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Applications of Broyden-based Input Space Mapping to Modeling and Design Optimization in High-Tech Companies in Mexico

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***Abstract* — One of the most powerful and computationally efficient optimization approaches in RF and microwave engineering is the space mapping (SM) approach to design. SM optimization methods belong to the general class of surrogate-based optimization algorithms. They are specialized on the efficient optimization of computationally expensive models. This paper reviews the Broyden-based input SM algorithm, better known as aggressive space mapping (ASM), which is perhaps the SM variation with more industrial applications. The two main characteristics that explain its popularity in industry and academia are emphasized in this paper: simplicity and efficiency. The fundamentals behind the Broyden-based input SM algorithm are described, highlighting key steps for its successful implementation, as well as situations where it may fail. Recent applications of the Broyden-based input space mapping algorithm in high-tech industries located in Mexico are briefly described, including application areas such as signal integrity and high-speed interconnect design, as well as post-silicon validation of high-performance computer platforms, among others. Emerging new applications in multi-physics interconnect design and power-integrity design optimization are also mentioned.**

***Keywords* — Broyden, DoE, equalization, eye diagram, high-speed interconnects, optimization, post-silicon validation, power integrity, signal integrity, space mapping, surrogate models.**

I. Introduction

RF and microwave engineers have relied for decades on numerical optimization techniques for modelling enhancement and design closure [1], [2]. However, classical optimization methods are rarely used in this arena [3], given the typically high computational cost associated to the microwave models, especially when full-wave electromagnetic (EM) simulators are employed, when multi-physics numerically-based simulators are required, or when a large scale system-level optimization is needed.

One of the most powerful and computationally frugal optimization techniques in RF and microwave engineering is the space mapping (SM) approach to design [4]. The family of SM optimization methods belong to the general class of surrogate-based optimization algorithms [5]. They are tailored to efficient optimization of computationally expensive models.

This paper reviews one of the first SM methods ever developed: the Broyden-based input SM algorithm, originally named as aggressive space mapping (ASM) [6]. This algorithm is the SM variation with more industrial applications publically documented [7], [8]. The two main characteristics that explain its popularity in industry and academia are its simplicity and efficiency. The fundamentals behind the Broyden-based input SM algorithm are illustrated, highlighting key steps for its successful implementation, as well as typical situations where it may fail. Particularly, recent applications of the Broyden-based input space mapping algorithm in high-tech industries located in Mexico are described, including application areas such as signal integrity and high-speed interconnect design, as well as post-silicon validation of high-performance server computer platforms, among others. Finally, some emerging new applications in multi-physics interconnect design, power delivery networks, and power-integrity optimization are briefly mentioned.

II. Broyden-based Input Space Mapping

A flow diagram for the Broyden-based input space mapping algorithm [9] is shown in Fig. 1. It assumes there are two models available for the design optimization problem: a coarse model, whose design parameters are in vector $\boldsymbol{x}_{\mathrm{c}}$ and the corresponding responses in vector $\boldsymbol{R}_{\mathrm{c}}$; and a fine model, whose design variables and responses are in $\boldsymbol{x}_{\mathrm{f}}$ and $\boldsymbol{R}_{\mathrm{f}}$, respectively. It is also assumed that evaluating $\boldsymbol{R}_{\mathrm{f}}(\boldsymbol{x}_{\mathrm{f}})$ is very accurate but much more expensive than evaluating $\boldsymbol{R}_{\mathrm{c}}(\boldsymbol{x}_{\mathrm{c}})$, whose evaluation is cheap but insufficiently accurate.

The algorithm starts by finding the optimal coarse model design, $\boldsymbol{x}_{\mathrm{c}}^{*}$, usually from direct optimization of the coarse model, as indicated in Fig. 1, using classical optimization methods that typically require many objective function evaluations, which can be afforded since evaluating $\boldsymbol{R}_{\mathrm{c}}(\boldsymbol{x}_{\mathrm{c}})$ is cheap. Here, $U(\boldsymbol{R}_{\mathrm{c}}(\boldsymbol{x}_{\mathrm{c}}))$ is a suitable objective function expressed in terms of the coarse model responses and the design specifications.

Matrix $\boldsymbol{B}$ shown in Fig. 1 is the so-called Broyden matrix: an approximation of the Jacobian of $\boldsymbol{x}_{\mathrm{c}}$ with respect to $\boldsymbol{x}_{\mathrm{f}}$. The algorithm initializes $\boldsymbol{B}$ and $\boldsymbol{x}_{\mathrm{f}}$, and then evaluates the fine

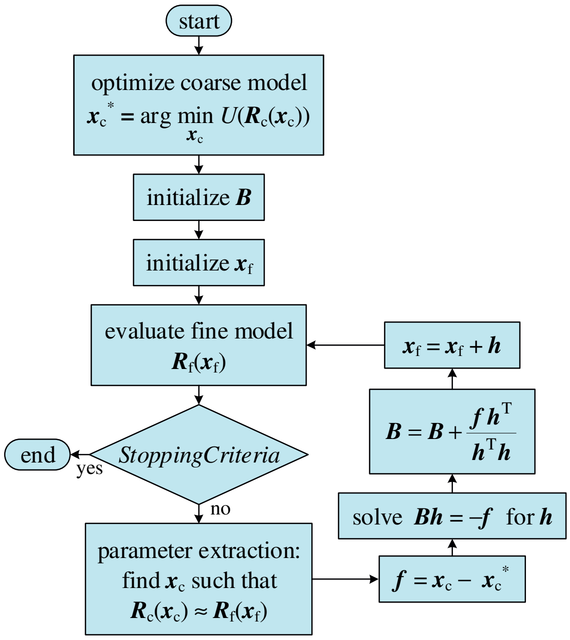

Fig. 1. Broyden-based input space mapping algorithm, originally named as aggressive space mapping (ASM) [6]. Taken from [7].

model at this initial design. If the stopping criteria are not met, the algorithm performs parameter extraction (PE), which consists of finding the coarse model design that makes the coarse model response as close as possible to the fine model response at the current iterate (see Fig. 1). It then calculates vector $\boldsymbol{f}$, which is the difference between the extracted parameters and the optimal coarse model design. Next it solves for $\boldsymbol{h}$ the linear system $\boldsymbol{Bh} = -\boldsymbol{f}$, from which a quasi-Newton step is obtained. It then updates $\boldsymbol{B}$ with Broyden's rank one formula, calculates the next iterate, and continues until the stopping criteria are met (see Fig. 1).

A. Initializing the Broyden Matrix

If $\boldsymbol{x}_f$ and $\boldsymbol{x}_c$ have the same nature and dimension, then the Broyden matrix is initialized by the identity matrix, $\boldsymbol{B} = \boldsymbol{I}$. However, if $\boldsymbol{x}_f$ and $\boldsymbol{x}_c$ have different nature or different dimension, the Broyden matrix can be initialized by

$$\boldsymbol{B} \approx \boldsymbol{J}(\boldsymbol{x}_c(\boldsymbol{x}_f)) = \frac{\partial \boldsymbol{x}_c}{\partial \boldsymbol{x}_f} \tag{1}$$

In practice, (1) is an approximation of the Jacobian calculated by finite differences around $\boldsymbol{x}_c^*$. Evaluating (1) in this manner has a very significant cost, since it implies at least $2n$ fine model evaluations and parameter extractions, where n is the number of fine model design variables ($\boldsymbol{x}_f \in \Re^n$).

B. Initializing the Fine Model Design

If $\boldsymbol{x}_f$ and $\boldsymbol{x}_c$ have the same nature and dimension, then the fine model design is initialized simply by $\boldsymbol{x}_f = \boldsymbol{x}_c^*$. However, if $\boldsymbol{x}_f$ and $\boldsymbol{x}_c$ have different nature or dimension, the fine model design can be initialized by solving the linear system

$$\boldsymbol{B}\boldsymbol{x}_f = \boldsymbol{x}_c^* \tag{2}$$

for $\boldsymbol{x}_f$, which has practically no cost once $\boldsymbol{B}$ is initialized.

C. Parameter Extraction (PE)

PE consists of a local alignment between the two models at the i-th iteration (see Fig. 1), which can be implemented as

$$\boldsymbol{x}_c^{(i)} = \boldsymbol{P}(\boldsymbol{x}_f^{(i)}) = \arg\min_{\boldsymbol{x}_c} \left\| \boldsymbol{R}_f(\boldsymbol{x}_f^{(i)}) - \boldsymbol{R}_c(\boldsymbol{x}_c) \right\| \tag{3}$$

Notice that performing a PE is equivalent to evaluating the implicit mapping function between the two models, $\boldsymbol{x}_c = \boldsymbol{P}(\boldsymbol{x}_f)$. Assuming a cheap coarse model, solving (3) by classical optimization methods (even by global meta-heuristic techniques) has a low computational cost, since the fine model response at the current iterate $\boldsymbol{R}_f(\boldsymbol{x}_f^{(i)})$ is already available.

PE is the most vulnerable part of the Broyden-based input SM algorithm, since multiple local minima in (3) may lead the algorithm to oscillations or divergence [10]. In those cases, three effective remedies can be implemented to increase PE uniqueness and consistency: a) multi-point parameter extraction [10], [11]; b) statistical parameter extraction [12] to escape from poor local minima; and c) statistical PE combined with penalty functions [13].

D. Implicit System of Non-linear Equations

As explained in [7], the Broyden-based input space mapping algorithm (see Fig. 1) is essentially equivalent to the Broyden method for solving the system of nonlinear equations

$$\boldsymbol{f}(\boldsymbol{x}_f) = \boldsymbol{P}(\boldsymbol{x}_f) - \boldsymbol{x}_c^* = \boldsymbol{0} \tag{4}$$

A solution of (4) is called a space mapped solution, $\boldsymbol{x}_f^{SM}$; it does not necessarily matches the fine model optimal design $\boldsymbol{x}_f^*$ [7], but ensures that $\boldsymbol{R}_f(\boldsymbol{x}_f^{SM})$ approximates the target $\boldsymbol{R}_c(\boldsymbol{x}_c^*)$.

Since this algorithm is equivalent to the Broyden method for solving (4), it is key for its success that $\boldsymbol{f}(\boldsymbol{x}_f)$ has at least one root, or an acceptable solution, which depends on the degree of similarity between the coarse and fine models. Its level of efficiency (i. e., the number of required iterations or fine model evaluations) depends on the level of non-linearity of the mapping function $\boldsymbol{P}(\boldsymbol{x}_f)$ [7].

III. Industrial Applications in Mexico

A. High-Speed Interconnect Design

Broyden-based input SM is used in [14] to minimize the return losses in a modern package interconnect. In this particular application, developed at Intel Guadalajara, both fine and coarse models are based on the finite-element method (FEM), to simplify and generalize the development of coarse models for complex interconnect structures. The FEM-based fine model uses a detailed geometry and very fine resolution meshing; it was first validated with respect to actual physical measurements. The FEM-based coarse model uses a simplified geometry with 2D metals, neglects all kinds of losses, and it is discretized using a very coarse mesh.

A section of the optimized package interconnect geometry is shown in Fig. 2. Its optimization in [14] required only 4 fine model evaluations, reducing the return losses with respect to the initially fabricated prototype, and yielding an enhanced package design.

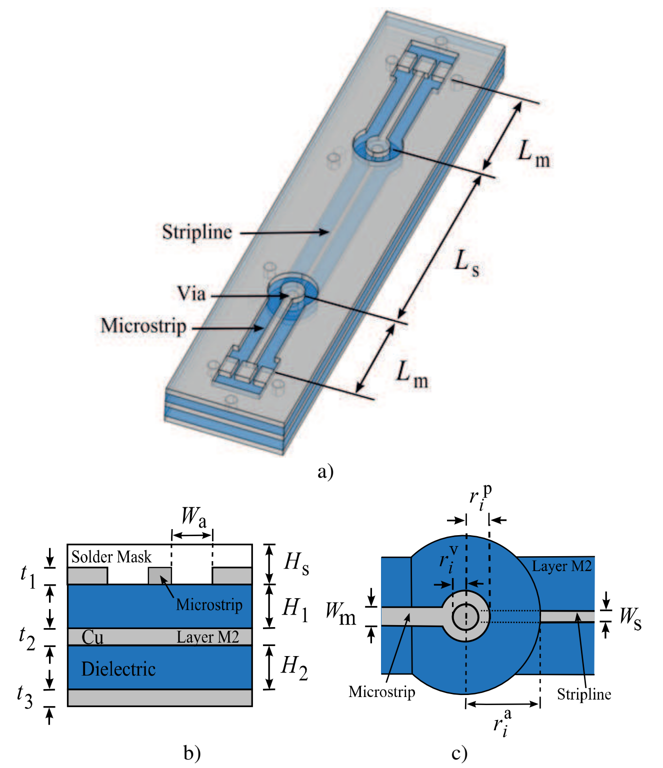

Fig. 2. Package interconnect: a) 3D view, b) cross-section view at the microstrip level, and c) top-view at a via. Taken from [14].

B. Coarse Meta-Modelling for Post-Silicon Validation of Computer Platforms

One of the most important challenges in post-silicon electrical validation of high-speed input/output (HSIO) links in modern computer platforms lies in the physical layer (PHY) tuning process, where equalization (EQ) techniques are used to cancel undesired effects induced by the channels [15] (see Fig. 3). Typical industrial practices for PHY tuning in HSIO links are very time consuming and require massive lab measurements [16]. They involve validating the electrical and EM performance *vs* specifications under different industrial interfacing standards, such as PCIE, SATA, USB, and Ethernet [17]. It is only in limited scenarios where classical direct optimization for tuning knobs equalization can be applied [18], [19].

An initial step towards accelerating the PHY tuning process in post-silicon validation consists of developing suitable surrogate models. In [20]-[21], several surrogate modeling methods and design of experiments (DoE) techniques are used to find a model that approximates the system with a very reduced set of data. In [22], a metamodeling approach based on neural networks is applied to efficiently simulate the effects of a receiver equalizer PHY tuning settings, as illustrated in Fig. 4, where several DoE techniques are also used to find a neural model capable of approximating the real system behavior without requiring a large amount of actual measurements, assessing the resultant surrogate performance by comparing with measured responses on a real industrial server HSIO link.

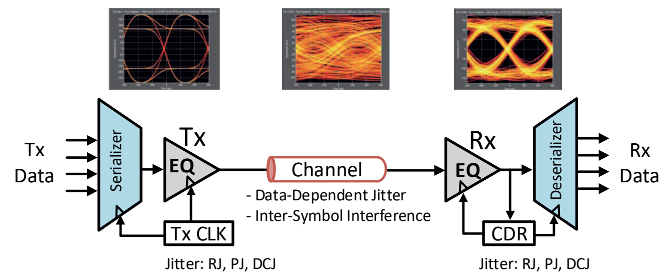

Fig. 3. Equalization (EQ) knobs at the transmitter (Tx) and receiver (Rx) are digitally tuned to compensate for undesired effects in high-speed links of computer platforms. Figure taken from [22].

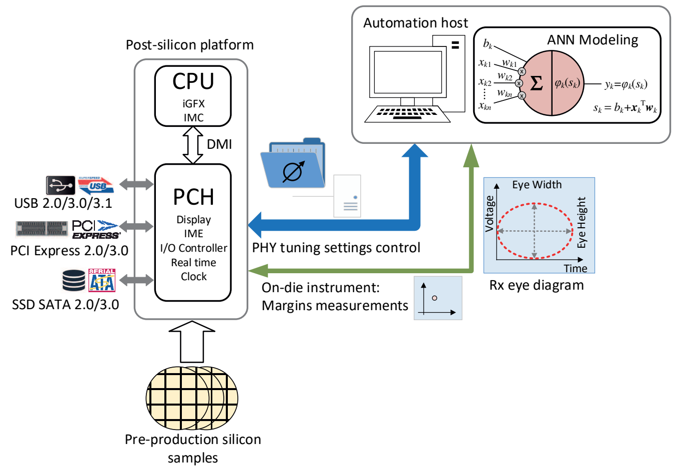

Fig. 4. HSIO server post-silicon configuration for metamodeling the Rx EQ tuning effects by using a coarse neural model. Figure taken from [22].

C. Space Mapping Optimization for Post-Silicon Validation of Computer Platforms

The Broyden-based input SM algorithm is used in [23] to efficiently optimize the PHY tuning receiver (Rx) equalization (EQ) settings for a SATA Gen 3 channel topology. As coarse model it uses a low-cost low-precision Kriging surrogate, and as the fine model it uses the actual measurement-based post-silicon validation platform. The experimental results, based on a real industrial validation platform, demonstrated the efficiency of SM to deliver an optimal eye diagram (see Fig. 5), showing a substantial performance improvement and accelerating the typical PHY tuning process, from days to a few hours [23].

D. Emerging Industrial Applications

We have identified some emerging new local industrial applications of SM-based modeling and design optimization. One of them is in multi-physics interconnect design, considering the coupled EM-thermo-mechanical performance [24], [25]. Another important area of potential industrial applications of SM-based methods is in power delivery networks and power-integrity optimization [26]-[28].

IV. Conclusion

We have reviewed in this paper the Broyden-based input SM algorithm as well as its main industrial applications in

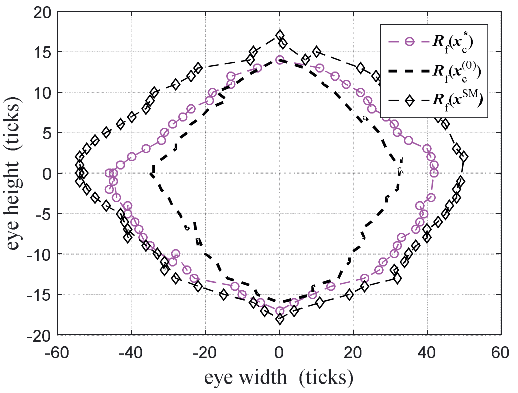

Fig. 5. Marginal eye diagrams: comparison between the system fine model responses at the initial Rx EQ coefficients, $\boldsymbol{x}_c^{(0)}$, at the optimal coarse model solution, $\boldsymbol{x}_c^*$, and at the space-mapped solution found, $\boldsymbol{x}^{SM}$. Taken from [22].

high-tech companies in Mexico that have been publically documented.

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