), p. 93. (My brackets)

[53] Simons (2017). p.1ff. (My emphasis)

[54] Cf. Feest & Sturm (2011).

[55] Rheinberger (2010), p. 2f. Cited in Sturm (2011), p. 7. (Final italics added.)

[56] "Historical epistemology ... meant a break from the Cartesian tradition which for centuries constituted the established point of reference for French philosophy. Here, historical epistemology could relate to a whole spectrum of anti-Cartesian currents, starting with ... Leibniz and Spinoza." Broady. (1991), p. 338. (My translation). In this context, Leibniz has been regarded as a precursor to Bachelard in his critique of Descartes:

> "In einer Buchbesprechung aus dem Jahr 1960 beschreibt Deguy das Verhältnis von Leibniz und Descartes in dem Sinne, daß Leibniz der « erste Bachelard » der cartesianischen Philosophie gewesen sei …". ["In a book review from 1960, Deguy describes the relationship between Leibniz and Descartes in the sense of Leibniz being the "first Bachelard" of Cartesian philosophy..."] Schmidgen, H. (2012), p. 29.

[57] These include *Essais de Théodicée* (pub. 1710), *Discourse de metaphysique* (pub. 1846) and the enormously influential *Nouveaux essais sur l'entendement humain* (pub. 1765) and *Monadologie* (pub. 1721 [Latin] ; 1840 [French]).

[58] For Leibniz, such "continual progression" concerned not only the epistemological-methodological realm but also the epistemological-ontological realm: "… because of the infinite divisibility of the continuum, there always remain in the depths of things parts which must yet be awakened and become greater and better, and, in a word, attain a better culture. And hence progress never comes to an end. (Leibniz (1697/1969), p. 491.)

[59] "... Poincaré was quite familiar with Leibniz's work. In 1880 the French edition of Leibniz's Monadology, prepared by Emile Boutroux, included a supplementary note at the end by Poincaré comparing Descartes' and Leibniz's conceptions of dynamics. He also collaborated in preparing the international edition of the works of Leibniz." (Heinzmann & Stump, 2017)

Couturat was a Leibniz scholar and one of the first to have access to the huge Leibnizian *Nachlass* in Hanover at end of the 19th Century. His publications of *La Logique de Leibniz* (1901) and *Opuscules et Fragments Inedits de Leibniz* (1903) were a major contribution to the Leibnizian revival in the early 20th Century. Furthermore, Couturat is clearly a precursor to a normative epistemological history:

> "His [Couturat's studies of] Leibniz could not be placed on the same footing as the studies produced by the Kantian school in France at the time. Its place lay in [Couturat's notion of] "perpetual parallelism" between the sciences and philosophy. Through his publication on Leibniz, Couturat was acting as an advocate and defender of modern ideas more than accomplishing the work of a historian. In a way, Leibniz enabled Couturat to turn against the Kantian thinking dominating French philosophy."

(Schmid (2012), p. 79.)

Levy-Bruhl, trained in philosophy and working in the field anthropology and ethnology, wrote a book on Leibniz and the German Aufklärung: "Germany from the time of Leibniz: An Essay on the Development of National Consciousness in Germany, 1700-1848". He was instrumental in having Bertrand Russell's "A Critical Exposition of the Philosophy of Leibniz" (1900) translated and published in French (La philosophie de Leibniz: exposé critique, 1908) and wrote the *Forward* to it.

Léon Brunschvicg was professor of philosophy at the Sorbonne where, incidentally, he was the supervisor for Simone de Beauvoir's master's thesis on the "Concepts according to Leibniz" (Cf. Simons, 1999, p. 189f.). In *Les Étapes de la philosophie mathématique*, he is especially interested in the reciprocal relationship between Leibniz's mathematics and metaphysics, and the philosophical implications of infinitesimal analysis as "a new stage in mathematical philosophy". (Cf. Loi, (1984).

[60] Bachelard was director of the Institut d'histoire des sciences at the Sorbonne from 1940-1955. Canguilhem succeeded him from 1955-1971. Who it was that first came up with this term *Épistémologie historique* seems to be a matter of contention. See e.g. Lobo-Guerrero (2016).

[61] See Gingras (2010); cf. Tiles (1987).

[62] Vagelli (2019), p. 106.

[63] Renn & Gutfreund (2020) P. 115. (My emphasis).

[64] Carrier (2012), p. 239.

[65] Fabry (2021), p. 5f. (Emphasis added)

[66] Hyder, 2003, p. 4)

[67] Cavaillès (1947/2021), p. 136. (Brackets added.)

[68] Turri (2011) p. 61f.

[69] Loraux's (2005); Vagelli (2019), p. 105.

[70] Atack (2016). Cf. Rood *et. al.* (2020).

[71] Gambarotto (2019), pp. 150 & 162.

[72] Schrödinger (1996) p. 18f.

[73] Cf. Renn (1995), p. 2: "universal norms of scientific rationality"; Tiles (1987), p. 152: "...the quest for an epistemological theory ... which will serve as a universal touchstone, applicable to any discipline at any period of time".

[74] Chimisso (2015), p. 6.

[75] Canguilhem (1983 [1968]), p. 184. Cited in Elden (2019), p. 120.

[76] Cf. Johnson-Laird (2010); Lakoff (1987). p. 7 states: "In this century [20th], *reason* has been understood by many philosophers, psychologists, and others as roughly fitting the model of formal deductive logic: Reason is the mechanical manipulation of abstract symbols which are meaningless in themselves, but can be given meaning by virtue of their capacity to refer to things either in the actual world or in its possible states."

[77] See e.g. Montgomery (1988); Seel (2012); Ubben & Heusler (2021).

[78] Craik (1943). Cf. Seel (2017).

[79] Topology, as a general theory of space – or general theory of spatial structures – is considered to be "more fundamental than logic as understood traditionally as a purely symbolic discipline quite unrelated to any spatial considerations" Cf. Mormann (2020).

[80] Hestenes (2015), p. 1.

[81] "Mental models are representational structures, which allow for inferences about complex objects even when only incomplete knowledge is available. Such representational structures are assumed to have a certain persistence, be shared by specific groups and located in specific (practical or theoretical) contexts. They can be extended when confronted with new areas of application, which in turn may lead to a reorganization of the very system of knowledge in question." Feest & Sturm (2011), p.10

See e.g. Johnson-Laird (2004, 2010); Johnson-Laird & Byrne (1991); Thagard (2010). See also Gärdenfors (2000). Note that we are not equating "mental models" and the commonly used notion of "conceptual models". E.g. Hestenes (2006), p. 10f, makes a crucial distinction between these two concepts. "Mental models are private constructions in the mind of an individual. They can be elevated to conceptual models by encoding model structure in symbols that activate the individual's mental model and corresponding mental models in other minds." See also Booth (2011) on the notion of "conceptual frame" as an "unconscious organizing principle and basic building block of thought." p. 58.

For "**topological** structure" see e.g. Mormann (2020)."… topology as a general theory of spatial structures may serve as a kind of conceptual toolkit at least as versatile and fruitful as traditional logic." (p. 28)

For "**typological** structure" see e.g. McKinney (1969). "… perceiving the world and structuring it by means of types and typologies, is ... an essential and intrinsic aspect of the basic orientation of actors to their situations. ... types and typologies are ubiquitous, both in everyday social life and in the language of the social sciences." [Abstract]

This even concerns symbolic notation systems. Cf. Landy & Goldstone (2007): "... some of the same cognitive resources involved in representing spatial relations and proximities are also involved in representing symbolic notations - in short, that formal notations are a kind of diagram." (p. 2033).

[82] Cf. Newell & Simon (1972); Simon & Newell (1971). For a historical background see Ohlsson (2012).

[83] Booth, 2011, p. 51. Cf Van Dyck & Heeffer (2014) p. 2: "Research from experimental psychology suggests that symbolism in mathematics acts in the same way as spatial representation schemes."

[84] Arthur Koestler, in his book *The Act of Creation* (1964 – duly cited by Turner & Fauconnier) coined the term *bisociation* to denote this "blending of concepts", drawn from two unrelated contexts or patterns of thought, into a new pattern.

[85] Cf. Conant & Trabasso (1964).

[86] See e.g. Lewis & Lawry (2016); Lieto & Pozzato (2018).

[87] Fauconnier & Turner (2002), p. 18.

[88] Ibid, p. 44.

[89] Fauconnier (2015) "The Encyclopedia of the Social and Behavioral Sciences" (u/Conceptual Blending). (Emphasis added.)

[90] Fauconnier and Turner (1996), p 113. (Emphasis added.)

[91] Adapted from Goguen & Harrell (2010), p. 158. This is the standard format of Fauconnier & Turner (1998, 2002).

[92] Miller (2000) p. 132ff & p 141.

[93] Cf. Sarma, G. (2015) for a more detailed discussion.

[94] The definition of "proof" (Gk. *apódeixis*; Latin *demonstrare*) in mathematics and logic has remained essentially the same from antiquity to the present. Cf. Rodin (2012) p. 9: "In today's logic the word 'proof' stands for a logical inference of *certain conclusion* from some *given premises*. In fact this is what by and large was meant by proof also by Aristotle and Proclus." [Italics added]

[95] *First intention* concepts target properties of or relations between "real" things (*in re*). Second intentions target properties of or relations between first intentions (*in intellectu*). Thus "formal logic" can be defined as the science of *second intentions* applied to *first intentions*.

[96] Note that pre-Euclidian geometry was not expressed as a fully articulated *axiomatic system*. However, it did represent a proper *deductive system* if we define this as "... the presentation of propositions as true through the ordered sequence of other propositions, in which each passage from two or more propositions to another (Le., their "conclusion") is effected according to set rules (Le., "laws of logic")..." [or rules of valid inference]. Knorr (1980), p. 147. It has even been questioned whether Euclid's Elements actually developed geometry "axiomatically". See e.g. Seidenberg (1975).

[97] Cf. Barney**,** 2012, p. 42. "[Plato is]… distinguishing "upwards" and "downwards" lines of argument, and insisting on the importance of the distinction. Upwards reasoning to a hypothesis will be a matter of finding non-deductive reasons to adopt some principle as "strongest" (Phae 100a4) or "sufficient" (Phae 101d8): presumably this is a matter of both explanatory power and inherent plausibility. ... Downwards reasoning from a hypothesis then takes the form of deducing its consequences and testing them for coherence, presumably by taking the hypothesis in conjunction with plausible auxiliary assumptions."

[98] Cf. Kisiel, 1979, p. 406f for a detailed discussion.

[99] Cf. Ferejohn (1991), p. 17: "Aristotelian science seems to be depicted ... as a sort of proto-Euclidian axiomatic system that starts from a relatively small set of starting-points or assumptions, and then proceeds by means of purely deductive (that is to say syllogistic) inference-chains to "prove" all of the explicanda pertinent to that science.

[100] See e.g. Salmieri et. al. (2014).

[101] Höffe (2003) p. 49. (Emphasis added.) Cf. Ferejohn (1991), pp. 16ff.

[102] Salmieri, et. al. (2014), p. 4

[103] Cf. de Jong & Betti (2010); Goldin (2013). For a discussion of how the "Aristotelian theory of science" and formal "synthetic" methods hindered the development of heuristically useful methods already in antique times, see Knorr (1980):

> "By the third century both philosophers and geometers had resolved the larger issues on the systematization of knowledge, as, respectively, the Aristotelian theory of science and the Euclidean compilation of geometry. The subsequent history of mathematics indicates that the success of this axiomatizing effort eventually served to discourage the creative forms of research which could have advanced mathematical knowledge." (p. 177f.)
>
> Cf. Helmig (2012): "... it is important to note that Aristotle's account of knowledge acquisition in *Met*. A 1 and *An*. *Post*. II 19 also provided a model for the Stoic doctrine of knowledge attainment. His comparison of the soul with an empty writing tablet (*grammateion*) in *De anima* III 4 corresponds to the Stoic view that the soul at birth is a *tabula rasa* (SVF II 83). Moreover, as with Aristotle, sense impressions are accumulating until we reach, via memory and experience, general concepts (SVF II 83). Finally, as with Aristotle, the Stoic theory of concept attainment has been dubbed empiricist." (p. 128).

[104] Topics, 100a25-b20 & De Sophisticis Elenchis, 2. Cf. Rescher (2007), p. 130ff.

[105] Höffe (2003), p. 36.

[106] For further discussion see Lloyd (1990)

[107] Cf. Kisiel (1980); Mckeon (1973).

[108] Laudan (1980). Cf. Mantzavinos (2016):

> "The development of classical methodology within the framework of classic rationalism is characterized by the fact that the ideal of certain scientific knowledge was tied to the search for a *rational heuristic*, an *ars inveniendi*, which was supposed to complement the *ars judicandi*, the rational art of proof and justification ....

This development went hand in hand with the exclusion of the *heuristic* altogether, since a new consensus emerged that no algorithmic form for generating new certain knowledge was possible. *Heuristic* has been excluded from methodology and has been largely assigned to the domain of the irrational." (p 173f)

[109] Laudan (1981) p. 183.

[110] Reichenbach (1938). Otte & de Barros (2016), p. 164. "The claim that there is a distinction between discovery and justification, together with the claim that *only the latter is the legitimate province of the philosophy of science*, was one of the cardinal principles of the Vienna Circle." (Emphasis added).

[111] Note that Karl Popper and Thomas Kuhn also promoted this position on slightly different grounds. Cf. Nickles (2008), p. 505: "For the logical empiricists and the Popperians, context of discovery, epistemically speaking, was something external to philosophy of science. Thus, ironically, their view cut itself off from the sources of innovation, the very thing that is supposed to drive inquiry. ... those philosophers who simply ceded [the] context of discovery to historiography, psychology, and sociology threw out the baby with the bathwater". (Nickles (2008), p. 505.

[112] Reichenbach (1949) p. 292.

[113] Wartofsky (1980), p. 1.

[114] Three of these are Lakatos & Musgrave (1970), Nickles (1978) and Nickles (1980). Already in his Cambridge Ph.D. (1961 – which was the basis for "Proofs and Refutations – The logic of mathematical discovery") Lakatos he distinguishes between "deductivist" and "heuristic" styles in mathematics:

"[The] deductivist style tears the proof-generated definitions of their 'proof-ancestors', presents them out of the blue, in an artificial and authoritarian way. It hides the global counter-examples which led to their discovery. Heuristic style on the contrary highlights these factors. It emphasizes the problem situation: it emphasizes the [non deductive] 'logic' which gave birth to the new concept." (Lakatos (1976), p. 153)

[115] Nickles, (2019), p. 171. (Emphasis added.)

[116] For a review of the contemporary debate see e.g. Hoyningen-Huene (2006).

[117] Pappus*: Treasury of analysis*, Book VII, cited in Heath (2013), p. 400 (Emphasis added). It is well known that the classical Greek geometers were great on demonstrating the synthetic method, but far less so in revealing their use of the analytic counterpart, to the point that one suspects that the "axioms" of geometry may simply have been assumed *per se*. Indeed, Plato was openly critical of the "the geometers" who uncritically accepted their initial assumptions.

[118] Antognazza (2018), p. 178f. Leibniz writes: "We have the synthesis when, moving on from the principles and examining in an orderly way the truths, we grasp some progressions and then we construct tables or even, sometimes, general formulae in which, later on, we may find the answers to what is required. Analysis, instead, traces back to the principles, from which the individual problem that has been proposed originates." (A VI iv 544)

[119] By its very nature, *sequential* synthesis & analysis is typically visualised as a horizontal "forwards" and "backwards" process, whereas *compositional* synthesis & analysis is visualised as a "downward" and "upward" process respectively.

[120] Pasini (1997), p. 35.

[121] During the past 2-3 decades there has been an spirited debate on the distinction between "Abduction" and "Inference to Best Explanation (IBE), i.e. whether abduction consists solely of the "hypothesis formation" phase, or also includes the "evaluation" phase, and other such issues, sometimes verging on hair-splitting and purely semantic divergences. See e.g. Park (2015) for an extended discussion.

[122] Magnani (2004, 2009).

[123] This is not to be confused with what Magnani (2004, 2009) calls *selective abduction*.

[124] "Peirce wished to show that reasoning *towards* a hypothesis is of a different kind than reasoning *from* a hy-

pothesis." (Fann 1970, p.4). But Peirce's theory of abduction is seen as being "... concerned with the reasoning which starts from data and moves towards hypotheses" (Ibid. p. 5). Plato's "higher" abduction, on the other hand, concerns starting from a hypothesis and moving towards a higher (or meta-) hypothesis (or higher principle) which can account for (i.e. justify) the initial hypothesis. (See §2.)

[125] There is an ongoing debate as to whether Peirce's abduction concerns only the hypothesis generation phase or whether it also includes the evaluation-consequence phase. He seems to have inferred both at different times.

[126] Note that this has nothing to do with *induction* as empirical generalization. As Carl Hempel notes:

"[There are]... no generally applicable 'rules of induction' by which hypotheses or theories can be mechanically derived or inferred from empirical data. The transition from data to theory requires creative imagination. Scientific hypotheses and theories are not *derived from observed facts*, but *invented in order to account for them*".... "And any 'rules of induction' will have to be conceived, in analogy with the rules of deduction, as canons of validation [i.e. *demonstrandi*] rather than of discovery." (Hempel, 1966, p. 205).

[127] Pietarinen (2018) p.1122

[128] Pape (1997), p. 202

[129] Serres (2017), pp. xxiiif.

[130] Khemlani et. al. (2018).

[131] Grosholz, (2007), p. 25.

[132] Bacon (1605/2014), Book II, xiii: 6.

[133] "[Real] Definitions are not arbitrary stipulations but rather hypotheses and, as all hypotheses, are means of discovery. Cellucci (2002), Ch. 36, cited in Damiani et. al. (2009), p 219.

[134] Ippoliti (2018), p. 13. (concerning Poincaré (1908).

[135] Ibid.

[136] Verboon (2014), p. 95.

[137] Greene's work has been reported – and expanded – by Brumbaugh (1961, 1965). Although Plato's verbal accounts are full of spatial metaphors, we have no certain knowledge of whether or not any actual diagrams were included in the original Platonic manuscripts – for the simple reason that no "original" copies of Plato's (or Aristotle's) exist today. The oldest Plato-manuscripts that presently exist date from 9th Century Byzantine sources. In many of these sources, commentators have either copied earlier examples and/or added their own interpretive logical and heuristic diagrams to the Platonic texts, in order to graphically visualise the classificatory structures of Plato's verbal accounts.

[138] Fig. 2.3a: On Augustine's "On Christian Doctrine" (1212) (British Library). Fig. 2.3b: From a 10th century manuscript of Boëthius' *De institutione arithmetica* (Einsiedeln, Stiftsbibliothek). From Marchese (2013).

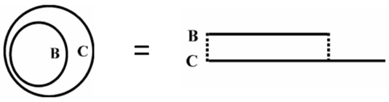
[139] Johansen (2014), p. 103.

[140] Kwasnik (1992), p. 63

[141] The principle involved in faceted classification is that an object is assigned (or defined by) multiple intersecting (or coordinate) categories which are orthogonal, symmetric and non-hierarchal. These are used as "tags" by which the object can be identified in a search. Interestingly, this is also called an *analytico-synthetic* scheme, which highlights the fact that this is a typological/morphological *modelling space*. Although faceted classification can also employ taxonomic structure *within* one or more of its "facets", its primary, "global" structure is typological. Cf. Levy (2011); Frické (2011). For a history of combinatory classification see Glymour, et. al. (1995); Schulte-Albert (1974, 1979).

[142] Note that the use of "Euler diagrams" did not start with Euler, who did not claim first use. First of all, such diagrams are implicate in Plato's spatial metaphors concerning class inclusion and concept blending (§3);

Ramon Llull is credited as being one of the earliest to explicitly use such diagrams in his texts (§4); and Leibniz made the first systematic study of them, showing how one could *inter alia* represent all valid syllogisms with them. He also invented an equivalent system base on line diagrams. Cf. Bennett (2015), p. 108.



[143] *Class disjunction* is what is called *differentia* in *species/differentia* schemes of syllogism. "... the power of Euler diagrams and semantic networks, both representing only two types of logical relation, *entailment* and *contrast/opposition*, has been demonstrated through examples of the representation of significant conceptual structures such as taxonomies..." Gaines (2010), p. 30f.

[144] Pavlinov (2011), Abstract. Cf. Doyle, & Patil (1991), p. 261f : "Taxonomic reasoning …is essentially equivalent to *logical entailment*, and so capable of supporting most forms of *logical inference*." (Emphasis added)

[145] McKinney (1969), Abstract. "… perceiving the world and structuring it by means of types and typologies, is ... an essential and intrinsic aspect of the basic orientation of actors to their situations. ... types and typologies are ubiquitous, both in everyday social life and in the language of the social sciences."

[146] As a set theoretical operation this is called a *cross product* or *cross join*; in mathematics a *Cartesian product*. For *concept* analysis and synthesis, Leibniz called this "real addition" and gave it the conjunctive operative symbol ⊕. See Swoyer (1994) for a detailed account.

[147] Kwasnik (1999), p. 80. (My brackets and emphasis.) "... faceted schemes are sometimes also called *synthetic* (or, to emphasize that the task of synthesis must be preceded by analysis of the relevant properties of the object, *analytico-synthetic)* schemes." Sperberg-McQueen (2004), p. 164f.

[148] Cf. Mineshima et. al. (2010) (Abstract) : "... a relation-based approach, where a diagram is defined in terms of topological relations (inclusion and exclusion) between circles and points.... a diagram construed as a set of topological relations corresponds to an implicational formula and the inference system based on such diagrams corresponds to a natural deduction system."

[149] Cf. Doty & Glick (1994).

[150] Serfati (2010), p. 103.

[151] Krämer (2016a), p. 163.

[152] Ibid. p.164. Cf. Vold & Schlimm (2019), p. 2: "... environmental states and processes can be seamlessly integrated with the functions of our neural states and processes, making the two—external and internal states and processes—equally essential for some aspects of cognitive life. On this view, cognitive processes are not merely 'scaffolded' by tools and structures in the environment – they are partially *constituted by*, rather than merely *causally dependent* on, external structures. ... Put more precisely, the information-bearing structures within the tool [e.g. a symbolic system – tpr] are the vehicles of genuine mental representations, as are the vehicles of neurally instantiated mental representations."

[153] Clark & Chalmers (1998), p. 8. Cited in Vold & Schlimm (2019), p. 4. The authors have also offered a softer expression of the Parity Principle as simply the reciprocal nature of internal and external elements of cognitive systems.

[154] Nesselmann (1842).

[155] For a detailed critique of Nesselmann's tripartite classification see Heeffer (2009, 2010).

[156] Krämer (2010), p. 20. (Author's emphasis). Cf. Valencia (2004).

[157] Cf. Krämer (2003), p. 522; Burgin, (2007)

[158] Mahoney (1980), p. 142. Cf. Burgin (2007), p.2: "Calculi combine languages, procedures and algorithms to provide means for derivation and generation of new entities from existing ones." Cf. da Silva (2010) for an

extended discussion.

[159] Johansen (2014), p. 91.

[160] Krämer (1993); see also Novaes (2011).

[161] Legris (2010), p. 3. Cf. Heeffer, (2008). Numerous historians of science, philosophy and mathematics have commented on this capacity:

[Serfati] "… symbolic writing indeed makes it possible to invent objects and concepts, mathematical and scientific." Serfati (2020), p. 359. For a deep dive into this issue see Serfati (2005), Part 2 on "Symbolic and Invention", pp. 249–406.

[Portides] "… successful representational models should be considered true enough not just because their predictions approximate the values of experimental measurements or because they bear similarity relations with what they represent, but also because they produce new knowledge about their target." Portides, (2014), p. 78.

[Dascal] "As a combinatorial system, mathematical symbolism permits the generation of formulas "blindly", i.e., independently of their meanings. This practice, which frees the mathematician from the traditional enslavement to a pre-ordained semantics, contributes powerfully to the modern idea of formalization. It also provides a new method of discovery, whose results are not submitted beforehand to semantic or logical "rational" constraints, and can therefore lead to innovative breakthroughs." Dascal (2008), p.4.

[Henley] "The most interesting mathematical theorems or concepts are often conjectured not by semantic modifications to existing concepts but from surprising syntactic patterns or analogies made by their formulae during the normal course of mathematics. This can lead to major discoveries going beyond the current semantics and would also give mathematical intuition a basis in proof since proof is itself syntactic." Henley (1995), p. 242.

[Eco] "In semiotic terms, what we have is a system of expression (made up of symbols and syntactic rules) such that, by associating the symbols with a content, various "states of things" (or of ideas) can be imagined. In order for the combinatory system to be most effective, however, it must be assumed that there are no restrictions on thinking all possible universes." Eco (2014), p. 387.

[Polanyi] "The process of reorganizing a conception for drawing new inferences from it can be formalized, by accepting as inferential operations certain rules for manipulating the symbols representing the states of affairs." [Polanyi 1962, p. 117] "... a mathematical formalism may be operated in ever new, uncovenanted ways, and force on our hesitant minds the expression of a novel conception." [Ibid., p. 114] (Cited in Gelfert (2011), p. 32.)

[162] Serfati (2010), p. 120.

[163] Magnani (2004) calls this "manipulative abduction" which is "... a kind of abduction, usually model-based, that exploits external models endowed with delegated (and often implicit) cognitive roles and attributes". (p. 242). … "The whole activity of manipulation is devoted to build various external epistemic mediators that function as an enormous new source of information and knowledge. Therefore, manipulative abduction represents a kind of redistribution of the epistemic and cognitive effort to manage objects and information that cannot be immediately represented..." Note that manipulative abduction not only emphasizes discovery via the manipulation of symbolic systems but also via the manipulation of material/technical and social systems (Ibid, p. 233). This was something that Leibniz stressed as well, given his lebensmotto theoria cum praxi, and the fact that he promoted not only symbolic logic but also its mechanical counterpart: designed and built the first calculating machine that performed all four arithmetic operations.

[164] Legris (2010), p. 3: "… symbolic knowledge is also related in a critical sense with *concept formation***.** One of the more important epistemological features of symbolic knowledge, already present in Leibniz, consists in introducing calculi with symbols without an intended reference for 'imaginable' objects, that cannot be grasped by sensible intuition. Although the reference of such symbols can be understood as 'fictitious' entities, they play an essential epistemological role in calculi: It is by means of them that an authentic new knowledge is obtained. A 'constitutive' aspect of symbolic knowledge has thus been asserted (s. Krämer 1992). This would be the case of Leibniz's infinitesimal calculus and differential equations, where symbols are in-

troduced without any denotative function, but in order to solve mathematical problems through a calculus. An example is the notion of infinitesimal as a '*fiction bien fondée*' (well founded fiction)".

Note that this seems to bridge the perennial question of to what extent mathematical concepts are *creations* of the human mind or, alternatively, "real" entities which are *discovered*. Leibniz *created the notion* of an infinitesimal (d*x*), and then *discovered its consequences* within the algebraic symbolic system.

[165] For further discussion of 1 and 3, see Lagris (2010).

[166] "...an isomorphic relation between a substructure of the conceptual model in question and some empirical conceptualisation (model) of relevant experimental data." (Ruttkamp (1998) p. 197.)

[167] Lehrer & Scauble (2000), p. 43. Cf. Steiner (2000) for "latent information".

[168] A stellar example is Leibniz's invention and introduction of the *infinitesimal* (d*x*), and the differential operator d/d*x*, into algebraic structure. "[Leibniz's] infinitesimal calculus is the supreme example, in all of science and mathematics, of a system of notation and terminology so perfectly mated with its subject as to faithfully mirror the basic logical operations and processes of that subject." Edwards (1979), p. 232.

[169] Dascal (2008), p. 4.

[170] For a deep dive into this issue, see Pombo (1987). Cf. Esquisable, (2010), p 2: "… although Viéte and Descartes introduced a revolutionary change in the mathematical practice of the time by means of the new methods of symbolic representation, it was for them mainly a methodological device for renewing and simplifying the resolution of mathematical problems. It was Leibniz who extracted the philosophical consequences of the new style of "symbolic language" and tried to show its philosophical significance. Such theoretical efforts, that go from semiotic and epistemological reflections up to the design of specific symbolic systems (calculi) for mathematics (for example, infinitesimal calculus and the diverse essays on geometric characteristics and *analysis situs*), and logic (essays on logical calculus), are centered on the concept of symbolic knowledge or 'blind thought', with which Leibniz tried to emphasize the function and value that semiotic systems in general have for human knowledge."

[171] Leibniz (1678) *Specimen analyseos novæ*, cited in Knobloch (2010), p. 291. (Emphasis added)

[172] Letter to Tschirnhaus, in Leibniz (1969), p. 193f. (Emphasis and brackets added)

[173] Krämer (2003, 2016)

[174] Scholz (2005, 2012).

[175] Kanamori (2009) p. 474.

[176] Swoyer (1991).

[177] Clark and Chalmers (1998).

[178] Hurley (1998); Menary, (2010).

[179] Vold & Schlimm (2020).

[180] From Contessa (2007). Bolinska (2013): "A vehicle is an *epistemic representation* of a given target system if and only if it is a tool for gaining information about some aspect of this system for its user(s)." Cf. Krämer (2016a) on "Epistemic diagrams".

[181] Cited in Stjernfelt (2011), p. 309. Stjernfelt points out the experimental and inventive character of *theorematical reasoning* such that we can make the distinction between Peirce's *corollarial* (apodictic) and *theorematic* reasoning as that between *deductive logic* and *heuristics*. Indeed, Peirce sees *theorematic* reasoning as the logico-mathematical counterpart to retroductive (i.e. abductive, i.e. hypothesis formulation) reasoning in philosophy and science (c.f. Pietarinen & Bellucci, 2014).

Cf. Eisele (1982), p. 337. "Some iconic or symbolic form is adopted to represent the given premises. If the assumed conclusion cannot be drawn directly from the iconic diagram or symbolic equation, a modification of the *form* suggests itself and is tried. Further modifications of the modification may be needed to reach the desired end. This process leads to new relations between parts not originally mentioned, but now helpful in nearing the conclusion." Eisele goes on to say that Peirce admits of no way to "logically" account for how one finds or chooses successful "modifications" or "transformations"; i.e. that the process is abductive."

Cf. Ketner (1985), p. 409. "Here, then, is the true importance of Peirce's corollarial/theorematic reasoning distinction — it makes significant contributions toward showing that mathematics and logic are observational, experimental, hypothesis-confirming sciences, in which one makes hypotheses about, and observes and experiments upon [manipulates], diagrams [i.e. symbolic systems] according to a distinctive [syntactic-based] method." Ketner (1985), p. 409.

[182] Fabry (2021), p. 7.

[183] Wigner (1960).

[184] See e.g. Magnani, et. al. (1999); Magnani, et. al. (2002); Magnani, (2014); Magnani, et. al. (2016).

[185] Bolinska (2013), p. 119. (Emphasis added)

[186] Cf. Stegmann (2021): "Some diagrams can also be employed as "representational models" ... i.e. as systems that aim to represent the world and, in addition, enable scientific investigations to be carried out on the model, rather than on reality itself. ... As representational models, diagrams are manipulated in lieu of their representational targets." (p. 2675.)

[187] The whole argument is: "Abduction is the process of forming an explanatory hypothesis. It is only logical operation which introduces any new idea; for induction does nothing but determine a value, and deduction merely evolves the necessary consequences of a pure hypothesis. Deduction proves that something **must** be; Induction shows that something **actually is** operative; Abduction merely suggests that something **may be.** Its only justification is that from its suggestion deduction can draw a prediction which can be tested by induction, and that, if we are ever to learn anything or to understand phenomena at all, it must be by abduction that this is to be brought about. No reason whatsoever can be given for it, as far as I can discover; and it needs no reason, since it merely offers suggestions. A man must be downright crazy to deny that science has made many true discoveries. But every single item of scientific theory which stands established today has been due to Abduction." (Peirce (1931-35) 5.171-2.

[188] Upmeier zu Belzen et. al. (2021), p. 1.