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Examining the Effectiveness of Explicit, Systematic Mathematics Vocabulary Instruction for Students with Learning Difficulties and Disabilities in a Specialized Setting

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EXAMINING THE EFFECTIVENESS OF EXPLICIT, SYSTEMATIC

MATHEMATICS VOCABULARY INSTRUCTION FOR STUDENTS

WITH LEARNING DIFFICULTIES AND DISABILITIES

IN A SPECIALIZED SETTING

by

Kristen R. Rolf, M. Ed.

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ABSTRACT

Investigating the Effectivness of Explicit, Systematic Mathematics Vocabulary Instruction for Students with Learning Difficulties and Disabilities in a Specialized Setting

by

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Utah State University, 2022

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Many students in the United States are not proficient in mathematics. Researchers have called for focusing on the language of mathematics as a way to bolster students' mathematics achievement. Mathematics vocabulary is one area that may impact students' understanding of and engagement with mathematics. This dissertation investigated the implementation, effectiveness, and social validity of an explicit, systematic, manualized mathematics vocabulary intervention for teaching mathematics vocabulary necessary for fourth grade and beyond.

This study randomly assigned 30 students (11-14 years old) to treatment and control conditions. Three teachers at a private school for students with learning difficulties and disabilities located in a unban center in the Pacific Northwest administered standardized mathematics vocabulary and mathematics achievement measures to their students as pre-tests and taught mini-lessons to students assigned to the treatment condition. Teachers administered the standardized mathematics vocabulary measure as a post-test. A research assistant and I observed each teacher six times using a researcher-created fidelity checklist. The teachers shared their perceptions of the intervention via a social validity survey.

I analyzed the observation data using descriptive statistics. Overall, results show that the teachers implemented the lessons as intended. I analyzed the assessment data using descriptive statistics, *t*-tests, and correlations. Results show students assigned to the treatment condition scored significantly higher on the post-test than students assigned to the control condition ($p < .001$). The effect size ($g = 1.99$) indicates that the intervention had a strong effect. Additionally, results show that mathematics achievement and teacher did not moderate the effectiveness of the intervention. I analyzed the social validity data using descriptive statistics and thematics analysis. Results suggest the teachers found the intervention acceptable, easy to use, and plan to use it or something similar to teach mathematics vocabulary in the future.

This study provides evidence that an explicit, systematic program for teaching mathematics vocabulary is feasible, effective, and acceptable to teachers. Future research could investigate the implementation, effectiveness, and social validity of the intervention with other groups of students in other settings (e.g. 4th grade general education classroom).

(307 pages)

PUBLIC ABSTRACT

Investigating the Effectivness of Explicit, Systematic Mathematics Vocabulary Instruction for Students with Learning Difficulties and Disabilities in a Specialized Setting

Kristen R. Rolf, M. Ed.

Many students in the United States are not proficient in mathematics. Mathematics vocabulary is one area that may impact students' understanding of and engagement with mathematics. This dissertation investigated the implementation, effectiveness, and teacher's perceptions of a program for teaching mathematics vocabulary necessary for fourth grade and beyond.

This study randomly assigned 30 students (11-14 years old) to receive mathematics vocabulary instruction or not. Three teachers at a school for students with learning difficulties and disabilities administered mathematics vocabulary and mathematics achievement tests to all of their students before teaching the program to 17 of the students. A research assistant and I observed the teachers, and all of the teachers shared their perceptions of the lessons via a survey.

Results show that the teachers taught the lessons as intended and that the students who received the lessons did better on the post-test than students who did not receive the lessons. Results from the survey suggest the teachers found the intervention acceptable, easy to use, and plan to use it or something similar to teach mathematics vocabulary in the future.

DEDICATION

To all the students looking for the right words

and all the teachers guiding them along the way

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Chapter I Introduction

The *National Assessment of Educational Progress (NAEP)* shows that many students in the United States are not proficient in mathematics. Results from the 2019 administration of the *NAEP* show that overall mathematics achievement of students in fourth grade has largely remained unchanged for the past ten years. Although scores have fluctuated slightly over the past decade, the current results reflect only a one-point increase compared to 2009. Additionally, only 41% of the students who took the exam in 2019 met or exceeded the proficiency requirements. This is a slight increase from 2009 when only 39% of the students achieved proficiency (National Center for Education Statistics [NCES], 2019a).

Results are equally discouraging for fourth-grade students identified with disabilities. Only 17% of students identified with disabilities in fourth grade scored at or above the proficient benchmark in 2019. In contrast, 45% of students without disabilities in fourth grade met or exceeded the standard for proficiency. Similar to the overall results for fourth-grade mathematics, the results of students with disabilities has remained relatively stable over the last decade. In 2009, 19% of students with disabilities in fourth-grade met or exceeded the proficiency benchmark (NCES, 2019b).

Notably, the 2019 *NAEP* also asked fourth-grade teachers employed in public schools across the nation about resources for teaching mathematics. Fifty-three percent of the surveyed teachers reported that a lack of adequate instructional materials was problematic. Of this 53%, 32% of the teachers reported that inadequate instructional materials were a “minor problem,” 15% of the teachers reported that they were a “moderate problem,” and 6% of the teachers reported that a lack of adequate instructional materials was a “severe problem” (NCES, 2019a). In other words, approximately one-fifth of the fourth-grade teachers across the nation reported that they do not have access to the instructional materials necessary to adequately teach mathematics. This lack of adequate instructional materials may partially explain the pattern of *NAEP* mathematics results seen over the past decade. Logically, teachers who do not have access to adequate instructional materials are unlikely to be able to deliver the type of mathematics instruction called for by leading mathematics education organizations (e.g. The National Council of Teachers of Mathematics [NCTM]) and measured by assessments like *NAEP*.

Calls for Focus on Language in Mathematics

The last two decades have seen increasing attention paid to the language of mathematics and its influence on mathematics understanding and achievement. In 2000, The National Council of Teachers of Mathematics (NCTM) highlighted the importance of mathematical language in *Principles and Standards for School Mathematics*. In addition to describing six principles for quality mathematics instruction (i.e., equity, curriculum, teaching, technology, learning, and assessment), the document presents standards for students in pre-kindergarten through twelfth grade. The standards are presented as content standards (i.e., number and operations, algebra, geometry, measurement, and data analysis and probability) and process standards (i.e., problem solving, reasoning and proof, communication, connections, and representations). Embedded throughout the process standards, in particular, is an emphasis on communicating mathematically. Students are expected to ask, reflect, engage in mathematical conversations, justify their answers, use mathematical arguments and rationales, and precisely communicate results orally and in writing. More recently, NCTM reiterated their position on the importance of communicating mathematically with the release of *Principles to Actions: Ensuring Mathematical Success for All* (2014).

A decade after NCTM released *Principles and Standards for School Mathematics* (2000), the National Governors Association released the *Common Core State Standards Mathematics* (*CCSSM*). The *Standards for Mathematical Practice* that accompany the grade-level instructional standards echo NCTM's earlier call for students to communicate mathematically in classrooms. Specifically, students are expected to have mathematical conversations, explain problems, describe, listen and/or read mathematical arguments, and critique arguments orally or in writing using precise mathematical terminology (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). As of this writing, 46 of the 56 states and U.S. territories had adopted the CCSSM indicating that the majority of students in the U.S. are expected to communicate mathematically with precision (Common Core State Standards Initiative, 2020).

Although there are a number of components of mathematics language (e.g. classroom discourse, syntax, gesturing, etc.), several researchers have identified mathematics vocabulary as one component of mathematics language that is critical for communicating mathematically and, more broadly, successfully learning mathematics (Garbe, 1985; Hardcastle & Orton, 1993; Miller, 1993; Milligan, 1983; Monroe & Orme, 2002; Oldfield, 1996; Powell et al., 2020; Riccomini et al., 2008; Slavit & Ernst-Slavit, 2007; Thompson & Rubenstein, 2000). The critical role that mathematics vocabulary plays in understanding and engaging in mathematics may be related to its dual function in mathematics. Mathematics vocabulary not only provides a name or a label for concepts, procedures, and items, it is also deeply interconnected with the concepts the words represent (Heath, 2010; Livers & Bay-Williams, 2014). For example, when teaching the word “numerator,” a teacher may take two approaches. The first approach would be to teach students to identify the top number in a fraction as the numerator. While this may be a straightforward way to teach students to identify or label which part of a fraction is a numerator, it does not provide any instruction about the meaning of “numerator.” The second approach would involve teaching students the concept of numerator (i.e. numerator represents parts of a whole) in addition to teaching students to identify or label numerators. Researchers note that effectively teaching vocabulary in mathematics appears to involve teaching the concepts that the words represent and that students who have limited mathematics vocabulary may also be more likely to have a weak understanding of the related concepts (Garbe, 1985; Leung, 2005; Miller, 1993; Raiker, 2002; Riccomini et al., 2008; Slavit & Ernst-Slavit, 2007; Thompson & Rubenstein, 2000).

As early as 1978, Preston speculated that the language of mathematics, particularly its technical vocabulary, may impede student understanding and, therefore, achievement in mathematics. Experts have offered a number of explanations for this. Perhaps most obviously, mastery of mathematics vocabulary is critical for solving problems that are presented textually (e.g., word or story problems). Students who have not mastered mathematics vocabulary may not understand what these problems are asking of them, how to identify an appropriate strategy for solving them, and how to communicate the solution to a teacher or other students (Burton, 1988; Heinrichs, 1987; Schumacher & Fuchs, 2012; Thompson & Rubenstein, 2000). Another less obvious consequence of not mastering mathematics vocabulary is that students may not fully benefit from material presented in textbooks or instruction from teachers. Students who lack mathematics vocabulary may not understand modeling provided by teachers or explanations read in textbooks (Capps & Pickreign, 1993; Heinrichs, 1987; Preston, 1978). As a result, they may lack understanding of mathematical concepts and use inappropriate procedures to solve various types of mathematical problems (Anghileri, 1995; Karp et al., 2014). Additionally, teachers use verbal and written communication during mathematics instruction to monitor students' understanding (Lubinski & Otto, 2002; Padula et al., 2002; Thompson & Rubenstein, 2000). Teachers of students who have not mastered the requisite mathematics vocabulary are unable to fully monitor their students' understanding. As a result, they are limited in their ability to adjust instruction and meet the unique needs of each student. Successfully understanding and using mathematics vocabulary empowers students to engage in mathematics, improves their access to instruction, and facilitates accurate data-based decision making by teachers during mathematics lessons (Chan, 2015; Kostopoulos, 2007; Miller, 1993; Rubenstein & Thompson, 2002; Whitin & Whitin, 1997).

Recent attempts have been made to quantify the relation between mathematics vocabulary and mathematics achievement described in the paragraph above. Bowie (2016) administered a state-mandated mathematics achievement test to 131 students in eighth grade in one state along with a researcher-created mathematics vocabulary assessment. The author reported a positive correlation ($r = 0.67$; $p < .001$) between the two variables. Powell and Nelson (2017) administered a standardized, norm-referenced general vocabulary assessment, a standardized, norm-referenced mathematics calculation assessment, and a researcher-created mathematics vocabulary assessment to 104 students in first grade. They conducted regression analyses on the obtained data and found that general vocabulary and mathematics calculation performance were positively associated with mathematics vocabulary performance.

In a similar study, Powell et al. (2017) investigated the relationship between general vocabulary, mathematics computation, and mathematics vocabulary among upper elementary-aged students. They administered a standardized, norm-referenced general vocabulary assessment, a standardized, norm-referenced mathematics computation assessment, and a researcher-created mathematics vocabulary assessment to 65 students in third grade and 128 students in fifth grade. They found positive correlations between mathematics vocabulary and general vocabulary, as well as mathematics vocabulary and mathematics computation, for students in both grades.

Unfortunately, mastering the vocabulary of mathematics appears challenging for many students. Table 1 presents unique features associated with learning and using mathematics vocabulary. Many, if not all, of the features presented in the table have been identified by other researchers as challenges or obstacles students face when learning mathematics vocabulary (Adams et al., 2005; Barrow, 2014; Chan, 2015; Gillam et al., 2016; Jourdain & Sharma, 2016; Moschkovich, 2002; Padula et al., 2002; Powell et al., 2020; Roberts & Truxaw, 2013; Rubenstein & Thompson, 2002; Smith & Angotti, 2012). I present these as *features* as opposed to *challenges* because some may also serve as supports to students in specific contexts. The first column of Table 1 identifies the specific features of mathematics vocabulary. The second column identifies the contexts in which students may encounter these features (e.g., general English, within mathematics, other content areas, languages other than English). The third column provides examples of each feature within the identified context(s). In the paragraphs that follow I elaborate on the features, contexts, examples, and how they may be challenging or supportive of learning mathematics vocabulary.

Table 1

Features of Mathematics Vocabulary

Feature	Content Area	Examples
Shared meaning of one word	General English and mathematics (e.g., add)	**English:** *Add* the dirty clothes to the laundry pile. **Mathematics:** *Add* two to the ten you already have.
	Other content areas and mathematics (e.g. table)	**Science:** Periodic *Table* of Elements **Mathematics:** Data *table*
Shared meanings of different words (synonyms)	Within mathematics	"Find the *factors* of 18" means the same as "Find the *multipliers* and *multiplicands* of 18."
Auditorily similar words	Homophones in English (e.g., sum and some)	**Sum:** The *sum* of the addition problem is 12. **Some:** She started with 12, lost *some*, and ended with 8.
Different meanings of one word (i.e., polysemous words)	General English and mathematics (e.g., table)	**English:** Put the chess *set* on the *table.* **Mathematics:** Find the mean of the first *set* of data in the *table*.

	Other content areas and mathematics (e.g., divide)	**History:** The Continental *Divide* marks where rivers flow east or west in North America. **Mathematics:** *Divide* 12 into four groups.
	Within mathematics (e.g., square)	**Geometry:** “Square” as the name of a two-dimensional shape **Operations:** To multiply a number by itself
	Nominalization within mathematics (e.g., sum)	**Noun:** Find the *sum* of the problem. **Verb:** *Sum* the numbers to find the answer.
Translating between languages	Cognates in languages other than English (e.g., area and el área)	“Area” in English and “el área” in Spanish share a meaning and origin and are visually and auditorily similar
	Languages other than English and mathematics (e.g., translating table into Spanish)	Table can be translated into Spanish as “mesa” (e.g., dinner table) or “tabla” (e.g., data table)

Symbols and diagrams	General English and mathematics	**English:** Symbols often represent (e.g., sounds, brands); diagrams less common **Mathematics:** Symbols direct (e.g., + indicates to add); more frequent use of diagrams
	Other content areas and mathematics	**Geography:** Symbols represent; diagrams are illustrative **Mathematics:** Symbols direct (e.g., - indicates to subtract); diagrams often convey numerical relations
Technical definitions	Within mathematics	A square is a plane figure with four equal-length sides and four right angles.
Classroom factors	Use of informal vocabulary during instruction	“One point five” instead of “one and five tenths”
	Inconsistent use of appropriate vocabulary	Referring to a rhomboid as a rhombus

Lack of time to practice	Students often encounter and use mathematics vocabulary only during a portion of mathematics instruction.

Shared Meaning of One Word

Perhaps the easiest type of mathematics vocabulary term for students to learn are words that share meaning across different contexts. These are words that students are likely already familiar with because they encounter them in everyday English. The meanings of these words are the same in general English as they are in mathematics, so students are able to easily learn and use them during mathematics. “Add” is an example of such a word. A direction such as “Add these oranges to this bowl of fruit” carries the same meaning in everyday English as the direction to “Add two manipulatives to the group of five manipulatives.” Similarly, some words have the same meaning across different academic content areas. These are words that students are likely to encounter in other content areas. “Table” is an example of such a word. During science instruction, students may encounter the Periodic Table of Elements or construct a table of data from an experiment. During mathematics instruction, students may use data from a table to solve problems. “Table” conveys the same meaning in both contexts. This feature of mathematics vocabulary may be supportive for developing an understanding of mathematical concepts because students are readily able to use words mastered in other contexts in the context of mathematics. This feature may also present a challenge for students because not all words share meanings across contexts. After learning mathematical vocabulary words that do share meanings across contexts, students may overgeneralize and believe that all words used during mathematics have the same meaning(s) as everyday English or other content areas.

Synonyms

Arguably, a slightly more difficult feature associated with learning mathematics vocabulary may be different words that share the same meaning within mathematics (i.e., synonyms). “Factor”, “multiplier,” and “multiplicand” are examples of this feature. “Multiplier” and “multiplicand” each refer to specific numbers that are multiplied together in a multiplication problem (the specific name of each number depends on its position within the problem). “Factor” is a more general term used to refer interchangeably to both numbers being multiplied. Similar examples may be found in geometry. A square, for example, is a type of rhombus, but not every rhombus can be categorized as a square. The hierarchical organization of shapes means that shapes belonging to a subordinate category may be called by multiple names while the shape(s) in the superordinate category may not be referred to by the same number of names. When words possessing this feature of mathematics vocabulary are used interchangeably during instruction, students may think the teacher is referring to multiple distinct concepts and become confused.

Auditorily Similar Words

Homophones may be the next most difficult feature when learning mathematics vocabulary. Homophones are words that sound similar but are spelled differently and have different meanings. *Sum* and *some* are examples of homophones that occur in mathematics and everyday English. In the same lesson, a teacher may state that the “*sum* of the addition problem is 12,” and then go on to describe a new problem saying, “Pearl had 12 pieces of candy. She gave away *some*. How many does she have left?” This example illustrates not only the different meanings of the homophones “sum” and "some," but shows how "sum" can be used when talking about an addition problem and "some" can be used when talking about a problem that requires subtraction. Homophones, especially when used during oral instruction, may be challenging because students must depend on contextual clues about their meanings. Additionally, familiarity with one homophone does not guarantee that students will infer the meaning of the related homophone. In the example described above, a student familiar with "sum" may infer that "some" in the second problem is related to addition and choose the incorrect operation for solving the problem.

Polysemous Words

The next most difficult feature of mathematics vocabulary may be polysemy. Polysemous words are words that sound and are spelled the same but have different (although related) meanings depending on the context. There are four situations related to mathematics in which polysemy may be challenging for students. The first is polysemous words that occur in everyday English and mathematics. These are everyday words that students use frequently in non-mathematical contexts, but they have specific meanings that differ from their everyday meanings when used mathematically. The challenge is that they sound and look the same, so students may struggle with determining the mathematical meaning. "Table" is an example of a polysemous word with meanings that differ between everyday English and mathematics. In everyday English, "table" usually refers to the piece of furniture found in many homes. In mathematics, "table" often refers to a visual display of data.

The second situation is polysemous words that occur in other content areas and mathematics. These are words that students may have learned in other content areas but have mathematical meanings that differ from what the students previously learned. Similar to the first situation, the challenge associated with this feature is that students may struggle with learning the mathematical meaning(s) of polysemous words because they sound and look the same as terms used in other content areas. “Divide” is an example of a polysemous word with different meanings in mathematics and another domain. In mathematics, “divide” is associated with separating a whole into a specified number of groups. In history or geography, “The Continental Divide” is the name of the geographical feature that locates where rivers flow to the east or west in North America. Although the meanings of “divide” are related in both domains, “The Continental Divide” in history or geography may be taught as the name of an object with little reference to its association with the term “divide” in mathematics (i.e., The Continental Divide separates the whole continent into two parts).

The third situation is polysemous words that occur within different branches of mathematics. Square, for example, may be used to name a shape. It may also refer to multiplying a number by itself. These words are challenging for students because determining their meaning does not simply depend on comparing the mathematical meaning of a word to its other meanings; these words require students to discriminate between two mathematical meanings of the same word.

The fourth situation, nominalization, occurs when words function as nouns or verbs depending on the spoken or written context. In the context of mathematics, these words may be difficult because students must discriminate their mathematical meaning based on the grammatical structure of the written or verbal communication. The use of “sum” to refer to the result of adding is an example of nominalization within mathematics. Using the verb form, students may be directed to "sum five and seven to find the answer" but using the nominalized (noun) form, they may be told that the "sum of the problem is 12."

Despite the potential obstacles presented by each of the four forms of polysemy described above, polysemy may also be a supportive feature of mathematics vocabulary. Polysemous words have different, but related, meanings. Teachers may take advantage of this feature to help students understand new mathematics vocabulary terms. "Table" as a piece of furniture and "table" as a visual display of data both usually share the characteristics of being flat and rectangular. "Divide" shares similar meanings across history, geography, biology, mathematics, and other content areas. Teachers who explicitly link the new meaning of a term to its already known related meanings (rather than merely teaching the term as a name) may help students generalize the shared meaning across domains. During mathematics instruction, teachers may take advantage of polysemous words to deepen students' understanding of relations between branches of mathematics. For example, the plane shape called "square" can serve as a model of a squared number (i.e., 4^2 can be modeled with a drawing of a square with sides that are four units in length). Also, teachers who explicitly teach the different functions of a word (e.g., "sum" as a noun and verb) may support students to better understand and engage in mathematics instruction.

Translating between Languages

Opportunities to translate mathematics vocabulary words between languages is a feature of mathematics vocabulary instruction that may be especially relevant to students whose first language is not English. There are two topics that need attention when translating mathematics vocabulary between languages. The first topic is cognates. Cognates are words from different languages that share the same origin. They are often visually and auditorily similar. Area and el área are examples of cognates in English and Spanish that occur in mathematics. They both derive from Latin and are auditorily and visually similar. Teaching the meaning of area to a student whose dominant language is Spanish (or another Romance language) will probably be much easier for the student and teacher if the teacher takes advantage of the common roots of mathematics terms shared by English and other languages. The second topic requiring attention when translating mathematical words between languages is using the correct word in each language. The English word “table,” for example, is used to describe a piece of furniture or a visual for organizing information (e.g., data table). In Spanish, two distinct words may be used to translate table: “mesa” and “tabla.” “Mesa” refers to the piece of furniture, and “tabla” refers to the visual for organizing information. Mistranslating words during mathematics instruction could easily confuse students.

Symbols and Diagrams

The presence and frequent use of symbols and diagrams may be one of the more challenging features of mathematics vocabulary for students. Symbols used in mathematics are often unique to mathematics and unlike symbols encountered in other everyday contexts. Similarly, symbols used during mathematics instruction often direct students to do something. Many symbols encountered by students in everyday life do not direct but represent. Logos, for example, represent brands that students may or may not want to associate with. Letters are another example of symbols that students frequently encounter. Letters do not direct a specific action but represent sounds that are put together to form words. In contrast, students are expected to act in response to a symbol in mathematics. The plus sign, for example, is a symbol the directs students to complete a specific operation. Students need to learn the symbols that are unique to mathematics and learn the procedures required to carry out the directives conveyed by the symbols. Similarly, mathematics uses diagrams that may not be like diagrams encountered by students in everyday life or other content areas. Students must learn to read and interpret the mathematical diagrams they encounter.

Technical Definitions

Finally, two overarching features affect all of the other features described above. The first is the technical nature of mathematics vocabulary. Mathematics vocabulary words have precise meanings. Often, the words used to describe the meaning of a particular mathematics vocabulary word may also be technical and unfamiliar to students. The level of precision inherent in mathematical definitions combined with unfamiliar words (or applications of words in new contexts) may make crafting student-friendly definitions challenging for teachers and learning new mathematics vocabulary words challenging for students. Additionally, researchers of mathematics vocabulary instruction have given seemingly contradictory guidance on the level of precision of language necessary during mathematics instruction. A number of researchers have asserted that requiring students to use technical language from the time a concept is first introduced is burdensome and unnecessary (Adams, 2003; Anghileri, 1995; Blais, 1995; Thompson & Rubenstein, 2000; Whitin & Whitin, 1997), while others have asserted that using technical language throughout instruction is necessary to support understanding of mathematical concepts (Hughes et al., 2016; Powell et al., 2019). This binary framing of language, and particularly definitions of vocabulary words, in mathematics classrooms may interfere with teachers' confidence in developing student-friendly definitions and is not supported by empirical research. A more productive approach to defining and using mathematics language in the classroom may be to use informal language that is accessible to students when introducing a concept and teaching and requiring the use of the more technically precise word(s) after students have shown a certain level of mastery with the newly introduced concept (Adams, 2003; Davis, 2008; Gough, 2007). What is considered informal language in this approach depends on student characteristics (e.g. grade level, background knowledge and experience in mathematics and English, etc.). For example, a teacher of students in kindergarten may describe a square as a shape with four sides that are the same length. A teacher of students in fifth grade, however, may describe the same square as a shape with four equal sides and four right angles. A teacher of secondary students may describe a square as a plane shape with four equal sides and four right angles. The definition increases in its complexity, precision, and technical language as the students progress through mathematics. This approach allows instruction of new concepts to move forward in a time efficient manner, avoids distracting students with unfamiliar words while learning new concepts, is flexible in response to student characteristics, and ensures that students grow in their understanding and use of mathematics language over time.

Classroom Factors

The second overarching feature relates to factors associated with the delivery of instruction that may make learning mathematics vocabulary more challenging. Bair and Mooney (2013) and Powell et al. (2019) note that teachers often use unnecessarily imprecise mathematics vocabulary. For example, teachers may refer to the number 1.5 using the words “one point five” instead of “one and five-tenths.” The latter phrase is preferable because it more accurately conveys the value represented by each digit and the entire number and does not introduce an inappropriate level of complexity (as described in the previous paragraph). While imprecise vocabulary may be useful when initially introducing a concept to reduce the learning demands placed on the students, teachers need to be mindful of modeling and teaching the use of grade-appropriate mathematics vocabulary to ensure students communicate mathematical ideas effectively before moving on to new topics (Adams, 2003; Blais, 1995; Davis, 2008; Gough, Leung, 2005; 2007; Thompson & Rubenstein, 2000). Teachers may also inconsistently use appropriate mathematics vocabulary. Rhomboid and rhombus, for example, are two related terms that are easily confused. A parallelogram with adjacent sides that are not equal lengths is a rhomboid, and a parallelogram with equal-length sides is a rhombus. Teachers may inadvertently confuse students if similar, related terms are used incorrectly. Finally, students may lack adequate time to practice using mathematics vocabulary. This is because mathematics vocabulary is unique to mathematics and differs from the vocabulary associated with the discourses of other content areas and everyday English. Typically, students only encounter and have an opportunity to use mathematics vocabulary during mathematics instruction (Capps & Pickreign, 1993). Wilkinson (2018) calls for teachers to create opportunities for students to use mathematical language in mathematics classrooms. However, the frequency and intensity of practice necessary for students to master mathematics vocabulary likely exceeds the number of practice opportunities that may be provided by teachers who heed this call. For students to participate actively in a class or small-group mathematics discussion using mathematics vocabulary, they must first have practiced the word(s) sufficiently for quick comprehension in the role of listener and timely recall and accurate use in the role of speaker. In order to get to this level of familiarity with the relevant words, students are likely to need frequent, targeted practice opportunities with specific feedback (Baumann et al., 2003; Beck, 2013). Relying on students to communicate mathematically (verbally or in writing) during instruction is unlikely to provide the practice needed to master mathematics vocabulary and engage in mathematics instruction. Instead, providing students with intense practice during targeted mathematics vocabulary instruction is more likely to enable students to communicate mathematically and engage in mathematics instruction. Students who are not given adequate practice with new mathematics vocabulary terms are unlikely to master the vocabulary necessary to access and engage in mathematics instruction or communicate mathematically (Kostopoulos, 2007; Miller, 1993; Powell et al., 2019; Riccomini et al., 2008; Whitin & Whitin, 1997; Wilkinson, 2018).

Notably, the features of mathematics vocabulary presented in Table 1 and described in the paragraphs above are fluid. More than one feature may be associated with any given mathematics vocabulary term and these associations are not fixed. In other words, depending on the context, one word may be associated with multiple features at one point within a lesson but associated with one or more other features at another point within a lesson. Consider “table.” “Table” appears in Table 1 as an example of words that share the same meaning between other content areas and mathematics, polysemous words that differ in meaning between everyday English and mathematics, and words that can be easily mistranslated. Depending on the context of the lesson, the teacher may need to anticipate errors due to the different meanings of “table” in mathematics and everyday English while taking advantage of the shared meaning of “table” in science and mathematics and remembering to accurately translate “table” in the mathematics sense as “tabla” for the students who speak Spanish fluently and are learning English.

The unique challenges associated with acquiring mathematics vocabulary underscore the need for students to receive high-quality instruction in this area and lend a new perspective to the *NAEP* results regarding instructional materials. Approximately one-fifth of the fourth-grade teachers in the U.S. report that a lack of access to adequate mathematics instructional materials was a “moderate” or “severe” problem (NCES, 2019a). Given the complexities of teaching mathematics vocabulary, it is logical to extend the *NAEP* results and conclude that teachers are inadequately supported to teach mathematics vocabulary. During a recent review of four of the most popular elementary mathematics instructional programs, Barnes and Stephens (2019) found that the vocabulary instruction embedded in the curricula varied considerably in the number of words taught, the difficulty of the terms, the number of instructional strategies employed to teach each word, and opportunities for practice and review. The results of this review, combined with the *NAEP* results, suggest that more teachers might consider a lack of access to adequate instructional materials a problem if asked specifically about materials for providing mathematics vocabulary instruction. Students who experience the instructional programs with weaker vocabulary instruction are not supported to have the same access to instruction, engagement in instruction, or mastery as students who experience instructional programs with more robust vocabulary instruction. Teachers who are supplied with an instructional program that includes relatively weaker vocabulary instruction are left to design their own instruction and fill in the gaps themselves. Teachers would likely benefit from instructional programs that account for the challenges associated with mathematics vocabulary and are designed using evidence-based principles of instructional design.

Direct Instruction

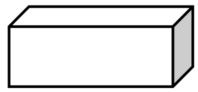
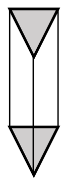
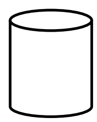
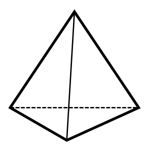
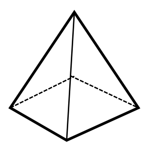
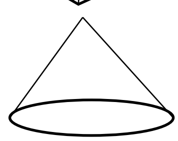
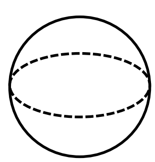
Direct Instruction (DI) is a system for teaching based on *Theory of Instruction: Principles and Applications* (Engelmann & Carnine, 1982/2016). Its goal is to teach advanced academic content to diverse learners (Rolf & Slocum, 2021). DI is characterized by analysis of the domain to be taught, careful example selection, intentional juxtaposition of examples, instructional formats that support clear communication between the teacher and learners and a systematic reduction of scaffolding, abundant practice opportunities, judicious review of previously learned material, and ongoing data-based decision making that allows teachers to respond to the unique needs of each individual student (Engelmann & Carnine, 1982/2016; Rolf & Slocum, 2021; Watkins & Slocum, 2004). Direct Instruction (note the capital letters) contrasts with direct instruction, a term frequently used as a synonym for explicit instruction or effective instruction, in that direct instruction does not necessarily include all of the features inherent in DI (Archer & Hughes, 2011; Hempenstall, 2004; Rosenshine, 2008). In the following paragraphs, I will describe each of the critical features of DI in more detail.

Domain Analysis

DI programs begin with a detailed analysis of the domain to be taught (e.g., beginning reading, mathematics, language, etc.). Prior to writing a single lesson, the developers of the programs consider the domain and identify any "big ideas," patterns, strategies, concepts, or rules that they can incorporate into the instructional design (Carnine, 1992; Engelmann et al., 1992; Kame'enui et al., 2002). For designers of DI programs, the goal of this activity is to design instruction that results in teaching for generalization rather than teaching unnecessarily isolated, or segmented, concepts and skills (Slocum & Rolf, under review). In beginning reading, for example, domain analysis reveals that many English words useful for beginning reading instruction may be read by teaching students the most regular sounds for each letter, to attend to each letter in a word, and a sounding out strategy. Once learned, students can use this strategy to read any number of previously unknown words. A less generative domain analysis may result in teaching that relies too heavily on teaching students to memorize individual words and ignores phonics. Convection is an example of a "big idea" that emerges when one analyzes the domain of earth science. Convection explains a number of topics in earth sciences: weather, plate tectonics, the water cycle, etc. A less generative domain analysis may result in content that treats each topic as though it is unrelated to the other topics. Domain analysis in mathematics reveals that certain strategies are useful for solving any number of problems (e.g., algorithms, number family arrows for problem-solving), and that the traditional content in some topics may be presented in an alternate way that is less cumbersome for students. For example, students are often taught seven different equations for determining the volume of three-dimensional shapes (one equation for each shape). Domain analysis reveals that slight variations of one formula reliably produce the same results (i.e., the volume of rectangular prisms can be calculated by multiplying the area of the base (B) by the height (h), and the volume of other three-dimensional shapes can be calculated by multiplying a fraction of B by h; Carnine, 1992; Kame'enui et al., 2002; Stein et al., 2018). Table 2 presents the traditional formulas, the three-dimensional shapes, and the alternative formula.

Table 2

Formulas for Volume of Three-Dimensional Figures

Shape		Traditional Formula	Alternate Formula
Rectangular Prism		lwh	Bh
Triangular Prism		$\frac{1}{2}lwh$	Bh
Cylinder		$\pi r^2 h$	Bh
Triangular Pyramid		$\frac{1}{6}lwh$	$\frac{1}{3}Bh$
Rectangular Pyramid		$\frac{1}{3}lwh$	$\frac{1}{3}Bh$
Cone		$\frac{1}{3}\pi r^2 h$	$\frac{1}{3}Bh$
Sphere		$\frac{4}{3}\pi r^3$	$\frac{2}{3}Bh$

Note. l = length; w = width; h = height; B = Area of the base; r = radius.

Analysis of a domain may produce any number of outcomes depending on the goals and values of the instructional designer(s). For example, Graham (1999) reported that some instructional designers choose to design spelling instruction around words that students frequently miss, words that students choose to learn, words that adhere to a theme (e.g., holidays, occupation-related words, school-related words, science words, etc.), and words that follow a specific pattern (e.g., they're, there, and their). Although themes are present in each of these approaches, they are not the types of patterns, strategies, or big ideas that result in teaching for generalization. In the area of mathematics, a domain analysis grounded in the belief that students must discover and create their own knowledge may result in instructional programs that direct students to invent their own strategies for performing calculations or try multiple algorithms for calculating one type of operation. The teaching that results from this type of domain analysis may produce instruction that is not as clear, efficient and reliable as possible. The goal of DI is to identify the domain analysis that is going to allow for the most efficient and effective delivery of instruction to the learners (Carnine, 1992; Engelmann & Carnine, 1982/2016; Kame'enui et al., 2002; Slocum & Rolf, under review). The analysis of the domain drives all of the instructional design that follows.

Example Selection

Careful example selection is one of the hallmarks of DI (Johnson, 2020). Whether a program addresses an early language concept like defining the term "under" or teaches students to solve complex mathematics problems, the selection of examples to present to the learners is critical for establishing the bounds of the concept (Engelmann & Carnine, 1982/2016; Johnson, 2020; Watkins & Slocum, 2004). Consider providing initial instruction to learners regarding the term “under.” The examples presented to the learners must show the range of instances of “under”, as well as the boundaries of “under”, without leading the learners to conflate the term “under” with some other meaning. If, for example, a teacher attempts to teach “under” by holding a ball under a table, under a piece of paper, and under a clipboard, the students may confuse “under” with flat objects and/or the ball. A more useful set of examples would be to hold a ball under a table, a piece of chalk under a cup, and a clipboard under a paperclip. In addition, the teacher would hold the objects at varying distances to establish that “under” is not related to how close one object is to another. The goal is to demonstrate the full range of the concept “under.”

An important facet of example selection is choosing appropriate non-examples. The selection of non-examples is critical for helping to establish the limits of the definition of a concept and avoid confusing students (Engelmann & Carnine, 1982/2016; Johnson, 2020; Watkins & Slocum, 2004). Non-examples of “under” would include placing the objects above, next to, in front of, or behind each other. Minimally different example and non-example pairs are especially useful for establishing the bounds of a concept. Continuing with the illustration of teaching “under,” a minimally different non-example would be holding the same ball next to the same table while maintaining a consistent distance between the ball and table. This demonstration would be minimally different from the previously described example of “under” using the ball and table because it employs all of the same objects and maintains the same distance between the objects. The only difference between the two objects is the relative position that results in the ball being under or not under the table. The logic of minimally different pairs of examples and non-examples extends to more advanced topics taught in all subjects at any grade (Engelmann & Carnine, 1982/2016).

Juxtaposition

After adequate examples and non-examples are identified, the sequence for presenting all of the examples to the learners in order to define and establish the limits of the concept needs to be determined (Engelmann & Carnine, 1982/2016; Twyman, 2020b). Critical to successful sequencing of examples if the juxtaposition of minimally different pairs of examples and non-examples. In the case of teaching "under," one positive example could be presenting a clipboard under a paperclip. The minimally different negative example would be placing the clipboard next to the paperclip. This negative example is minimally different because it uses the same objects as the previous positive example but slightly changes their position. This slight transformation that turns the positive example into a negative example helps define the concept and its limits for the students. It clarifies that "under" is not the clipboard, the paperclip, any of their associated physical traits, or the distance between them; "under" is the relative position of the clipboard to the paperclip. Following this minimally different negative example, the teacher would present another negative example that is not minimally different. This negative example may be presenting a pencil that is next to a desk. The teacher would continue by presenting a mix of positive and negative examples to the students for additional practice. Although variations in how examples and non-examples are juxtaposed occur, the goal is always to define the concept and its range as effectively and efficiently as possible (Engelmann & Carnine, 1982/2016).

Instructional Formats

Analysis of the domain, identifying positive and negative examples, and sequencing examples are the backbone of any DI program. In order to effectively use the products of these activities to positively impact learners, DI programs include instructional formats. Instructional formats provide the structure for the activities within each lesson and result in clear communication between teachers and students. Clear communication is central to the development of DI programs because it results in students learning as quickly as possible while minimizing the possibility of confusing students as much as possible (Engelmann & Carnine, 1982/2016; Twyman, 2020a). Instructional formats embody the domain analysis, exemplification, and juxtaposition of examples previously discussed and present a framework for engaging in clear communication that results in student learning (Engelmann & Carnine, 1982/2016; Johnson, 2020; Slocum & Rolf, under review; Twyman, 2020b; Watkins & Slocum, 2004). Specific features of the instructional formats include a teaching script, the sequenced examples and non-examples previously described, and multiple practice items (Rolf & Slocum, 2021; Watkins & Slocum, 2004).

In addition to supporting clear communication during instruction, the scripts play an important role in systematically reducing the scaffolding of instruction. When a concept is introduced for the first time, the associated script includes frequent prompts to support students. As time passes and students demonstrate mastery, these prompts are systematically faded from the scripts and the script includes less scaffolding. Over the course of many lessons, students transition from being highly supported by the teacher to independently engaging in the instructional task. Without the scripts, teachers may easily over- or under-scaffold for their students and instructional time is not used as efficiently. The scripts provide the language and fading that are necessary for successfully teaching students to mastery (Rolf & Slocum, 2021; Watkins & Slocum, 2004).

Abundant practice opportunities with active student responding is another feature of DI embedded in the instructional formats. During initial teaching of a concept, the intentionally sequenced examples and non-examples described above are presented to the students to define the concept and its boundaries (Engelmann & Carnine, 1982/2016; Twyman, 2020b). After initial teaching, frequent practice opportunities are presented to the students. In order to provide frequent practice opportunities to all of the learners in a group, many of the practice items at the elementary-level are delivered orally and are designed to be answered with a unison group response. This gives the most possible practice opportunities to all of the students and increases the likelihood that the students attend to the entire lesson. Students are also given targeted individual turns to assess progress. These usually occur at the end of an exercise and are designed to ensure that all students are mastering the material (Rolf & Slocum, 2021; Slocum & Watkins, 2004).

The practice opportunities also serve to provide strategic review of previously learned concepts. Mass practice immediately following initial teaching of a concept is not enough to bring students to mastery. Students need to engage in distributed practice of concepts across time. The practice items included in the instructional formats ensure that this practice occurs. Depending on the concept, this practice may be using the previously learned skill as a component skill for a more advanced skill, or it may be included in exercises with the explicit purpose of reviewing previously learned material (Carnine et al., 2017; Engelmann & Carnine, 1982/2016; Stein et al., 2018).

Data-based Decision Making

The detailed design of DI programs combined with the presence of instructional formats and all of the prescriptive features may give the impression that DI programs are rigid, unadaptable, and unresponsive to the unique needs of individual students. This could not be further from the truth. Critical to the successful implementation of any DI program, and interwoven throughout each program, are procedures for making data-based decisions that drive instruction. At the program-level, each DI program provides guidance to teachers about where to begin instruction using placement tests. DI programs are leveled, and the prerequisite skills necessary for success are encapsulated in each level’s placement test; students who meet the criteria for a level’s placement test are likely to possess the prerequisite knowledge necessary to succeed in that level of the program. To determine a more nuanced placement within a level of a program, the mastery tests given approximately every 10 lessons can be administered prior to beginning instruction. These procedures allow the teacher to individualize instruction prior to teaching any lessons. Rather than assuming that a student has the skills necessary to engage in a certain level of a program based on age or grade, the teacher can provide more targeted, individualized instruction. This uses instructional time efficiently because the teacher avoids teaching content that a student has already mastered or teaching content that is beyond the student’s current skill-level (as evidenced by a lack of prerequisite skills). The explicit guidance about placement in DI programs contrasts with many core literacy and mathematics programs that do not provide any guidance to teachers regarding placement within a program (Carnine et al., 2017; Engelmann et al., 2008; Engelmann et al., 2012; Stein et al., 2018).

As briefly stated above, DI programs include mastery tests that are to be administered approximately every ten lessons. These mastery tests are designed to assess mastery of material learned in the previous lessons. In addition to providing data about students' current performance, they provide an additional opportunity for individualization. Students who perform above criteria on the mastery tests are eligible for skipping specified exercises and future lessons. This is done to avoid wasting instructional time on providing more practice than students require. The mastery tests also specify remediation for students who perform below criteria on the mastery tests. Remediation usually involves re-teaching specified lessons and/or exercises, depending on the concepts included in the mastery test (Engelmann et al., 2008; Engelmann et al., 2012). The explicit provision of remediation serves as another opportunity to individualize a program to meet each student's unique needs.

DI programs also include provisions for collecting and analyzing data within lessons. The unison group responses support this purpose. When students respond in unison, teachers are able to quickly determine if students are answering correctly or incorrectly. If students answer correctly, the data (i.e., the group response) indicates that the teacher should present the next item. If the students answer incorrectly, the data indicates that the teacher needs to intervene. The intervention for answering incorrectly involves specified error-correction procedures. Typically, the teacher stops the group immediately, models the correct response, tests the students for the correct response, and then provides a delayed test (Engelmann et al., 2008; Engelmann et al., 2012; Rolf & Slocum, 2021; Watkins & Slocum, 2004). Although highly specified, this procedure supports teachers to meet the unique needs of the students by providing timely reteaching of the specific concept that was problematic, additional practice on the problematic concept, and fine-tuned re-assessment of the problematic concept. If the students continue to make errors, the procedure may be repeated as many times as is beneficial to support student learning. This procedure is highly responsive to the specific instructional needs of the group (Rolf & Slocum, 2021).

Research Supporting Direct Instruction

Decades of research have shown DI to be effective. Project Follow Through was a federal program begun in the 1960s that examined the results of implementing a variety of instructional programs in diverse schools across the U.S.. Results showed that students in schools that implemented DI programs scored higher on academic, conceptual skills (e.g., reading comprehension, problem solving), and self-esteem measures than students who were in comparison schools or schools that implemented programs based on other instructional models (Kennedy, 1978). Over the decades that followed, many researchers continued to report positive effects of various interventions based on design principles found in *Theory of Instruction* (Engelmann & Carnine, 1982/2016). In the area of mathematics, for example, Darch et al. (1984) found that students in fourth grade taught to use an explicit mathematics problem solving strategy designed using DI principles outperformed students taught to use a more traditional strategy. Moore and Carnine (1989) found that secondary students with disabilities who received instruction via videodisc using DI design principles to teach ratio and proportion outperformed similar students at immediate posttest and maintained gains at a delayed posttest two weeks later. Kelly et al. (1990) also used a videodisc program designed according to DI principles to teach fractions to students in high school who were diagnosed with learning disabilities.

The students assigned to the experimental group outperformed the control group. Brent and DiObilda (1993) found that students in second grade who received DI programs generally performed just as well or better on standardized assessments than comparable students in the same school district who received traditional instruction. Tarver and Jung (1995) found that students in second-grade who were taught using a DI mathematics program performed better on a standardized mathematics assessment than their peers in the same school who received traditional mathematics instruction. Additionally, students in the experimental group scored higher on an attitudinal survey created by the researchers than students who were assigned to the control group. Parsons et al. (2004) found that secondary mathematics students with low mathematics performance made significant gains on a standardized mathematics assessment from pre- to post-test after experiencing a DI mathematics intervention delivered by peer-tutors.

Within the last few years, Stockard et al. (2018) published a meta-analysis cataloging 549 reports of studies on the effectiveness of DI programs in all subject areas (e.g. reading, mathematics, writing, language, science, etc.). Not wanting to exclude any meaningful research, the authors included dissertations, masters theses, technical reports, and other non-published reports, in addition to articles published in peer-reviewed journals between 1966 and 2016. After excluding reports that could not be located, combined results of DI with another intervention, did not provide sufficient information for calculating effects, failed to include comparisons involving a non-DI group, or had other quality issues related to research design, the authors examined 3,999 effects, 413 designs, 328 studies, and 393 reports. Their analysis produced effect estimates for all of the studies included in the meta-analysis as well as subject-specific subgroups (i.e., reading, mathematics, writing, etc.). They found an overall effect estimate of 0.6 (SE 0.06) for all of the included studies, and an effect estimate of 0.75 (SE 0.12) for all of the mathematics-related studies. Both effect estimates were statistically significant at $p < .001$. Their results suggest that DI consistently increases student achievement across academic domains (e.g., reading, mathematics, language) for a range of diverse learners.

Explicit Vocabulary Instruction

Evidence suggests that explicit vocabulary instruction results in increased vocabulary and improved comprehension (Jenkins et al., 1984; Jenkins et al., 1989; McKeown et al., 1985; Stahl & Fairbanks, 1986). Although a DI intervention focused exclusively on vocabulary acquisition has yet to be written (vocabulary instruction is embedded in DI programs like *Reading Mastery – Signature Edition* [Engelmann et al., 2008] and *Language for Learning* [Engelmann & Osborn, 1998], among others), experts in the instructional design principles found in *Theory of Instruction* (Engelmann & Carnine, 1982/2016) have made a number of recommendations for teaching vocabulary across content areas. Specifically, Carnine et al., (2017) recommend teaching students to use context clues, to use a dictionary, the meanings and applications of morphemes, and providing instruction using semantic mapping, modeling through the use of examples and non-examples, and synonyms incorporated into student-friendly definitions. Using context clues involves teaching students to use the words surrounding an unknown word to determine its meaning. Using a dictionary involves explicitly teaching students how to look up the meaning(s) of unknown words up in a dictionary and interpret their meaning(s). Knowing and applying the meanings of morphemes involves teaching students to recognize smaller parts of words (e.g. prefixes, suffixes, base words), the meaning(s) associated with the morphemes, and how to put the meanings of the morphemes together to determine the likely meaning of the unknown word. Semantic mapping involves graphically organizing and displaying information to develop knowledge and understanding of a concept. Modeling through the use of examples and non-examples involves presenting positive and negative examples of a concept that define and show the limits of the concept (as was described in the sections on exemplification and juxtaposition above). Providing student-friendly definitions using synonyms involves presenting technical definitions using terms that students have already mastered.

Other researchers have made similar recommendations regarding vocabulary instruction. Baumann et al. (2003) recommended teaching students to use strategies for independently determining the meanings of unknown words (e.g., context clues and morphemic analysis) and explicitly teaching students the meanings of specific unknown words using synonyms and/or student-friendly definitions that build on prior knowledge and semantic mapping. Archer and Hughes (2011) recommend selecting a limited number of high-impact words that students can practice repeatedly after being taught. They recommend teaching students the meanings of the selected words using student-friendly definitions, definitions found in-text or in the glossary, using morphemes, taking advantage of cognates, and always presenting a series of examples and non-examples. Beck et al. (2013) recommend introducing three to five high-impact words per lesson using student-friendly definitions and examples that students can apply immediately and providing additional practice with each of the words over the course of several days.

Notably, Carnine et al. (2017), Baumann et al. (2003, Archer and Hughes (2011), and Beck et al. (2013) make a number of similar recommendations about providing explicit vocabulary instruction. First, they recommend selecting a limited number of meaningful words to teach. Second, they recommend providing student-friendly definitions using synonyms that are already familiar to the students. Third, they recommend presenting a series of examples and non-examples when teaching the definition of the new word. Finally, they recommend providing ample practice opportunities when the word is first introduced and during subsequent lessons.

Explicit Vocabulary Instruction in Mathematics

The recommendations described above summarize general guidelines for providing explicit vocabulary instruction during reading or content-area lessons (e.g. science & social studies). In line with the increased focus on the language of mathematics previously noted, some researchers have made specific recommendations for providing vocabulary instruction in the context of mathematics. Foremost, multiple researchers agree that teachers need to consistently model correct mathematics vocabulary usage for their students (Bair & Mooney, 2013; Hughes et al., 2016; Karp et al., 2014; Miller, 1993; Powell et al., 2019; Raiker, 2002; Wilkinson, 2018), and students need to write about mathematics regularly (Barrow, 2014; Miller, 1993; Rubenstein & Thompson, 2002). Similar to the general vocabulary instruction recommendations described above, Smith and Angotti (2012) provide guidance for selecting a limited number of high-impact words to teach. Milligan (1983) described teaching students to create flashcards that identify the meanings of morphemes found in mathematics vocabulary terms. Rubenstein and Thompson (2002) suggest using word walls, graphic organizers, and teaching morphemes. They also note that students need many opportunities to practice using mathematics vocabulary because most mathematics vocabulary words are not used regularly outside of the mathematics classroom. Roberts and Truxaw (2013) echo the recommendations to use word walls and semantic maps or graphic organizers. Barrow (2014) also suggests using gestures and movement. The author notes that students need to apply newly learned words immediately, practice words frequently, and that teaching topically-related words (e.g. inch, foot, mile) together may improve understanding. Chan (2015) recommends building on pre-existing knowledge when introducing new vocabulary words and the strategic use of antonyms. Gillam et al. (2016) suggest that speech-language pathologists support mathematics vocabulary acquisition by teaching specific words using student-friendly definitions and providing opportunities for students to use the words by explaining their meaning and writing narrative essays.

Possible limitations of the recommendations for providing general vocabulary instruction and mathematics vocabulary instruction are worth noting. From the recommendations for general vocabulary instruction, teaching students to use context clues and the dictionary may not be feasible in mathematics classrooms. Unknown words encountered during mathematics lessons are unlikely to be accompanied by enough text for students to successfully guess at their meanings. Additionally, mathematical definitions are often technical and precise – two characteristics that are not conducive to using context clues to determine meaning (Reehm & Long, 1996). The technicality of mathematical terms also impedes the use of dictionary definitions. Students who do not know the meaning of a mathematical term are unlikely to understand the words used to define the unknown term in the dictionary. Regarding the recommendations specific to mathematics vocabulary instruction, it should be noted that few of these recommendations are based on empirical research of mathematics vocabulary instruction. Most of the recommendations for teaching mathematics vocabulary are found in practitioner journals and lean heavily on general vocabulary acquisition research. In mathematics classrooms, word walls, mathematics journals, narrative essays, semantic maps or graphic organizers, and morpheme instruction may be difficult to implement consistently and may not produce the desired results. Word walls are problematic because they may represent a passive form of exposure to vocabulary terms. A teacher may create a beautiful word wall that students do not use as a resource unless they are explicitly taught to do so. Even then, the onus of responsibility is typically on students to engage with the word wall. Mathematics journals and narrative essays are not introductory activities and may not provide frequent or targeted enough practice for all students. Additionally, they may require more instructional time than is usually available, may present additional challenges for students with disabilities, and generally do not allow for quick feedback to students. Semantic maps or graphic organizers may be useful for providing deep initial instruction, but are typically time-intensive and do not provide opportunities for frequent practice with specific words or allow for timely performance feedback. Finally, morphemes may provide students with a generalizable strategy that they can use to determine possible meanings of unknown words, but using morphemes is unlikely to provide students with the precise definition of a mathematical term. The most feasible recommendations for providing mathematics vocabulary instruction that produces lasting, positive impacts on student learning appear to be to teach a small number of high-impact words in each vocabulary lesson, to use student-friendly definitions that incorporate synonyms, to include positive and negative examples that illustrate the concept and its boundaries, and to provide immediate application followed by multiple practice opportunities distributed across several days (Archers & Hughes, 2011; Barrow, 2014; Baumann et al., 2003; Beck et al., 2013; Carnine et al., 2017; Gillam et al., 2016; Hebert & Powell, 2016; Jenkins et al., 1989; McKeown et al., 1985; Padula et al., 2002; Raiker, 2002; Riccomini et al., 2008; Rubenstein & Thompson, 2002; Smith & Angotti, 2012; Stahl & Fairbanks, 1986; White et al., 1990).

Chapter II Literature Review

In order to learn about the characteristics of existing mathematics vocabulary interventions and their effectiveness, I systematically reviewed studies that reported the effectiveness of mathematics vocabulary interventions in elementary and secondary school settings. The following research questions guided this review:

1. What instructional strategies have researchers investigated for teaching mathematics vocabulary across kindergarten through twelfth grade?

2. How effective are the investigated instructional strategies?

A doctoral student and I searched the Academic Search Ultimate, Education Source, ERIC, PsycINFO, ASHAWire, Educational Full Text, ProQuest Digital Dissertation, and Teacher Reference databases using a combination of search terms related to mathematics and vocabulary instruction. Tables A1, A2, A3, A4, A5, A6, A7, and A8 in Appendix A present all of the search terms and the combinations of terms that were used in each database. We reviewed the articles in two stages. During the first stage, we conducted a title and abstract review of all of the database-identified studies. During the second stage, I conducted a full-text review of all of the remaining articles.

Inclusion Criteria

The doctoral student and I used five criteria to identify studies for this systematic review. We included all intervention studies published in English in peer-reviewed journals that addressed the effects of an intervention related to mathematics vocabulary. We considered a study related to mathematics vocabulary if it included an independent and/or dependent variable that addressed mathematics vocabulary. For example, a study was included if the intervention was intended to affect mathematics achievement generally, but the researchers measured at least one outcome specific to mathematics vocabulary. Additionally, we included studies that used an intervention designed to improve mathematics vocabulary even if the outcome measures did not specifically capture results related to mathematics vocabulary. We included studies with participants in kindergarten through twelfth grade regardless of the country or setting in which the intervention was delivered (e.g. general education classroom, special education classroom). Because our intervention in designed to be delivered in English, we excluded studies that reported the results of interventions that were delivered using a language other than English. In order to identify as many relevant studies as possible, we did not limit our search by date.

Coding of Studies

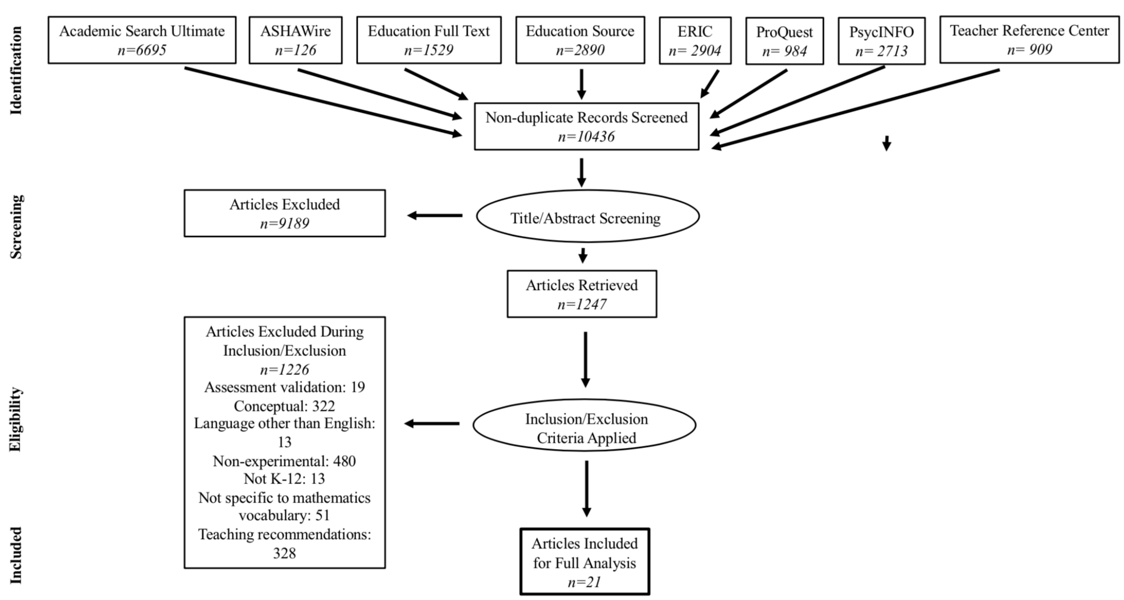
The doctoral student and I created a coding sheet to extract relevant information from the studies, including: study characteristics, participant characteristics, intervention characteristics, intervention effects, outcome measures, and methods for analyzing results. After finalizing the coding sheet, I coded all of the included studies. Elements of the coding sheet that were not addressed in a study were coded as "not stated" or "unclear." I applied qualitative techniques to synthesize the data extracted from all of the included studies.

Results

Our search returned 10,436 records. After removing duplicates and screening titles and abstracts, we were left with 1,657 unique records. We excluded 19 articles that exclusively addressed assessments for mathematics vocabulary and 1,023 that did not investigate the effectiveness of interventions related to mathematics vocabulary. We identified 21 studies for further analysis after applying all inclusion criteria. Figure 1 provides a visualization of our search process (Liberati et al., 2009). The studies were published between 1983 and 2019, with over half of the studies published after 2009. Twelve of the studies took place in the general education classroom during whole-group instruction. Only nine of the studies reported if their participants were or were not identified with any disabilities. Fourteen of the studies reported outcomes related to mathematics achievement, and 13 of the studies reported outcomes related specifically to mathematics vocabulary. Thirteen of the 21studies included elementary-aged participants, six of the studies included participants in middle school, and two of the studies included participants in secondary schools not located in the United States.

Figure 1

PRISMA Diagram



Studies in Secondary Settings

Eight studies were conducted in secondary settings (i.e. six in middle schools and two in secondary schools not located in the United States). A summary of these studies is presented in Table 3. The number of participants in each study ranged from 3 to 1000. Three of the articles reported the participants' ages (Fore et al., 2007; Hott et al., 2014; Root & Browder, 2019). The participants in these three studies were all 12 to 14 years old. The other five articles did not provide the participants' ages but did report grade levels, which ranged from grades six through eight and Form Two (studies conducted in Kenya). The majority of the articles reported that the studies included students identified with disabilities. Fore et al. (2007) included students diagnosed with learning disabilities, Hott et al. (2014) included students diagnosed with emotional/behavior disorder, Root & Browder (2019) included students diagnosed with autism spectrum disorder, and Karuza (2014) included students with disabilities but did not report specific diagnoses. Four of the articles reported that the studies took place in a general education classroom (Johnson, 2011; Jackson & Phillips, 1983; Karuza, 2014; Wanjiru & O-Connor, 2015), three of the articles reported that the studies took place in special education settings (Fore et al., 2007; Hott et al., 2014; Root & Browder, 2019), and one article did not provide information about the setting of the study (Wasike, 2006).

Table 3

Characteristics of Secondary Interventions and Studies

Study	Intervention	Dependent Variable(s) MA	Dependent Variable(s) MV	Design	*n*	Participant Age/Grade	Special Services	Interventionist	Effect Size
00. Fore et al. (2007)	Definition + sentences		X	Multiple baseline	6	12-13 y.o.	Special Education	Teacher as researcher	NA
01. Hott et al. (2014)	Peer-tutoring		X	Multiple baseline	6	12-14 y.o.	Special Education	School personnel	NA
02. Jackson & Phillips (1983)	Vocabulary activities	X	X	Treatment v. control – post	191	7th grade	Not stated	School personnel	Unknown[a]
03. Johnson (2011)	Vocabulary activities	X		Mixed Methods - pre/post	93	8th grade	Not stated	Teacher as researcher	Unknown[a]

04. Karuza (2014)	DARTS	X		Comparative secondary analysis	~1000	6-8th grades	11% Special Education	School personnel	Unknown[a]
05. Root & Browder (2019)	Flashcards	X	X	Multiple baseline	3	6-7th grades	Special Education	Researcher	NA
06. Wanjiru & O-Connor (2015)	Frayer model v. definitions	X		Pre/post with control	216	Form Two (Secondary)	Not stated	School personnel	Frayer > definition: $g = 0.66$[b]
07. Wasike (2006)	Socialized Mathematical Language Module	X		Solomon Four Group	156	Form Two (Secondary)	Not stated	Not stated	Unknown[a]

Note. MA = Mathematics achievement; MV = Mathematics vocabulary. [a] The authors did not provide an effect size or report the details necessary to calculate an effect size. [b] I calculated the effect size for Wanjiru and O-Connor (2015) using means and standard deviations reported for girls and boys and then calculated the mean of the two groups’ effect sizes to determine the overall effect size.

Secondary Interventions

A variety of interventions were investigated in secondary settings. Fore et al. (2007) investigated the effects of having students write sentences using definitions for mathematics terms. Hott et al. (2014) examined the effects of peer-tutoring. Jackson and Phillips (1983) and Johnson (2011) both investigated the effects of vocabulary activities (the authors did not provide descriptions of what the activities entailed). Root and Browder (2019) investigated the effects of using flashcards as a mathematics vocabulary intervention. Wanjiru and O-Connor (2015) compared the effects of using the Frayer model (Frayer et al., 1969) and teaching definitions. Karuza (2014) and Wasike (2006) examined the effects of multi-component interventions that included a mathematics vocabulary component on mathematics achievement. Karuza (2014) analyzed the effects of the DARTS program on the mathematics achievement of approximately 1000 students in California. DARTS stands for data collection and analysis, assessment, rescue assignments, translations (mathematics vocabulary), and story problems. Wasike (2006) provided the Socialized Mathematics Language Module to 156 secondary students in Kenya.

Table 4 presents a summary of characteristics associated with each intervention. The first column indicates the setting in which the intervention was delivered. The second, third, and fourth columns show if the students were provided with a definition as part of the intervention. The color of the circle shows who served as the interventionist. A white circle indicates that school personnel (e.g. teacher, paraeducator, therapist) provided the instruction. A dark gray circle indicates that a teacher who also served as the primary researcher provided the instruction. A black circle indicates that a researcher delivered the instruction, and a light gray circle indicates that the report did not provide enough detail to determine the exact role of the interventionist. The numerals in the circles correspond to the record numbers found on Table 3.

Table 4

Secondary Level - Intervention Characteristics

Setting	Definitions Not Provided	Definitions Provided	Unclear
Whole Class	04 06		02 03
Small Group			
Special Education	01 05	00	
Not stated			07

Note. Interventionist denoted by the shaded circles; = school personnel; = teacher as researcher; = researcher; = unclear.

Only one of the interventions clearly provided definitions to the students (Fore et al., 2007). Four of the interventions did not provide definitions to the students (Hott et al., 2014; Karuza, 2014; Root & Browder, 2019; Wanjiru & O-Connor, 2015), and three of the interventions were not described in enough detail to determine if definitions were or were not provided to the students (Jackson & Phillips, 1983; Johnson, 2011; Wasike, 2006).

Four of the interventions were delivered to large groups of students in general education settings (Jackson & Phillips, 1983; Johnson, 2011; Karuza, 2014; Wanjiru & O-Connor, 2015), three of the interventions were delivered in special education settings (Fore et al., 2007; Hott, 2014; Root & Browder, 2019), and the setting in which one intervention (Wasike, 2006) was delivered was not described in enough detail to determine. Notably, none of the interventions were used to provide supplemental instruction to small groups of students outside of special education settings. The intervention that provided definitions to students was delivered in a special education classroom (Fore et al., 2007).

Most of the interventions were delivered by school employees. Four of the interventions were delivered by teachers (Hott et al., 2014; Jackson & Phillips, 1983; Karuza, 2014; Wanjiru & O-Connor, 2015), two were delivered by teachers who were also researchers (Fore et al., 2007; Johnson, 2011), and one (Root & Browder, 2019) was delivered by researchers. Wasike (2006) did not provide information about the interventionist. Table 4 shows that most of the interventions related to mathematics vocabulary that have been studied do not provide definitions to students and an almost equal number have investigated the effects of interventions designed to be implemented in general education and special education settings.

Study Designs and Main Findings

Table 5 provides a summary of the design characteristics of the secondary studies. The first column shows whether the researchers used a group or single-case design. The second column indicates that researchers measured effects on mathematics vocabulary as the dependent variable. The third column indicates that researchers measured effects on mathematics achievement as the dependent variable. The final column indicates that the researchers measured effects on both mathematics vocabulary and mathematics achievement as the dependent variables. The squares show that the researchers created their own assessments to measure the dependent variables, and the circles show that the researchers used pre-existing standardized exams. The numerals within the shapes correspond to the record numbers found in Table 3.

Table 5

Secondary Level – Study Design Characteristics

	Dependent Variable		
Design	Mathematics Vocabulary	Mathematics Achievement	Both
Group		○ 03 ○ 04 □ 06 □ 07	□ 02
Single Case	□ 00 □ 01		□ 05

Note. Shapes indicate type of assessment; □= researcher-created assessment; ○ = standardized assessment.

Three of the studies used the single-case multiple baseline design (Fore et al., 2007; Hott et al., 2014; Root & Browder, 2019). The other researchers used a variety of group designs. Jackson and Phillips (1983), Wanjiru and O-Connor (2015) and Wasike (2006) used designs that included treatment and control groups. Jackson and Phillips (1983) and Wasike (2006) compared their treatment group to a control group using only a post-test. Wanjiru and O-Connor (2015) compared their groups using pre- and post-tests. Johnson (2011) employed a mixed methods design using pre- and post-tests without a control group, and Karuza (2014) conducted a secondary data analysis on existing school district data.

Two of the studies measured the effects of interventions on mathematics vocabulary exclusively (Fore et al., 2007; Hott et al., 2014). Both of these studies took place in special education settings and used researcher-created assessments. Fore et al. (2007) found that students answered more vocabulary-related questions after being taught using a concept model than when they looked up definitions of words and wrote sentences about them. Hott et al. (2014) found that peer-tutoring and academic self-monitoring resulted in increased scores on mathematics vocabulary quiz and cumulative test items.

Four of the studies measured the effects of interventions on general mathematics achievement exclusively (Johnson, 2011; Karuza, 2014; Wanjiru & O-Connor, 2015; Wasike, 2006). Three of these studies took place in general education settings, and the setting of the fourth study is unclear. Johnson (2011) and Karuza (2014) used state standardized achievement exams. Johnson (2011) found that direct instruction of mathematics vocabulary on the Ohio Achievement Assessment resulted in increased student scores on the same assessment. Similarly, Karuza (2014) found that teaching mathematics vocabulary from the California Standards Test as one component of an intervention package resulted in increased student scores. Wanjiru and O-Connor (2015) and Wasike (2006) used researcher-created assessments. Wanjiru and O-Connor (2015) found that students who were instructed using a variation of the Frayer model outperformed students who were taught using only definitions. Wasike (2006) found that students who experienced the "Socialized Mathematical Language" (p. 79) module outperformed students who did not.

Two studies measured effects on mathematics vocabulary and mathematics achievement. Jackson and Phillips (1983) used a researcher-created assessment with a group design and found that students who engaged in vocabulary activities in a general education setting outperformed students in computation and vocabulary who did not engage in the same activities. Root and Browder (2019) used a researcher-created assessment with a single-case design and found that students' performance increased compared to baseline when taught the meaning of mathematics vocabulary words and a schema-based strategy for solving word problems in a special education setting. In sum, six of the eight studies used researcher-created assessments to measure outcomes (Fore et al., 2007; Hott et al., 2014; Jackson & Phillips, 1983; Root & Browder, 2019; Wanjiru & O-Connor, 2015; Wasike, 2006), and all of the studies conducted in secondary education settings obtained positive results.

Discussion of Secondary Studies

The studies conducted in secondary settings indicate that a variety of interventions may be useful for improving mathematics vocabulary. Additionally, providing instruction specific to mathematics vocabulary may result in improved mathematics achievement. Six of the eight studies used interventions implemented by school personnel, suggesting that educators are capable of implementing interventions related to mathematics vocabulary that improve student performance.

The studies conducted in secondary settings have limitations that need to be considered when interpreting their results. First, three of the eight studies included fewer than six participants, making generalization about the effectiveness of these interventions difficult. Second, four of the eight articles reported that students who received special education services were included as participants. The remaining four studies did not indicate if participants received any special services (e.g. special education, language services, Title I). Third, each study measured effects using different assessments, and six of the eight studies used researcher-created assessments. Finally, three of the studies took place in special education settings, and the remaining five studies occurred in general education, whole-class settings. None of the secondary studies investigated the effects of interventions related to mathematics vocabulary designed to be implemented with small groups of students in addition to the core instruction received in general education. Despite the limitations, the studies indicate that implementing interventions related to mathematics vocabulary may enhance mathematics vocabulary and general mathematics achievement.

Studies in Elementary Settings

A summary of the thirteen studies conducted in elementary settings (i.e. kindergarten through sixth grade) is presented in Table 6. The number of participants in each study ranged from two to 2,348. Two of the articles reported the participants' ages (Parsons et al., 2005; Topping et al., 2003). Parsons et al. (2005) reported the participants as being eight and nine years old, and Topping et al. (2003) reported that the participants were seven and eleven years old. The other eleven articles did not provide the participants' ages but did report grade levels. Three of the studies included participants in kindergarten (Hassinger-Das et al., 2015; Jennings et al., 1992; Williams, 2019). One study (Powell & Driver, 2015) included participants in first grade. Two studies included participants in second grade (Cohen et al., 2015; Kostos & Shin, 2010). One study included participants in third grade (Petersen-Brown, 2019). Four studies included participants in fourth grade (Botes & Mji, 2010; Bruun et al., 2015; Monroe & Pendergrass, 1997; Petersen-Brown, 2019). Two studies included participants in fifth grade (Botes & Mji, 2010; McAdams, 2012), and one study included participants in sixth grade (Botes & Mji, 2010).

Table 6

Elementary Level – Study Design Characteristics

Study	Intervention	Dependent Variable(s) MA	Dependent Variable(s) MV	Design	*n*	Participants' Age/Grade	Special Services	Interventionist	Effect Size
Implicit Definition Interventions									
08. Cohen et al. (2015)	Mathematical reasoning language		X	Treatment v. control – post	384	2nd grade	Not stated	School personnel	Formal vocabulary count: g = 0.539[a] Formal vocabulary used correctly: g = 0.39[a]

09. Jennings et al. (1992)	Children's literature	X	X	Treatment v. control – pre/post	61	Kindergarten	Not stated	School personnel	Unknown[d]
10. Kostos & Shin (2010)	Mathematics journals and three mini-lessons	X	X	One condition – mixed methods	16	2nd grade	Not stated	Teacher as researcher	Math achievement and explanation: Unknown[d] Journal writing: $g = 0.60$[b, c]
11. Monroe & Pendergrass (1997)	Definition v. Frayer model		X	Two conditions – pre/post	58	4th grade	Not stated	Teacher as researcher	# of concepts: Frayer > Definition: $g = 0.51$

								# of applications: Definition > Frayer: g = 0.297
12. Parsons et al. (2005)	Word Wizard, family involvement	X	One condition – pre/post	2	8 & 9 y.o.	Vocabulary difficulties	School personnel	Unknown[d]
13. Topping et al. (2003)	Peer tutoring with board games	X	One condition – pre/post	27	7 y.o., 11y.o.	Not stated	Researcher	Use of mathematical words: g = 1.3[c] Strategic dialogue: g = 1.33[c]

Explicit Definition Interventions									
14. Botes & Mji (2010)	Learner companion	X		Treatment v. control – pre/post	2348	4th-6th grades	Not stated	School personnel	Unknown[d]
15. Bruun et al. (2015)	Journal writing and discussion v. Frayer model	X	X	Two conditions – pre/post	84	4th grade	ELL	Teacher as researcher	Unknown[d]
16. Hassinger-Das et al. (2015)	Children's literature v. number sense v. control	X	X	Three conditions – pre/post	124	Kindergarten	ELL	Researcher	Mathematics Vocabulary: SNC > number sense: g = 0.57

SNC > control: g = 0.51

Number Sense: Number sense > control: g = 0.21

Calculation: Number sense > control: g = 0.59

Number sense > SNC: g = 0.58

17. McAdams (2012)	Dictionary definition on graphic organizer	X		Treatment v. control – post	114	5th grade	33% at-risk	Teacher as researcher	Unknown[d]
11. Monroe & Pendergrass (1997)	Definition v. Frayer model		X	Two conditions – pre/post	58	4th grade	Not stated	Teacher as researcher	# of concepts: Frayer > Definition: *g* = 0.51 # of applications: Definition > Frayer: *g* = 0.297
18. Petersen-Brown (2019)	Compared practice intervals		X	Three conditions - post	62	3rd/4th grades	Not stated	Researcher	Interval conditions >

	using flashcards								massed: *g* = 0.63 Fixed > massed: *g* = 0.72
19. Powell & Driver (2015)	Addition tutoring v. addition + vocabulary v. control	X	X	Three conditions – pre/post	98	1st grade	Mathematics difficulties	Researcher	Vocabulary: Addition + vocab > control: *g* = 0.49 Addition > control: *g* = 0.64 Addition:

									Addition > control: g = 0.48
20. Williams (2019)	Explicit, small group instruction	X	X	One condition – pre/post	12	Kindergarten	IEP: 3 ELL: 2	Teacher as researcher	Achievement: g = 4.96[c] Vocabulary: Unknown[d]

Note. MA = Mathematics achievement; MV = Mathematics vocabulary. Please see text on p. 66 for procedures used to calculate effect sizes. Effect sizes for statistically significant results are reported. [a] I calculated effect sizes using means and standard deviations reported for “low level” and “high level” (p. 350) groups of students and then calculated the mean of the two groups’ effect sizes to determine the overall effect size for each outcome. [b] I calculated effect sizes for each journal topic and then calculated the mean effect size for all of the topics to determine the overall effect size. [c] Within-subjects effect size. [d] Not enough information provided to calculate an effect size.

Eight of the thirteen articles did not report whether the students were diagnosed with disabilities or received special services. Williams (2019) reported that three participants had an individualized education plan (IEP) and two were identified as English-language learners. Hassinger-Das et al. (2015) also reported that some participants were identified as English-language learners. Parsons et al. (2005) noted that participants had vocabulary difficulties, McAdams (2012) indicated that one-third of participants were labeled “at-risk” based on behavior, attendance, or academic data, and Powell and Driver (2015) described their participants as having mathematics difficulties.

Nine of the studies took place in a general education classroom (Bruun et al., 2015; Cohen et al., 2015; Jennings et al., 1992; Kostos & Shin, 2010; McAdams, 2012; Monroe & Pendergrass, 1997; Petersen-Brown, 2019; Topping et al., 2003; Williams, 2019). The interventions under investigation were provided as part of core mathematics instruction available to all students. Three of the included studies investigated supplemental instruction provided in addition to the core mathematics instruction (Hassinger-Das et al., 2015; Parsons et al., 2005; Powell & Driver, 2015). One article did not report the intervention in enough detail to determine the instructional tier (Botes & Mji, 2010). None of the studies investigated interventions implemented in special education settings or designed exclusively for students diagnosed with disabilities. More detailed information about the interventions delivered in elementary settings is provided in the following section.

Elementary Interventions

The thirteen studies conducted in elementary settings can be divided into two categories based on how the students obtained definitions of the mathematics vocabulary terms. Six of the thirteen studies investigated the effects of interventions that did not explicitly provide definitions to students (Cohen et al., 2015; Jennings et al., 1992; Kostos & Shin, 2010; Monroe & Pendergrass, 1997; Parsons et al., 2005; Topping et al., 2003). Throughout this chapter, I describe this group of studies as having implicit definitions. Eight of the studies investigated the effects of interventions that did provide definitions to students (Botes & Mji, 2010; Bruun et al., 2015; Hassinger-Das et al., 2015; McAdams, 2012; Monroe & Pendergrass, 1997; Petersen-Brown et al., 2019; Powell & Driver, 2015; Williams, 2019). I describe this group of studies as having explicit definitions throughout this chapter. One study (Monroe and Pendergrass, 1997) falls into both categories because one condition included the provision of definitions to the students and the other did not. Table 7 presents a summary of the interventions provided in elementary settings. In addition to grouping the studies on the basis of providing definitions to participants, the table provides a brief description of the interventions, their durations, and notes the instructional tier in which the interventions were administered. The numerals in the left-most column provide the record number for each study.

Table 7

Elementary Level - Intervention Duration and Instructional Tier

	Intervention(s)	Duration	Instructional Tier			
			Core	Supplemental	Special Education	Unclear
Implicit Definition Interventions						
08. Cohen et al. (2015)	Mathematical reasoning language	Unclear	X			
09. Jennings et al. (1992)	Children’s literature	Unclear	X			
10. Kostos & Shin (2010)	Math journals and three mini-lessons	Unclear	X			
11. Monroe & Pendergrass (1997)	Definitions v. Frayer model	10 school days	X			

12. Parsons et al. (2005)	Word Wizard, family involvement	Five-ten hours		X		
13. Topping et al. (2003)	Peer tutoring with board games	Five hours	X			
Explicit Definition Interventions						
14. Botes & Mji (2010)	Printed dictionary of terms in students' home languages	Unclear				X
15. Bruun et al. (2015)	Journal writing and discussion v. Frayer model	Five weeks	X			
16. Hassinger-Das et al. (2015)	Explicit instruction incorporating children's literature	Eight weeks		X		
17. McAdams (2012)	Dictionary definition on graphic organizer	Entire school year	X			

11. Monroe & Pendergrass (1997)	Definitions v. Frayer model	10 school days	X	
18. Petersen-Brown et al. (2019)	Compared practice intervals using flashcards	21 days		X
19. Powell & Driver (2015)	Tutoring with explicit instruction	Eight weeks		X
20. Williams (2019)	Explicit instruction	Six weeks	X	

Elementary Interventions with Implicit Definitions. A variety of interventions were implemented in the group of studies that did not provide definitions to students as part of instruction. Cohen et al. (2015) compared the effects of the standard mathematics curriculum and a curriculum that emphasizes written communication of mathematical reasoning on the mathematics vocabulary and mathematical writing of 384 students in second grade. Jennings et al. (1992) read stories from children's literature and provided manipulatives and props related to the stories to 61 students in kindergarten in Arkansas. Kostos and Shin (2010) delivered an intervention that paired mathematics journals with three teacher-directed mini-lessons in one second grade classroom. Monroe and Pendergrass (1997) compared the effects of using modified Frayer models with one group of fourth grade students and definition-only instruction with another group of fourth grade students. This study appears in both groups of interventions because the authors did not indicate whether definitions were provided to the students in the Frayer model condition. Parsons et al. (2005) taught two students ten steps to becoming a "word wizard" (p. 46) and provided family involvement activities. Topping et al. (2003) investigated the effects of a structured peer-tutoring program using mathematics board games on the self-concept, frequency of use of mathematics terms, frequency of use of terms related to game procedures or strategies, frequency of praise, and length of utterance in 27 students in primary school in Scotland.

Table 8 presents a summary of characteristics associated with the implementation of each intervention. The first column indicates the setting in which the intervention was delivered. The second, third, and fourth columns show if the students were provided with a definition as part of the intervention. The color of the circle shows who served as the interventionist. A white circle indicates that school personnel (e.g. teacher, paraeducator, therapist) provided the instruction. A dark gray circle indicates that a teacher who also served as the primary researcher provided the instruction. A black circle indicates that a researcher delivered the instruction, and a light gray circle indicates that the report did not provide enough detail to determine the exact role of the interventionist. The numerals in the circles correspond to the record numbers found on Tables 6 and 7.

Table 8

Elementary Level - Intervention Characteristics

Setting	Implicit Definitions	Explicit Definitions	Unclear
Whole Class	08 09 10 11 13	11 15 17 18 20	
Small Group	12	16 19	
Special Education			
Not stated		14	

Note. Interventionist denoted by the shaded circles; = school personnel; = teacher as researcher; = researcher.

Five of the implicit definition interventions were delivered to large groups of students in general education settings (Cohen et al., 2015; Jennings et al., 1992; Kostos & Shin, 2010; Monroe & Pendergrass, 1997; Topping et al., 2003). One was delivered individually to two students as a supplement to core instruction received in the classroom

(Parsons et al., 2005). None of the interventions were delivered in special education settings or designed exclusively for students diagnosed with disabilities.

Most of these interventions were delivered by school employees. Three of the interventions were delivered by educators (Cohen et al., 2015; Jennings et al., 1992; Parsons et al., 2005), two were delivered by teachers who were also researchers (Kostos & Shin, 2010; Monroe & Pendergrass, 2011), and one was delivered by researchers (Topping et al., 2003). Four of the articles do not report the time spent delivering the interventions to the participants (Cohen et al., 2015; Jennings et al., 1992; Kostos & Shin, 2010). The remaining articles report that no more than 10 hours were spent delivering instruction to the participants (Monroe & Pendergrass, 1997; Parsons et al., 2005; Topping et al., 2003).

Elementary Study Designs and Main Findings – Implicit Definition Interventions. Table 9 provides a summary of the design characteristics of the studies of implicit definition interventions. The first column describes the design of the studies. The second column indicates whether researchers measured effects on mathematics vocabulary as the dependent variable, the third column indicates whether researchers measured effects on mathematics achievement as the dependent variable, and the final column indicates whether the researchers measured effects on both mathematics vocabulary and mathematics achievement as the dependent variables. The squares show that the researchers created their own assessments to measure the dependent variables, and the circles show that the researchers used pre-existing standardized exams. A circle within a square indicates that the researchers used a combination of researcher-created mathematics vocabulary and standardized general mathematics achievement measures. A triangle overlaid on top of a circle indicates that the researchers used a combination of standardized general vocabulary and researcher-created mathematics achievement measures. The numerals within the shapes correspond to the record numbers found in Tables 6 and 7.

Table 9

Elementary Level, Implicit Definition Interventions – Research Design Characteristics

Design	Dependent Variable: Mathematics Vocabulary	Dependent Variable: Mathematics Achievement	Dependent Variable: Both
Two or more conditions, pre/post	11	09*	
Two or more conditions, post-test only	08		09*
One-condition, pre/post	12 13		10

Note. Shapes denote type of assessment; = researcher-created assessment; = standardized assessment(s); = researcher-created vocabulary assessment and standardized mathematics assessment(s); = standardized general vocabulary assessment and researcher-created mathematics assessment.

* Study 09 appears in multiple cells because one assessment was used to measure mathematics achievement during pre/post-testing and others were used only to measure mathematics vocabulary and mathematics achievement during post-testing.

The studies used a variety of group designs. Monroe and Pendergrass (1997) and Jennings et al. (1992) assigned participants to two or more conditions and compared pre- and post-test scores to analyze effects. Notably, Jennings et al. (1992) used an additional assessment that was only administered as a post-test. Cohen et al. (2015) also assigned participants to two or more conditions but only administered a post-test to document effects. Kostos and Shin (2010), Parsons et al. (2005), and Topping et al. (2003) used a within-subjects design to compare pre- and post-test scores for participants assigned to one condition.

All six of the studies measured the effects of interventions on mathematics vocabulary (Cohen et al., 2015; Jennings et al., 1992; Kostos & Shin, 2010; Monroe & Pendergrass, 1997; Parsons et al., 2005; Topping et al., 2003). Jennings et al. (1992) and Kostos and Shin (2010) also measured effects on mathematics achievement. Five of the studies took place in general education settings (Cohen et al., 2015; Jennings et al., 1992; Kostos & Shin, 2010; Monroe & Pendergrass, 1997; Topping et al., 2003), and one provided supplemental instruction to students in addition to the core mathematics instruction (Parsons et al., 2005). Three used only researcher-created assessments (Cohen et al., 2015; Monroe & Pendergrass, 1997; Topping et al., 2003), and three used a combination of researcher-created and standardized assessments (Jennings et al., 1992; Kostos & Shin, 2010, Parsons et al., 2005). Jennings et al. (1992) and Kostos and Shin (2010) used researcher-created mathematics vocabulary and standardized general mathematics achievement assessments. Parsons et al. (2005) used a researcher-created mathematics achievement measure in combination with two standardized general vocabulary assessments.

Table 6 presents effect sizes for the implicit definition interventions. None of the articles reported effect sizes for outcomes related to mathematics vocabulary or mathematics achievement. Where possible, I calculated effect sizes using reported means and standard deviations (Stangroom, 2020). All effect sizes are reported as Hedges' *g*.

Cohen et al. (2015) found that implementing a curriculum that emphasized using the language of mathematical reasoning resulted in the increased frequency of formal mathematics vocabulary used by second grade students when explaining their reasoning in writing ($g = 0.539$) and improved accuracy when using formal mathematics vocabulary in written responses ($g = 0.39$) compared to students who received the standard mathematics curricula used by their school districts. Jennings et al. (1992) found that kindergarten students who received children's literature incorporated into their mathematics lessons used significantly more mathematical terms during free play than students in the control condition. Not enough information to calculate an effect size was reported. Kostos and Shin (2010) found that using mathematics journals supplemented by three mini-lessons on mathematics-related topics resulted in improved mathematics journal writing by one group of students in second grade according to the scoring criteria presented in the *Saxon Math Teacher Rubric for Scoring Performance Tasks* (Larson, 2008; $g = 0.6$; effect size represents the mean of effect sizes for two different journal prompts). Monroe and Pendergrass (1997) compared using a Frayer model with providing definition-only instruction to 58 students in fourth grade. They found that students who experienced the Frayer model condition mentioned measurement concepts in mathematics journals more frequently ($g = 0.51$) and with more accuracy ($g = 0.297$) than students who received the definition-only instruction. Topping et al. investigated the effects of using mathematics board games during peer tutoring with 27 seven- and 11-year old students. They found that five pairs of students increased their use of mathematical terms (g = 1.3) and strategic dialogue (g = 1.33) by the end of the intervention. Parsons et al. (2005) provided an intervention that supplemented core mathematics instruction to two students. The researchers used a combination of researcher-created and standardized assessments. They found that the students' vocabulary knowledge improved from pre- to post-test using researcher-created assessments, but the students' vocabulary knowledge did not change from pre- to post-test on the standardized *British Picture Vocabulary Scale* (Dunn et al., 1982) or the *Test of Word Finding* (German, 1989).

Two studies also examined the effects of implicit definition interventions on mathematics achievement. The kindergarten students who received children's literature during mathematics instruction in Jennings et al. (1992) outperformed control students on the *Test of Early Mathematics Ability* (Ginsburg & Baroody, 1983) and the *Metropolitan Readiness Test* (not enough information provided to calculate effect sizes). The students in second grade who wrote in mathematics journals and received three mathematics mini-lessons improved their performance on an Illinois standardized mathematics assessment from pre- to post-test (Kostos & Shin, 2010; not enough information provided to calculate effect size).

Discussion of Elementary Studies – Implicit Definition Interventions. The studies conducted in elementary settings to evaluate implicit definition interventions indicate that a variety of interventions may be useful for improving students' mathematics vocabulary and mathematics achievement. Five of the six studies used interventions implemented by school personnel, suggesting that these interventions are feasible for improving student outcomes in real-world contexts.

The elementary implicit definition intervention studies have limitations that need to be considered when interpreting their results. First, although the range of grades and ages associated with elementary school (i.e. 5-12 years old, grades K-6) are included in these studies, approximately only one study took place at each grade level, and each study investigated the effects of a different intervention. Second, none of the six articles reported that students who received special education services were included as participants. Parsons et al. (2005) reported that students with "vocabulary difficulties" were included but did not specify if the participants were formally diagnosed with a disability. Third, each study measured effects using different assessments, and three of the six studies exclusively used researcher-created assessments. The other three studies used a combination of researcher-created and standardized assessments; Jennings et al. (1992) and Kostos and Shin (2010) used researcher-created mathematics vocabulary measures with standardized general mathematics achievement measures, and Parsons et al. (2005) used standardized general vocabulary assessments with a researcher-created mathematics assessment. None of the studies used a standardized mathematics vocabulary assessment to measure outcomes. Fourth, three of the six studies used a within-subjects design (Kostos & Shin, 2010; Parsons et al., 2005; Topping et al., 2003). Fifth, only two of the six studies investigated the effects of an intervention related to mathematics vocabulary on general mathematics achievement (Jennings et al., 1992; Kostos & Shin, 2010). Finally, none of the studies took place in special education settings; five studies occurred in general education, whole-class settings (Cohen et al., 2015; Jennings et al., 1992; Kostos & Shin, 2010; Monroe & Pendergrass, 1997; Topping et al., 2003), and one study provided supplemental instruction to two students (Parsons et al., 2005). Despite the limitations, the studies indicate that implementing interventions related to mathematics vocabulary may enhance mathematics vocabulary and general mathematics achievement.

Studies of Elementary Interventions with Explicit Definitions. Eight of the thirteen studies that investigated the effects of interventions in elementary settings provided explicit definitions to students (Botes & Mji, 2010; Bruun et al., 2015; Hassinger-Das et al., 2015; McAdams, 2012; Monroe & Pendergrass, 1997; Petersen-Brown et al., 2019; Powell & Driver, 2015; Williams, 2019). One study included one condition in which definitions were provided to the students and one condition in which definitions were not provided to students (Monroe and Pendergrass, 1997). I described this study in an earlier section, and provide additional information about this study later in this section.

Tables 6, 7, and 10 and Figure 2 summarize the studies of explicit definition interventions. Table 6 summarizes the design characteristics of the studies investigating the effectiveness of explicit definition interventions. The table provides information about the interventions, dependent variables, study design, participants, and interventionists in each study. Table 7 presents a summary of the explicit definition interventions provided in elementary settings. The table provides a brief description of the interventions, their durations, and notes the instructional tier in which the interventions were administered. The numerals in the left-most column provide the record number for each study. Table 10 presents research design characteristics of the studies of explicit definition interventions conducted in elementary settings. The left-most column describes the design, and the other columns indicate which dependent variables were included. The boxes indicate that the researcher(s) used researcher-created assessments, the circles indicate that the researcher(s) used standardized assessments, and a box surrounding a circle indicates that the researcher(s) used a researcher-created mathematics vocabulary measure and a standardized mathematics achievement measure. Figure 2 presents a visual of the effect sizes for the explicit definition interventions used in the studies that employed a between-subjects design. Because this group of studies is most closely related to my research interest (i.e. explicit mathematics vocabulary interventions for students in elementary settings), I will describe each of the elementary-level explicit definition studies in detail in the paragraphs that follow.

Table 10

Elementary Level, Explicit Definition Interventions - Research Design Characteristics

Design	Dependent Variable: Mathematics Vocabulary	Mathematics Achievement	Both
Two or more conditions, pre/post	$\boxed{11}$	$\boxed{14}$	$\boxed{15}$ 16 $\boxed{19}$
Two or more conditions, post-test only	$\boxed{18}$	17	
One-condition, pre/post			$\boxed{20}$

Note. Shapes denote type of assessment; $\square$ = researcher-created assessment; $\bigcirc$ = standardized assessment(s); $\boxed{\bigcirc}$ = researcher-created vocabulary assessment and standardized mathematics assessment(s).

Figure 2

Elementary Level, Explicit Definition Interventions – Between-subject Effect Sizes

Record	**Mathematics Vocabulary**									**Mathematics Achievement**								
						11												
						16	18								16			
			11		19	16	19	18				16		19	16			
ES	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8

Note. All effect sizes are Hedges' *g;* Multiple record identifiers in a cell indicate that more than one effect size was obtained for that outcome.

Williams (2019). Williams (2019) investigated the effects of using explicit, small group instruction on the mathematics vocabulary and mathematics achievement of 12 students in kindergarten in the southern region of the United States. The author reports in the dissertation that three of the participants received special education services and two of the participants were identified as English-language learners.

The teacher-researcher used a single-condition within-subjects design to measure the effects of providing explicit, small group instruction (details not provided) to students for six weeks. A researcher-created data sheet was used to measure change in mathematics vocabulary using pre- and post-intervention scores. The author described the data sheet as a checklist of vocabulary terms. The author reports that the participants' mathematics vocabulary improved as indicated by increased usage during instruction, in mathematics journals, and during assessments. Not enough information was provided to calculate an effect size. The researcher used the Georgia Kindergarten Inventory of Developing Skills (Georgia Department of Education, 2020) to measure effects on mathematics achievement. The students' scores improved significantly from pre- to post-test ($g = 4.96$). The author interprets the results as evidence that explicit, small group instruction increased mathematics achievement and improved students' confidence when using academic language. Limitations identified by the author include lack of a control group, small sample size, a change of interventionist mid-study, and frequent student illnesses. The author recommends that future researchers include a control group, increase the sample size, and investigate the effects of a small group intervention focused exclusively on mathematics vocabulary.

Petersen-Brown et al. (2019). Petersen-Brown et al. (2019) compared the effects of three different practice intervals when using flashcards to teach mathematics vocabulary terms to 62 students in third and fourth grades in one midwestern state. The authors did not report if any of the participants were diagnosed with a disability.

Participants were assigned to a fixed interval spaced practice group, an expanded interval spaced practice group, or a massed practice group. Data collection took place in two phases. In the first phase, participants were randomly assigned to the fixed interval spaced practice group or the expanded interval spaced practice group. Prior to beginning the second phase, the researchers decided to add a third condition – the massed practice group. The authors attempted to randomly assign participants to the three conditions but report that participants in the second phase were more likely to be assigned to the massed practice group because some participants were already assigned to fixed interval or expanded interval groups during the first phase. Twenty-two participants were assigned to the massed practice group, 19 participants were assigned to the fixed interval practice group, and 20 participants were assigned to the expanded interval practice group. All participants were taught eight mathematics vocabulary words from the Minnesota Comprehensive Assessment for one grade above their current grade level using flashcards. The flashcards were 4 x 6 inch index cards with a mathematical vocabulary term and a diagram written on one side and the corresponding definition written on the other. Each student had a second set of flashcards that were identical except for the diagram for practice and retention checks. All participants completed an initial teaching session, three practice sessions, and a retention check. The authors report that each session lasted between 15 and 20 minutes. The students in the fixed interval spaced practice group experienced an initial teaching session followed by practice sessions on days 7, 14, and 21 of the study. The students in the expanded interval spaced practice group experienced an initial teaching session followed by practice sessions on days 2, 9, and 21. The students in the massed practice group experienced three practice sessions immediately following their initial teaching session. All students participated in a final retention check seven days after their final practice session. Other than the intervals between the practice sessions, intervention procedures were the same for all of the students. The authors report that instruction and practice sessions occurred in one-to-one settings in addition to the core mathematics instruction. The researchers were responsible for delivering all instruction throughout the study.

The researchers measured effects on mathematics vocabulary using a researcher-created data sheet in which they recorded a binary score indicating whether a student retained the correct pronunciation of the term and its meaning when presented with flashcards at each practice session. They found that the students assigned to the interval groups outperformed students assigned to the massed practice group (g = 0.63). More specifically, students assigned to the fixed interval group significantly outperformed students assigned to the massed practice group (g = 0.72), while students in the expanded interval group were not significantly different than the students assigned to the massed practice group. They found no significant differences between the students assigned to the fixed or expanded interval groups after the final retention check. The authors conclude that spaced practice (either fixed or expanded) is more effective than massed practice. They acknowledge that non-random assignment of participants to conditions and possible differences between participants in each condition, the lack of general achievement data and baseline data specific to this study, and variations in the spacing schedule due to absences and school closures as limitations.

Botes and Mji (2010). Botes and Mji (2010) investigated the effects of "learner companions" (p. 127) on the mathematics achievement of students in fourth through sixth grades in South Africa. The authors describe the learner companions as similar to a printed dictionary; they present mathematical terms and visual representations in English, Afrikaans, IsiZulu, IsiXhosa, Setswana, and Sesotho so that students can access information in their primary language. The researchers supported the school personnel who were responsible for implementing the intervention for an undisclosed period to time. The authors did not report whether students with disabilities were included in the sample or if the intervention was provided as part of core mathematics instruction, supplemental instruction, or in the context of special education.

Botes and Mji (2010) employed a quasi-experimental between-subjects design with two conditions to investigate the effects of learner companions on mathematics achievement. Assignment of schools to each condition was mutually agreed upon by the researchers and the participants. Two thousand three hundred and forty-eight students participated in the study. One thousand one hundred and sixty-four students from 10 schools were assigned to the treatment condition, and 1,184 students from 10 different schools were assigned to the control condition. Treatment consisted of encouraging the students to use the learner companions, allowing the students freedom in how they chose to communicate, and using "interactive teaching strategies" daily (p. 130; the authors did not provide a description of the teaching strategies). Students assigned to the comparison condition did not have access to the learner companions and used English exclusively during their mathematics lessons (i.e. business as usual). The researchers administered a researcher-created pre- and post-test to measure the effect of the learner companions. Despite using a research design that would allow for comparisons between conditions, the authors only reported within-subjects results. They reported that the pre-test scores of students in the comparison condition were not significantly different than their post-test scores. The students in the treatment group, however, did experience a significant improvement from pre- to post-test (pre-test M = 9.89, post-test M = 10.88). The authors did not provide enough information to calculate an effect size.

The authors conclude that the results of the study indicate that learner companions help students whose first language is not English improve their mathematical vocabulary and, by extension, their mathematics achievement. They note that the study was limited in the following ways: 1) the researchers relied on the teachers to report how frequently the learner companions were used, 2) teachers often spoke different languages than their students, so they were unable to ascertain if students were accurately learning the terms in their home languages, and 3) teaching strategies may have varied between classrooms. The authors suggest researching the effectiveness of learner companions on a larger scale in the future.

Hassinger-Das et al. (2015). Hassinger-Das et al. (2015) compared the effects of incorporating children's literature into mathematics lessons using explicit instruction, a number sense intervention, and business-as-usual on the mathematics vocabulary and mathematics achievement of 124 students in kindergarten from the mid-Atlantic region of the United States. The authors report that 55% of the participants were identified as English-language learners, and 83% received free or reduced-price lunch.

In the children's literature condition, the researchers reviewed the Common Core State Standards for Mathematics in kindergarten and the mathematics curricula to identify appropriate mathematics vocabulary words to address during instruction. The researchers then created lessons based on stories with rich mathematical content. They focused on dialogic reading, direct instruction of vocabulary words, guided play, systematic review of previously learned words, and maintaining consistent definitions for terms across stories as described by Beck & McKeown (2001). During instruction, the instructor read each story multiple times, pointed out the specified vocabulary word, explicitly taught the vocabulary word and meaning to the students, led the children in applying their new knowledge by asking the children to identify additional examples of the word or participate in an activity, provided opportunities for guided-play, and then reviewed previously learned words using a board game. In the number sense condition, the instructors used an evidence-based number sense intervention. The business-as-usual condition involved typical classroom instruction and served as a control condition. All of the participating children received the core mathematics instruction available to all students. Students in the children's literature and number sense conditions received eight weeks of supplemental instruction. The lessons were taught to small groups of four students for thirty minutes three times per week by researchers. The students assigned to the control group engaged in typically scheduled non-mathematics activities while the students in the two experimental conditions received instruction.

Hassinger-Das et al. (2015) randomly assigned participants to one of three conditions. The 124 participants came from 17 kindergarten classes in four schools. Participants were randomly assigned to each condition and then randomly assigned to small groups at each school. The authors did not specify how many students were assigned to each condition. The researchers used standardized assessments to measure effects on both outcome variables. To measure effects on mathematics vocabulary, the researchers used the *Bracken Basic Concept Scale – Third Edition: Receptive: Quantity Subtest* (Bracken, 2006). To measure effects on mathematics achievement, the researchers used the Number Sense Brief (Jordan et al., 2010) and the calculation and applied problems subtests of the *Woodcock-Johnson III – Tests of Achievement* (WJII; Woodcock et al., 2007). All measures were administered as pre-, immediate post-, and delayed post-tests. The authors report that the students assigned to the children's literature condition significantly outperformed students assigned to the number sense condition (g = 0.57) and control condition (g = 0.51) on the *Bracken Basic Concept Scale* (Bracken, 2006) assessment at delayed post-test but did not show significant differences from the other conditions in the areas of number sense *(Number Sense Brief*; Jordan et al., 2010) and calculation *(WJIII*; Woodcock et al., 2007). Students assigned to the number sense intervention significantly outperformed students assigned to the control group (g = 0.21) on the *Number Sense Brief* (Jordan et al., 2010) and the calculation subtest of the *WJIII* (Woodcock et al., 2007) at immediate post-test (g = 0.59). Students assigned to the number sense condition also significantly outperformed students assigned to the children's literature condition on the calculation subtest of the *WJIII* (Woodcock et al., 2007) at immediate post-test (g = 0.58).

The authors conclude that the children's literature intervention was effective for improving mathematics vocabulary despite no significant differences between the groups being present at immediate post-test. They point to the significant positive differences between the children's literature condition and the other conditions at delayed post-test as evidence of the effectiveness of the intervention. They also note that the results related to general mathematics achievement deviate from earlier studies. They believe that this is the result of overlap between the content taught in the children's literature and number sense conditions and the mathematics curriculum used by the participating schools. The authors identify the short duration of the intervention, the possible interaction(s) between the experimental conditions and the schools' mathematics curriculum, and the presence of a possible reverse novelty effect (i.e. students in the control condition underperforming as a result of resenting that they are not a part of novel activities) as study limitations. They suggest that future researchers investigate combining more explicit numeracy instruction with mathematics vocabulary instruction and comparing the effects of using children's literature to teach mathematics vocabulary with more explicit methods.

McAdams (2012). McAdams (2012) investigated the effects on general mathematics achievement of using dictionary definitions and a graphic organizer to teach 114 students in fifth grade mathematics vocabulary words throughout the course of a school year. The author reported that 33% of the participants were identified as at-risk but did not indicate if students diagnosed with disabilities were included in the sample.

In the experimental condition, the teacher-researcher provided explicit instruction of mathematics vocabulary terms to students. The explicit instruction included having the students complete a graphic organizer to analyze a specific vocabulary term. The graphic organizer instructed the children to write each word three times, write a dictionary definition of the word, translate the dictionary definition into the student's own words, draw a picture about the word, and record a synonym, antonym, example, and non-example of the word. The teacher-researcher used the graphic organizer as part of core instruction throughout the school year. Students assigned to the control condition were taught vocabulary terms at the beginning of each instructional unit and did not have access to the graphic organizer.

McAdams (2012) employed a quasi-experimental design to compare the effects of using a graphic organizer to teach mathematics vocabulary with business-as-usual on general mathematics achievement. Intact classes were assigned to each condition. McAdams (2012) compared the two conditions using only the state standardized exam in mathematics as a post-test. The author reports that no statistically significant differences were found between the groups at post-test and did not provide enough information to calculate an effect size. Despite the lack of significant differences between the groups at post-test, the author believes the graphic organizer used in the experimental condition helped the students gain a deeper understanding of mathematical terms.

Bruun et al. (2015). Bruun et al. (2015) compared the effects of journal writing with discussion-time and a variation of the Frayer model on the mathematics vocabulary and mathematics achievement of 84 students in fourth grade in the United States. In the journal writing with discussion-time condition, the students were taught one or two mathematics vocabulary words per day for five days. The teacher taught the students the words by writing each word and definition and instructing the students to copy the word and definition into their mathematics journals. Then, the students wrote about their prior knowledge of the word(s). After writing the word(s), definition(s), and recording prior knowledge, the students discussed the word(s) with a classmate. The teacher presented multiple short discussion opportunities throughout each day. Additionally, the teacher reviewed previously learned mathematics vocabulary words throughout the study. In the modified Frayer model condition, the students had 30 vocabulary activity sheets comprised of a definition box, an example box, a non-example box, and a box for illustrations. Every day for five weeks, the students discussed a mathematics vocabulary word and definition provided by the teacher and completed one of the vocabulary activity sheets. In addition, the teacher reviewed previously learned mathematics vocabulary terms with students throughout the study. The teachers who provided the instruction in each condition were also the researchers responsible for conducting the study. The instruction provided in each condition was part of the core mathematics instruction that was available to all students. The authors did not report if students with disabilities were included in the sample but did indicate that an unspecified number of students identified as English-language learners were included.

Bruun et al. (2015) employed a quasi-experimental between-subjects design with two conditions to compare the effects of journal writing with class discussion and the modified Frayer model on mathematics vocabulary and mathematics achievement. Two classroom teachers who were also graduate students conducted this study and provided the instruction. Each of the teacher-researchers taught two classes of fourth grade students and were responsible for implementing one of the interventions. The authors did not specify how many students were assigned to each condition. The researchers administered researcher-created pre- and post-tests to measure the effects of the two interventions. The authors report that most students increased their scores on the mathematics achievement post-test and the mathematics vocabulary post-test. They note that five students, including two identified as English-language learners, did not improve their scores. Additionally, they report that the students were more confident when completing the post-tests and finished the post-tests quicker than the pre-tests. Students in the modified Frayer model condition increased their mathematics vocabulary scores by 17%, and the students in the journal writing condition increased their scores by 26%. The authors did not provide enough information to calculate effect sizes. The authors conclude that both methods of vocabulary instruction (i.e. journal writing with discussion and the modified Frayer model) positively affected students' mathematics vocabulary. They describe student motivation in both conditions and limited instructional time in the modified Frayer model condition as study limitations. Specifically, the authors felt that the students would have benefitted from more instructional time in the modified Frayer model condition but held the amount of instructional time constant to match the journal writing with discussion group.

Monroe and Pendergrass (1997). Monroe and Pendergrass (1997) compared the effects of providing mathematics vocabulary instruction using only definitions and using a Frayer model that incorporated a Concept of Definition graphic organizer (Schwartz, 1988, cited in Vacca & Vacca, 1996) paired with class discussion on the mathematics vocabulary of 58 students in fourth grade in the United States. The authors did not report if any of the participants were diagnosed with a disability. I previously described this study in the sections related to elementary interventions using implicit definitions. In this section, I will describe results in the context of the condition that used an intervention with explicit definitions.

The students assigned to the definition-only condition wrote definitions of measurement terms in their vocabulary journals as part of core mathematics instruction.

For most of the ten days of the study, the teacher-researcher provided the definitions to the students and discussed the terms with the students prior to instructing the students to write the words and definitions in their journals. For three or four days, the teacher-researcher and the students discussed the terms and worked together to compose a definition for each term. The authors note that five to ten minutes per day were allocated to providing definition-only instruction. Students in the modified Frayer model condition were guided in summarizing the measurement terms using the modified Frayer model by the teacher-researcher. The authors report that sometimes the entire class discussed and defined a term using the modified Frayer model written on butcher paper and sometimes the teacher-researcher recorded relevant comments made by individual students on the modified Frayer model and then led the class in a discussion of the comments recorded throughout the lesson. Instruction in the modified Frayer model condition lasted for five to ten minutes for each of the ten days of the study.

The participants were randomly assigned to each condition; 28 students were assigned to the definition-only condition, and 30 students were assigned to the modified Frayer model condition. The researchers used the students' vocabulary journals as a pre- and post-assessment. They coded the number of measurement concepts mentioned, the number of concepts with measurement content, the number of accurate concepts, the number of measurement applications, and the number of additional concepts mentioned that were not explicitly taught during the study. The authors' description of the analysis suggests that a rubric was used to categorize the coded entries but a rubric was not included in the report. The researchers conducted a multivariate analysis of variance to further examine the coded data. Students in the modified Frayer model condition wrote a statistically significant greater number of measurement concepts in their journals post-intervention ($g = 0.51$). Interestingly, the students in the definition-only condition more accurately applied the measurement terms in their journal entries ($g = 0.297$). The researchers did not find significant differences between the groups related to the other codes.

Despite the absence of significant differences between the groups for many of the categories and the students in the definition-only group outperforming the students in the modified Frayer model group in application of the terms, the authors interpret the results as evidence that the modified Frayer model improved students' mathematical vocabulary. The authors suggest that the results may have been different if the students had been given more time to write in their journals or if the instructions for journal-writing had been more explicit. The authors also note that the limited amount of time available to provide instruction, the possibility of unbalanced groups despite random assignment to each condition, and the limited experience of the teacher-researcher as limitations. The authors suggest that future research examine the effectiveness of the modified Frayer model with other mathematical content.

Powell and Driver (2015). Powell and Driver (2015) compared the effects of supplemental small-group addition tutoring, small group addition tutoring with a vocabulary component, and a control condition on the mathematics vocabulary and mathematics achievement of 98 first grade students in the mid-Atlantic region of the United States. The authors did not report if any of the participants were diagnosed with a disability but did indicate that the participants were identified as having "mathematics difficulties" (p. 224). They identified students as having mathematics difficulties if they scored zero or one (out of 25) points on the researcher-created screening assessment. In addition to the 67 students the researchers identified as having mathematics difficulties, the researchers were also able to randomly select 26 out of 93 students who scored two points.

Participants were randomly assigned to one of the three conditions. Thirty-eight students were assigned to the addition tutoring condition. Students in this condition experienced a flashcard activity, a tutor-led, scripted lesson, and a timed paper-and-pencil review activity. Thirty-nine students were assigned to the condition that received addition tutoring with a vocabulary component. Students in this condition experienced all of the activities that the students in the addition tutoring condition experienced as well as being introduced to or reviewing a vocabulary word each day and being asked questions during the lesson about the meaning of key vocabulary terms. Thirty-three students were assigned to the control condition. Students assigned to this condition did not receive any mathematics tutoring during the study. Students assigned to the experimental conditions participated in 15 tutoring sessions that occurred approximately three times per week across two months and lasted for 10 to 15 minutes each session. Research assistants served as tutors throughout the study.

The researchers used Addition Fluency (Fuchs et al., 2003) and Vocabulary (Powell & Driver, 2013) (a mathematics vocabulary assessment) to measure the effects of the tutoring conditions on mathematics vocabulary and mathematics achievement. Students assigned to the addition with vocabulary condition performed significantly better than students assigned to the control condition on the mathematics vocabulary measure (g = 0.49) as did students assigned to the addition tutoring group (g =. 0.64).

There were no significant differences between the students in the addition with vocabulary tutoring condition and the students in the addition tutoring condition. Students assigned to the addition tutoring group also significantly outperformed students assigned to the control group on the mathematics achievement measure (g = 0.48). There were no statistically significant differences in mathematics achievement (addition fluency) between the students in the addition with vocabulary tutoring group and the students assigned to the other groups.

The authors interpret these findings as evidence of the usefulness of explicit mathematics vocabulary instruction. They do acknowledge that the greater effect size for the students in the addition tutoring group on the vocabulary and the addition fluency assessments was unexpected and may indicate that intensive, structured instruction in a specific area of mathematics results in improved mathematics vocabulary without embedded mathematics vocabulary instruction. Other possible reasons for the unexpected results that they suggest include not devoting enough time to the mathematics vocabulary instruction, similarities in the shape-sorting activity they included in the addition tutoring condition and the directions for the addition fluency assessment, and a lack of sensitivity to growth in the vocabulary assessment.

The authors note that weather-related school closures were a limitation of this study. Some students did not see their tutors for 10 consecutive days because of the weather. They note that the results may have been different if the students were able to receive tutoring more consistently and for at least eight weeks. They also recognize the lack of maintenance data, relying on only one researcher-created assessment for each dependent variable, and assessing students using written responses exclusively as limitations. For future research, they recommend implementing interventions for longer periods of time, collecting maintenance data, using standardized assessments to measure the outcome(s), and allowing alternative methods for students to respond. The authors also note that additional research is needed on the most appropriate instruction framework(s) for teaching mathematics vocabulary. Despite reporting that they followed the recommendations of earlier vocabulary researchers, the students who received vocabulary instruction in this study did not perform as well as students who received only the addition tutoring on both the mathematics vocabulary and mathematics achievement measures. They posit that teaching mathematics vocabulary requires a different instructional framework than what past researchers have presented.

Discussion of Studies of Elementary Interventions with Explicit Definitions. The studies conducted in elementary settings to evaluate explicit definition interventions indicate that a variety of interventions and instructional approaches may be useful for improving students' mathematics vocabulary and mathematics achievement. The variability in interventions and study design also presents a number of limitations that make drawing conclusions from this body of research difficult. First, every study investigated a unique intervention. Five of the eight studies relied heavily on definition-oriented instruction (Botes & Mji, 2010; Bruun et al., 2015; McAdams, 2012; Monroe & Pendergrass, 1997; Petersen-Brown et al., 2019). Two of these five studies employed modified Frayer models (Bruun et al., 2015; Monroe & Pendergrass, 1997). Only three of the eight studies investigated interventions that incorporated principles of explicit instruction (Hassinger-Das et al., 2015; Powell & Driver, 2015; Williams, 2019). Each of these three studies implemented a different intervention. The duration of the interventions also varied. One study lasted an entire school year (McAdams, 2012). Four of the eight interventions were implemented for five to eight weeks (Bruun et al., 2015; Hassinger-Das et al., 2015; Powell & Driver, 2015; Williams, 2019). Petersen-Brown et al. (2019) reported that their study took place across 21 days, but they did not specify if these were school days or calendar days. Monroe and Pendergrass (1997) stated that their study lasted ten school days. Botes and Mji (2010) were unclear about the length of their intervention. The authors did not describe the interventions in enough detail to support replication of the studies or use of the interventions in school settings.

Second, variability is also seen in participant and setting characteristics. Four of the eight studies took place in general education settings as part of core mathematics instruction (Bruun et al., 2015; McAdams, 2012; Monroe & Pendergrass, 1997; Williams, 2019). Three of the studies provided supplemental instruction. None of the studies took place in special education settings or provided instruction exclusively to students diagnosed with disabilities. Three of the articles report including students identified as English-language learners as participants (Bruun et al., 2015; Hassinger-Das et al., 2015; Williams, 2019). McAdams (2012) reported that 33% of the participants were identified as “at-risk,” and Powell and Driver (2015) included students they labeled as having “mathematics difficulties.” Williams (2019) was the only author from this group of studies to explicitly state that their sample included participants who received special education services.

The grade-levels of the participants also varied widely. Two of the studies included participants in kindergarten (Hassinger-Das et al., 2015; Williams, 2019). Powell and Driver (2015) included participants in first grade. None of the researchers included participants in second grade. Petersen-Brown (2019) included participants in third and fourth grades, and Botes and Mji (2010) included participants in fourth through sixth grades. Bruun et al. (2015) and Monroe and Pendergrass (1997) also included participants in fourth grade, for a total of four out of the eight students that included participants in fourth grade. McAdams (2012) included participants in fifth grade. Although four of the eight studies included participants in fourth grade (Botes & Mji, 2010; Bruun et al., 2015; Monroe & Pendergrass, 1997; Petersen-Brown, 2019), all of the researchers implemented different interventions. Bruun et al. (2015) and Monroe and Pendergrass (1997) both implemented a modified Frayer model, but synthesis of the two studies is limited because each study used a different variation of the modified Frayer model.

Third, five of the eight studies relied exclusively on researcher-created assessments (Botes & Mji, 2010; Bruun et al., 2015; Monroe & Pendergrass, 1997; Petersen-Brown et al., 2019; Powell & Driver, 2019). This is problematic because researcher-created assessments are not externally validated and may be more likely to show positive effects than standardized assessments due to overalignment between the intervention and assessment(s) (What Works Clearinghouse, 2020). Two of the studies used standardized assessments. McAdams (2012) used one state's standardized mathematics assessment, and Hassinger-Das et al. (2015) used the *Bracken Basic Concept Scale – Third Edition: Receptive: Quantity Subtest* (Bracken, 2006), the *Number Sense Brief* (Jordan et al., 2010), and the *WJIII* (Woodcock et al., 2007). One study used a combination of researcher-created and standardized assessments. Williams (2019) used a researcher-created data sheet to measure the effects of explicit instruction delivered to small groups on mathematics vocabulary and the Georgia Kindergarten Inventory of Developing Skills to measure effects on mathematics achievement. The preference for using researcher-created assessments when researching interventions related to mathematics vocabulary and the lack of consistency between studies when standardized assessments are used impedes synthesizing the results of this body of literature.

Fourth, only five of the eight studies compared the effects of two or more conditions using pre- and post-tests (Botes & Mji, 2010; Bruun et al., 2015; Hassinger-Das et al., 2015; Monroe & Pendergrass, 1997; Powell & Driver, 2015), and only one of these studies used standardized assessments (Hassinger-Das et al., 2015). Two of the studies compared the effects of two or more conditions using only post-tests (McAdams, 2012; Petersen-Brown et al., 2018), and Williams (2019) employed a within-subjects group design using only pre- and post-tests. Using a within-subjects or between-subjects post-test only design is a limitation because doing so creates an opportunity for additional threats to external validity that a between-subjects pre-/post-test design is less likely to encounter (e.g. confounding variables, differences between groups before intervention).

Fifth, only four of the eight studies measured effects of the interventions on both mathematics achievement and mathematics vocabulary (Bruun et al., 2015; Hassinger-Das et al., 2016; Powell & Driver, 2015; Williams, 2019). Two of the studies only measured effects on mathematics achievement (Botes & Mji, 2010; McAdams, 2012), and two of the studies only measured effects on mathematics vocabulary (Monroe & Pendergrass, 1997; Petersen-Brown et al., 2018). Again, Hassinger-Das et al. (2015) is the only study that used standardized assessments to measure outcomes related to mathematics achievement and mathematics vocabulary. Williams (2019) used a standardized assessment to measure effects on mathematics achievement but not on mathematics vocabulary. This is a limitation because we assume that improving mathematics vocabulary will improve mathematics achievement. Without measuring the effects of an intervention on both domains using externally validated measures, we are unable to determine if this assumption is correct.

Sixth, all of the interventions in this group of studies was implemented by an interventionist with additional research training except one. Botes and Mji (2010) reported on the effects of an intervention implemented by school personnel. Three of the studies investigated interventions conducted by teachers who were completing the studies as part of requirements for an advanced degree (Bruun et al., 2015; McAdams, 2012; Williams, 2019). The teachers' participation in an advanced degree program suggests access to additional training, support, and resources that may not be available to all teachers. Although the authors did not specify that one of the researchers was a graduate student, Monroe and Pendergrass (1997) reported on a study conducted by a classroom teacher and a researcher from a nearby university. The remaining three studies included interventions that were conducted by researchers or research assistants (Hassinger-Das et al., 2015; Petersen-Brown, 2019; Powell & Driver, 2015). The lack of studies involving typically-resourced school personnel as interventionists makes drawing conclusions about the effectiveness of the interventions in real-world settings challenging. The positive effects reported may relate to the interventionists more than the interventions.

Finally, none of the studies systematically investigated the social validity of the interventions with practitioners or students, and only three of the studies documented implementation fidelity (Hassinger-Das et al., 2015; Petersen-Brown, 2019; Powell &

Driver, 2015). Hassinger-Das (2015) provided interventionists with scripted lessons and audio recorded all lessons. An undergraduate research assistant then checked a random sample of one-third of the lessons for each interventionist against the lesson scripts. All of the interventionists demonstrated at least 90% fidelity across all of the lessons. Petersen-Brown (2019) provided training and required interventionists to demonstrate 100% fidelity before allowing them to provide instruction. Interventionists used a 17-step checklist during teaching sessions and a 3-step checklist during practice and retention sessions to ensure fidelity. Observers used the same checklists to monitor fidelity of 18.9% of the teaching sessions and 14.9% of the practice and retention sessions. The interventionists demonstrated at least 99.99% fidelity across all sessions. Powell and Driver (2015) audio recorded all sessions. A research assistant then randomly selected 9.8% of the sessions to check using a 24-item checklist. The interventionists demonstrated over 97% fidelity across all sessions. Notably, all of the studies that collected fidelity data used interventions that were implemented by researchers or research assistants. None of the studies that used interventions implemented by practitioners collected fidelity data.

Rationale for the Proposed Study

CCSS-mathematics and NCTM call for elementary students to be able to communicate mathematically. Examples of communicating mathematically involve explaining reasoning and defending answers verbally and in writing. Students are unable to communicate mathematically and fully access mathematics instruction if they do not know, understand, and accurately use mathematics terminology (Garbe, 1985; Hardcastle & Orton, 1993; Miller, 1993; Milligan, 1983; Monroe & Orme, 2002; Oldfield, 1996;

Powell et al., 2020; Riccomini et al., 2008; Slavit & Ernst-Slavit, 2007; Thompson & Rubenstein, 2000). Mathematics vocabulary interventions that incorporate explicit instruction have yielded positive results (Hassinger-Das et al., 2015; Powell & Driver, 2015). However, continued research in this area is needed due to a lack of replication, infrequent use of standardized assessments to measure effects, lack of reported effect sizes, sparse implementation fidelity data, and an absence of social validity findings. One major gap in the literature is the lack of research investigating supplementary mathematics vocabulary interventions implemented with groups of students with learning difficulties and disabilities. Additionally, the majority of mathematics vocabulary interventions have been implemented by teacher-researchers or external researchers. In order to help bridge the research-to-practice gap, researchers need to examine the impact of practitioner-delivered supplementary mathematics vocabulary interventions for students with learning difficulties and disabilities. Therefore, the purpose of this study is to investigate the effects of an explicit, systematic supplementary mathematics vocabulary intervention on the mathematics vocabulary knowledge of students with learning difficulties and disabilities in a specialized setting. Through this study, I will aim to answer the following research questions:

1. What are the effects of a manualized, explicit, and systematic mathematics vocabulary intervention implemented by practitioners on the mathematics vocabulary of students with learning difficulties and disabilities in a specialized setting?
2. Will general mathematics achievement moderate any effects of the intervention on mathematics vocabulary performance at post-test?

3. With what level of fidelity will practitioners implement the intervention?

4. What are the implementing practitioners' perceptions of the intervention?

Chapter III Methods

School

I partnered with a private school for students with learning disabilities and difficulties in an urban center in the Pacific Northwest. The school employs a model drawn from Applied Behavior Analysis that involves regularly collecting student data, making data-based decisions about student learning, flexibly grouping students as their academic needs change to maximize student learning, and using research-based instructional programs and practices to build fluency and mastery across academic and behavioral domains. The school does not place students according to the traditional age-based system employed in public schools. Rather, students are placed in instructional groups of 10-12 students based on their current performance. The school uses publisher- and school-created placement tests aligned to the instructional programs used for each academic domain to assess students' current performance and needs. The students are assigned to instructional groups for each academic domain based on the results of the placement tests. The membership of instructional groups is reviewed and adjusted multiple times throughout the school year using multiple metrics to maximize student learning. Students may receive instruction from one to four teachers during a typical school day, depending on their individual needs. The school currently employs 11 teachers; seven serve as full-time teachers, one is a part-time instructional coach, one is a part-time progress monitoring coordinator, one is a part-time instructional designer, and one is a full-time permanent substitute. The school currently serves 87 students who would be in grades 1-8 in a traditional school.

Setting of Mathematics Vocabulary Instruction

The mathematics vocabulary lessons were taught to students in addition to their typical mathematics instruction in three classrooms at the partner school. Table 11 provides detailed information about the typical mathematics instruction at the partner school. I obtained the information presented in the table as part of the demographic survey administered to the participating teachers via Qualtrics (more information provided below in measures section). All of the teachers reported teaching mathematics to their students five days per week in a typical week. Two of the teachers reported teaching mathematics for 81-90 minutes on a typical day, and one teacher reported teaching mathematics for 71-80 minutes on a typical day. All of the teachers reported using “Singapore Primary Math” when asked about core instructional programs used to support teaching mathematics, and one reported also using *Essentials for Algebra*. All of the teachers reported using school-created math facts and fluency programs when asked about supplemental instructional programs or resources for teaching mathematics. Additionally, two of the teachers mentioned *Spring Math*. The teachers responded with a range of minutes when asked to estimate the amount of time spent explicitly teaching mathematics vocabulary during a typical week. Teacher A reported spending 20 minutes per week explicitly teaching mathematics vocabulary, Teacher B reported spending 25 minutes per week, and teacher C reported spending no minutes per week. Interestingly, when prompted to describe how they typically teach mathematics vocabulary, Teacher A did not provide a response, Teacher B responded with “Model/Lead/Test,” and Teacher C responded with “as necessary.”

Table 11

Typical Mathematics Instruction

Survey Item	Teacher A	Teacher B	Teacher C
1. How many days do you teach mathematics in a typical week?	5	5	5
2. How many minutes do you spend teaching mathematics on a typical day?	71-80	81-90	81-90
3. Do you typically use instructional program(s)/curricula to teach mathematics?	Yes	Yes	Yes
4. Please list the instructional program(s)/curricula you typically use.	Singapore Primary Math; Essentials for Algebra	Singapore Primary Math	Singapore Math
5. Please indicate how you became aware of each instructional program.	Current school	Professional Development	Current school

6. Do you use any supplemental instructional programs/curricula/resources when teaching mathematics?	Yes	Yes	Yes
7. Please list the supplemental instructional programs/curricula/resources you use.	School-created math facts and fluency programs; Spring math	Keys Books	School-created math facts and fluency programs
8. Please indicate how you became aware of each supplemental instructional program/curricula/resource.	Current school	Professional Development	Current school
9. How many minutes do you explicitly teach mathematics vocabulary per week?	20	25	0
10. Please describe how you typically teach mathematics vocabulary.	No response	Model/Lead/ Test	As necessary

Participants

I collaborated closely with the administration of the partner school in determining which teachers and students to invite to participate in this study. In the early stages of the partnership, I described the intervention to the administrators, shared sample lessons, and outlined the prerequisite skills and knowledge necessary to benefit from the intervention. The administrators at the partner school were enthusiastic about implementing the intervention and identified existing classes that they believed would benefit from the instruction.

The classes identified by the administrators were three classes of students 11 to 14 years old. The administrators chose these classes because they believed the students had the necessary prerequisite skills and knowledge to benefit from the intervention and the content of the intervention would address gaps in learning demonstrated by the students on progress-monitoring and placement assessments, as well as classroom assignments and instructional program assessments. I then invited the teachers assigned to those classes and all of their students to participate in the study. The only inclusion criterion for the teacher participants was recommendation from the school's administrators. The only exclusion criteria for student participants was verbal ability; due to the reliance on oral responses in the intervention, the students must be able to answer verbally. None of the students assigned to the identified classes were non-verbal, so we did not apply this exclusion criteria.

Study Personnel

Principal Investigator

I, a doctoral candidate in the Department of Special Education and Rehabilitation Counseling specializing in Special Education, served as lead-author during the development of the intervention. I have been dual-certified for K-8 elementary education and P-12 special education in the state of Washington since 2007. During my time in education, I have served as a substitute teacher, K-5 Resource Teacher, a middle school humanities teacher, an instructional coach, field supervisor and field coordinator for students obtaining K-8 elementary education and P-12 special education dual-certification, and an adjunct lecturer in an elementary education/special education dual-certification program. During this study, I took responsibility for training the teachers in all study procedures, conducting the majority of the fidelity checks, providing feedback to the teachers after fidelity checks, managing data collection, scoring all pre and post-assessments, and analyzing the data produced during the study.

Research Assistants

One research assistant assisted me during the study. The research assistant is currently a second-year doctoral student in the Department of Special Education and Rehabilitation Counseling specializing in Special Education. He has a background in Applied Behavior Analysis, has experience coaching teachers and conducting research, and has been involved in writing the mathematics vocabulary lessons. He contributed to the development of study procedures, conducted fidelity checks, provided feedback to teachers after fidelity checks, and independently double-scored 50% of pre- and post-tests.

Interventionists

Three full-time classroom teachers from the partner-school served as the interventionists during the study. They were fully trained in study procedures, assessment administration, and intervention implementation prior to beginning the study. The partner-school's full-time substitute was also fully trained in study procedures, assessment administration, and intervention implementation. The full-time substitute assisted with administering the pre- and post-tests during the study and was available to teach mathematics vocabulary lessons if any of the classroom teachers were absent. None of the classroom teachers were absent during the study, so the full-time substitute teacher did not teach any lessons. More detailed demographic data for the participating teachers is presented in Table 12 of chapter 4.

Dependent Variables

The primary dependent variable was mathematics vocabulary performance as measured by *Mathematics Vocabulary – Grade 3* (Powell, 2016). Students were asked to independently answer a variety of questions designed to measure their understanding of vocabulary words from different strands of mathematics (e.g., geometry, number sense, operations, measurement, etc.) using various response forms (e.g., matching, labeling, simple drawing, etc.). Teachers' implementation and perceptions of the social validity of the mathematics vocabulary lessons were secondary dependent variables.

Measures

Teacher Demographic Survey

I requested that the teachers complete a demographic survey via Qualtrics after obtaining informed consent. The survey included items that asked teachers to report their name, gender, highest level of education, and number of years as a teacher. The survey also included items intended to obtain more information about typical mathematics (including mathematics vocabulary) instruction. The teachers were allowed to skip any item(s) they wished. A copy of the survey may be found in Appendix B.

Student Demographic Survey

After obtaining informed consent and youth assent, I requested that the parents/guardians of the participating students complete a demographic survey. The demographic surveys were distributed to parents/guardians using hard copies and via email. I chose to distribute surveys in both ways so the parents/guardians could choose their preferred method of responding. The survey included items that asked the respondents to report their child's name, teacher, age, gender, race/ethnicity, special education status and qualifying category, if applicable, parents/guardians' highest level of education, parents/guardians' marital status, parents/guardians' annual income, and language(s) spoken at home. The respondents were allowed to skip any item(s) they wished. A copy of the survey may be found in Appendix C.

Mathematics Achievement

Teachers administered *Proficient Math 4_Winter* (University of Oregon, 2014) as a pre-test (visit www.easycbm.com for more information and sample test items). It is the winter form of a series of three standardized, norm-referenced benchmarking assessments designed for administration to students in fourth grade in fall, winter, and spring. It is comprised of 40 items that assess general mathematics achievement in alignment with Common Core State Standards (University of Oregon, 2016). The assessment is group-administered and untimed. Items are multiple choice and scored as 0 if incorrect and 1 if correct. Forty points are possible.

Mathematics Vocabulary

Teachers administered *Mathematics Vocabulary – Grade 3* (Powell, 2016) as the pre- and post-test (Appendix E). It is comprised of 45 items and sub-items that assess student understanding of mathematics vocabulary words commonly found in core mathematics programs for third grade and the Common Core State Standards – Mathematics (CCSS – M). The assessment is group-administered. Students have twenty minutes to independently answer as many items as possible. Items are scored as a 0 if incorrect and a 1 if correct. Forty-five points are possible.

Powell (2016) designed *Mathematics Vocabulary – Grade 3* as a way to assess the mathematics vocabulary knowledge of students in third grade. To create the assessment, Powell reviewed the glossaries of the three most-popular core mathematics instructional programs in the U.S. and the CCSS-M. Words were included on the assessment if they appeared in the glossary of one of the reviewed programs or in the CCSS-M. The author of the assessment also included words not found in the glossaries or the CCSS-M but deemed necessary for engaging in mathematics instruction in third grade.

Mathematics Vocabulary – Grade 3 was used by Powell et al. (2017) to analyze the associations between general vocabulary, mathematics vocabulary, and mathematics computation. They report Cronbach's α of .92 for the sample of students in third grade who participated in the study. Additional reliability and validity data are unavailable. The proposed study presents an opportunity to investigate the validity of *Mathematics Vocabulary – Grade 3* as a pre-test and outcome measure.

Although *Mathematics Vocabulary – Grade 3* was designed to measure the mathematics vocabulary of students in third grade, this assessment was appropriate for this study because it is designed to measure vocabulary performance for the grade-level of mathematics vocabulary addressed in the lessons. Additionally, I am unaware of any other research-validated mathematics vocabulary assessments appropriate for this study.

Social Validity

I obtained social validity data from the teachers using two sets of survey items. The first set of items was administered as part of the demographic survey and may be found in Appendix B. These items requested information about teachers current mathematics vocabulary instructional practices (i.e. time spent explicitly teaching mathematics vocabulary and approaches for teaching mathematics vocabulary) and their perceptions of the importance of mathematics vocabulary. The items addressing the teachers' perceptions of the importance of mathematics vocabulary were Likert-type items that used a five-point scale (1-strongly disagree to 5-strongly agree). The items included: 1) Mathematics vocabulary is critical for students to understand mathematics instruction, 2) Mathematics vocabulary is critical for students to participate in mathematics instruction, 3) Mathematics vocabulary is critical for students to engage with mathematics in and out of the classroom, and 4) Students need to master mathematics vocabulary before moving to the next grade.

Teacher Acceptability of Intervention

I measured teacher acceptability of the instruction using an online survey with a mix of Likert-type items that used a five-point scale (1-strongly disagree to 5-strongly agree) and open-ended items. A copy of the survey may be found in Appendix F. Likert-type survey items included: 1) The amount of time required to teach the mathematics vocabulary lessons was reasonable, 2) The lessons were clearly written and easy for me to understand, 3) The mathematics vocabulary words included in the lessons are necessary for students to understand and engage with mathematics, 4) My students received enough practice using the mathematics words during the lessons, 5) Choral responding is an effective way to provide multiple practice opportunities when teaching mathematics vocabulary, 6) My students were engaged during the mathematics vocabulary lessons, and 7) My students enjoyed the mathematics vocabulary lessons. Open-ended items included: 1) What did you like most about the mathematics vocabulary lessons? Why?, 2) What would you change about the mathematics vocabulary lessons? Why?, 3) Are there additional words that you think need to be included in the lessons? If so, record them and provide a brief explanation for the need to include each word, 4) Are there words that you think should be removed from the lessons? If so, please record them and provide a brief explanation for each word, and 5) Would you use these or similar lessons to teach mathematics vocabulary again in the future? Why or why not?

Independent Variable

The independent variable was an intervention that includes 47 words and/or concepts taught over the course of twenty-two mini-lessons. I developed the intervention with assistance from two faculty members and two doctoral students. We drafted an initial version of the program during the 2019-2020 and 2020-2021 academic years. We then field-tested portions of the program in four inclusive general education classrooms during the spring of 2021. We collected pre- and post-test data using *Mathematics Vocabulary – Grade 3* (Powell, 2016) to examine the effects of the instruction, observation data to investigate implementation and gain student feedback, and survey data to gain teacher feedback. We used all of the data throughout the summer and fall of 2021 to revise and finish the program (Rolf et al., 2022).

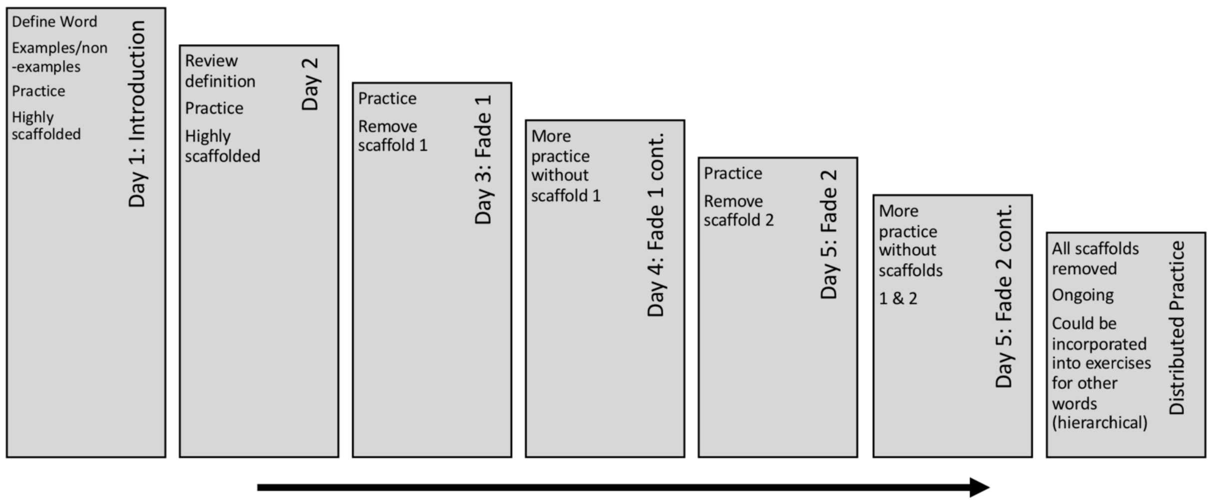
The program is designed for teaching one mini-lesson per day to students who have completed third-grade mathematics instruction. Each lesson was designed to require no more than 15 minutes. To select words for inclusion in the intervention, the research team reviewed the *Mathematics Vocabulary – 3rd Grade* assessment created by Powell (2016). We also added a small number of words/concepts that serve as prerequisites for the words identified by Powell (2016) or that the research team deemed critical. Please see Appendix G for a list of the included words in the order that they are introduced.

The mathematics vocabulary lessons are designed for students who have completed third-grade mathematics instruction. We elected to focus on students beyond third grade because we believe there are two ideal times to teach mathematics vocabulary: when a concept or procedure is first introduced and taught or after the initial teaching as review or remediation. The relation between the conceptual nature of mathematics, its procedures, and its vocabulary make pre-teaching mathematics vocabulary difficult, if not impossible. Without some level of background knowledge, fully teaching a concept and/or its related procedures becomes necessary when attempting to pre-teach mathematics vocabulary. In recognition of that characteristic of mathematic vocabulary, we elected to create lessons designed to serve as review or remediation. The lessons assume that the students have encountered the words and their associated concepts before but have not mastered them. The goal of the instruction, therefore, is to build on the prior knowledge of the students in a way that refines their understanding of each mathematics vocabulary word, provides ample practice using the words in context, and results in mastery of the included words.

We designed the intervention according to the instructional design principles described by Engelmann and Carnine (1982/2016). As a result, each word is explicitly and systematically taught in a student-friendly manner. Figure 3 illustrates the general process for introducing a word, providing application practice, and reviewing.

Figure 3

Process for Introducing and Teaching a Word Across Lessons



When first introduced, the meaning of each word is explained using student-friendly language. Typically, three examples and/or non-examples are presented using careful teacher wording designed to promote student understanding efficiently and avoid adding unrelated difficulty to the task. The purpose of these initial examples and non-examples is modeling the application of a word and/or definition. Multiple intentionally scaffolded examples and non-examples follow with the purpose of guiding students through practicing the application of a word and/or definition.

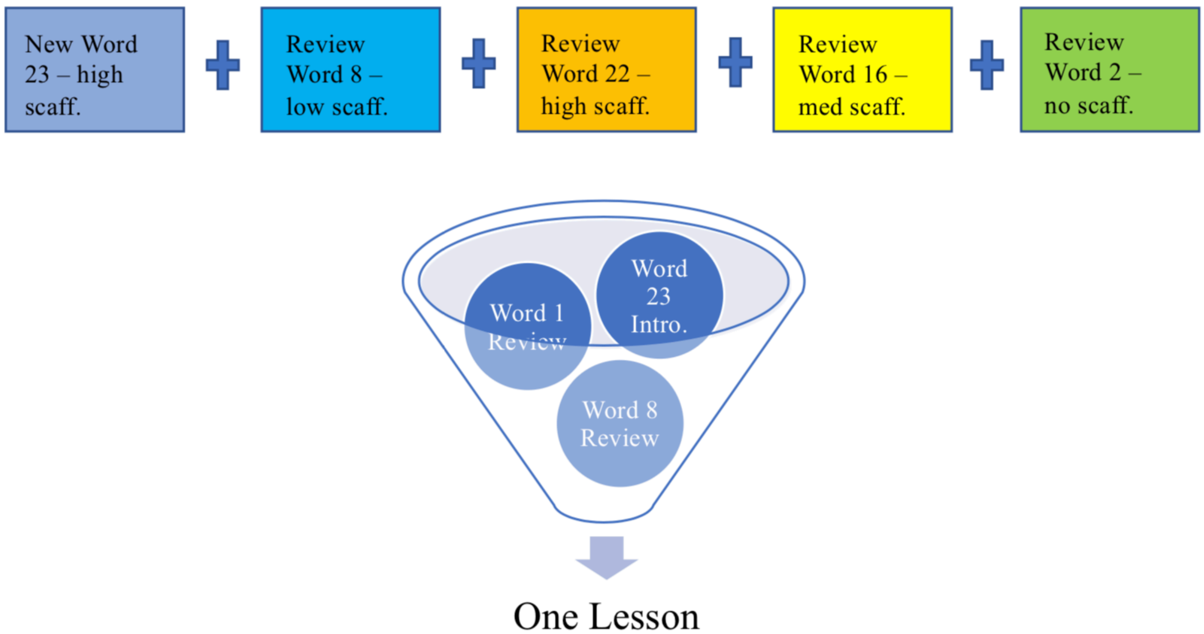
Multiple practice and review exercises are included in lessons that follow the introduction of each word. The scaffolding included in the introductory exercise for each word is systematically removed to increase students' independence over a number of days (typically two to four days depending on the word). After all of the scaffolding is removed, each word frequently appears in review exercises throughout the duration of the intervention. By taking advantage of the relations between mathematical concepts included in the intervention (e.g. rhombus, parallelogram, and quadrilateral), we provide review exercises that efficiently use instructional time to review multiple previously taught concepts.

Figure 4 illustrates the process for building each lesson after designing the instruction described above (i.e., introduction through complete removal of scaffolding for each word). Each lesson is comprised of multiple exercises. One exercise provides instruction for one or more words. An exercise may introduce a new word, provide review for a previously introduced word, or do both. Multiple items are found within each exercise. An item is a question or prompt given by the teacher with the purpose of eliciting a response from the students.

First, we identified introductory exercises that must be included in each lesson. Then, we identified exercises that must be included in each lesson to facilitate the removal of scaffolding for recently introduced concepts. Then, we identified review exercises for words that were previously mastered that must be included. Throughout this process, we attended to relations between concepts and designed exercises that use and reinforce those relations. For example, “Review Word 2” in the figure below may represent “rhombus,” one of the first words introduced in the intervention. “Review Word 8” may represent “parallelogram,” and “New Word 23” may represent “quadrilateral.” When designing instruction to introduce “quadrilateral,” we included “rhombus” and “parallelogram” as review, thereby maximizing instructional time and reinforcing relations between concepts. The other words included in the lesson (represented by “Review Word 22” and “Review Word 16”) may be unrelated to the new concept(s) being introduced but are included to ensure frequent review. The figure below also notes the varying levels of scaffolding that may be present in each exercise depending on how recently a concept was introduced. The exercises written for each of the concepts identified for inclusion in any given lesson were logically combined to create each lesson.

Figure 4

Building a Lesson



Contents of the Intervention

The intervention includes 47 words organized into six strands (i.e., geometry, measurement, number composition, fractions, data, and operations). Appendix H presents the words organized by strand and sub-strand. The geometry strand is further divided into three sub-strands: two-dimensional shapes, three-dimensional shapes, and components of shapes. The operations strand is further divided into four additional sub-strands: addition, subtraction, multiplication, and division. Organizing the words by strand and sub-strand highlights relations between terms (both within and between strands) that build student understanding and increase the efficiency of instruction. In the following sections, I describe our instructional design approach for each strand and sub-strand.

Data

The data strand consists of four words (tally chart, pictograph, bar graph, and dot plot). We rely primarily on a visual identification approach supplemented with a brief explanation in this strand to maximize the efficiency of the intervention. For example, when introducing "tally chart," we present a picture of a tally chart and explain that the tally chart helps keep track of things that have been counted; one line (i.e. tally) is recorded for each counted item. We then proceed to provide additional examples and non-examples before asking the students to identify if an item is or is not a tally chart. To confirm students' understanding of the purpose of a tally chart, each example of a tally chart is followed by a simple interpretation question (e.g. "How many students chose candy?"). The instruction for pictograph, bar graph, and dot plot follow this same general pattern while also incorporating review of previously introduced data terms. For example, dot plot is the last word introduced in the data strand. Non-examples of dot plot include tally charts, pictographs, and bar graphs. Rather than only having the students identify if something is or is not a dot plot, we have the students name the non-examples. This feature ensures that the students continue to review previously learned words.

Fractions

The fraction strand includes five words: fraction, denominator, numerator, unit fraction, and equivalent fractions. Our approach to instructional design varied with each word. The first word introduced is “fraction.” When designing instruction for the word “fraction,” we focused on developing conceptual understanding of fractions in addition to visual identification of fractions. Specifically, we include instruction that the bottom numeral of a fraction always tells how many parts are in a whole, and the top numeral always tells how many parts are used. We provide illustrations of fractions to support developing conceptual understanding and numerical representations to support identification of fractions.

Within a few lessons of introducing “fraction,” we introduce “denominator” and then “numerator.” We elected to avoid teaching “numerator” and “denominator” with “fraction” to allow the students to only focus on one concept and word at a time. We elected to separate the introduction of “denominator” and “numerator” to avoid unnecessary confusion for the students. “Denominator” and “numerator” sound similar which may lead to confusion for the students if the terms are introduced to close to each other. Upon introducing “denominator” and “numerator,” the lessons stop referring to the parts of the fractions as “top numeral/number” and “bottom numeral/number” and start referring to them using their technical names. We employed a substitution approach when designing instruction for these two terms. Because the students receive instruction on the conceptual meaning of the parts of the fraction when they complete the lessons teaching “fraction,” the most straightforward path is to teach the students that the “bottom number/numeral” is actually called the “denominator,” and the “top number/numeral” is actually called the “numerator.” Additional teaching of concepts related to these elements of fractions is unnecessary because the students will receive generalizable concept instruction during the previous “fraction” lessons.

We use a visual identification approach supplemented with rule-application to teach “unit fraction.” A unit fraction is a fraction with a numerator of one, making them easy to identify. The lesson provides this definition and multiple examples and non-examples of a unit fraction. The students then engage with practice opportunities for identifying a unit fraction. In this case, the examples and non-examples provide most of the instruction in identifying a unit fraction. The rule supplements the reliance on modeling through examples and non-examples. We took this approach because unit fractions are relatively simple to identify and the conceptual teaching the students will receive during the prior lessons on “fraction” still applies.

We returned to a more conceptually-oriented approach to instructional design when writing lessons for “equivalent fractions.” The lessons for “equivalent fraction” provide an illustration of two fractions with accompanying numerical representations. The students are then led through identifying if the two fractions are equivalent or not equivalent. This approach is necessary to ensure the students understand the meaning of equivalent fractions. The alternative to this approach would be to teach the students to complete computations to determine if two numerically-represented fractions are equivalent. We felt the computational approach would not adequately address the meaning of “equivalent fractions” and, because of the necessity of teaching students to compute equivalent fractions, would fall outside of the objectives of this intervention.

Geometry

The geometry strand includes 19 words and/or concepts organized into three sub-strands (two-dimensional shapes, three-dimensional shapes, and components of shapes). Our approach to instructional design varied with each of the sub-strands. We combined a rule-application approach and a visual identification approach when designing lessons for the two-dimensional sub-strand. This approach involves providing a student-friendly definition, or rule, for identifying a shape, providing an example of the shape, and guiding students through applying the rule to correctly identify the shape as an example or non-example. For example, the introductory lesson for “rhombus” defines rhombus as a shape with four sides that are the same length, shows a rhombus, and guides students through applying the definition to an example to verify that it is a rhombus. After modeling with a series of examples, the instruction includes non-examples randomly presented throughout the lesson. The scripting allows the teacher to systematically support the students in application of the rule to determine if a shape is or is not a rhombus. This dual approach (rule application and visual identification) to instructional design supports the students to learn to visually identify a shape and communicate the features of the shape. These dual outcomes are especially important for the two-dimensional sub-strand because of the hierarchical nature of the included shapes. Many of the shapes can be called by more than one name (e.g., a rhombus is also a parallelogram, quadrilateral, and a polygon). As the lessons progress, the students use the rules and visuals to identify all of the relevant names of shapes.

We relied on a visual identification approach exclusively when designing instruction for the components of shapes and three-dimensional shapes sub-strands. We did this to maximize the efficiency of the intervention. We include far fewer three-dimensional shapes in the intervention, and teaching hierarchical relations among the three-dimensional shapes is unnecessary at this point in students' development. The research team determined that the more time-consuming rule-application approach was unnecessary, based on the features of three-dimensional shapes that are included in the intervention. Similarly, components of shapes (i.e., face, vertex, edge) may be efficiently and appropriately taught without relying on a rule-application approach at this stage in students' development.

Measurement

The measurement strand includes four words (perimeter, area, angle, and right angle). We employ a visual identification approach for instruction in this strand and focus on developing students' conceptual understanding related to each term. For example, we highlight the perimeter and area on shapes to demonstrate the concepts of perimeter and area but do not teach procedures for calculating perimeter and area. Similarly, we teach students to discriminate angles from lines, line segments, and rays, and to discriminate between right angles and angles, but do not teach students how to measure angles.

Number Composition

The number composition strand includes expanded form and standard form. The lessons use a visual identification approach to teach students to identify numbers presented in standard and expanded forms. Initially, the lessons present the students with a number written in standard form and label it as such (e.g., 48). The lessons then present the same number in expanded form alongside the number presented in standard form and label it as such (e.g., 40 + 8). The lessons follow this pattern with additional examples before guiding the students in identifying numerals as written in standard or expanded form.

Operations

The operations strand includes 13 words that span addition, subtraction, multiplication, and division. We combine a visual identification approach with a rule-application approach for instruction of words in this strand. We designed the lessons to teach students to label parts of mathematics problems (e.g., addend, sum, factor, quotient, etc.) and explain what each part communicates using student-friendly language (e.g., "A sum is what you get when you add.").

Instructional Delivery

Lesson Formatting

We systematically designed the lessons to provide explicit introductions, frequent practice opportunities, and abundant review of each word. To accomplish this, we carefully sequenced the introduction of the words throughout the program. We also carefully attended to the sequencing of examples and non-examples during the introductory lesson for each word. We ensure frequent practice opportunities by requiring the students to apply and say the words during the introductory lesson. Then, the word appears in subsequent lessons for additional practice and review. During this period of practice and review, the scaffolding employed when first introducing the word is systematically removed. This takes anywhere from two to six lessons, depending on the particular word being taught. After the scaffolding is completely removed, the word continues to appear in review exercises at least every fourth lesson. By following this careful approach to instructional design, we ensure that each word is clearly introduced and practiced and applied frequently as a result of systematic ongoing review.

To facilitate the delivery of instruction to the students and ensure the elements described in the preceding paragraph were implemented as intended, we included lesson scripts. A sample lesson script is included in Appendix I. Teacher wording is bold. Directions for the teachers are italicized. Visuals of a group of students or a single student are in green and alert the teacher to signaling for a unison group response or asking an individual to respond. Student responses are purple and on the right side of the lesson. Visuals are included at the point in the lesson when they are needed along with a slide number. Teachers used GoogleSlides to present the accompanying visuals to the students.

Student Responses

The lessons rely heavily on oral responses from the students. As short lessons designed to ensure students have mastered critical mathematics vocabulary words necessary for upper-elementary mathematics instruction, it is important that the lessons be quick and easy to implement. Relying on oral responses allows teachers to provide instruction without using valuable classroom time to manage student materials.

The lessons frequently include two types of student responses: group unison (or choral) responses and individual responses. Group unison responses are ideal because they provide the most practice to the greatest number of students. Rather than having one student answer a question while the others sit passively, group unison responses require all students to be attentive and engaged throughout each lesson. Teachers obtain group unison responses with a signal to the students that it is time to respond. Effective signals vary by teacher but often include snapping, dropping a hand, tapping, or lightly clapping. Questions with straightforward, short, and obvious answers are appropriate for group unison responses.

The lessons employ individual responses when the questions or items are not suitable for a group unison response. Typically, the lessons include individual responses when students are asked to explain their thinking in response to questions such as "Why?," "Why not?," or "How do you know?" These types of questions are not suitable for group unison responses because of the varied responses they are likely to produce.

Correcting Errors

Errors are an expected part of learning anything new. As such, the lessons anticipate student errors by providing specific error correction procedures. Generally, correcting errors involves modeling the correct response, testing students by presenting the item again, and testing students on the missed item again later in the lesson (i.e., delayed test).

Study Design

I conducted a quasi-experimental study with randomization occurring at the level of the student. Figure 5 provides a visual of the study activities and the order in which they occurred. All teachers agreed to teach the mini-lessons, and students within each instructional group were randomly assigned to treatment or control conditions. *Proficient Math 4_Winter* (University of Oregon, 2014) and *Mathematics Vocabulary – 3rd Grade* (Powell, 2016) were administered as pre-tests to all participating students regardless of their assignment to treatment or control conditions. The students assigned to the treatment condition received the mathematics vocabulary instruction in addition to their typical mathematics instruction, and the students assigned to the control condition engaged in alternate mathematics activities in addition to their typical mathematics instruction. The partner school developed the alternate mathematics activities. They primarily were designed to build procedural fluency and/or problem-solving through modeling and were not intended to teach or practice mathematics vocabulary. Students assigned to the control condition completed the alternate mathematics activities at the same time students in the treatment condition received the mathematics vocabulary instruction. Students assigned to the treatment condition received the intervention in their usual classrooms, and students assigned to the control condition engaged in the alternate activities in a different room at the school. After all of the mathematics vocabulary lessons were taught, all students completed *Mathematics Vocabulary – 3rd Grade* (Powell, 2016) as a post-test.

Figure 5

Study Design

Teacher Social Validity Survey 1	**Condition**	**MA Test**	**MV Test 1**	**Intervention**	**MV Test 2**	**Teacher Social Validity Survey 2**
	Treatment	X	X	MV Intervention	X	
	Control	X	X	Alternative Activities	X	

Note. MA = Mathematics Achivement; MV = Mathematics Vocabulary

Data Analysis

Research Questions One and Two: 1) What are the effects of a manualized, explicit, and systematic mathematics vocabulary intervention implemented by practitioners on the mathematics vocabulary of students with learning difficulties and disabilities in a specialized setting?, and 2) Will general mathematics achievement moderate any effects of the intervention on mathematics vocabulary performance at post-test?

Prior to answering the first and second research questions, I compared the treatment and control conditions to ensure comparability between the conditions demographically and in regards to mathematics achievement and mathematics vocabulary pre-test performance. I used Fisher's exact test to make demographic comparisons between conditions because some of the expected values for a Chi-square test of independence were less than five (Kim, 2017). I used *t*-tests to compare the means of the treatment and control conditions on the mathematics achievement and mathematics vocabulary pre-tests.

I used Ordinary Least Squares multiple regression to answer Research Questions One and Two. Given the nested nature of the data (i.e. students assigned to teachers within a school), a multi-level model would be appropriate for analyzing the data. Multi-level models require a larger sample size than was available for this study (Hox, 2018). Although not as sensitive to the clustering of the students, Ordinary Least Squares multiple regression is still a useful tool for analyzing the data with minimal variance (Field, 2018).

To answer Research Question One, used a bottom-up approach when fitting models. First, I fit the intercept-only model. Then, I added condition assignment as a predictor. I was curious if teacher influenced the results, so I fit a model with teacher as the only predictor. Then, I fit a model with condition assignment and teacher as covariates. Finally, I fit a model with an interaction between condition assignment and teacher. I examined the adjusted R^2 values to determine the best model for the data.

To answer Research Question Two, I again used a bottom-up approach when fitting models. As with Research Question One, I first fit the intercept-only model and then the model that included condition assignment as the only predictor. I fit a third model with mathematics achievement pre-test score as the only predictor. I then fit a model with condition assignment and mathematics achievement pre-test score as covariates before fitting a final model with an interaction between condition assignment and mathematics achievement. I conducted all analyses for these research questions using R (R Core Team, 2022).

Research Question 3: With what level of fidelity will practitioners implement the intervention?

To answer Research Question 3, we rated teachers on their implementation of six lessons using a fidelity checklist (Appendix D), and we collected data on the time required to teach lessons. I used descriptive statistics to analyze the implementation data, including the mean and range of time required to teach each observed lesson by teacher. I calculated the scripted student response rate for each observed lesson by dividing the number of scripted response opportunities by the number of minutes required to teach the lesson. I also calculated the mean percent of lesson components fully implemented across the six observed lessons for each teacher and as a group.

Research Question 4: What are the implementing practitioners' perceptions of the intervention?

To answer Research Question 4, I analyzed the social validity data by calculating the mean, median, and range of the responses to the Likert-type items on both sets of social validity items (see Appendices B and G). I used thematic analysis to analyze data from the open-ended items. Thematic analysis involves identifying themes in textual data and is appropriate for written responses to open-ended questions (Glesne, 2016).

Experimental Procedures

Recruiting Participants

Teachers. The executive and assistant director of the partner-school identified in-tact instructional groups they believed would benefit from the intervention. They arranged for me to contact the potential teacher participants. After receiving USU Institutional Review Board (IRB) and school approval, I met with the potential teacher participants to provide more information about the study and distribute hard copies of informed consent documents. I then emailed the teachers a demographic survey (Appendix B) that included a link to provide informed consent. The only inclusion criterion for the teacher participants was recommendation from the partner school's administrators.

Students. After receiving USU IRB and school approval, I trained the assistant director in obtaining informed consent and youth assent from potential participants. The assistant director of the partner-school then distributed a cover letter, informed consent, and youth assent documents to the parents/guardians of all potential student participants via email and hard copies. The email version included a link to a Qualtrics survey for providing informed consent. I elected to distribute the recruitment documents via email and hard copy to provide parents/guardians the opportunity to respond in their preferred mode. We invited all students assigned to the instructional groups identified by the executive and assistant director to participate. The assistant director forwarded two reminder emails from me to the parents/guardians of students who did not return the informed consent and youth assent forms. The first reminder email was sent approximately one week after the initial invitation was distributed, and the second reminder email occurred approximately one week later.

The parents/guardians who provided informed consent via the Qualtrics survey were invited to complete the demographic survey (Appendix C) immediately after their child(ren) provided their assent. The parents/guardians of the students who returned hard copies of the informed consent and youth assent forms received email invitations to complete the demographic survey via Qualtrics at a later time. The assistant director forwarded an email from me to the parents/guardians for whom we did not have demographic information. One follow-up email was sent to the non-responding parents/guardians approximately one week after the initial email invitation was sent.

Group Assignment

Randomization occurred at the level of the student. Students within in-tact instructional groups were randomly assigned to treatment or control conditions by the assistant director of the partner-school. Shortly before beginning the intervention, four students initially assigned to the treatment condition tested positive for COVID-19 and had to quarantine. The assistant director randomly swapped these four students with four students who were initially assigned to the control condition.

Training

Assessment Administration. I trained the participating teachers prior to the administration of the *Mathematics Vocabulary – 3rd Grade* (Powell, 2016) and *Proficient Math 4_Winter* (University of Oregon, 2014) assessments. Training took place in person at the partner-school on a professional development day and took approximately 30 minutes. I oriented all of the participating teachers to the study's purpose and procedures. I then trained them in administration of the two pre-tests. Key points of the training included the standardized, scripted instruction to be read to students, appropriate test accommodations, time limits for completion, and procedures for returning the completed assessments to me.

Intervention Content and Delivery. I trained all participating teachers on the intervention content and delivery in person at the partner-school during a professional development day. The training required approximately 90 minutes. I provided an overview of the mathematics vocabulary instruction, highlighted features of each lesson (e.g., objectives, estimated instructional time, organization), provided a rationale for using scripted lessons, modeled and provided time to practice obtaining group unison responses and correcting errors effectively, and pre-corrected possible instructional delivery and/or student response errors.

Pre-test Administration

I used *Proficient Math 4_Winter* (University of Oregon, 2014) as a pre-test measure of general mathematics achievement. *Proficient Math 4_Winter* is a standardized, norm-referenced benchmarking assessment designed to measure students general mathematics performance in the winter of an academic year. After receiving training on test administration, the teachers administered the untimed pre-test to all of their students prior to teaching any of the mathematics vocabulary mini-lessons, in accordance with the test administration procedures described above.

I used *Mathematics Vocabulary – 3rd Grade* (Powell, 2016) as a pre-test measure. *Mathematics Vocabulary – 3rd Grade* is a standardized, research-validated assessment designed to measure students' mathematics vocabulary performance. After receiving training on test administration, the teachers administered the pre-test to their students prior to the start of the mathematics vocabulary instruction and in accordance with the test administration procedures described above.

Intervention Administration

The participating teachers taught the mathematics vocabulary mini-lessons to their students assigned to the treatment condition in addition to the typical mathematics instruction (described above) they provided to all of their students. The teachers taught one mini-lesson per day throughout the intervention period. The students assigned to the control condition engaged in alternative mathematics fluency activities (e.g., math facts, procedural fluency with basic operations, modeling word problems using bar models) in a separate room while the students assigned to the treatment condition participated in the mathematics vocabulary instruction. No mathematics vocabulary instruction occurred for the students assigned to the control condition. The teacher who supervised the students assigned to the control condition was the permanent substitute who was trained in all study procedures.

Post-test Administration

Mathematics Vocabulary – 3rd Grade (Powell, 2016) served as the outcome measure. The teachers administered the assessment to all of their students, following the same administration procedures as the pre-test.

Social Validity Survey Distribution

I emailed the teachers an invitation to complete the social validity survey after they administered the post-test. The invitation included a link to complete the survey via Qualtrics.

Treatment and Assessment Fidelity Procedures

Treatment Fidelity. The research assistant and I conducted fidelity checks in-person using a researcher-created checklist. (Please see Appendix D.) The checklist includes items related to time required to teach each lesson, presenting all lesson components, obtaining student responses, correcting errors, presentation style, and teacher adaptations of the lessons. Additionally, the checklist includes a section for open-ended notes for providing additional context. I selected six lessons (27%) for observation per teacher (i.e., lessons two, six, eight, thirteen, seventeen, and twenty-one). I attempted to spread the observations across the intervention period in order to obtain implementation data representative of multiple points in the program and to facilitate providing performance feedback to the teachers. I conducted observations for lessons two, six, eight, thirteen, and seventeen, and the research assistant conducted observations for lesson twenty-one. All three implementing teachers were observed during each observation. Due to challenges presented by traveling out-of-state and COVID-19 restrictions, I was unable to collect interobserver agreement data during fidelity checks.

Interrater Agreement on Mathematics Achievement Assessments. I scored 100% of the assessments, and the research assistant independently double-scored 50% (n = 15) of the pre-tests using a scoring key. I used a random number generator to randomly select the pre-tests for double-scoring. Prior to independently double-scoring, I trained the research assistant on scoring procedures. There were 40 possible agreements for each double-scored mathematics achievement test. Our initial rate of agreement was 99.67%. We came to consensus on all disagreements.

Interrater Agreement on Mathematics Vocabulary Assessments. I scored 100% of the assessments, and the research assistant independently double-scored 50% (*n* = 15) of the pre- and post-tests using a scoring key. I used a random number generator to randomly select the pre- and post-tests for double-scoring. Prior to independently double-scoring, I trained the research assistant on scoring procedures. He then independently scored two practice assessments. I compared his scores to mine and calculated agreement (number of agreements/number of possible agreements). There were 45 possible agreements for each double-scored mathematics vocabulary test. Our initial agreement was 97.78%. We discussed and came to consensus on all disagreements. Because our initial agreement was above 90%, we proceeded with independently double-scoring the remaining pre- and post-tests. Our initial rate of agreement for the remaining pre-tests was 97.78%, and our initial rate of agreement for the post-tests was 98.37%. We came to consensus on all disagreements.

Chapter IV Results

The purpose of this study was to investigate the effects, teacher-implementation, and social validity of a manualized, explicit, and systematic intervention for teaching mathematics vocabulary necessary for fourth grade and beyond. Specifically, the study addressed the following questions:

1. What are the effects of a manualized, explicit, and systematic mathematics vocabulary intervention implemented by practitioners on the mathematics vocabulary of students with learning difficulties and disabilities in a specialized setting?

2. Will general mathematics achievement moderate any effects of the intervention on mathematics vocabulary performance at post-test?

3. With what level of fidelity will practitioners implement the intervention?

4. What are the implementing practitioners' perceptions of the intervention?

First, I present demographic data for the participating teachers and students. Next, I present data on the implementation of the intervention (RQ 3), followed by data on the effects of the mathematics vocabulary intervention on students' mathematics vocabulary performance (RQ 1) and the influence that general mathematics achievement at pre-test may have on the effects of the intervention (RQ 2). I conclude the chapter by presenting data on the teachers' perceptions of the intervention (RQ 4).

Participants

Teachers

Three teachers from the partner school served as interventionists and responded to the social validity survey. Table 12 provides demographic information for these teachers. Two teachers self-identified as female, and one teachers self-identified as male. Teachers reported teaching for four to 19 years with a mean of 12.67 years of experience. Two teachers reported completing a bachelor's degree, and one reported completing a master's degree. The students in each teacher's class were randomly assigned to treatment or control conditions meaning that all teachers in the study were responsible for implementing the intervention.

Table 12

Teacher Characteristics

	n	%
Total	3	100
Gender		
Female	2	67.7
Male	1	33.3
Years of Experience		
0-4	1	33.3
5-9	1	33.3
10-14	0	0

15-19	1	33.3
Highest Level of Education		
Completed Bachelors	2	67.7
Bachelors plus credits	0	0
Completed Masters	1	33.3
Masters plus credits	0	0
Completed doctorate	0	0

Students

I invited all students in the classes identified by the school's administration to participate in the study (n = 32). The assistant director distributed a cover letter explaining the study, the informed consent document, and the youth assent document to all parents/guardians via email and hard copies. I chose to use both methods of obtaining informed consent and youth assent so the parents/guardians could choose their preferred method of responding (i.e., complete a survey or sign and return a document). Thirty students agreed to participate by providing informed consent and youth assent.

Despite the study taking place during the COVID-19 pandemic, student absences were minimal. Overall, students missed an average of 2.29 days of school with a range of zero to 10 days. Students assigned to the control condition missed an average of 2.17 days of school (range of zero to 10), and students assigned to the treatment condition missed an average of 2.38 school days (range of zero to eight).

Table 13 provides detailed demographic information for the students who participated. Parents/guardians responded to demographic surveys for 83.3% (25 of the

30) participating students. Students' ages ranged from 11-14 years with a mean of 13 years. Most of the students identified as male (*n* = 15, 60%) and White (*n* = 17, 68%). Two thirds (*n* = 20, 67%) of the students qualified for special education and had an individualized education plan (IEP). Of the students with an IEP, 16 (64%) were identified with ADHD and nine (36%) with Learning Disabilities. All of the students spoke English at home (*n* = 25, 100%), and one student (3.3%) also spoke German at home. A majority of the students' parents reported being married (*n* = 21, 84%). All of the parents reported completing a bachelor's degree or higher; 12 (48%) completed a bachelor's degree only, six (30%) completed a master's degree, and seven (28%) completed a doctoral degree. Most of the respondents reported an annual household income of $200,000.00 or more (*n* = 17; 68%).

Table 13

Student Characteristics

	Treatment *n* (%)	Control *n* (%)	Total *n* (%)
Total	17	13	25 (100)
Age (years)			
11	0	2 (8.0)	2 (8.0)
12	3 (12.0)	1 (4.0)	4 (16.0)
13	6 (24.0)	4 (16.0)	10 (40.0)
14	6 (24.0)	3 (12.0)	9 (36.0)
No response	2 (8.0)	3 (12.0)	5 (20.0)

Gender			
Female	5 (20.0)	3 (12.0)	8 (32.0)
Male	10 (40.0)	5 (20.0)	15 (60.0)
Abinary	0	1 (4.0)	1 (4.0)
No response	2 (8.0)	4 (16.0)	6 (24.0)
Race/Ethnicity			
African American/Black	0	1 (4.0)	1 (4.0)
American Indian/Alaska Native	0	0	0
Asian	2 (8.0)	0	2 (8.0)
Hispanic/Latino	1 (4.0)	0	1 (4.0)
Middle Eastern	0	0	0
Native Hawaiian/Pacific Islander	0	0	0
White	10 (40.0)	7 (28.0)	17 (68.0)
Multiracial	2 (8.0)	2 (8.0)	4 (16.0)
No response	2 (8.0)	3 (12.0)	5 (20.0)
IEP			
No	2 (8.0)	3 (12.0)	5 (20.0)
Yes	13 (52.0)	7 (28.0)	20 (80.0)
No response	2 (8.0)	3 (12.0)	5 (20.0)
IEP Qualification Category[a]			
ADHD	11 (44.0)	5 (20.0)	16 (64.0)
Autism Spectrum Disorder	2 (8.0)	0	2 (8.0)
Deaf/blindness	0	0	0

Deafness	0	0	0
Developmental delay	0	0	0
Emotional disturbance	0	0	0
Hearing impairment	0	0	0
Intellectual disability	0	0	0
Learning Disability	6 (24.0)	3 (12.0)	9 (36.0)
Orthopedic Impairment	0	0	0
Other Health Impairment	0	1 (4.0)	1 (4.0)
Speech or Language Impairment	2 (8.0)	1 (4.0)	3 (12.0)
Traumatic Brain Injury	0	0	0
Visual Impairment	1 (4.0)	0	1 (4.0)
Other	0	0	0
No response	2 (8.0)	3 (12.0)	5 (20.0)
Language Spoken at Home			
English	17 (68.0)	13 (52.0)	25 (100)
German	1 (4.0)	0	1 (4.0)
No response	2 (8.0)	3 (12.0)	5 (20.0)
Parents' Marital Status			
Divorced or separated	2 (8.0)	1 (4.0)	3 (12.0)
Domestic partnership	0	0	0
Married	12 (48.0)	9 (36.0)	21 (84.0)
Never married/single	1 (4.0)	0	1 (4.0)
Widowed	0	0	0

Other	0	0	0
No response	2 (8.0)	3 (12.0)	5 (20.0)
Parents' highest level of education			
Some high school	0	0	0
Completed high school	0	0	0
Some college	0	0	0
Completed associate's degree	0	0	0
Completed bachelor's degree	8 (32.0)	4 (16.0)	12 (48.0)
Completed master's degree	4 (16.0)	2 (8.0)	6 (24.0)
Completed doctoral degree	3 (12.0)	4 (16.0)	7 (28.0)
No response	2 (8.0)	3 (12.0)	5 (20.0)
Annual household income (dollars)			
0-9525	0	0	0
9525-38700	0	0	0
38701-82500	1 (4.0)	1 (4.0)	2 (8.0)
82501-157500	0	3 (12.0)	3 (12.0)
157501-200000	2 (8.0)	1 (4.0)	3 (12.0)
200001-500000	9 (36.0)	4 (16.0)	13 (52.0)
500001 or more	3 (12.0)	1 (4.0)	4 (16.0)
No response	2 (8.0)	3 (12.0)	5 (20.0)

Note. [a] The total number of responses are greater than 25 as a result of respondents selecting multiple options.

Table 14 presents results of Fisher's exact tests. Fisher's exact test is appropriate when expected values for a Chi-square test are less than five (Kim, 2017). There were not any significant differences between the treatment and control conditions in the areas of gender, race/ethnicity, IEP, IEP service-area category, language spoken at home, parents' marital status, parents' highest level of education, and parents' annual income. An independent samples *t*-test indicated that the mean ages of the students in the treatment and control conditions were not significantly different (t (1, 23) = -1.0507 [-1.1875, 0.3875; 95% CI], p = .30, g = 0.43).

Table 14

Fisher's Exact Tests for Demographic Data

Demographic	*p*
Gender	.48
Race/ethnicity	.64
IEP	.36
IEP Service Category	.82
Language	1.0
Parents' Marital Status	1.0
Parents' Highest Level of Education	.58
Parents' Income	.27

Implementation of the Intervention

In this section, I will present data on the implementation of the intervention by the participating teachers. The research assistant and I observed each of the participating teachers during six lessons for a total of 18 observations. I conducted 15 of the observations, and the research assistant conducted three of the observations. We collected implementation data using an observation tool found in Appendix D. We collected data on the number of minutes required to teach each lesson and the estimated student response rate for each lesson (number of scripted responses divided by the number of minutes to teach the lesson). We chose these metrics in order to assess the feasibility of the intervention as a relatively quick, supplemental resource to accompany core mathematics instruction. We also rated teachers on their implementation of the lessons (described in more detail below) and collected qualitative data to provide context when interpreting the implementation fidelity ratings. Below, I present data on the time required to teach each lesson, the estimated student response rate for each lesson, and the results of the fidelity items.

All of the teachers taught one lesson per school day from January 12, 2022 to February 14, 2022. None of the teachers were absent for any of the scheduled mathematics vocabulary lessons during the intervention period. Figure 6 shows the amount of time in minutes that each teacher required to teach lessons two, six, eight, 13, 17, and 21, as well as the mean number of minutes required to teach each lesson. Teacher A required a mean of 8.67 minutes to teach each lesson, Teacher B required a mean of 15.17 minutes to teach each lesson, and Teacher C required a mean of 12.33 minutes to teach each lesson. As a group, the teachers required an average of 12.06 minutes to teach

each lesson. Notably, the average amount of time required to teach each lesson initially increased and then decreased after lesson eight. Also, Teacher A consistently required the least time to teach the lessons, and Teacher B usually required the most time to teach the lessons. The exception to this pattern is lesson 13; Teacher B required two minutes less than Teacher C.

The red line at 15 minutes represents the time we expected each lesson to require. The teachers taught most of the observed lessons within the estimated amount of time. Lesson six took longer than expected for Teachers B and D. Open-ended notes taken during the observations indicate that Teacher B allowed irrelevant material (e.g. jokes, off-topic discussions) between items throughout the lesson, and that Teacher C frequently redirected off-task student behavior. Lesson eight also took longer than expected for Teacher B. Open-ended notes from this observation indicate that Teacher B again spent time on irrelevant material during the lesson and repeated items as a result of students' inattention.

Figure 6

Time to Teach Each Lesson in Minutes

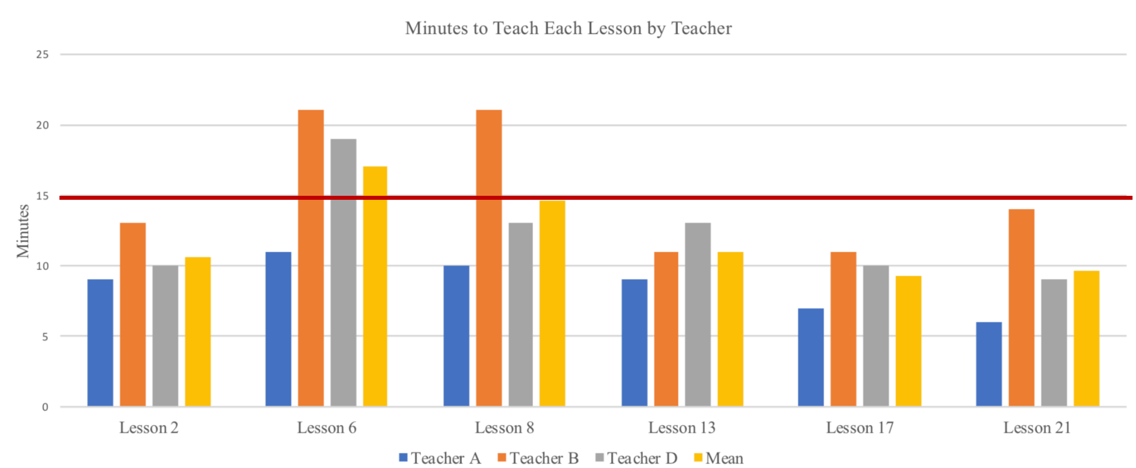
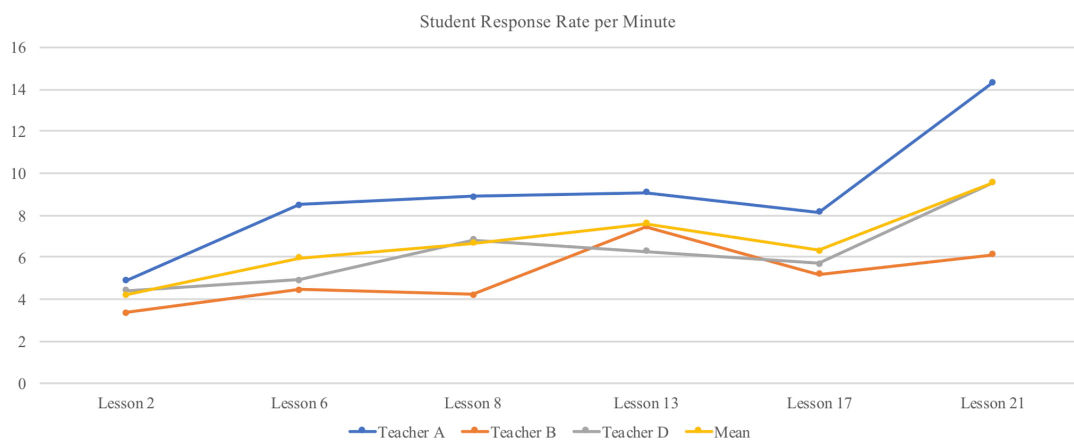


Figure 7 shows the student scripted response rate obtained by each teacher across the six observed lessons, as well as the mean student scripted response rate for each lesson. Each lesson was designed to be taught at a pace of at least seven student scrupted responses per minute. We calculated the student scripted response rate by dividing the scripted number of student responses in a lesson by the number of minutes the teacher required to teach the lesson. Teacher A obtained an average of 8.98 student scripted responses per minute, Teacher B obtained an average of 5.14 student scripted responses per minute, and Teacher C obtained an average of 6.28 student scripted responses per minute. As a group, the teachers obtained an average of 6.8 student scripted responses per minute across all observed lessons. With the exception of lesson 17, the average number of student scripted responses per minute increased as the intervention progressed.

Figure 7

Student Scripted Response Rate per Minute



Implementation Fidelity

As stated above, the research assistant and I used an observation tool that included a fidelity checklist (Appendix D) to collect implementation data. Items 1 through 16 used a Likert-type scale of 1-3 and addressed implementation fidelity. We rated the teachers as always, never, or sometimes implementing the items as intended during the observed lesson, and we took open-ended notes to provide context for the ratings. Items 17 through 21 used a Likert-type scale of 1-5 and were also intended to provide context for the implementation data.

Table 15 provides results from the fidelity checks for items 1 through 16. The leftmost column provides a description of the item on the fidelity checklist record. The three columns to the right provide specific results for each teacher. The numerals in these columns indicate the percent of observed lessons that we observed the teacher always implementing the item as intended. The rightmost column provides a mean percent across teachers for each row. The table is divided into sections that correspond to the sections of the fidelity checklist record. The final row of each section provides mean percentages across items in that section for each teacher and a grand mean for that section. The final row of the table provides grand totals for each teacher as well as a grand mean.

Table 15

Implementation Fidelity: Percent of Lessons Implemented as Intended

Item	Teacher A	Teacher B	Teacher C	Mean
Number of observations	6	6	6	6
Lesson Components				
All exercises presented	100	100	100	100
All visuals presented	100	100	100	100
All items in each exercise presented	100	100	83.33	94.44
Reasonably adhered to lesson script	100	66.67	83.33	83.33
Total (%)	100	91.67	91.67	94.44
Student Responses				
Obtained unison group responses (UGR)	100	100	100	100
All students participated in UGR	100	100	100	100
UGR clear and on signal	100	83.33	33.33	72.22
Provided individual turns as called for	100	100	100	100
Avoided only calling on volunteers	100	100	100	100
Total (%)	100	96.67	86.67	94.44
Error Correction Procedures				
Corrected all errors immediately	75	100	100	91.67

Involved all students in error corrections	25	50	50	41.67
Modeled correct responses/asked appropriate guiding questions	75	66.67	100	80.56
Provided immediate test	75	33.33	83.33	63.89
Provided delayed test	25	66.67	33.33	41.67
Total (%)	55	63.33	73.33	63.90
Lesson Presentation				
Prepared to teach lesson	100	66.67	83.33	83.33
Used a comprehensible rate of speech	100	100	100	100
Used an engaging/expressive tone of voice	100	100	83.33	94.44
Total (%)	100	88.89	88.89	92.59
Grand mean (%)	88.75	85.14	85.14	86.34

All teachers implemented the intervention with acceptable overall levels of fidelity. Teacher A had the highest level of fidelity across all observations, followed by Teacher C, and then Teacher B. As a group, the teachers implemented items under the headings “Lesson Components,” “Student Responses,” and “Presentation” as intended during more than 90% of the observed lessons. The items included in “Lesson Components” and “Student Responses” were implemented with the highest level of fidelity relative to the other sections on the fidelity checklist record. The items in the

"Lesson Components" section include presenting all of the exercises, visuals, and items in each lesson, as well as reasonably adhering to the scripts for each lesson. Teachers B and D had lower levels of implementation fidelity than Teacher A in this section (91.67%, 91.67%, and 100%, respectively). Teacher B tended to deviate from the scripted lesson in ways that were unacceptable more than the other teachers. On two separate occasions Teacher B demonstrated a lack of understanding of the mathematical concepts and provided unscripted information to the students that may have been confusing. Teacher C deviated from the script by allowing her students to respond with "yes" rather than saying the targeted vocabulary word.

The items in "Student Responses" address obtaining unison group responses from students and providing individual turns as called for in the lessons. Teachers B and D had lower levels of fidelity for this section than Teacher A (96.67%, 86.67%, and 100%, respectively). During the first observation, Teacher C appeared less familiar than the other teachers with signaling to obtain unison group responses (despite demonstrating using a signal to obtain a group unison response during training) and did not hold students accountable for answering in unison in response to her signal. I provided feedback and coaching via email after the observation. Teacher C's implementation in this area improved after receiving coaching. She often pre-corrected her students to answer on signal at the beginning of observed lessons and held students accountable for not answering in unison on signal more frequently. Despite these changes, her students did not answer on signal throughout all of the observed lessons. The pattern demonstrated by the teacher and students suggests that the teacher may not have taught the students to answer on signal to a sufficient level prior to starting the intervention and/or the teacher may not have required the students to answer in unison in response to her signal when not being observed. Teacher B also did not hold his students accountable for answering in unison on signal during the first observed lesson. I provided coaching via email, and he began pre-correcting his students before starting each observed lesson and consistently holding them accountable for answering in unison on signal.

The items included in “Presentation” were implemented with the next highest level of fidelity. The items in this section address teaching the material with familiarity, expression, and energy. Teacher A implemented the items in this section with the highest level of fidelity (100%). Teachers B and D both appeared unfamiliar with the lessons (e.g., rereading items with a confused tone, pausing during a lesson to read ahead, stumbling over scripted directions) during one or two of the observations and implemented the items with fidelity during 88.89% of the observed lessons.

The teachers implemented the items in the “Error Corrections” section with the lowest levels of fidelity. This section addresses correcting errors using a specific error correction strategy (i.e. model, test, delayed test). As a group, the teachers were most likely to correct all errors immediately and model the correct response or ask appropriate guiding questions in response to a student error. The teachers inconsistently provided an immediate test to the students as part of an error correction during the observed lessons. The teachers were least likely to involve all students in error corrections or provide a delayed test as part of an error correction.

Teacher A had the lowest levels of fidelity for this section (55.0%). She was most consistent with correcting all errors immediately, modeling the correct answer or asking appropriate guiding questions, and providing an immediate test of the missed item. In all of these areas, she taught one observed lesson in which she did not implement the items with intended levels of fidelity. The components of the error correction procedure that she implemented with the least fidelity were involving all students in error corrections and providing a delayed test. In three of the four lessons in which she had the opportunity to correct student errors, Teacher A involved only the student who made the error in the error correction (rather than the entire class), and she did not provide a delayed test.

Teacher B had the next highest level of fidelity for “Error Corrections” (63.33%). Teacher B corrected errors immediately during all six observed lessons. He provided a model and a delayed test for errors during four of the six observations, and involved all students in error corrections during three of the observations. (He involved only the student who made the error in the error correction during the other three lessons.) Interestingly, he only provided immediate tests (i.e. presenting the item again immediately after correcting the error) during two of the observed lessons. Often, Teacher B modeled the correct response to the missed item, skipped back a few items in the lesson, and then began again. This followed the procedures for immediately correcting an error and providing a delayed test but not for providing an immediate test to make sure the students attended to the correction.

Teacher C had the highest levels of implementation fidelity for this section (73.33%). Teacher C immediately corrected all errors and provided a model or asked appropriate guiding questions across all six observations, and she provided an immediate test of the missed item during five of the observations. As with Teacher B, Teacher C involved all students in error corrections in only three of the observed lessons. She only provided a delayed test as part of error corrections during two of the six observed lessons.

Mathematics Vocabulary

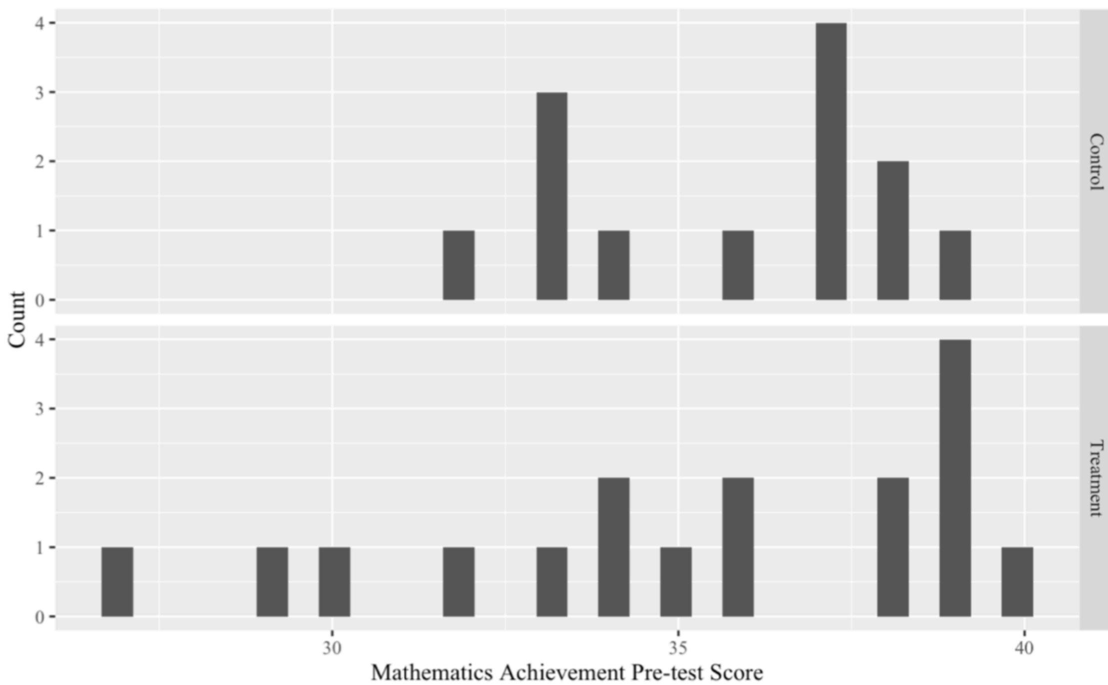
In this section I report effects of the intervention on mathematics vocabulary performance. First, I present data on the comparability of the students assigned to the treatment and control conditions in the areas of mathematics achievement and mathematics vocabulary. I then share results of the mathematics vocabulary post-test and compare the results of the treatment and control conditions. Finally, I explore the influence that mathematics achievement at pre-test may have on the effectiveness of the intervention by presenting data on the association between mathematics achievement and mathematics vocabulary before and after the intervention for both conditions.

Pre-test Outcomes

As presented above, the students assigned to the treatment and control conditions were comparable demographically. They were also comparable in their mathematics achievement and mathematics vocabulary pre-test scores. Figure 8 shows the mathematics achievement pre-test score distribution for each condition. The students assigned to the control condition (n = 13) scored a mean of 35.7 (SD 2.4) on the mathematics achievement pre-test. The students assigned to the treatment condition (n = 17) scored a mean of 35.2 (SD 3.9). Although at pre-test, the students assigned to the treatment condition had a lower mean and larger standard deviation than the students assigned to the control condition, the two conditions were not significantly different in mathematics achievement and the magnitude of difference was small, t (28) = 0.4172, p = .68, g = 0.15.

Figure 8

Mathematics Achievement Pre-test Score Distributions by Condition



The distributions of the mathematics achievement pre-test scores are further illustrated by Figure 9. Each condition is represented by a boxplot with a dot plot overlay. The figure shows that the control group scores are more tightly grouped and have a higher median than those for the treatment group.

Figure 9

Boxplots of Mathematics Achievement Pre-test Scores by Condition

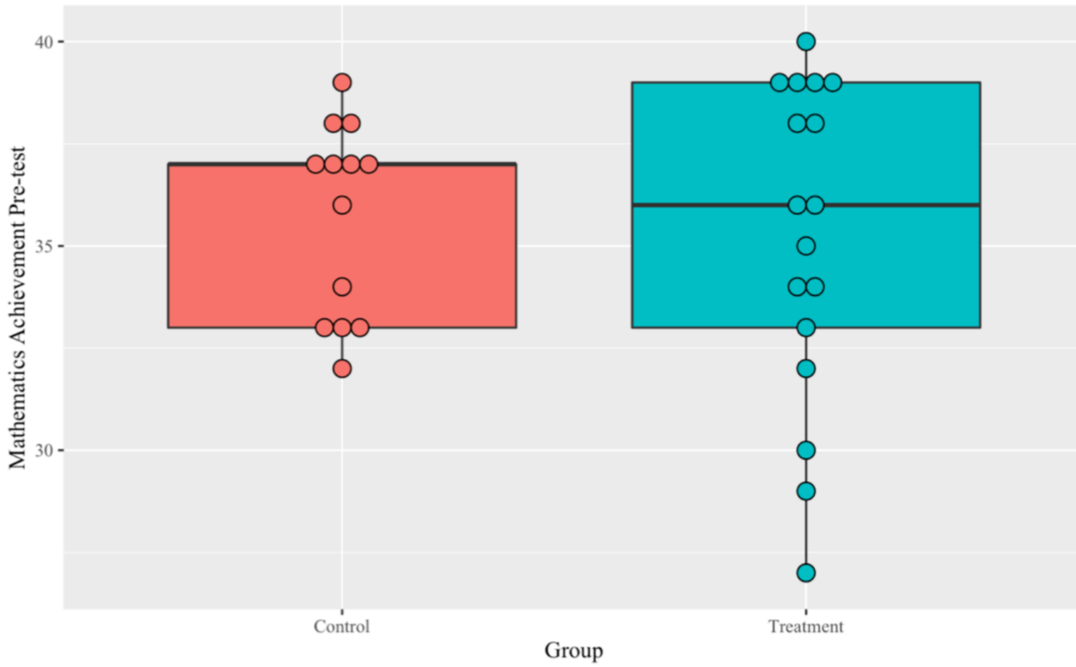


Figure 10 shows the mathematics vocabulary pre-test score distribution for each condition. The students assigned to the control condition had a mean score of 32.3 (SD 5.6), and the students assigned to the treatment condition had a mean score of 33.4 (SD 5.8). Although the students assigned to the treatment condition had a mean score 1.1 points greater than the students assigned to the control condition and their standard deviation is slightly lower, the conditions were not significantly different in mathematics vocabulary at pre-test and the magnitude of the difference is small, *t* (28) = -0.5342, *p* = .60, *g* = 0.19.

Figure 10

Mathematics Vocabulary Pre-test Score Distributions by Condition

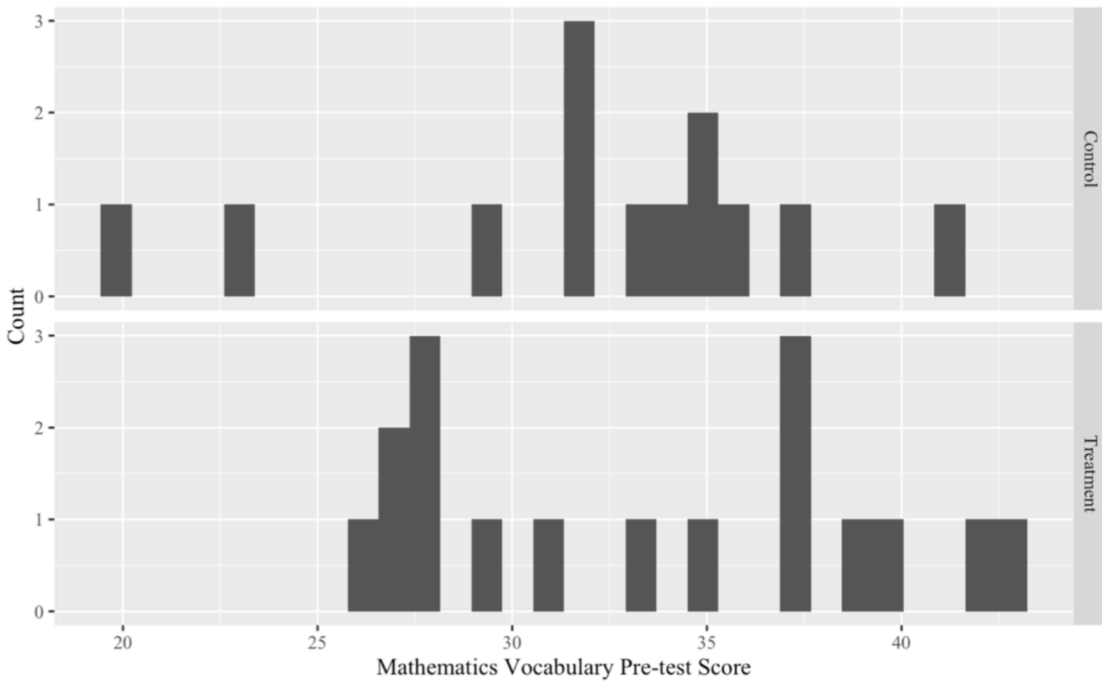
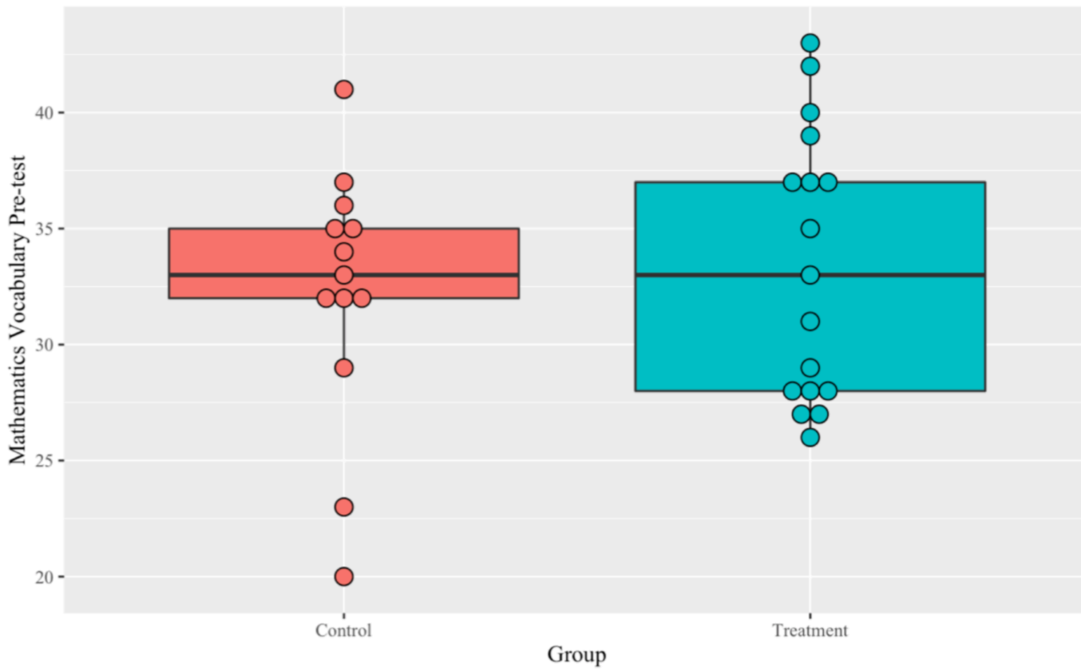


Figure 11 provides another illustration of the mathematics vocabulary pre-test distributions for both conditions. As with the boxplots illustrating mathematics achievement pre-test scores in Figure 9, the boxplot for the control condition in Figure 11 shows a much tighter distribution for mathematics vocabulary pre-test scores than the boxplot for the treatment condition. Notably, the boxplot for the control condition shows three outliers; two below the first quartile and one above the fourth quartile. The two outliers below the first quartile show that two students in the control condition had lower mathematics vocabulary pre-test scores than anyone in the treatment condition.

Figure 11

Boxplots of Mathematics Vocabulary Pre-test Scores by Condition



Post-test Outcomes

On the mathematics vocabulary post-test, the mean score of the students assigned to the treatment condition was higher than the mean score of the students assigned to the control condition and the magnitude of the difference was very large (g = 1.99). Figure 12 shows the post-test score distributions for the students assigned to each condition. The treatment condition had a mean 8.5 points higher than the mean of the control condition, and 14 of the 17 (76%) students assigned to the treatment condition scored within two points (i.e., greater than 95%) of the maximum score. Additionally, a single outlier represents the only score below 39 (86.67% of the total points available on the assessment) in the treatment condition. In contrast, the scores for the students in the control condition are fairly evenly distributed, and none of the students assigned to the control condition scored within two points of the maximum score.

Figure 12

Mathematics Vocabulary Post-test Score Distributions by Condition

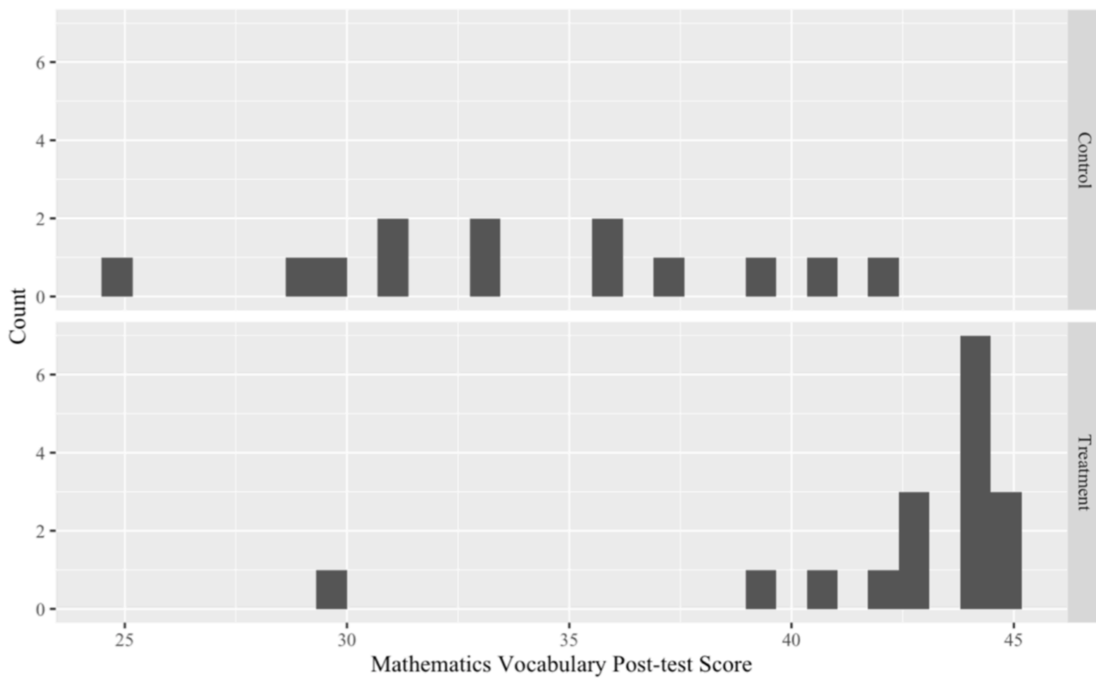


Figure 13 further illustrates the differences between the treatment and control conditions on the mathematics vocabulary post-test. Two additional outliers in the treatment condition are visible in Figure 13 that are not as clearly identifiable in Figure 12. The three outliers in the treatment condition represent students who scored 30 (66.67%), 39 (86.67%), and 41 (91.11%) points on the mathematics vocabulary post-test, respectively. These outlying scores further emphasize the tendency of the students assigned to the treatment condition to score at or near the maximum score possible on the mathematics vocabulary post-test. This tendency is also made clear by the compression of the treatment condition's quartiles above 41 points. In contrast, the control condition's quartiles are spread across a much broader range of scores.

Figure 13

Boxplots of Mathematics Vocabulary Post-test Scores by Condition

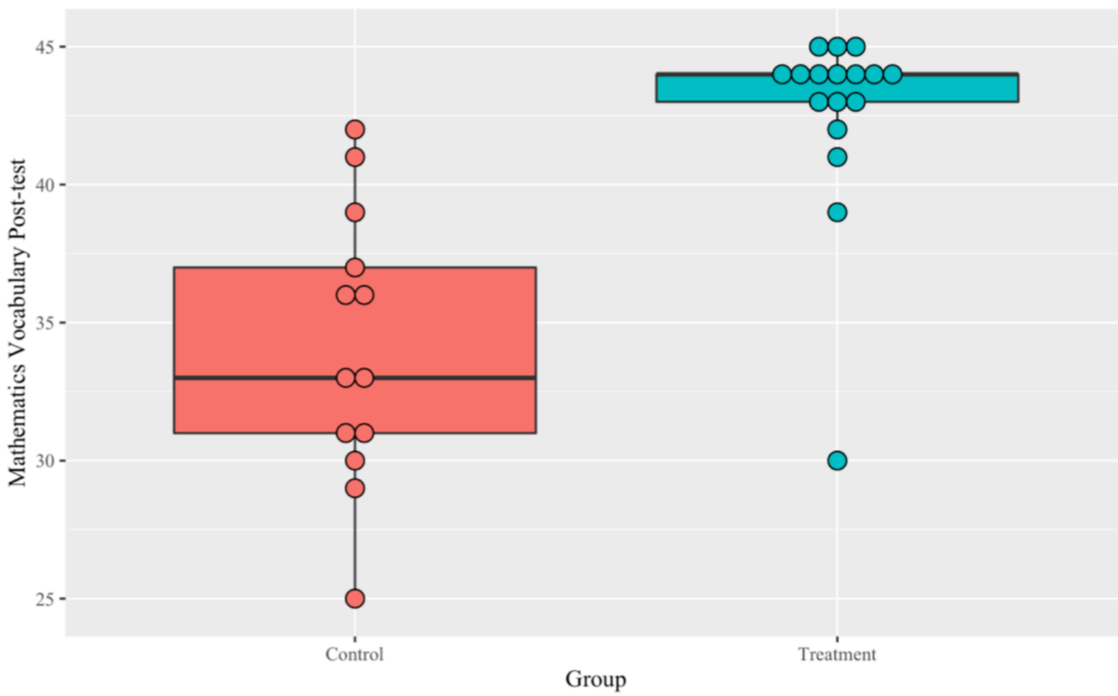


Figure 14 provides a detailed look at the changes experienced by each student from mathematics vocabulary pre- to post-test. The red circles represent each student's pre-test score, and the black circles represent each student's post-test score. The black line indicates the direction (positive or negative) and amount of change from pre- to post-test. The figure provides another illustration of the clear measurement ceiling on both post-test and change scores for the treatment group. In the control group, eight students' performance improved from pre- to post-test, four students' performance decreased, and one student's performance remained stable. In contrast, in the treatment condition sixteen students' performance improved, only one decreased slightly, and none remained stable.

Figure 14 also suggests that mathematics vocabulary at pre-test did not influence the effectiveness of the intervention. Of the eight students who scored lowest on the mathematics vocabulary pre-test, five scored within two points of the maximum score. The remaining three all made gains, with one of the students gaining 10 points and another student gaining 11 points from pre- to post-test. The intervention appears to have made a positive difference for all of the lowest performing students that is comparable to most of the higher performing students.

Figure 14

Individual Pre-, Post-, and Change-scores in Mathematics Vocabulary by Condition

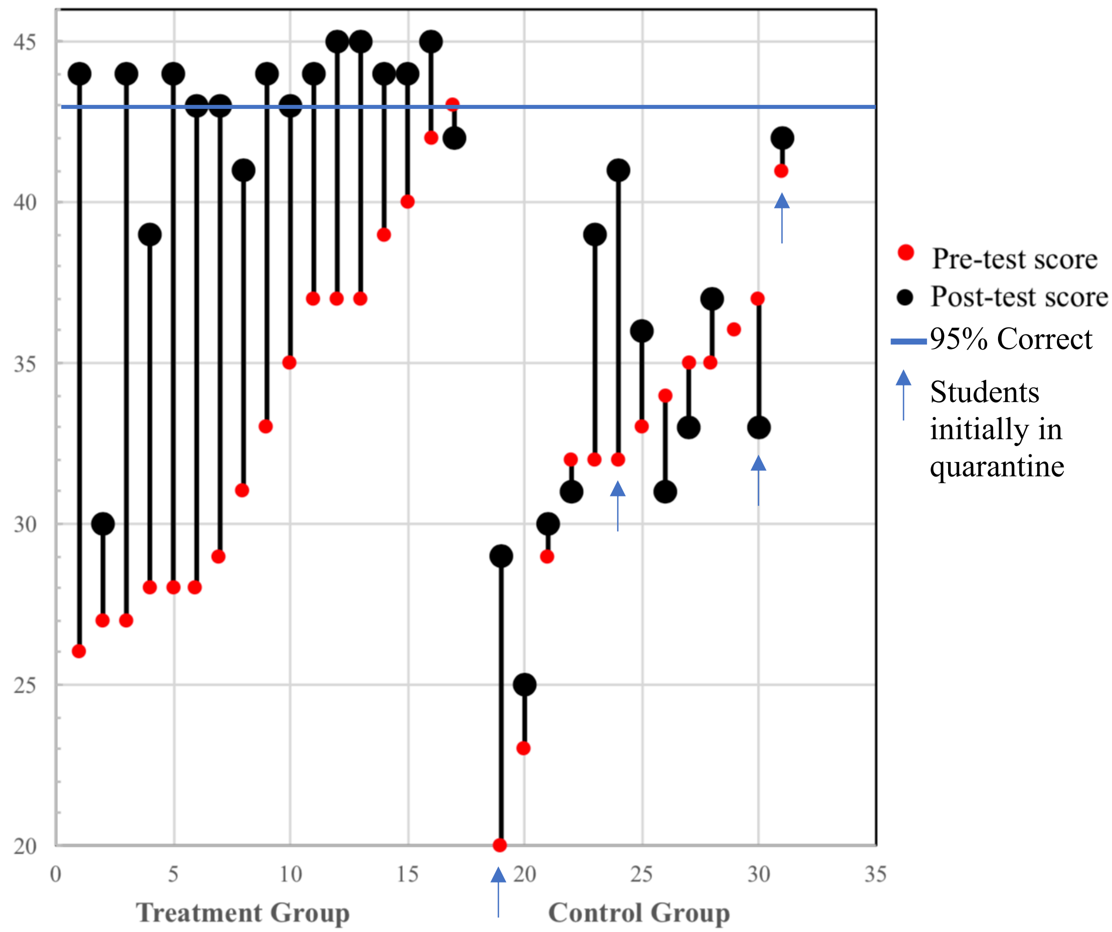
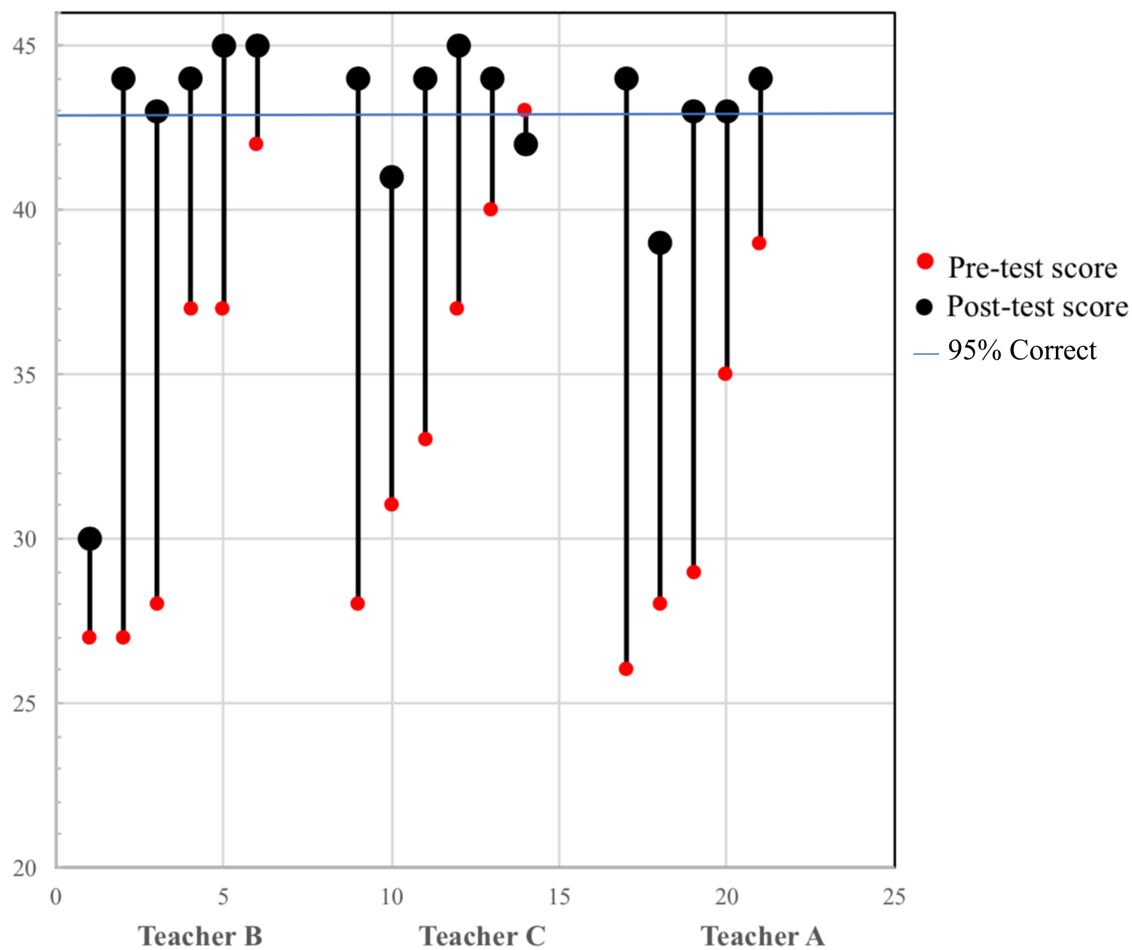


Figure 15 shows changes experienced by each student in the treatment condition from mathematics vocabulary pre- to post-test by teacher. The red circles represent each student's pre-test score, and the black circles represent each student's post-test score. The black line indicates the direction (positive or negative) and amount of change from pre- to post-test. The blue horizontal line shows a score equivalent to 95% correct on the post-test. The figure provides another clear illustration of the ceiling on both post-test and change scores for the treatment group. It also shows that pre- and post-test performance was fairly well-distributed among the teachers. Each teacher was assigned three students who scored below 35 (78%) on the pre-test and two students who scored between 35 and 40 (78%-89%) on the pre-test. The only difference between the groups is that two of the teachers each had one student who scored above 95% on the pre-test, and one of the teachers did not. At post-test, one teacher had zero students who scored below 95%, and two of the teachers each had only one student who did not score above 95%.

Figure 15

Individual Pre-, Post-, and Change-scores in Mathematics Vocabulary by Teacher



The results of the regression analysis confirm the findings of the visual analysis described above. Table 16 shows the results of fitting the regression models. Model 1 is the intercept-only model. Model 2 includes assignment to the treatment or control condition as the only variable. Model 3 includes teacher as the only variable. Model 4 includes condition assignment and teacher as covariates, and Model 5 investigates the interaction between condition assignment and teacher. R^2 values suggest that the models including condition assignment as a covariate (Models 2, 4, and 5) better explain the variance than the models that do not include condition as a covariate (Models 1 and 3). Adjusted R^2 is highest for the model that includes assignment condition as the only variable ($R^2 = 0.50$), suggesting that this is the most appropriate model for the data.

Table 16

Linear Regression Models

	Model 1: Null Model	Model 2: Condition Assignment	Model 3: Teacher Assignment	Model 4: Condition and Teacher	Model 5: Condition and Teacher Interaction
Intercept	38.90*** (1.09)	34.08*** (1.18)	38.18*** (1.85)	33.52*** (1.56)	34.20*** (1.99)
Condition		8.51*** (1.56)		8.54*** (1.59)	7.30* (2.69)
Teacher B			0.42 (2.68)	-0.05 (1.88)	-1.45 (2.98)
Teacher C			1.93 (2.76)	1.84 (1.93)	1.05 (2.98)
Group and Teacher B					2.45 (3.93)
Group and Teacher C					1.45 (4.01)
R^2	0	0.51	0.02	0.54	0.54
Adj. R^2	0	0.50	-0.05	0.48	0.45

Note. $n = 30$. *** $p < .001$. ** $p < .01$. * $p < .05$.

Table 17 shows the best-fitting model with confidence intervals. Assignment to the treatment condition was associated with significantly higher scores on the mathematics vocabulary post-test ($b = 8.51$, $p < .001$, 95 CI [5.45, 11.57].

Table 17

Final Linear Regression Model

Model	*b*	SE	95% CI	*p*
Model 2: Condition	8.51	1.56	5.45, 11.57	< .001

Note. $n = 30$.

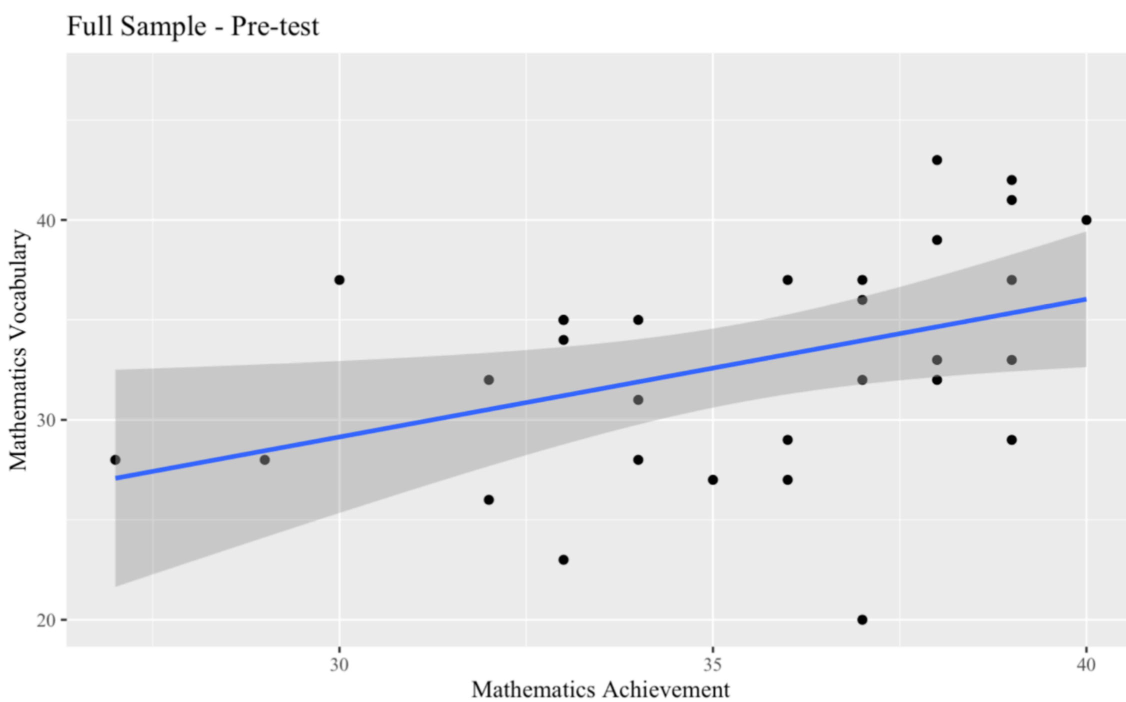
Associations between Mathematics Achievement and Mathematics Vocabulary

Investigating the associations between mathematics achievement and mathematics vocabulary at pre- and post-tests is important for increasing our understanding of the influence that general mathematics achievement may have on the effectiveness of the intervention. In order to learn more about the associations between mathematics achievement and mathematics vocabulary, I first analyzed the relation between mathematics achievement and mathematics vocabulary in the absence of the intervention. Specifically, I examined the correlation between mathematics achievement and mathematics vocabulary at pre-test for the full sample and both the control and treatment conditions separately, and between mathematics achievement and mathematics vocabulary at post-test for the control condition. I then determined the correlation between mathematics achievement and mathematics vocabulary at post-test for the treatment condition. Finally, I used ordinary least squares multiple regression to further investigate the influence that mathematics achievement at pre-test may have on the effectiveness of the intervention.

Figure 16 shows the relation between mathematics achievement and mathematics vocabulary for the full sample at pre-test. Each student's score is represented by a black dot, and the regression line summarizes the association between the two variables. The figure indicates an association between mathematics achievement and mathematics vocabulary for the full sample at pre-test (r = .40). This association is further quantified by regressing mathematics achievement and mathematics vocabulary pre-test scores, b = 0.69, p < .05, 95 CI [0.11, 1.27].

Figure 16

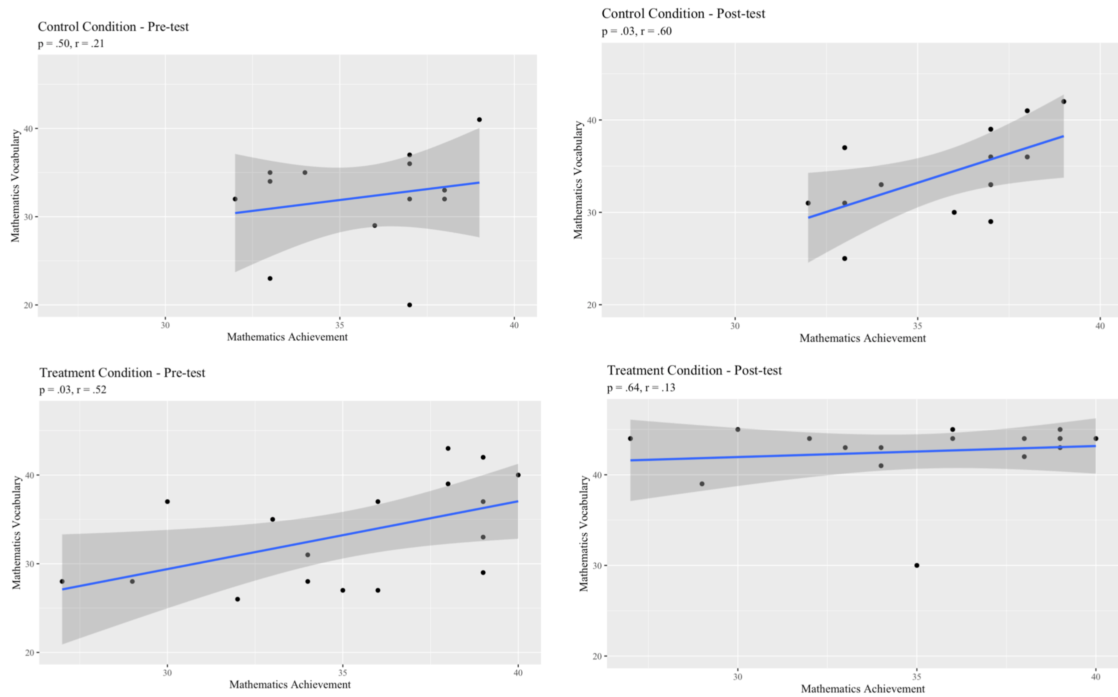
Relation between Mathematics Achievement and Mathematics Vocabulary Scores at Pre-test for Full Sample



In order to further investigate the possible relation between mathematics achievement and mathematics vocabulary, I examined the correlation between the two by condition and at each testing point (pre and post). Figure 17 shows the relation between mathematics achievement and mathematics vocabulary scores for each condition at pre- and post-tests. As with the figure above, the panels show each student's score using dots and a regression line that summarizes the association of the mathematics achievement and mathematics vocabulary scores. The panel in the upper-left presents the association between the pre-test scores for students in the control condition. The panel in the upper-right presents the association between the pre-test scores for the students in the treatment condition. The panel in the lower-left presents the association between the mathematics achievement pre-test and mathematics vocabulary post-test scores for students in the control condition, and the panel in the lower-right presents the association between the mathematics achievement pre-test and mathematics vocabulary post-test scores for students assigned to the treatment condition.

Figure 17

Relations between Mathematics Achievement and Mathematics Vocabulary Scores



Although the regression lines (i.e. associations) seen in the panels for the untreated groups (i.e. Control Condition – Pre-test, Treatment Condition – Pre-test, Control Condition – Post-test) differ from each other, they are similar to the pattern for the full sample at pre-test seen in Figure 16. An association between mathematics achievement and mathematics vocabulary appears to be present in the untreated groups. Pearson's product-moment correlation confirms the statistically significant associations for the treatment condition at pre-test (r = .52) and the control condition at post-test (r = .60). Notably, Pearson's product-moment correlation does not indicate an association between mathematics achievement and mathematics vocabulary at pre-test for the control condition (r = .21). This may be the result of one outlier representing a higher mathematics achievement score and the lowest mathematics vocabulary pre-test score for that condition.

The panel for the treatment condition at post-test in Figure 17 (lower right) is starkly different than the other panels. The previously noted ceiling effect in mathematics vocabulary scores is clearly apparent. This ceiling effect minimizes the likelihood of any strong association between mathematics achievement and mathematics vocabulary after treatment.

Table 18 shows the results of fitting the regression models to further investigate the relation between mathematics achievement score at pre-test and mathematics vocabulary score at post-test. Model 1 is the intercept-only model. Model 2 includes assignment to the treatment or control condition as the only variable. Model 3 includes mathematics achievement pre-test score as the only variable. Model 4 includes condition assignment and mathematics achievement pre-test score as covariates, and Model 5 investigates the interaction between condition assignment and mathematics achievement pre-test score. Adjusted R^2 values suggest that the models including condition assignment as a covariate (Models 2, 4, and 5) better explain the variance than the models that do not include condition as a covariate (Models 1 and 3). Adjusted R^2 is highest for the model that includes the interaction between condition assignment and mathematics achievement pre-test score (Adjusted $R^2 = 0.58$), suggesting that this is the most appropriate model for the data.

Table 18

Linear Regression Models Investigating the Influence of Mathematics Achievement on Mathematics Vocabulary

	Model 1: Null	Model 2: Condition Assignment	Model 3: MA	Model 4: Condition Assignment and MA	Model 5: Condition Assignment and MA Interaction
Intercept	38.90 *** (1.09)	34.08 *** (1.18)	29.73 * (12.02)	21.11 * (8.39)	-10.99 (17.01)
Condition		8.51 *** (1.56)		8.70 *** (1.53)	49.30 * (19.12)
MA			0.26 (0.34)	0.36 (0.23)	1.26 * (0.48)
Condition and MA					-1.14 * (0.54)
R^2	0.00	0.51	0.02	0.55	0.62
Adjust R^2	0.00	0.50	-0.01	0.52	0.58

Note. $n = 30$. MA = Mathematics Achievement. *** $p < .001$. ** $p < .01$. * $p < .05$.

Table 19 shows the best-fitting model with confidence intervals. This model indicates a significant interaction between condition assignment and mathematics achievement pre-test score when predicting mathematics vocabulary post-test scores. A simple slopes analysis shows that students assigned to the treatment condition had much less variation in the mathematics vocabulary post-test scores (M = 0.12, SE = 0.25) than the students assigned to the control condition (M = 1.26, SE = 0.48), regardless of mathematics achievement pre-test score.

Table 19

Final Linear Regression Model for Interaction between Mathematics Achievement and Mathematics Vocabulary

	b	SE	95% CI	*p*
Condition	49.30	19.12	11.82, 86.78	< .05
Mathematics Achievement	1.26	0.48	0.33, 2.20	< .05
Condition and MA Interaction	-1.14	0.54	-2.19, -0.09	< .05

Note. *n* = 30. MA = Mathematics Achivement

Overall, the evidence suggests that there may be an association between mathematics achievement and mathematics vocabulary for untreated students. The ceiling effect on mathematics vocabulary post-test scores for students assigned to the treatment conditions limits the association between mathematics achievement and mathematics vocabulary at post-test. The low correlation suggests that the mathematics vocabulary post-tests scores do not depend on mathematics achievement. In other words, the effectiveness of the intervention does not appear to be influenced by mathematics achievement at pre-test. The mathematics achievement pre-test scores are generally well-distributed in both conditions as are the mathematics vocabulary pre-test scores. Students who began the mathematics vocabulary intervention with lower mathematics achievement scores than their peers generally responded to the intervention similarly to their peers with relatively higher mathematics achievement scores. For example, two of the three lowest performers scored above 95% on the mathematics vocabulary post-test (see Figure 14).

Social Validity

The three teachers who were primarily responsible for implementing the intervention completed a social validity survey. I presented the social validity items to the teachers across two surveys. The demographic survey presented to the teachers at the beginning of the study included social validity items that addressed their current instruction and perspectives of mathematics vocabulary. After fully implementing the intervention, I sent the second survey to the teachers. This survey consisted entirely of social validity items and addressed their perspectives of the intervention. The demographic and social validity surveys may be found in Appendices J and D.

Table 20 shows the amount of time the teachers reported explicitly teaching mathematics vocabulary each week prior to teaching the mathematics vocabulary lessons and the instructional strategies they employed. Two of the teachers reported spending at least 20 minutes per week explicitly teaching mathematics vocabulary, and one teacher reported not spending any time on it. Interestingly, the teacher who reported spending 20 minutes per week teaching mathematics vocabulary did not provide a description of any instructional strategies used, and the teacher who reported not spending any time teaching mathematics vocabulary responded with "as necessary."

Table 20

Time Spent Explicitly Teaching Mathematics Vocabulary and Instructional Strategies Employed

Survey Item	Respondent		
	Teacher A	Teacher B	Teacher C
How many minutes do you explicitly teach mathematics vocabulary per week?	20	25	0
Please describe how you typically teach mathematics vocabulary.	No response	Model/Lead/Test	As necessary

Table 21 presents results from the survey items that used a Likert-type scale of one to five (1 = definitely disagree, 5 = definitely agree) to gather information about teachers' perceptions of the importance of mathematics vocabulary prior to implementing the intervention. All of the teachers agreed that mathematics vocabulary is critical for students to understand mathematics instruction, participate in mathematics instruction, and engage with mathematics in and out of the classroom. One teacher disagreed that students need to master mathematics vocabulary before advancing to the next grade.

Table 21

Teachers' Perspectives on the Importance of Mathematics Vocabulary

Survey Item	Respondent			Mean (Range)
	Teacher A	Teacher B	Teacher C	
1. Mathematics vocabulary is critical for students to understand mathematics instruction.	4	5	5	4.67 (4-5)
2. Mathematics vocabulary is critical for students to participate in mathematics instruction.	5	5	5	5 (5)
3. Mathematics vocabulary instruction is critical for students to engage with mathematics in and out of the classroom.	5	5	5	5 (5)
4. Students need to master mathematics vocabulary before advancing to the next grade.	2	5	5	4 (2-5)

Table 22 presents results of the second social validity survey which teachers completed after fully implementing the intervention (completed anonymously). The items included in this table used a Likert-type scale (1 = strongly disagree, 5 = strongly agree) to obtain teachers' perceptions of the intervention. The leftmost column provides the survey item, the center three columns provide each respondent's responses, and the rightmost column provides the mean.

Table 22

Teachers Perceptions of the Mathematics Vocabulary Intervention

Survey Item	Respondent			Mean (Range)
	1	2	3	
1. The amount of time required to teach the mathematics vocabulary lessons was reasonable.	5	4	5	4.67 (4-5)
2. The lessons were clearly written and easy for me to understand.	5	5	4	4.67 (4-5)
3. The mathematics vocabulary words included in the lessons are necessary for students to understand and engage with mathematics.	5	4	5	4.67 (4-5)
4. My students received frequent practice using the mathematics words during the lessons.	5	4	5	4.67 (4-5)
5. Choral responding is an effective way to provide multiple	5	3	5	4.33 (3-5)

practice opportunities when teaching mathematics vocabulary.				
6. My students were engaged during the mathematics vocabulary lessons.	3	3	4	3.33 (3-4)
7. My students enjoyed the mathematics vocabulary lessons.	1	3	NR	2 (1-3)

Note. NR = No response

The teachers also responded to open-ended questions about their perceptions of the intervention. When asked what they liked most about the mathematics vocabulary lessons, the teachers responded as follows.

1. "Examples and non-examples of terms, lots of practice opportunities"
2. "Slideshow was very user-friendly"
3. "Practicing vocabulary to fluency as knowing these terms are important because they are ubiquitous throughout math instruction."

When asked what they would change about the mathematics vocabulary lessons, the teachers responded as follows.

1. "Instead of introducing fractions without vocab, use vocab from the beginning"
2. "Introduce numerator/denominator terms earlier"
3. "More examples (far out and close in) and more choices for responses so that process of elimination can't be used."

When asked what words they recommend adding, one teacher suggested adding the following words to the intervention: “factor,” “multiple,” “greatest common factor,” and “lowest common multiple” because these “are the most common errors I see.” The same teacher also suggested adding “base and height when explaining area/perimeter, esp [sic] in the context of finding height of a triangle.” Another teacher echoed the suggestion to include “factor” and “multiple.” This teacher also suggested adding “reciprocal,” “cancel/simplify/reduce,” “composite figures,” “ratios,” and “prime vs. composite numbers.” When asked what words they recommend removing from the lessons, none of the teachers recommended removing any words, but one teacher suggested changing wording in the script of the introductory lesson of “parallelogram” from “and are both sets of opposite sides parallel” to “opposite sides parallel.” The survey concluded by asking if the teachers would use these or similar lessons in the future. All three teachers said that they would use these or similar lessons again. When asked why they would use the program again, one teacher did not provide a response, one teacher responded with “…building fluency with vocabulary knowledge,” and the third teacher replied “…this is a very effective program.” The teachers did not elaborate on any of the Likert-type items in their open-ended responses.

Chapter V Discussion

The purpose of this study was to investigate the effects on student learning, teacher-implementation, and social validity of a manualized, explicit, and systematic intervention for teaching mathematics vocabulary typically introduced in grades K-3 and necessary for fourth grade and beyond. First, I discuss the results of the study organized by research question. Then, I present implications for practice and research. I conclude with limitations of the current study and directions for future research.

Research Question 1: What are the effects of a manualized, explicit, and systematic mathematics vocabulary intervention implemented by practitioners on the mathematics vocabulary of students with learning difficulties and disabilities in a specialized setting?

The effects of the intervention on student learning were investigated with students at a private school for students with learning difficulties and disabilities. The students were randomly assigned to treatment and control conditions. At pre-test, the students in both conditions were equivalent demographically and in the areas of mathematics achievement and mathematics vocabulary. The quantitative analysis indicates that the mathematics vocabulary lessons positively affected the mathematics vocabulary performance of the students assigned to the treatment condition. Students who were assigned to the treatment condition had statistically significant higher scores on the mathematics vocabulary post-test when compared to students assigned to the control condition. The difference between the students assigned to the treatment and control conditions translates to an effect size of $g = 1.99$.

Cohen (1988) provided the following general guidelines for interpreting effect sizes: 0.2 is a small effect, 0.5 is a medium effect, and 0.8 is a large effect. Gravetter and Wallnau (2014) echoed this guidance for interpreting effect sizes in the behavioral sciences. Hill et al. (2008) argued that effect sizes need to be interpreted in their proper context (i.e. in relation to studies in similar fields, with similar participants) and provided evidence that effect sizes for education interventions typically range from a low of 0.07 to a high of 0.51 depending on factors like grade level, topic, and type of test. Compared to other studies investigating the effectiveness of various approaches to mathematics vocabulary instruction, the effect size obtained in this study is quite large. The studies presented in Table 6 (chapter two) that assigned students to different conditions and included pre- and post-tests (n = 6) range in effect size from g = 0.297 to 0.57. The effect on mathematics vocabulary performance seen in this study far exceeds the effect of any other comparably designed mathematics vocabulary instruction studies of which I am aware.

Furthermore, the pre- and post-test distributions of students assigned to the treatment condition as seen in Figure 15 suggest that teacher did not influence the effectiveness of the intervention. The students in the treatment condition were well-distributed among the three teachers at pre-test, with each of the teachers having approximately the same number of relatively low, mid, and high performers on the mathematics vocabulary pre-test. The students were also well-distributed among the teachers at post-test; one teacher had zero students who scored below 95% and two teachers each had only one student who scored below 95%. The rest of the students all scored above 95%. These results are evidence that the effectiveness of the intervention was likely not dependent on an individual teacher. This is especially interesting and important, given the observed variability in implementation (e.g. use of error correction procedures, time to teach each lesson, student response rate) across teachers. The regression models reported in Table 16 confirms this analysis.

A closer look at the distribution of the post-test scores for the students in each condition (Figure 14) provides further support for concluding that the intervention was highly effective. Five of the students assigned to the control condition scored lower on the mathematics vocabulary post-test than they did on the pre-test, one student did not make any gains, and none of the students approached the maximum score on the test. The highest score on the post-test in the control condition was 93%, and that student had a score of 91% on the mathematics vocabulary pre-test. In contrast, all except one of the students assigned to the treatment condition made gains from pre- to post-test, and 10 of the 17 students assigned to the treatment condition scored within two points of the maximum score (96% - 100%), resulting in a clear ceiling effect. The one student who did not make gains was within two points of the maximum score on the mathematics vocabulary pre-test and within three points on the post-test.

Our goal in developing the intervention was to teach all of the included mathematics vocabulary words/concepts to all the students. We developed the intervention not only in alignment with instructional design principles common to Direct Instruction, but also in alignment with the philosophy of Direct Instruction. This philosophy demands that instruction meets the needs of all students with the necessary prerequisite skills/knowledge and enables them to master the material (Engelmann, 2014). As such, our approach to designing and testing the effectiveness of the intervention manifests itself in the tendency of the treatment condition's post-test scores to cluster at the high end of the distribution. The ceiling effect in the treatment condition's post-test scores shows that most students assigned to the treatment condition learned what we endeavored to teach them – evidence that we approached our goal. Further, this ceiling effect provides strong, direct evidence that the program was effective for students with a wide range of mathematics vocabulary performance at pre-test. For example, five of the eight lowest performers on the mathematics vocabulary pre-test in the treatment condition scored at or above 95% on the posttest, and two of the remaining three participants improved their scores by 22 and 24 percentage points (see Figure 14). From an instructional design perspective, this is evidence that the program is appropriate for students with varying levels of prior mathematics vocabulary knowledge.

Research Question 2: Will general mathematics achievement moderate any effects of the intervention on mathematics vocabulary performance at post-test?

The intervention was designed to supplement core mathematics instruction with the purpose of ensuring that all students have mastered vocabulary typically introduced in grades K-3 so they can meaningfully understand and participate in mathematics instruction at the fourth-grade level and beyond. To achieve this purpose, we assumed that students would have been introduced to the words/concepts included in the intervention but may not have achieved mastery in their receptive and/or expressive use of them. This assumption depends on the general mathematics achievement of the students being at least at the same level as a student who has completed third grade. Therefore, we were concerned that mathematics achievement at pre-test may impact the students' ability to benefit from the intervention.

One way to examine this possible association is with correlations. I began by calculating correlations between mathematics achievement and mathematics vocabulary at pre-test for the full sample to see if these variables were related in the absence of an intervention. I found evidence for a significant association so I calculated correlations between the variables at pre-test by condition. The evidence suggests an association between general mathematics achievement and mathematics vocabulary in the untreated participants. If mathematics achievement at pre-test moderated the effectiveness of the intervention for some students, we would expect to see a correlation between the two variables at mathematics vocabulary post-test. Results, as presented in Figure 17, revealed a very small and non-significant correlation for the treatment condition at post-test ($r = .13$, $p = .64$). Where there had been a significant, moderate correlation between mathematics achievement and mathematics vocabulary at pre-test for the treatment condition, there was no longer a significant or strong correlation, suggesting that the intervention was not differentially effective for students with higher or lower general mathematics achievement. The lack of correlation between mathematics achievement and mathematics vocabulary post-test in the treatment condition also contrasts with the significant, moderate correlation between the two variables present in the control condition at post-test.

Notably, as discussed in chapter four, regression analysis revealed an interaction between assignment to the treatment or control condition and mathematics achievement at pre-test (see Table 19). A simple slope analysis showed that students assigned to the treatment condition had much less variation in mathematics vocabulary post-test scores than students assigned to the treatment condition, regardless of performance on the mathematics achievement pre-test. These results suggest, as illustrated in Figure 17, that an association between mathematics achievement and mathematics vocabulary may be present for untreated students but not for treated students.

The lack of a correlation between mathematics achievement and mathematics vocabulary for the treatment condition at post-test is related to the ceiling effect on the post-test. A significant correlation is highly unlikely given the clustering of the treatment groups posttest scores near the maximum. Again, we interpreted this as evidence of the effectiveness of the intervention for students with varying levels of prior general mathematics knowledge. This suggests that we achieved our goal of creating an intervention that would meet the needs of diverse students. Many of the students who received the intervention have been diagnosed with disabilities, and all of them have experienced learning difficulties. Additionally, they are all well-behind their same-age peers in mathematics achievement. Regardless, the intervention produced ceiling-level performance, even for students with relatively lower pre-test scores. The results suggest that the mathematics vocabulary intervention is capable of teaching all objectives to a wide range of students. Such instructional programs are necessary for addressing common and persistent mathematics achievement deficits (NCES, 2019b) and directly address a need for such materials identified by teachers (NCES, 2019a).

Research Question 3: With what level of fidelity will practitioners implement the intervention?

We endeavored to create a supplemental mathematics vocabulary intervention that would be easy for teachers to implement in a relatively short amount of time. We created an intervention for teaching over 40 words/concepts that is designed to be taught for 15 minutes per day across 22 school days. We estimated a student response rate of 7 responses per minute when creating each lesson to help ensure each lesson could reasonably be taught in 15 minutes. The implementation data collected during the study suggest that the intervention requires relatively little instructional time, provides frequent practice opportunities for students, and is feasible for implementation as a supplemental intervention that accompanies core mathematics instruction.

The teachers successfully taught the lessons every school day. Most of the time, the teachers required less than 15 minutes to teach each lesson. Across the six observed lessons, the teachers required a mean of just over 12 minutes to teach each lesson. Teacher A never required more than 11 minutes to teach each observed lesson, and Teacher D only required more than 15 minutes to teach a lesson during one observation. Teacher B required more than 15 minutes to teach two observed lessons. Both of these lessons required 21 minutes; more than we estimated but still feasible for using as a supplement to core mathematics instruction.

We designed the lessons to be an active experience for the students by producing frequent responses. We relied primarily on unison group (or choral) responding and individual turns. Through unison group responding and individual turns, the students encounter many opportunities to practice what they have learned. The unison group responses ensure that every student participates in every lesson and engages with the practice opportunities. The individual turns allow the teachers to probe students' understanding more deeply.

Student scripted response rate data show that teachers were able to use the program to provide many practice opportunities for their students in relatively little instructional time. The student scripted response rates are closely associated with the amount of time each teacher required to teach the observed lessons. As a group, they obtained a mean of 6.8 student responses per minute. The fastest teacher (Teacher A) had the highest number of student responses per minute. Likewise, the slowest teacher (Teacher B) had the lowest number of student responses per minute. It should be noted that, due to the way we calculated the student scripted response rate (number of scripted student responses divided by the number of minutes required to teach the lesson), the student scripted response rates reported in this study are underestimated. Teachers added unscripted response opportunities by implementing the correction procedures, adding individual turns, and repeating items. Due to the nature of the fidelity checklist, we were unable to collect data that accounts for all of the additional response opportunities in this study. Despite being an underestimate, the student scripted response data provides confirmation that teachers can implement the intervention in the amount of time and with the pacing that we intended (i.e. 15 minutes per lesson with approximately 7 student responses per minute). Additionally, the data show that teachers are able to use the intervention to provide abundant practice for their students in relatively little instructional time.

The implementation fidelity rating data suggest that teachers can implement the intervention with acceptable levels of fidelity given relatively little training and ongoing coaching. Training consisted of approximately two hours of orienting the teachers to the study’s procedures, the design of the intervention, and the delivery of the instruction. It should be noted that the participating teachers work at a school that regularly uses Direct Instruction programs like *Reading Mastery* and *Essentials for Algebra*. As a result of using these types of programs, the teachers were already familiar with instructional delivery approaches employed in the mathematics vocabulary intervention (e.g. signaling to gain group unison responses and following specific error corrections). Their level of familiarity with implementing explicit, systematic, and scripted instruction may mean that they required less time in training than other teachers would.

Coaching consisted of providing the teachers with a copy of the fidelity checklist prior to implementing the intervention and sending follow-up emails after each observation to each individual teacher. In the follow-up emails, I praised each teacher for implementation-related strengths and identified one or two areas for focus. I also provided one or two specific suggestions related to the area(s) of focus. The teachers responded to this feedback and coaching favorably and improved their implementation, especially around signaling to obtain unison group responses. Again, these results may be unique to this group of teachers. Being familiar with implementing similar instructional programs for different academic domains, the participating teachers may have required less feedback and coaching than teachers in other settings would.

Additionally, the implementation data suggest that the effectiveness of the intervention may be robust to certain adaptations made by the implementing teachers. Although the teachers implemented with high levels of fidelity in most areas, they consistently implemented error correction procedures with lower levels of fidelity. Most of the teachers (a) stopped instruction to correct all errors immediately and (b) provided a model or asked appropriate guiding questions; however, implementation was more variable in (c) involving all students in error corrections, (d) providing an immediate test of the correction, and (e) providing a delayed test. It appears that the effectiveness of the intervention is not dependent on fully adhering to error correction procedures – at least for these students. The inclusion of multiple practice items and frequent review of each word/concept throughout the intervention may provide enough instruction and practice that most students reach mastery without full and consistent error corrections.

Research Question 4: What are the implementing practitioners' perceptions of the intervention?

The participating teachers completed two social validity surveys. I administered the first prior to training the teachers on the study procedures and intervention. This survey attempted to gain insight into their perspectives on the importance of mathematics vocabulary and their current teaching practices related to mathematics vocabulary. I administered the second social validity survey after teachers implemented the intervention and administered the post-test to their students.

Results from the first survey show that teachers overwhelmingly report that mathematics vocabulary is critical for understanding, participating in, and engaging with mathematics in and out of the classroom. Two of the three teachers reported that mastering mathematics vocabulary is necessary before advancing to the next grade. Interestingly, the teachers reported that they taught mathematics vocabulary from zero to 25 minutes each week. When asked how they typically teach mathematics vocabulary, the teacher who reported spending 20 minutes per week teaching mathematics vocabulary did not provide a response, and the teacher who reported spending zero minutes per week on mathematics vocabulary responded with "as necessary." Overall, results from the first social validity survey suggest that teachers believe mathematics vocabulary is important but may be inconsistent in their approach to teaching mathematics vocabulary. This supports the validity of the instructional objectives and the need for an instructional program.

The second social validity survey included Likert-type and open-ended items directly related to the intervention. The teachers overwhelmingly agreed that the amount of time required to teach the lessons was reasonable, the lessons were clearly written and easy to understand, the words included in the lessons were necessary for students to understand and engage with mathematics, and that students received frequent practice with the words/concepts during the lessons. The teachers were a bit more variable in their responses regarding the effectiveness of unison group responding as a way to provide multiple practice opportunities; two strongly agreed and one neither agreed nor disagreed. Feedback from the teachers on these items suggests that the teachers find the intervention easy to implement in relatively little instructional time and that the intervention provides adequate practice for students.

The teachers were relatively less positive when asked about their students' engagement during the lessons. One teacher agreed that the students were engaged and two neither agreed nor disagreed. Given the multiple practice opportunities provided during each lesson and the teachers' agreement on the survey that students received frequent practice opportunities, these responses were surprising. They may be attributable, however, to differing ideas about the meaning of "engaged" in the context of mathematics instruction. Baroody et al. (2016) notes that multiple models exist for defining engagement in a mathematics classroom and that measures of engagement may also vary by reporting method and respondent. I interpreted the term to mean "actively participating in the lesson," but the teachers may have defined it as being engaged in discussion and/or problem-solving (Webb et al., 2014). This may be an example of a poorly worded survey item that is not obtaining the relevant data.

The teachers were also less positive when asked about their students' enjoyment of the lessons. One teacher neither agreed nor disagreed that the students enjoyed the lessons, one teacher disagreed, and one teacher did not respond. Based on student involvement and general lack of off-task behavior during the observations, this result was also surprising. Additionally, results from the field-test in the spring of 2021 show that the students provided feedback for improving the intervention but do not suggest that the students did not enjoy the lessons. This may be because the current participants were older than the originally intended intervention recipients and the participants of the spring 2021 field-test. The intervention was designed for students in fourth grade (typically 9 to 10 years old), but the participants of this study were 11 to 14 years old. This occurred because we invited the administrators of the partner school to determine which classes would benefit from the instruction. Because the partner school serves students with learning difficulties and disabilities who are generally behind their same-age peers academically, the students invited to participate in this study were older than the recipients we had in mind when developing and field-testing the intervention. What was acceptable to the younger students in the field-test may not be acceptable to older students. This underscores the importance of involving all stakeholders in intervention development and implementation (Fixsen et al., 2019). Also, we should note that this item reflected teachers' estimates of student enjoyment; we did not ask the students for their opinions directly. Future research may address this issue.

When responding to the open-ended survey items asking about what they would keep in the lessons, what they would change, what words they would add or remove, and if they would use the intervention again in the future and why, the teachers identified the amount and type of practice as a strength, as well as the ease of using GoogleSlides to present examples and non-examples. The teachers provided feedback about redesigning the fractions exercises so that the words *numerator* and *denominator* are taught earlier. They also suggested adding several words/phrases to the program. Notably, all of the words/phrases they suggested are associated with concepts typically taught after fourth grade. Again, this may be the result of implementing the intervention with students who are older than typical fourth grade students. The administrators of the partner school choose the invited classes because they believed the intervention would be beneficial for them and "fill holes." It may be that the students encountered more advanced topics and skills during their core mathematics instruction. The words/phrases suggested by the teachers may be appropriate for students who participated in this study but may be too advanced for the targeted recipients. The teachers' feedback in this area may indicate a need for a similar intervention designed for middle school students.

Notably, all three of the teachers responded that they would use these or similar lessons again. This willingness to implement the intervention again in the future corresponds with responses to most of the Likert-type items on the social validity survey but is in stark contrast to the items about student engagement and enjoyment of the lesson. It seems the teachers feel the benefits of implementing the intervention outweigh any potential lack of student engagement or enjoyment. Still, the teachers' responses to those items draw attention to a need to create an intervention designed for and acceptable to older students and the importance of involving students of comparable ages in the development process.

The dissonance between the teachers' reported plans to use the intervention again and their perceptions about student engagement and enjoyment needs to be interpreted with caution due to the small sample size. One teacher reported that the students did not enjoy the intervention, one reported neutrally, and one did not respond. It also underscores the need to more directly measure student engagement and enjoyment. Qualitative data obtained during an earlier field-test of the intervention show that students provided suggestions for changing the intervention but did not express a lack of enjoyment. On the contrary, many said that they liked the amount of practice included in the exercises and enjoyed using their voices to respond (Rolf et al., 2022). Notably, the students who provided that feedback were in fourth and fifth grades and approximately 9 to 11 years old, making them more representative of the intended recipients than the sample of the current study. Additionally, we used direct methods to procure their feedback rather than the indirect methods used in the current study.

Implications for Practice and Research

The intervention under investigation in the current study includes a number of improvements on earlier approaches to providing mathematics instruction. Additionally, the design of the current study addresses many of the limitations noted in prior mathematics vocabulary research (see Chapter 2). In the following sections, I discuss the improvements on mathematics vocabulary instruction and research represented by this study and their implications for practitioners and researchers. Then, I acknowledge the limitations of the current study and provide recommendations for future research.

Improvements on Prior Mathematics Vocabulary Instruction

As noted in Chapter 2, the extant mathematics vocabulary research shows that a number of approaches to mathematics vocabulary instruction may improve students' mathematics vocabulary. Of the eight studies of elementary-level mathematics vocabulary content that included explicit definitions (see Table 6), five utilized definition-oriented instruction (Botes & Mji, 2010; Bruun et al., 2015; McAdams, 2012; Monroe & Pendergrass, 1997; Petersen-Brown et al., 2019). Two of these five employed modified Frayer model designs (Bruun et al., 2015; Monroe & Pendergrass, 1997), and each of these studies used Frayer models in a different way. Only three studies used interventions that clearly incorporated principles of explicit, systematic instruction (Hassinger-Das et al., 2015; Powell & Driver, 2015; Williams, 2019), and each of these three studies investigated a different intervention. To my knowledge, Powell and Driver (2015) were the only researchers who investigated the use of a manualized intervention.

The variety of instructional approaches investigated and the apparent lack of a manualized interventions reveals a practical problem for practitioners. Even if practitioners have access to the research and are willing to select and implement an intervention that is empirically-supported, they have little to no guarantee that what they implement is what was studied unless the intervention is manualized. For example, two teachers may use the same Frayer model template to teach the same words in different manners. Without specifying what the teachers say and do, and what the students are expected to say and do, the two teachers may have very different lessons. As a result, the outcomes of the lessons for their two classes may be very different. Manualizing an intervention increases the likelihood that practitioners will implement the intervention in a way that is similar to the implementation that occurred during the study, thus increasing the likelihood of obtaining similar effects of the intervention (Fixsen et al., 2019).

One important implication of this study is that this manualized, explicit, systematic mathematics vocabulary instructional program efficiently and effectively teaches students mathematics vocabulary. The quantitative results confirm that this intervention improves the mathematics vocabulary of students with learning difficulties and disabilities when implemented by practitioners in real-world settings (i.e. classrooms). The qualitative results reveal that the teachers involved in this study found the intervention acceptable and would be willing to use it or something similar to teach mathematics vocabulary in the future. Taken together, the quantitative and qualitative results provide evidence of the feasibility, acceptability, and effectiveness of the intervention. Practitioners can use this intervention to improve their students' mathematics vocabulary with relatively little instructional or planning time. Recipients of this intervention benefit from increased understanding of mathematics vocabulary and may experience greater access to general mathematics instruction (Garbe, 1985; Hardcastle & Orton, 1993; Monroe & Orme, 2002; Powell et al., 2020).

Another important implication relates to the instructional design of the intervention. The instructional design of the intervention was inspired by the principles described by Engelmann and Carnine (1982/2016). It is characterized by the careful sequencing of examples and non-examples when initially introducing concepts, intentional and gradual reduction of scaffolding, systematic mass and distributed practice, and high levels of student engagement. In *Mathematics Vocabulary for Fourth Grade* (Rolf et al., 2021), we thought deeply about the order of introduction for the words/concepts included in the program. We carefully selected examples and non-examples when designing the introductory instruction for each word/concept in order to define the bounds of the word/concept while maintaining clarity of instruction for the students. We systematically and gradually reduced the scaffolding associated with each word/concept across lessons and carefully planned review exercises for each word/concept to ensure students' independent mastery. We achieved active student involvement in the lessons and used instructional time efficiently by relying heavily on unison group responses throughout the intervention. Using unison group responses allowed us to provide a large amount of practice for every student for each word/concept in a short amount of time. It also allowed teachers to ensure that all students were engaged in the instruction. We also included individual turns strategically in many of the lessons to allow for deeper thinking (e.g. "Why?" and "How do you know?" questions). Finally, we involved teachers and students throughout the instructional design process (from initial conception and brainstorming to field-testing to making small adjustments to the script in the current study). The current intervention is the result of merging the instructional design principles outlined by Engelmann and Carnine (1982/2016) with feedback from teachers, students, and other experts in the field of mathematics education (Rolf et al., 2022). Powell and Driver (2015) encountered unexpected results in their study of mathematics vocabulary instruction. In reflecting upon the lack of mathematics vocabulary growth documented in the students who received their mathematics vocabulary intervention, they noted that effectively teaching mathematics vocabulary may require a different instructional framework than has been previously investigated. To my knowledge, none of the interventions/instructional approaches for teaching mathematics vocabulary in the prior research use the aforementioned instructional design principles or were developed with feedback from teachers and students. The qualitative and quantitative results of this study suggest that this approach to designing mathematics vocabulary instruction has promise, both for individuals interested in designing effective instruction but also for practitioners looking for an effective instructional program for teaching mathematics vocabulary.

Improvements on Prior Mathematics Vocabulary Studies

The manualization of the intervention under investigation in this study also represents an improvement related to study design. As stated above, only one of the eight studies investigating approaches to elementary-level mathematics vocabulary instruction that included explicitly-taught definitions investigated the effects of a manualized intervention (Powell & Driver, 2015; see Table 6). Not only does this create a replication problem for practitioners, it creates a replication problem for researchers. Researchers are unable to replicate prior studies when they do not know what the interventions involved.

As described in the previous section, the same materials or approaches (e.g., Frayer models, definitions) may be used in different studies in very different ways. The results of the studies may depend on how the approaches or materials were used as well as the actual approaches/materials themselves. Without clear descriptions (such as a manualized intervention) it is very difficult, if not impossible, to replicate and verify or extend the earlier findings. By manualizing the intervention under investigation in this study, we paved a path for future researchers to replicate and extend the current findings.

The current study also provides an important contribution to the existing body of research in terms of participants. None of the eight studies previously noted took place in specialized settings or reported including only students with learning difficulties and/or disabilities. McAdams (2012) and Powell and Driver (2015) reported including students "at-risk" or with "mathematics difficulties," and Williams (2019) reported including students who received special education services. Additionally, all but one of the studies investigated mathematics vocabulary instruction provided by individuals with additional research training (e.g. researchers, doctoral students, trained research assistants). Botes and Mji (2010) were the only authors to report the effects of an intervention delivered by school personnel without additional research training.

The design of the current study and choice of measures are additional improvements on the prior, similar elementary-level mathematics vocabulary research. The current study randomly assigned students to treatment and control conditions. All students completed research-validated mathematics achievement and mathematics vocabulary pre-tests prior to the start of the intervention, and all students completed a research-validated mathematics vocabulary post-test after the intervention concluded.

Only five of the eight prior studies compared the effects of two or more conditions using pre- and post-tests (Botes & Mji, 2010; Bruun et al., 2015; Hassinger-Das et al., 2015; Monroe & Pendergrass, 1997; Powell & Driver, 2015). Only one of these five used research-validated, standardized assessments to measure the effects (Hassinger-Das et al., 2015). The design and choice of measures in the current study are more rigorous and may instill more confidence in the results than many of the prior studies.

Only three of the prior studies investigating elementary-level mathematics vocabulary instruction that incorporated explicitly-taught definitions documented implementation fidelity (Hassinger-Das et al., 2015; Petersen-Brown, 2019; Powell & Driver, 2015). Notably, the interventions under investigation in these studies were implemented by researchers or research assistants, meaning that none of the prior studies documented implementation fidelity for typical practitioners. The current study is an improvement in this area because we documented the implementation fidelity of real teachers in real classrooms. The data collected during the observations shows that teachers can implement the intervention with acceptable levels of fidelity with relatively little training and provides evidence of the usability of the intervention in real-world settings.

Finally, none of the similar prior studies systematically investigated the social validity of the approaches to elementary-level mathematics vocabulary instruction with explicitly-taught definitions. The current study is unique in that I obtained social validity data on teachers' perceptions of mathematics vocabulary prior to beginning the intervention as part of the demographic survey. After concluding the intervention, I obtained additional social validity data from the teachers about the importance of mathematics vocabulary instruction and the intervention.

Limitations and Future Directions

As with all research, this study has limitations to acknowledge. First, this study took place in a private school in an urban setting for students with learning difficulties and disabilities. The school follows a model rooted in Applied Behavior Analysis that may not be found in typical public schools throughout the United States and maintains a lower teacher to student ratio than is common in most public schools. Additionally, the teachers have training in Applied Behavior Analysis and Direct Instruction that teachers in typical schools in the United States may not have. The students who attend this school and participated in this study are also different from many public-school students in the United States in that they were predominately White, male, and of a higher socioeconomic status. Additionally, they were early adolescents (11-14 years old) performing academically substantially below their same-age peers and many had IEPs. The participants in this study do not represent the intended recipients of the intervention (i.e. students in fourth grade). As a result, it is possible that the outcomes of this study do not generalize to students in fourth grade or their teachers. Future research could investigate the implementation, effectiveness, and social validity of the intervention in a setting that more closely reflects the typical school in the United States and includes a more culturally, linguistically, and economically diverse sample of students in fourth grade.

Second, although the sample size was adequate for detecting effects of the intervention, it is small. Results from this study may not generalize to larger populations.

Future research could endeavor to include a larger sample of students and use a multi-level model or cluster-robust standard errors to account for the nested nature of the data (Hox, 2018; McNeish et al., 2017).

Third, the implementation data suggest that the intervention may be robust to certain adaptations from the teachers. As a group, the teachers implemented error correction procedures with the lowest levels of fidelity out of all of the fidelity checklist items. Despite these relatively low levels, most of the students who received the intervention made sizable gains from pre- to post-test. We did not collect data on the number of errors made during each lesson or what types of errors were made. However, based on informal observations, we do know that the students did not make any errors during a small number of the observed lessons, and they made few errors during the other lessons. It may be that all of the error correction procedures are not critical for obtaining positive effects or that consistent and complete implementation of the error correction procedures is not necessary. Future research could document teachers' implementation of the intervention, including their adaptations to the intervention, and record more detailed information about students' errors and error rates.

Fourth, the teachers' responses to the items about engagement and student enjoyment on the second social validity survey suggest areas that require further investigation. I was unable to explore those topics further as a part of this study. Future research could survey the teachers more frequently, include more open-ended survey questions and/or follow-up interviews, and solicit feedback systematically from the students via surveys and/or interviews. Future research could also endeavor to develop a mathematics vocabulary intervention for middle school mathematics. Students as well as teachers could be involved in the development process through field-testing lessons and providing feedback via surveys and focus groups.

Finally, we were unable to investigate generalization of learning from the intervention to other settings (i.e. lessons from the core mathematics instructional program, mathematics partner and/or group work) or more complex tasks (e.g. problem solving) in the current study. The primary objective of the current study was to investigate the effectiveness of the intervention for teaching the targeted content. Now that we have evidence showing that it is highly effective, future research could address more distal questions regarding generalization. For example, future studies could document mathematics vocabulary used by the students in writing or orally in other settings before and after implementing the intervention to see if students generalize the knowledge gained from the intervention to other more naturalistic contexts.

Conversations with practitioners and researchers throughout the development and testing of the intervention, as well as data analysis, inspired additional directions for future research. For example, the range of mathematics vocabulary pre-test scores suggests that students are likely to vary in their mastery of the words/concepts currently included in the program, with some not needing the instructional program. The development of a placement test that suggests a starting point for students may be useful. A placement test would allow teachers to save instructional time by skipping lessons and/or exercises that focus on words/concepts already mastered by the students. Feedback from the teachers regarding words and phrases to add to the intervention suggests a need for an intervention geared toward middle school mathematics vocabulary. Likewise, students served by speech-language pathologists may benefit from an explicit, systematic mathematics vocabulary intervention. Future research could address the development of such interventions with input from practitioners and students.

Developing a technology-enhanced version of the program may also be useful. Such a variation may allow students to proceed at an individualized pace without the need for teacher-directed instruction and/or a teacher who is highly skilled in implementation of Direct Instruction-style instructional programs. It may also allow researchers to investigate the influence of specific instructional design features on the effectiveness of the intervention. For example, the current study revealed that teachers implemented correction procedures with the lowest levels of fidelity compared to other fidelity checklist items. Despite the lower-levels of implementation fidelity in this area, the students still made impressive gains. A technology-enhanced version of the intervention would allow researchers to control the frequency and/or type of error corrections while holding other elements of the instructional delivery constant. This could allow researchers to learn more about the importance of error correction procedures for diverse students.

Conclusion

Researchers have argued that mathematics vocabulary is necessary for students to learn, participate in, and engage with mathematics (Garbe, 1985; Hardcastle & Orton, 1993; Monroe & Orme, 2002; Powell et al., 2020). Despite its assumed importance, relatively few studies have examined the effectiveness of mathematics vocabulary instruction, especially for students with learning difficulties and disabilities (Fore et al., 2007; Hott et al., 2014; Karuza, 2014; McAdams, 2012; Parsons et al., 2005; Powell & Driver, 2015; Root & Browder, 2019; Williams, 2019). The purpose of this study was to examine the effectiveness, implementation, and social validity of a manualized, explicit, and systematic intervention for teaching fourth grade mathematics. Results show that the students who received the intervention made significant gains when compared to the students who did not receive the intervention. The effect size of $g = 1.99$ is very large and far exceeds the effect sizes reported in other comparable studies investigating the effectiveness of mathematics vocabulary instruction (Botes & Mji, 2010; Bruun et al., 2015; Hassinger-Das et al., 2015; Jennings et al., 1992; Monroe & Pendergrass, 1997; Powell & Driver, 2015). Further, almost all students learned almost all the content of the intervention. Teachers found the intervention acceptable, and implementation data suggests that the intervention is an efficient way to teach students mathematics vocabulary. Taken together, the results suggest that *Mathematics Vocabulary for Fourth Grade* (Rolf et al., 2021) may be effective and acceptable for improving students' mathematics vocabulary. Additionally, the results suggest that this approach to instructional design may be useful when designing mathematics vocabulary instruction for students in other grades. Future research to document the effects of the intervention for students from diverse backgrounds with and without disabilities in multiple grades would be beneficial, as would research on the relation between implementation factors (such as fidelity levels) and student achievement.

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Appendices

Appendix A. Literature Review Search Terms

Table A1

Academic Search Ultimate Database Abstract Search Terms and Results

Search ID	Term	Number of Search Results
1	math*	244726
2	DE "MATHEMATICAL ability"	2610
3	DE "MATHEMATICAL enrichment"	122
4	DE "MATHEMATICAL linguistics"	393
5	DE "MATHEMATICAL literacy"	86
6	DE "MATHEMATICAL notation"	1362
7	DE "MATHEMATICAL symbols (Typefaces)"	5
8	DE "MATHEMATICS"	71909
9	DE "MATHEMATICS terminology"	209
10	DE “LANGUAGE & mathematics”	28
11	DE “REMEDIAL mathematics teaching”	74
12	DE “MATHEMATICS education”	12414
13	S1 or S2 or S3 or S4 or S5 or S6 or S7 or S8 or S9 or S10 or S11 or S12	303113
14	vocab*	15778
15	DE "VOCABULARY"	8395
16	DE “VOCABULARY education”	731
17	“academic lang*”	378
18	register*	99472
19	discourse	83773

20	terminology	15967
21	grammar	11051
22	gesture	8645
23	symbol*	61949
24	syntax	7083
25	S14 or S15 or S16 or S17 or S18 or S19 or S20 or S21 or S22 or S23 or S24	297753
26	S13 and S25	6695

Note. Search was conducted on 12/4/2019. Results were limited to peer-reviewed reports published in English.

Table A2

ASHAWire Database Abstract Search Terms and Results

Search ID	Term	Number of Search Results
1	math*	105
2	numer*	128
3	math* or numeracy	227
4	S1 or S2 or S3	460
5	vocab*	688
6	“academic lang*”	0
7	register*	81
8	discourse	318
9	terminology	60

10	grammar	526
11	gesture	224
12	symbol*	160
13	syntax	489
14	S5 or S6 or S7 or S8 or S9 or S10 or S11 or S12 or S13	2148
15	S4 and S14	126

Note. Search was conducted on 12/6/2019. Results were limited to peer-reviewed reports published in English.

Table A3

Education Full Text Database Abstract Search Terms and Results

Search ID	Term	Number of Search Results
1	math*	28751
2	DE "Mathematical ability"	106152
3	DE “Remedial mathematics teaching”	304910
4	DE “Mathematics education”	559376
5	S1 or S2 or S3 or S4	31990
6	vocab*	6334
7	DE “Vocabulary”	44836
8	DE “Vocabulary education”	548012
9	“academic lang*”	347
10	register*	2466

11	discourse	11202
12	terminology	1164
13	grammar	2005
14	gesture	537
15	symbol*	4548
16	syntax	643
17	S6 or S7 or S8 or S9 or S10 or S11 or S12 or S13 or S14 or S15 or S16	28987
18	S5 and S17	1529

Note. Search was conducted on 12/4/2019. Results were limited to peer-reviewed reports published in English.

Table A4

Education Source Database Abstract Search Terms and Results

Search ID	Term	Number of Search Results
1	math*	59799
2	DE "Mathematical ability"	2886
3	DE “Remedial mathematics teaching”	67
4	DE “Mathematics education”	19694
5	S1 or S2 or S3 or S4	64550
6	vocab*	13322
7	DE “Vocabulary”	8435
8	DE “Vocabulary education”	1315

9	"academic lang*"	599
10	register*	7014
11	discourse	25293
12	terminology	2917
13	grammar	8995
14	gesture	1275
15	symbol*	11140
16	syntax	2496
17	S6 or S7 or S8 or S9 or S10 or S11 or S12 or S13 or S14 or S15 or S16	72378
18	S5 and S17	2890

Note. Search was conducted on 12/4/2019. Results were limited to peer-reviewed reports published in English.

Table A5

ERIC Database Abstract Search Terms and Results

Search ID	Term	Number of Search Results
1	math*	41531
2	DE "Mathematical Aptitude"	411
3	DE "Mathematical Concepts"	6900
4	DE "Mathematical Linguistics"	63
5	DE "Mathematics"	5278
6	DE "Mathematics Achievement"	7682

7	DE "Mathematics Activities"	2060
8	DE "Mathematics Curriculum"	2705
9	DE "Mathematics Education"	12074
10	DE "Mathematics Instruction"	23419
11	DE "Mathematics Materials"	460
12	S1 or S2 or S3 or S4 or S5 or S6 or S7 or S8 or S9 or S10 or S11	53538
13	vocab*	9835
14	DE "Vocabulary"	4390
15	DE “Vocabulary Development”	6388
16	DE “Vocabulary Skills”	869
17	“academic lang*”	471
18	register*	2137
19	discourse	19163
20	terminology	1567
21	grammar	3976
22	gesture	1591
23	symbol*	5559
24	syntax	1326
25	S13 or S14 or S15 or S16 or S17 or S18 or S19 or S20 or S21 or S22 or S23 or S24	45959
26	S12 and S25	2904

Note. Search was conducted on 12/4/2019. Results were limited to peer-reviewed reports published in English.

Table A6

ProQuest (Digital Dissertations) Database Abstract Search Terms and Results

Search ID	Term	Number of Search Results
1	math*	38087
2	vocab*	15408
3	"academic lang*"	421
4	register*	18987
5	discourse	71993
6	terminology	6449
7	grammar	9726
8	gesture	6259
9	symbol*	42714
10	syntax	6814
11	S2 or S3 or S4 or S5 or S6 or S7 or S8 or S9 or S10	161186
12	S1 and S11	1093

Note. Search was conducted on 12/6/2019. Results were limited to Master's theses and doctoral dissertations published in English.

Table A7

PsycINFO Database Abstract Search Terms and Results

Search ID	Term	Number of Search Results
1	math*	33575
2	DE “Mathematical Ability”	4665
3	DE "Mathematics"	11053
4	DE "Mathematics (Concepts)"	2186
5	DE "Mathematics Achievement"	3709
6	DE "Mathematics Education"	6515
7	S1 or S2 or S3 or S4 or S5 or S6	42306
8	vocab*	14959
9	DE "Vocabulary"	12183
10	“academic lang*”	239
11	register*	20669
12	discourse	31940
13	terminology	5385
14	grammar	5141
15	gesture	6228
16	symbol*	25554
17	syntax	3460
18	S8 or S9 or S10 or S11 or S12 or S13 or S14 or S15 or S16 or S17	114737
19	S7 and S18	2713

Note. Search was conducted on 12/4/2019. Results were limited to peer-reviewed reports published in English.

Table A8

Teacher Reference Center Database Abstract Search Terms and Results

Search ID	Term	Number of Search Results
1	math*	22596
2	vocab*	1752
3	"academic lang*"	128
4	register*	478
5	discourse	5158
6	terminology	443
7	grammar	750
8	gesture	223
9	symbol*	1541
10	syntax	159
11	S2 or S3 or S4 or S5 or S6 or S7 or S8 or S9 or S10	10165
12	S1 and S11	909

Note. Search was conducted on 12/6/2019. Results were limited to peer-reviewed reports published in English.

Appendix B. Teacher Demographic Survey

Teacher Demographic Survey Questions

Basic Demographic Information

1. What is your name?
2. At what school do you work?
3. What is your gender?
4. How many years have you been a teacher and/or paraprofessional?
5. Please indicate your highest level of education:
    - High School
    - Currently completing an associate degree
    - Completed associate degree
    - Currently completing a bachelor degree
    - Completed bachelor degree
    - Bachelor degree plus additional credits
    - Completed master degree
    - Master degree plus additional credits
    - Completed doctoral degree

Typical Mathematics Instruction

6. How many days do you teach mathematics in a typical week?
    - 0
    - 1
    - 2
    - 3

- 4
- 5

7. How many minutes do you spend teaching mathematics on a typical day?
   - 0-10
   - 11-20
   - 21-30
   - 31-40
   - 41-50
   - 51-60
   - 61-70
   - 71-80
   - 81-90
   - 91 or more
8. Do you typically use instructional program(s)/curricula to teach mathematics?
   - If so, please list the instructional program(s)/curricula you typically use (e.g., GoMath, Everyday Math, Eureka Math).
   - For each instructional program/curricula/resource listed, please indicate how you became aware of each. (e.g., district-provided, heard about it from another teacher, internet, professional development, conference)
9. Do you use any supplemental instructional programs/curricula/resources when teaching mathematics?
   - If so, please list the supplemental instructional programs/curricula/resources you use.

- For each supplemental instructional program/curricula/resource listed, please indicate how you became aware of each. (E.g., district-provided, heard about it from someone else, internet, professional development, conference)

10. How much time in minutes do you spend explicitly teaching mathematics vocabulary during a typical week?
11. Describe how you typically teach mathematics vocabulary.

Mathematics Vocabulary Social Validity – Perception of the Problem

Please indicate your agreement to the following statements using a scale of 1 to 5. (1 = definitely disagree, 2 = somewhat disagree, 3 = neither agree nor disagree, 4 = somewhat agree, 5 = definitely agree)

12. Mathematics vocabulary is critical for students to understand mathematics instruction.
13. Mathematics vocabulary is critical for students to participate in mathematics instruction.
14. Mathematics vocabulary is critical for students to engage with mathematics in and out of the classroom.
15. Students need to master mathematics vocabulary before advancing to the next grade.

Appendix C. Student Demographic Survey

Student Demographic Questions

1. What is your child's name?
2. Who is your child's classroom teacher?
3. How old is your child (in years)?
4. What is your child's gender?
5. Please select your child's race/ethnicity. (Choose all that apply.)
    a. American Indian and/or Alaskan Native
    b. Asian
    c. Black or African American
    d. Latino and/or Hispanic
    e. Middle Eastern
    f. Native Hawaiian and/or Pacific Islander
    g. White
    h. Other
        i. If other, please indicate.
6. Does your child qualify for special education services? (Y/N)
    a. If yes, under what category does your child qualify? (Please select all applicable categories.)
        i. ADHD
        ii. Autism Spectrum Disorder
        iii. Deaf-blindness
        iv. Deafness
        v. Developmental Delay

vi. Hearing Impairment

vii. Emotional Disturbance

viii. Intellectual Disability

ix. Learning Disability

x. Orthopedic Impairment

xi. Other Health Impairment

xii. Speech or Language Impairment

xiii. Traumatic Brain Injury

xiv. Visual Impairment, including blindness (E.g., ADHD, learning disability, intellectual disability, speech, OT/PT)

xv. Other

7. What is the primary language spoken at your child's home?
8. What is the parents/caregivers' marital status?
   a. Married
   b. Never married/single
   c. Divorced or separated
   d. Widowed
   e. Domestic partnership
   f. Other
9. What is the parents/caregivers' highest level of education?
   a. Some high school
   b. Completed high school
   c. Some college

d. Completed associate degree

e. Completed bachelor degree

f. Completed master degree

g. Completed doctoral degree

10. What is your annual household income?

a. 0 - $9,525

b. $9,526-$38,700

c. $38,701-82,500

d. $82,501-$157,500

e. $157,501-$200,000

f. $200,201-$500,000

g. $500,501 or more

Appendix D. Fidelity Checklist

Mathematics Vocabulary Lesson Fidelity

Directions to Observer: Bring a copy of the lesson to the observation and refer to it during the lesson and while completing this form. Record your name, the teacher's name, the lesson number, and the lesson start time at the beginning of the observation. After the observation, record the time the lesson ended, calculate the number of minutes the lesson required and the estimated student response rate for the lesson, answer items 1-21, and record open-ended notes.

Observer Name: **Date:**

Teacher Name: **Lesson #:**

Lesson Start Time: **Lesson End Time:**

Observed Lesson Time (Mins.): **# of Scripted Response Opps:**

Estimated Response Rate (Divide the # of scripted response opps by minutes to teach):

Answer the following items by circling Y (yes), N (no), or S (sometimes). Only circle one letter for each item.

Lesson Components

1. Did the teacher present all exercises in the lesson? **Y** **N**

1a. If no, which exercises did the teacher skip? (Record each skipped exercise's number.)

2. Did the teacher present all of the appropriate visuals throughout the lesson? **Y** **N**

2a. If no, which visuals did the teacher skip? (Record each skipped visual's number.)

3. Did the teacher present all items in each exercise? **Y** **N**

3a. If no, which exercisess were not presented in their entirety? (Record each partially presented exercise's number.)

4. Did the teacher reasonably adhere to the script? (i.e., used correct terms, presented all questions/prompts, presented questions/prompts in their scripted order)	**Y**	**N**
Student Responses		
5. Did the teacher obtain unison group responses when called for throughout the lesson?	**Y**	**N**
6. Did all students participate in unison group responses throughout the lesson?	**Y**	**N**
6a. If no, did the teacher correct the students to ensure participation from all students in unison group responses?	**Y S**	**N**
7. Were the unison group responses clear and on signal?	**Y S**	**N**
7a. If no, did the teacher correct to ensure the students answered clearly and on signal?	**Y S**	**N**
8. Did the teacher provide all individual turns as called for in the lesson?	**Y**	**N**
8a. Did the teacher only call on volunteers?	**Y S**	**N**
Correcting Errors		
9. Did the teacher correct all errors immediately?	**Y**	**N**
10. Did the teacher involve all students in error corrections?	**Y S**	**N**
11. Did the teacher model correct responses or ask appropriate guiding questions as part of correcting errors?	**Y S**	**N**
12. Did the teacher test students as part of correcting errors by presenting the missed item again right away?	**Y S**	**N**
13. Did the teacher provide a delayed test by presenting the missed item again later in the lesson?	**Y S**	**N**
Presentation		
14. Did the teacher appear familiar with the lesson and prepared to teach it?	**Y**	**N**

15. Did the teacher use a comprehensible rate of speech when presenting the lesson? **Y** **N**

16. Did the teacher use an engaging and expressive tone of voice throughout the lesson? **Y** **N**

Rate the following items on a scale from 1 (not at all) to 5 (very frequently).

17. The teacher **redirected students from off-task behavior** throughout the lesson.

1 2 3 4 5
(not at all) (very frequently)

18. The teacher **added relevant material** to the lesson.

1 2 3 4 5
(not at all) (very frequently)

19. The teacher **added irrrelevant material** to the lesson.

1 2 3 4 5
(not at all) (very frequently)

20. The teacher **added additional individual turns** throughout the lesson.

1 2 3 4 5
(not at all) (very frequently)

21. The teacher **repeated items** during the lesson.

1 2 3 4 5
(not at all) (very frequently)

Observer Notes:

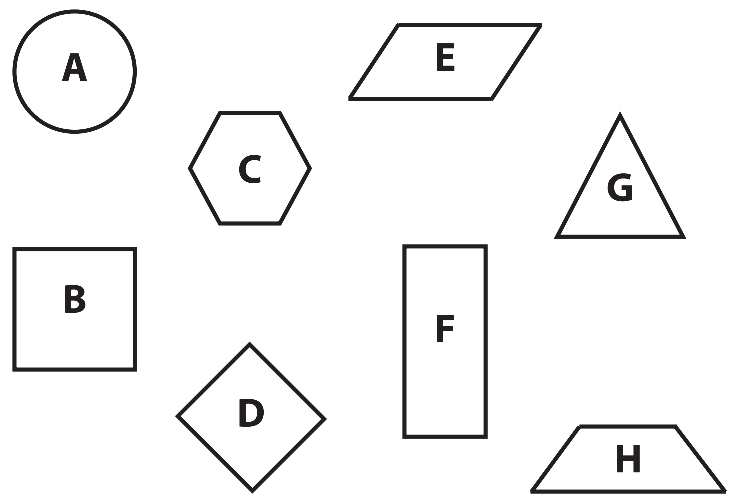
Appendix E. *Mathematics Vocabulary – 3rd Grade*

Mathematics Vocabulary - Grade 3

Answer the questions. Try the easy problems first, then go back and try the harder problems.

1. Match the letter of each shape with the name.

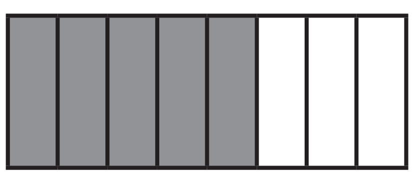
circle	
triangle	
rectangle	
parallelogram	
rhombus	
square	
trapezoid	



2. Write an **odd** number.

Write an **even** number.

3. Write a **fraction** for the picture.



4. In the box, draw a **line**.

In the box, draw a **line segment**.

5. Write 537 in **expanded form**.

6. Write a **unit fraction**.

7. Draw an **array** for 4 times 2.

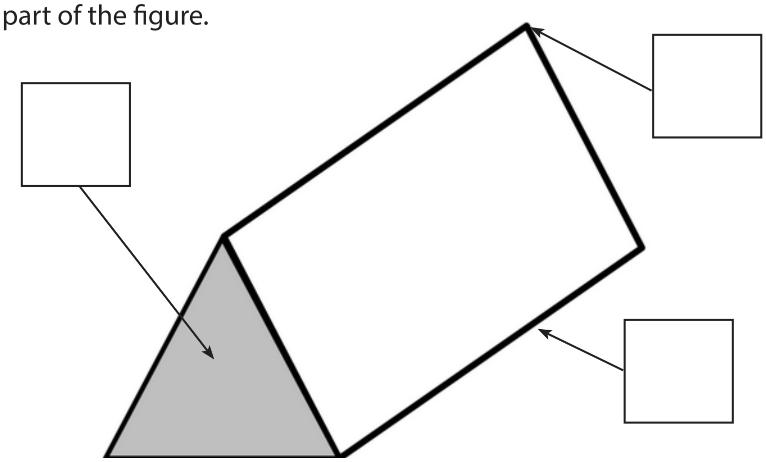
8. Match the letter with each part of the figure.

A **edge**

B **face**

C **side**

D **vertex**



9. Draw a **polygon**.

11. Draw a **right angle**.

12. Write an **equation**.

13. Write *three-hundred, twenty-five* in **standard form**.

14. Mark the **perimeter** of the shape.

Mark the **area** of the shape.

15. Draw a **quadrilateral**.

16. Circle the set of **equivalent fractions**.

A. $\frac{3}{4} = \frac{3}{8}$

B. $\frac{3}{4} = \frac{8}{12}$

C. $\frac{3}{4} = \frac{6}{8}$

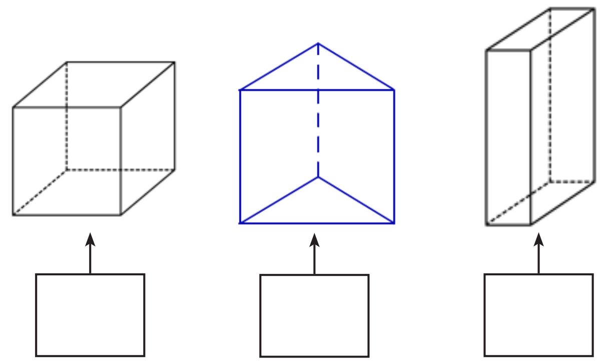
17. Write the letter of each shape.

A cube

B rectangular pyramid

C rectangular prism

D triangular prism



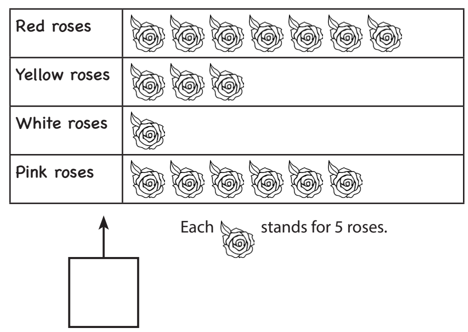
18. Write the letter that matches each graph.

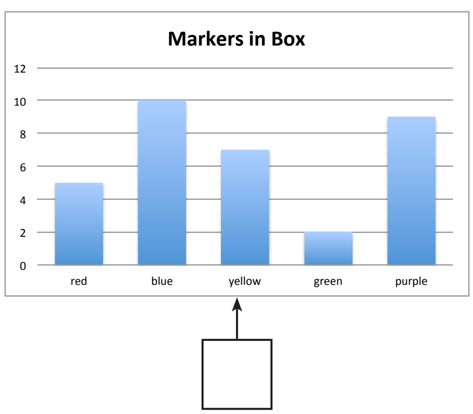
A bar graph

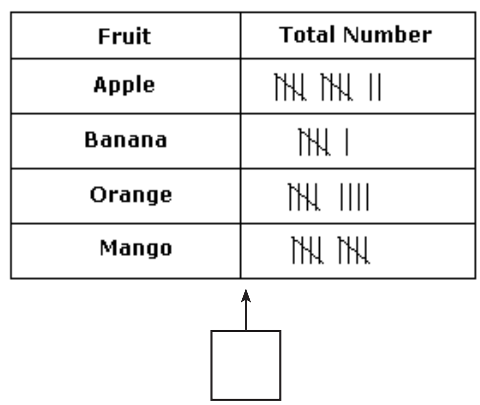
B dot plot

C pictograph

D tally chart







19. Draw an **angle**.

20. Write the letter for each part of a number sentence.

A **addend**

B **difference**

C **dividend**

D **divisor**

E **factor**

F **minuend**

G **product**

H **quotient**

J **sum**

$5 + 6 = 11$

$24 \div 4 = 6$

$3 \times 9 = 27$

$14 - 5 = 9$

21. What is the name of this?

$$6 \overline{)125}\quad 20\ R5$$

22. Write the **numerator**.

$\frac{3}{8}$

Write the **denominator**.

$\frac{6}{9}$

23. Draw a shape with three **sides**.

STOP

Appendix F. Teacher Social Validity Survey

Please respond to the following items using a scale of 1 (strongly disagree) to 5 (strongly agree):

1. The amount of time required to teach the mathematics vocabulary lessons was reasonable.
2. The lessons were clearly written and easy for me to understand.
3. The mathematics vocabulary words included in the lessons are necessary for students to understand and engage with mathematics.
4. My students received frequent practice using the mathematics words during the lessons.
5. Choral responding is an effective way to provide multiple practice opportunities when teaching mathematics vocabulary.
6. My students were engaged during the mathematics vocabulary lessons.
7. My students enjoyed the mathematics vocabulary lessons.

Please provide responses to the questions below.

8. What did you like most about the mathematics vocabulary lessons? Why?
9. What would you change about the mathematics vocabulary lessons? Why?
10. Are there additional words that you think need to be included in the lessons? If so, record them and provide a brief explanation for the need to include each word.
11. Are there words that you think should be removed from the lessons? If so, please record them and provide a brief explanation for each word.

12. Would you use these or similar lessons to teach mathematics vocabulary again in the future? Why or why not?

Appendix G. Mathematics Vocabulary Words by Order of Introduction

Word/Concept	Lesson Introduced
Shape Orientation and Size	1
Rhombus	1
Rectangular prism	1
Tally chart	2
Parallel	2
Face	3
Dividend	3
Area and perimeter	4
Greater than and less than	4
Edge	5
Pictograph	6
Quotient	6
Fraction	6
Parallelogram	7
Angle	7
Cube	7
Equation	8
Vertex	9
Bar graph	9
Sum and addend	10
Denominator	10

Standard and expanded forms	11
Dot plot	12
Trapezoid	12
Divisor	13
Triangular prism	13
Line	14
Numerator	14
Difference	15
Closed Shape	16
Quadrilateral	16
Array	16
Unit fraction	17
Line segment	17
Factor and product	18
Rectangular and triangular pyramids	18
Polygon	19
Equivalent fractions	19
Remainder	20
Right angle	20

Appendix H. Mathematics Vocabulary Words by Strand (Alphabetized)

Word/Concept	Lesson Introduced
Data Strand	
Tally chart	2
Pictograph	6
Bar graph	9
Dot plot	12
Fractions Strand	
Fraction	6
Denominator	10
Numerator	14
Unit fraction	17
Equivalent fractions	19
Geometry Strand – Two-dimensional Shapes Sub-strand	
Shape orientation and Size	1
Rhombus	1
Parallel	2
Parallelogram	7
Trapezoid	12
Line	14
Closed Shapes	16
Quadrilateral	16
Line segment	17
Polygon	19

Geometry Strand – Components of Shapes Sub-strand	
Face	3
Edge	5
Vertex	9
Geometry Strand – Three-dimensional Shapes Sub-strand	
Rectangular prism	1
Cube	7
Triangular prism	13
Rectangular pyramid	18
Triangular pyramid	18
Measurement Strand	
Area	4
Perimeter	4
Angle	7
Right angle	20
Number Composition Strand	
Standard and expanded forms	11
Operations Strand	
Greater than and less than	4
Equation	8
Operations Strand – Addition Sub-strand	
Addend	10
Sum	10

Operations Strand – Division Sub-strand	
Dividend	3
Quotient	6
Divisor	13
Remainder	20
Operations Strand – Multiplication Sub-strand	
Array	16
Factor	18
Product	18
Operations Strand – Subtraction Sub-strand	
Difference	15

Appendix I. Sample Lesson

Scripted Student Responses: 54

Lesson Five

Objectives

Introduce:
- Edge

Review:
- Face
- Tally Chart
- Area & Perimeter
- Greater Than & Less Than
- Dividend
- Parallel
- Rhombus

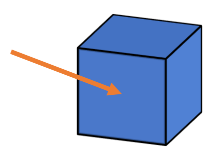
Instructional Time: 10 minutes

Materials:
- Powerpoint 5

Exercise 1: Edge – Introduction; Face - Review

General Error Correction: Model, test, delayed test

1. **Let's talk about parts of shapes. [Advance slide.]**

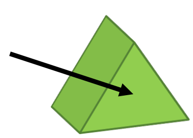


Slide 5.1

2. **Is this arrow pointing to a face or not?** — Face

---

3. **[Advance slide.]**

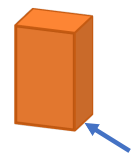


Slide 5.2

4. **Is this arrow pointing to a face or not?** — Face

Scripted Student Responses: 54

5. **[Advance slide.]**

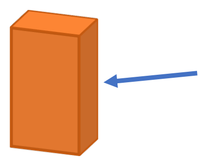


Slide 5.3

6. **Is this arrow pointing to a face or not? (** Not

7. **Correct, it's not a face. This arrow is pointing to an edge. The edge is the straight part where two faces meet.**

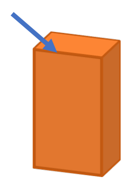
8. **[Advance slide.]**



Slide 5.4

9. **This arrow is also pointing to an edge.**

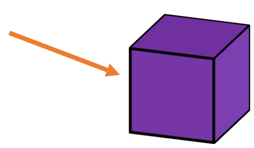
10. **[Advance slide.]**



Slide 5.5

11. **This arrow is also pointing to an edge.**

12. **I'm going to show you more. You tell me if each arrow is pointing to an edge or a face. [Advance slide.]**

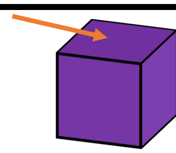


Slide 5.6

13. **Is this arrow pointing to an edge or face?** Edge

Scripted Student Responses: 54

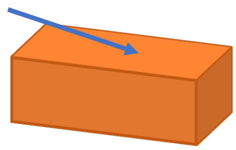
14. **[Advance slide.]**



Slide 5.7

15. **Is this arrow pointing to an edge or face?** Face

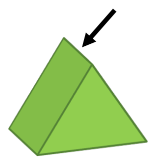
16. **[Advance slide.]**



Slide 5.8

17. **Is this arrow pointing to an edge or face?** Face

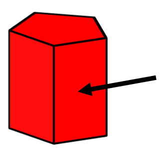
18. **[Advance slide.]**



Slide 5.9

19. **Is this arrow pointing to an edge or face?** Edge

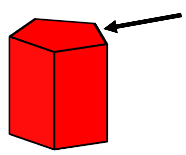
20. **[Advance slide.]**



Slide 5.10

21. **Is this arrow pointing to an edge or face?** Face

22. **[Advance slide.]**



Slide 5.11

23. **Is this arrow pointing to an edge or face?** Edge

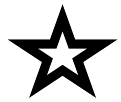
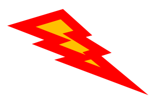
You learned edge and practiced face!

Scripted Student Responses: 54

Exercise 2: Perimeter and Area - Review

General Error Correction: Model, test, delayed test

1. **Let's review perimeter and area. [Advance slide.]**  Slide 5.13	
2. **Perimeter is the distance around a shape. What color is the perimeter of this shape? (**	Orange
3. **What does the perimeter tell us?**	The distance around the shape
4. **Area is how much space is inside a shape. What color is the area of this shape?**	Purple
5. **What does the area tell us?**	How much space is inside the shape
6. **[Advance slide.]**  Slide 5.14	
7. **What color is the perimeter of this shape?**	Blue
8. **What color is the area?**	Red
9. **[Advance slide.]**  Slide 5.15	
10. **What color is the perimeter of this shape?**	Green
11. **What color is the area?**	Blue

12. **[Advance slide.]** 	Slide 5.16	
13. **What color is the perimeter?**		Black
14. **What color is the area?**		White
15. **[Advance slide.]** 	Slide 5.17	
16. **What color is the perimeter?**		Blue
17. **What color is the area?**		Yellow
18. **[Advance slide.]** 	Slide 5.18	
19. **What color is the perimeter?**		Red
20. **What color is the area?**		Orange
21. **What does the perimeter tell us?**		The distance around a shape
22. **What does the area tell us?**		How much space is inside a shape
You reviewed area and perimeter!		

Scripted Student Responses: 54

Exercise 3: Tally Chart - Review

General Error Correction: Model, test, delayed test

1. **Let's practice tally chart. I'll show you charts. You tell me if each is a tally chart or not. [Advance slide.]**

Colors			
1	6	4	13

Slide 5.20

2. **Is this a tally chart or not?** — Not

3. **[Advance slide.]**

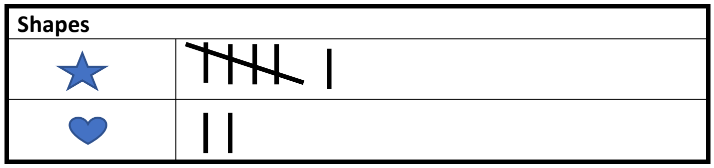
Food	
	| | | |
	| | |

Slide 5.21

4. **Is this a tally chart or not?** — Tally chart

5. **How many chose ice cream?** — Three

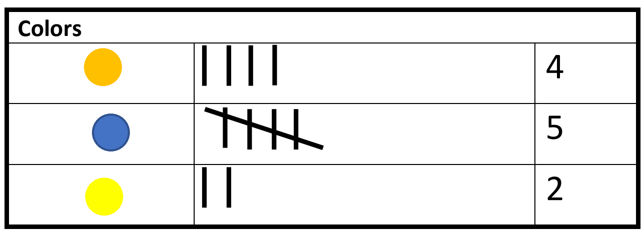
6. **[Advance slide.]**



Slide 5.22

7. **Is this a tally chart or not?**	Tally chart
8. **How many hearts were there?**	Two

9. **[Advance slide.]**



Slide 5.23

10. **Is this a tally chart or not?**	Tally chart
11. **How many liked yellow?**	Two

12. **[Advance slide.]**

Slide 5.24

13. **Is this a tally chart or not?**	Not

You practiced tally charts!

Exercise 4: Less Than and Greater Than – Review

General Error Correction: Model, test, delayed test

1. **Let's practice less than and greater than symbols. [Advance slide.]**

>

Slide 5.26

2. **Is this less than or greater than?** — Greater than

---

3. **[Advance slide.]**

>

Slide 5.27

4. **Is this less than or greater than?** — Greater than

---

5. **[Advance slide.]**

<

Slide 5.28

6. **So is this less than or greater than?** — Less than

---

7. **[Advance slide.]**

>

Slide 5.29

8. **So is this less than or greater than?** — Greater than

---

9. **[Advance slide.]**

<

Slide 5.30

10. **So is this less than or greater than?** — Less than

You reviewed greater than and less than!

Exercise 5: Dividend - Review

General Error Correction: Model, test, delayed test

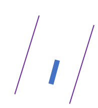
1. **Let's review dividend. What do we call the number that needs to be divided?**	Dividend
2. **Look at this.** **[Advance slide.]**	
$$\begin{array}{r} 10 \\ 4\overline{)\,40} \end{array}$$ Slide 5.32	
3. **This says forty divided by four equals ten. What number is the dividend?**	Ten
4. **What is the ten called?**	Dividend
5. **Let's look at another one.** **[Advance slide.]**	
$$\begin{array}{r} 7 \\ 2\overline{)\,14} \end{array}$$ Slide 5.33	
6. **This says fourteen divided by two equals seven. What number is the dividend?**	Fourteen
7. **What is fourteen called?**	Dividend
8. **Here's another one.** **[Advance slide.]**	
$$\begin{array}{r} 40 \\ 5\overline{)\,200} \end{array}$$ Slide 5.34	
9. **This says two hundred divided by five equals forty. What number is the dividend?**	Two hundred
10. **What is two hundred called?**	Dividend

Scripted Student Responses: 54

11. **Here's another one.** **[Advance slide.]** $$\begin{array}{r} 16 \\ 2\overline{)32} \end{array}$$ Slide 5.35	
12. **This says thirty-two divided by two equals sixteen. What number is the dividend?**	Thirty-two
13. **What is thirty-two called?**	Dividend
14. **[Advance slide.]** $$\begin{array}{r} 3 \\ 2\overline{)6} \end{array}$$ Slide 5.36	
15. **This says six divided by two equals three. What number is the dividend?**	Six
16. **What is six called?**	Dividend
17. **What does the dividend tell us?**	The number that needs to be divided
You practiced dividend!	

Exercise 6: Parallel - Review

General Error Correction: Model, test, delayed test

1. **Let's practice parallel.** **[Advance slide.]**  Slide 5.38	
2. **Are these lines parallel or not?**	Parallel
3. **[Advance slide.]**  Slide 5.39	
4. **Are these lines parallel or not?**	Not

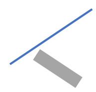
Scripted Student Responses: 54

5. **[Advance slide.]**

Slide 5.40

6. **Are these lines parallel or not?** Parallel

7. **[Advance slide.]**



Slide 5.41

8. **Are these lines parallel or not?** Not

You reviewed parallel!

End of Lesson 5

Appendix J. Permission to Use *Mathematics Vocabulary – 3rd Grade*

Wednesday, April 13, 2022 at 8:40:50 AM Mountain Daylight Time

Subject: [EXT] Re: Mathematics Vocabulary Assessment Question
Date: Wednesday, April 13, 2022 at 8:18:04 AM Mountain Daylight Time
From: Sarah Powell
To: Kristen Rolf

Oh yes, that's fine to include it. Do you need a copy?

Sarah

On Tue, Apr 12, 2022 at 11:26 PM Kristen Rolf <kristen.rolf@usu.edu> wrote:

> Hi Sarah-
>
> I'm getting ready to submit my dissertation, and my committee would like a copy of your math vocab assessment included as an appendix. Would that be alright with you? I completely understand if it's not – please just let me know either way.
>
> I hope you're doing well and that the semester is wrapping up nicely!
>
> Kristen R. Rolf, M. Ed.
>
> Doctoral Candidate
>
> Department of Special Education and Rehabilitation Counseling
>
> Utah State University

--
Sarah R. Powell, Ph.D.
Associate Professor
Department of Special Education
The University of Texas at Austin
@sarahpowellphd

CURRICULUM VITAE

April 12, 2022

Kristen R. Rolf, M.Ed.
Department of Special Education & Rehabilitation Counseling 253.905.8121
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2865 Old Main Hill
Logan, Utah 84322-2865

EDUCATION

Utah State University – Logan, UT
Ph.D. anticipated, 2022
Advisors: Timothy A. Slocum and Kaitlin Bundock
Disability Disciplines – Special Education
Fellowship Recipient: Multidisciplinary Program to Train Leaders in Evidence-Based Practice and Implementation Science
Dissertation: *Investigating the Effectiveness of Explicit, Systematic Mathematics Vocabulary Instruction for Students with Learning Difficulties and Disabilities in a Specialized Setting*

University of Washington Tacoma – Tacoma, WA
M.Ed., 2009
K-8 Elementary Education Certification, 2007
P-12 Special Education Certification, 2007
Secondary English Endorsement, 2007

Pacific Lutheran University – Tacoma, WA
B.A., 2003
English, emphasis in Literature

PUBLICATIONS

Peer-reviewed

Pinkelman, S. E., **Rolf, K. R.,** Landon, T., Detrich, R., McLaughlin, C., Peterson, A., and McKnight-Lizotte, M. (2022). Curriculum adoption in U.S. schools: An exploratory, qualitative analysis. *Global Implementation Research and Applications, 2*(1). doi: 10.1007/s43477-022-00039-2

Rolf, K. R. & Slocum, T. A. (2021). Features of Direct Instruction: Interactive lessons. *Behavior Analysis in Practice, 14*(3), 793-801. doi: 10.1007/s40617-021-00613-4

Slocum, T. A. & **Rolf, K. R.** (2021). Features of Direct Instruction: Content analysis. *Behavior Analysis in Practice, 14*(3), 775-784. doi: 10.1007/s40617-021-00617-0

Bundock, K., Callan, G., Longhurst, D., **Rolf, K. R.**, Benney, C., & Brunson McClain, M. (2021). Mathematics intervention for college students with learning disabilities: A pilot study targeting rate of change. *Insights on Learning Disabilities, 18*(1), 1-28. Retrieved from https://files.eric.ed.gov/fulltext/EJ1295246.pdf

Rhine, S., Driskell, S. O. S., **Rolf, K. R.,** Bundock, K. & Hurdle, Z. (2021). PK-8 mathematics digital curriculum adoption process: Analysis across states. *Curriculum and Teaching Dialogue, 23*(2), 213-228.

Rolf, K. R., Pinkelman, S. E., & Bundock, K. (2021). Reviewing tools for evaluating K-12 instructional materials through an implementation lens. *Global Implementation Research and Applications, 1*(1), 5-16. doi: 10.1007/s43477-020-00005-w

Manuscripts Under Review and in Preparation

Bundock, K., **Rolf, K. R**., Hornberger, A. & Holiday, C. The impact of professional development on mathematics co-teachers' teaching practices, perceptions, and student outcomes. *(Under review).*

Hager-Martinez, K., Bundock, K., & **Rolf, K. R.** Social validity of multiple methods of performance feedback on preservice teaching: Supervisor, preservice teacher self-evaluation, and peer evaluation. *(Under review).*

Bundock, K., Callan, G., McClain, M.B., Benney, C., **Rolf, K.R**., Burton, A., & Harris, B. The effects of a rate of change intervention on the achievement of middle and high school students with or at risk for learning disabilities. *(In preparation).*

Rolf, K. R., Peterson, A., Bundock, K., & Slocum, T. How do we teach students to talk about mathematics?: A systematic review. *(In preparation).*

Rolf, K. R., Slocum, T. A., Bundock, K., & Peterson, A. Features of mathematics vocabulary instruction. *(In preparation).*

Reports

Rolf, K. R. (2020). The adoption of curricula in K-12 schools: An exploratory qualitative analysis. The Wing Institute. https://www.winginstitute.org/about-student-research

Rolf, K. R. (2019). State department of education support for implementation issues faced by school districts during the curriculum adoption process. The Wing Institute. https://www.winginstitute.org/about-student-research

Books

Stein, M., Kinder, D., **Rolf, K.**, Silbert, J., & Carnine, D. (2018). *Direct Instruction Mathematics* (5th ed.). Pearson.

Book Chapters

Pinkelman, S. E., Bundock, K., & **Rolf, K. R.** (2020). Supporting students with Autism Spectrum Disorder in schools through multi-tiered system of supports. In M. B. McClain, J. D. Shahidullah, & K. R. Mezher (Eds.), *Interprofessional care coordination for pediatric Autism Spectrum Disorder: Translating research into practice.* Springer.

Stein, M., Kinder, D., Rasplica, W., **Rolf, K.**, & Bellamy, T. (2017). Project RTI. In J. Goeke & K. Mitchem, K. Kossar (Eds.), *Redesigning Special Education Teacher Preparation: Challenges and Solutions*. Routledge/Taylor and Francis.

Instructional Materials

Rolf, K. R., Slocum, T. A., & Wieszciecinski, P. (2021). *Mathematics Vocabulary for Fourth Grade.* Author.

Rolf, K. (2018). *Instructor's Manual for Direct Instruction Mathematics – 5th Edition.* Pearson.

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Rolf, K.R., & Bundock, K. (2019). Reviewing mathematics curriculum evaluation tools through an implementation lens. In Otten, S., Candela, A. G., de Araujo, Z., Haines, C., & Munter, C. (Eds.). *Proceedings of the 41st Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 133). St. Louis, MO.

Bundock, K., & **Rolf, K.R.** (2018). The effects of a state-implemented co-teaching training on students' mathematics achievement scores. In Hodges, T. E., Roy, G. J., & Tyminski, A. M. (Eds.). *Proceedings of the 40th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 440). Greenville, SC.

PRESENTATIONS

Peer-reviewed Presentations

Rolf, K. R., Peterson, A., Wieszciecinski, P., & Bundock, K. (February, 2022). Using the Plan-Do-Study-Act Cycle to Design Mathematics Vocabulary Instruction. Poster presented at the Pacific Coast Research Conference, San Diego, CA.

Rolf, K. R., Peterson, A., Wieszciecinski, P., & Bundock, K. (January, 2022). Using the Plan-Do-Study-Act Cycle to Improve Instruction. Poster presented to the Council for Exceptional Children, Orlando, FL.

Rolf, K. R. & Peterson, A. K. (March, 2021). Using Implementation Science to improve student outcomes across tiers of instruction. Paper presented to the Council for Exceptional Children, Virtual conference.

Rolf, K. R. Peterson, A. K., & Bundock, K. (March, 2021). Reviewing effective interventions for teaching mathematics vocabulary across tiers. Poster presented to the Council for Exceptional Children, Virtual conference.

Rolf, K. R., Peterson, A. K., & Bundock, K. (February, 2021). A systematic review of mathematics vocabulary interventions across instructional tiers. Poster presented at the Pacific Coast Research Conference, Virtual conference.

Rhine, S., Driskoll, S., **Rolf, K. R.**, Bundock, K., & Hurdle, Z. (October, 2020). PK-8 mathematics digital curriculum selection process: Analysis across states. Paper presented to the Society for Information Technology & Teacher Education, Virtual conference.

Rolf, K.R., Pinkelman, S. E., & Bundock, K. (May, 2020). Reviewing state-created curriculum evaluation tools through an implementation lens. Poster presented to the Association for Behavior Analysis International, Virtual conference.

Rolf, K.R., Pinkelman, S. E., & Bundock, K. (February, 2020). Reviewing state-created curriculum evaluation tools through an implementation lens. Poster presented at the Pacific Coast Research Conference, San Diego, CA.

Rolf, K.R. & Bundock, K. (February, 2020). Reviewing 50 years of disproportionality research: Predictor variables and analysis methods. Poster presented at the Pacific Coast Research Conference, San Diego, CA.

Rolf, K.R., & Bundock, K. (November, 2019). *Reviewing mathematics curriculum evaluation tools through an implementation lens.* Poster presented at the Psychology of Mathematics Education – North America Conference, St. Louis, MO.

Pinkelman, S. E., **Rolf, K. R.**, McLaughlin, C., Detrich, R., & Landon, T. J. (September, 2019). *Adoption of programs in U.S. schools: A qualitative analysis.* Poster presented at the Global Implementation Conference, Glasgow, Scotland.

Bundock, K. & **Rolf, K. R.** (November, 2018). *The effects of a state-implemented co-teaching training on students' mathematics achievement scores.* Poster presented at the Psychology of Mathematics Education – North America Conference, Greenville, SC.

Rolf, K. & Bundock, K. (April, 2018). *The effects of a state-implemented co-teaching training on students' mathematics achievement scores.* Poster presented at the Utah State University Student Research Symposium, Logan, UT.

Stein, M., Bellamy, T., Kinder, D., Rasplica, W., & **Rolf, K**. (February, 2018). *Improving teacher preparation through successful university/school partnerships: Lessons learned.* Paper presented to the Council for Exceptional Children, Tampa, FL.

Rolf, K. & Bundock, K. (February, 2018). *The effects of a state-implemented co-teaching training on students' mathematics achievement scores.* Poster presented at the Pacific Coast Research Conference, San Diego, CA.

Invited Presentations

Slocum, T. A. & **Rolf, K. R.** (May, 2020). *Features of Direct Instruction: Analysis of the domain and effective interaction.* Paper presented to the Association for Behavior Analysis International, Virtual conference.

Rolf, K. (February, 2017). *Identifying and planning supports for academic language demands in content areas.* Guest lecture presented to graduate students in TEDUC 527 Content Literacy at University of Washington Tacoma, Tacoma, WA.

Rolf, K. (January, 2017). *Lesson planning in Language Arts.* Guest lecture presented to graduate students in TEDUC 554 Topics in Literacy: Language Arts at University of Washington Tacoma, Tacoma, WA.

Professional Development

Rolf, K. R. (October, 2019). *easyCBM Math* – Progress Monitoring. Professional development training provided to teachers at Bear River Charter School, Logan, UT.

Rolf, K. R. (September, 2019). The implementation of *easyCBM Math* for benchmarking and progress monitoring. Professional development training provided to teachers at Bear River Charter School, Logan, UT.

Rolf, K., & Beard, D. (March, 2017). 1:1 coaching on delivery of *Reading Mastery* and *Corrective Reading*. Professional development training provided to teachers and paraprofessionals at Midland Elementary School, Tacoma, WA.

Rolf, K. & Beard, D. (December, 2016). *Reading Mastery and Corrective Reading: Fidelity Checks.* Professional development training provided to teachers and paraprofessionals at Midland Elementary School, Tacoma, WA.

Rolf, K. & Beard, D. (October, 2016). *Direct Instruction Reading.* Professional development training provided to teachers and paraprofessionals at Midland Elementary School, Tacoma, WA.

Rolf, K. & Beard, D. (September, 2016). *Direct Instruction Reading.* Professional development training provided to teachers and paraprofessionals at Midland Elementary School, Tacoma, WA.

Rolf, K. (August, 2016). *Reading Mastery.* Professional development training provided to newly hired special education teachers from the Franklin Pierce School District, Tacoma, WA.

Rolf, K. (May, 2016). 1:1 coaching on delivery of *Reading Mastery* at Central Avenue Elementary School, Tacoma, WA.

Rolf, K. (April, 2016). 1:1 coaching on delivery of *Reading Mastery* at Central Avenue Elementary School, Tacoma, WA.

Rolf, K. (March, 2016). 1:1 coaching on delivery of *Reading Mastery* at Central Avenue Elementary School, Tacoma, WA.

Rolf, K. (February, 2016). 1:1 coaching on delivery of Connecting Math Concepts and *Reading Mastery* at Central Avenue Elementary School, Tacoma, WA.

Rolf, K. and Beard, D. (February, 2016). *Connecting Math Concepts* and *Corrective Math.* Professional development training provided to math specialists from the Franklin Pierce School District, Tacoma, WA.

Rolf, K. (November, 2015). *Reading Mastery Signature Edition Grade 1.* Professional development training provided to teachers and paraprofessionals from the Sumner School District, Sumner, WA.

Rolf, K. (November, 2015). 1:1 coaching on delivery of *Connecting Math Concepts* and *Reading Mastery* at Central Avenue Elementary School, Tacoma, WA.

Rolf, K. and Beard, D. (November, 2015). *Connecting Math Concepts* and *Corrective Math.* Professional development training provided to math specialists from the Franklin Pierce School District, Tacoma, WA.

Rolf, K. and Beard, D. (October, 2015). *Connecting Math Concepts* and *Corrective Math.* Professional development training provided to math specialists from the Franklin Pierce School District, Tacoma, WA.

Rolf, K. (October, 2015). *Reasoning and Writing.* Professional development training provided to middle school teachers and paraprofessionals from the Franklin Pierce School District, Tacoma, WA.

FUNDING

Funded Grants

2019 **$300.00** - Graduate Student Travel Award, Utah State University
$5000.00 - Graduate Research Funding Program in Evidence-based Education, The Wing Institute

2018 **$300.00** - Graduate Student Travel Award, Utah State University
2017 **$300.00** - Graduate Student Travel Award, Utah State University

Grant Submissions

$1,307,770.00 – Collaborative Author. PI: Shumway, J.F., Co-PIs: Bundock, K., & Moyer-Packenham, P.S. (August, 2019). *MathVision Interventions: Improving First-Graders' Visual Number System Knowledge.* Education Research Grant: Science, Technology, Engineering, and Mathematics (STEM), Project Type: Development and Innovation. U.S. Department of Education Institute of Education Sciences (84.305A). Not funded.

HONORS AND AWARDS

Distinguished Scholar
Cambridge Center for Behavioral Studies, 2021-2022

Doctoral Student Researcher of the Year Award
Department of Special Education and Rehabilitation,
Utah State University, 2019

Graduate Student Teacher of the Year Award
Department of Special Education and Rehabilitation,
Utah State University, 2018

School of Education Distinguished Alumni Award
University of Washington Tacoma, 2017

TEACHING

Undergraduate

SPED 5312 Mathematics Content, Applications & Co-teaching (Utah State University) - Teaching Assistant, Spring 2020

SPED 5340 Teaching Math to Students with Mild/Moderate Disabilities (Utah State University) – Teaching Assistant, Fall 2019

SPED 5310 Teaching Reading and Language Arts to Students with Mild/Moderate Disabilities (Utah State University) – Instructor of Record, Fall 2018

SPED 5310 Teaching Reading and Language Arts to Students with Mild/Moderate Disabilities (Utah State University) – Teaching Assistant, Fall 2017

SPED 5340 Teaching Math to Students with Mild/Moderate Disabilities (Utah State University) – Teaching Assistant, Fall 2017

Graduate

SPED 6300 Effective Practices with Culturally and Linguistically Diverse Populations (Utah State University) – Co-teacher, Fall 2021

TEDUC 554 Topics in Literacy: Language Arts (University of Washington Tacoma) – Adjunct Lecturer, Winter 2016

TEDSP 541 Reading Methods and Interventions (University of Washington Tacoma) – Adjunct, Lecturer, Fall 2015

PROFESSIONAL ORGANIZATIONS

Association for Behavior Analysis International – Jan. 2020 – Present

Council for Exceptional Children – Nov. 2017 – Present

- CEC Division for Culturally and Linguistically Diverse Exceptional Learners – Feb. 2019 – Present
- CEC Division for Learning Disabilities – Feb. 2019 – Present
- CEC Teacher Education Division – Feb. 2019 – Present
- Utah CEC – Nov. 2017 - Present

Global Implementation Society – May 2020 - Present

PROFESSIONAL SERVICE

Journal Reviews

Journal of Positive Behavior Interventions (Guest reviewer)

Conference Reviews

Teacher Education Division of the Council for Exceptional Children – 2021

Global Implementation Society – 2020

Consulting

Bear River Charter School, Logan, Utah – Spring 2019 – Present
Implementation of benchmarking and progress monitoring in mathematics in grades K-8

Other

Doctoral Student Representative to the Faculty
Disability Disciplines Doctoral Program,
Utah State University, Logan, Utah – 2020-2021

Kids' Church Teacher
Alpine Church
Logan, Utah – Fall 2018 - Present

EMPLOYMENT HISTORY

2017-Present	**Department of Special Education & Rehabilitation, Utah State University Logan, UT**	**Graduate Assistant**

- Co-author scholarly articles, book chapters, and conference presentations
- Leading a team of cross-disciplinary researchers in designing a mathematics vocabulary intervention
- Project manager for a qualitative study utilizing semi-structured interviews about the curriculum adoption process
- Led a content analysis of state-created curriculum evaluation tools
- Supported mixed methods evaluation of Utah's implementation of co-teaching in secondary classrooms
- Collaborated with faculty members at institutions across the country on the design, data collection, and analysis of a study utilizing surveys to gather information about the adoption of digital mathematics curricula in K-8 schools
- Contributed to the completion of IRB protocols
- Drafted interview and survey questions, contacted potential participants, and coded articles related to various projects
- Teaching assistant for undergraduate reading and mathematics methods classes and masters cultural and linguistic diversity class
- Reformatted and taught two sections of a reading methods class from a traditional face-to-face model to a hybrid model. This reformatting involved creating asynchronous presentations using Nearpod, facilitating practice during face-to-face sessions for on-campus and distance students, coaching and providing feedback to undergraduate students, revising and grading quizzes and

assignments, keeping up-to-date records using Canvas, and responding to students written and verbal requests.

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2015-2017	**University of Washington Tacoma and Franklin Pierce School District Tacoma, WA**	**UWT Field Coordinator**

- Collaborated with university faculty and school district personnel to place K-8 elementary education certification students in seven partner schools in two school districts
- Coordinated and supported mentor teachers, site coordinators, and field supervisors in each partner school
- Reviewed evaluation data and disseminated pertinent information to appropriate personnel
- Collaborated with university faculty, district personnel, and field supervisors to build connections between coursework and fieldwork and to support struggling teacher candidates
- Provided support and feedback to teacher candidates completing the edTPA in math or literacy for elementary education
- Designed and delivered professional development in reading and mathematics interventions to school district personnel

2015-2016	**University of Washington Tacoma Tacoma, WA**	**Part-time Lecturer**

- Planned and delivered instruction to M.Ed. students earning K-8 elementary education certification and K-12 special education certification in reading methods and interventions (fall 2015) and language arts (winter 2016)
- Evaluated assignments submitted by students and provided feedback in a timely manner
- Maintained accurate and up-to-date records using Canvas
- Responded to students written and verbal requests
- Met with students privately, as requested
- Supported students to complete the literacy edTPA for Elementary Education

2014-2015	**Franklin Pierce School District Tacoma, WA**	**UWT Field Supervisor**

- Supervised M.Ed. students earning K-8 elementary education and K-12 special education certification from the University of Washington Tacoma
- Provided coaching and written weekly observation reports for Direct Instruction lessons

- Reviewed lesson plans and provided positive and corrective feedback to teacher candidates
- Conducted formal evaluations of general education whole-group lessons and created individualized focus assignments based on teaching performance
- Provided professional development for district staff and UWT students

2013-2014	**Pearson**	**edTPA Scorer – Special Education**

- Completed edTPA scorer training and met qualification requirements
- Scored at least 12 special education portfolios per month during scoring fall, winter, and spring scoring sessions

2010-2012	**Lighthouse Christian School Gig Harbor, WA**	**6th Grade Teacher**

- Planned and delivered instruction in the areas of Language Arts, History, and Biblical Studies
- Designed and provided remedial instruction to K-8 students in the areas of reading and math
- Developed and implemented Speech & Debate curriculum for seventh and eighth grade students
- Member of school board committees to explore implementation of benchmarking assessments and progress-monitoring using curriculum-based measures, Social Studies Curriculum Review, and committee to plan science-themed summer camp

2007-2010	**Franklin Pierce School District Tacoma, WA**	**Special Education Teacher**

- Provided instruction in the areas of reading, math, writing, social skills, behavior, and adaptive living
- Case manager for approximately 20 students with special needs annually
- Conducted academic evaluations using standardized tests and curriculum-based measures and wrote individualized education plans
- Progress-monitored students weekly using curriculum-based measures and observations
- Collaborated with general education teachers, special education teachers, related service personnel, parents, and other staff regarding interventions, behavior, evaluations, and IEP meetings
- Supervised two full-time paraeducators and volunteers
- Member of building level problem-solving team, special education team, Response to Intervention team, and Safe and Civil team
- Member of district-level special education team, Response to Intervention team, and Special Education Parent Advisory Committee