

INTEGRATING THE AFFECTIVE DOMAIN WHEN INTERPRETING UNDERSTANDING IN MATHEMATICS: AN OPERATIONAL APPROACH

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We present here an integrative proposal allowing to interpret the different systems of the affective domain and its relationship with understanding when performing mathematical activity. A conceptualisation of the affective domain is advanced based on the emotional system, which serves as a central point of reference, and a functional perspective of understanding relating to the uses given to mathematical knowledge. We also provide a specific interpretive method and exemplify it with a practical case of a preservice elementary teacher engaged in solving a flat surface measurement task. To conclude, by incorporating affective phenomena into the interpretation, we found complementary reasons that accounted for the student's mathematical understanding.

INTRODUCTION

While progress has been made in research on the affective domain in Mathematics Education, a consensus has yet to be reached on its organisation, characterisation and assessment. Calls have been made for specific models to be elaborated that would include affective domain conceptualisations linked to the specific issues under study (Hannula, 2012; Schlöglmann, 2002). It is in this problematic context that we conducted the present study, which addresses understanding in mathematics. Focusing on the key role of emotions in the development of understanding in mathematics, we sought to elaborate an integrative framework of the various components of affect in mathematics, a major challenge recognised in our field of research.

We used a developing model that is based on the interpretation of students' mathematical understanding (Gallardo & Quintanilla, 2019; Quintanilla, 2019). At a theoretical level, we put forward a dialectical approach in which the affective domain has a systemic nature, in order to incorporate a number of consolidated results within a single common process. At the methodological level, we provide a specific qualitative method to observe and interpret students' mathematical understanding based on the affective dimension. To show the method's potential in practice, we implemented it in an empirical qualitative study with preservice elementary teachers who undertook the resolution of measurement tasks. By integrating the different components of the affective domain and the understanding of mathematical knowledge, we succeeded at characterising the student's distinctive cognitive and affective features within a single interpretative process as he performed mathematical activity in the classroom.

THEORETICAL FRAMEWORK

We conceive the affective domain as an autonomous, dynamic, cyclical and closed meta-system that arises from the interactions among the related main systems it is made of: (a) belief system, (b) motivational and behavioural system, (c) emotional system, (d) attitudinal system, and (e) values and norms. Emotions are a central reference of the affective domain and are directly linked to the rest of the components, acting as mediators among them.

Every emotional experience starts with a first unconscious phase during which the person seeks – though a cognitive evaluation, conditioned by the context – to establish whether a certain object (physical or mental) or event (real, evoked or imaginary) may become an emotionally competent stimulus (ECS) (Damasio, 1994). This natural or acquired stimulus has the ability to trigger a particular *emotion*. Emotion is always the product of a value judgment made by the individual's cognitive system based on innate genetic patterns or socially and culturally learned patterns (Nussbaum, 2001). Beliefs, motivation, values and norms also contributes to this cognitive assessment of whether an object is an ECS (Di Martino & Signorini, 2019; Rouleau, 2019). In addition, emotions usually manifest themselves through *emotional responses* in the form of facial expressions, body language, tone of voice, and verbal locutions that can be recognised by outside observers (Ekman, 1993).

In a second phase of the emotional experience, the person's awareness of the different physiological changes provokes new thoughts relative to the object or initial situation that generated them and also relative to one's general physical status. A *feeling* thus appears as a mental representation of emotion (Damasio, 1994). Feelings prolong the impact and effects of emotions and, when evaluated cognitively, enable the generation of new emotions following a dynamic and cyclical process (meta-emotion). In addition, feelings predispose the subject to *action* by creating consciously adapted responses. Changes in emotional states also form stable affective patterns that are intertwined with cognition, creating contexts that are conducive to the individual taking action and in which beliefs act as systems of applied rules (Beswick, 2018; Goldin, 2004). Attitudes, on the other hand, constitute a tendency towards a specific type of action and contribute to shaping a person's identity. Again, the motivational system intervenes in the cognitive evaluations of this phase of the emotional experience, decisively influencing the subsequent course of action. In short, the specific actions associated with a given emotion in a particular context are triggered by a decision-making process link to the self-regulating and self-controlling nature of the emotion itself (Goldin, 2004; Lazarus & Lazarus, 1994). This facet allows to manage the emotion's *external representations* according to the interests of the person and the surrounding social and cultural norms, and finally, to generate the voluntary behaviours considered appropriate in each situation (Nussbaum, 2001).

From this perspective, the relationships between the different systems that compose the affective domain are established and transit through the emotional system. Thus,

norms regulate emotional experience by influencing how people value objects within their social group (Nussbaum, 2001). This system of values and norms can modify attitudes and beliefs with the mediation of the emotional system. The attitudinal system, on the other hand, is based on repeated emotional experiences and it is these experiences that link beliefs to attitudes. Beliefs, in turn, are directly related to meta-affect, self-regulation, and motivation, processes that can also modify the belief system itself (Di Martino & Signorini, 2019; Goldin, 2004; Hannula, 2012). Finally, objectives and motivations, such as impulses to act, can lead to specific attitudes and beliefs, behaviour being the most reliable manifestation of motivation (Rouleau, 2019).

Regarding our approach to understanding in mathematics, we assume that individuals understand a mathematical knowledge in so far as they are able to use it, in any of its possible forms, in situations where the knowledge makes sense and contributes to a resolution. It is a functional view based on the uses of mathematical knowledge that students implement during their classroom activity. The actions deployed in the concrete situation, including the uses of mathematical knowledge, are directly related to a decision process in which emotions are key. The student's various accumulated emotional experiences, resulting from their experiences in the mathematics classroom, directly influence their future decisions, actions and uses in the classroom. We thus recognise the existence of mental processes that are strongly linked to the emotions underlying the decisions about the uses of mathematical knowledge and which explain the student's understanding. Therefore, we characterise understanding in mathematics as an intellectual activity of an affective nature that enables the person to elaborate observable, adapted and contextualised responses, involving a recordable and interpretable usage of mathematical knowledge.

METHODOLOGY

In recent years, we have been developing an interpretive method which we call *the hermeneutic circle of understanding in mathematics* (Gallardo & Quintanilla, 2019). This method allows us to interpret simultaneously both the affective traces that accompany actions and motivate them, and the traces of understanding displayed by students when they solve problems. We applied our method in a qualitative study in which we interpreted the preservice elementary teachers' understanding of measurement based on their various manifestations of affect.

Participants and context

The participants were 20 volunteer preservice teachers enrolled in their fourth year of their *Degree in Primary Education* at the University of Málaga. They studied the *Didactics of Measurement* subject during the second semester of the 2017-2018 academic year. The participants solved measurement tasks in their ordinary classroom, within the usual classroom schedule and together with the rest of their classmates. Our method was implemented over nine weeks, during the subject's two weekly hours of practice.

Mathematical tasks

The selection was made based on each representative task of the different phases involved in the mathematical foundation of magnitude measurement (identification of magnitudes, conservation and comparison of quantities of magnitude, choice of measurement units, quantification and use of measurement instruments and arithmetisation). Non-equivalent tasks were used, the joint resolution of which would allow us to characterise the understanding of measurement. We illustrated the study using the records generated by one of the participating teachers (Antonio) when solving a task focused on the surface measurement of flat figures (Figure 1).

How large is the surface of the following figures?

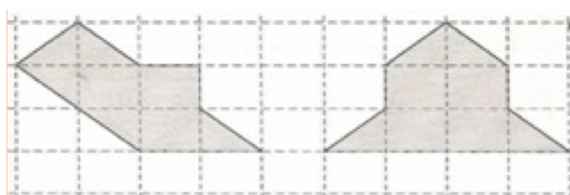


Figure 1: Flat Figure Surface Measurement Task.

Data collection

Each episode was conducted over three consecutive phases in which we used different data collection instruments.

Phase 1. Each participant was given the measurement tasks and a brief conversational interview took place. We addressed three main themes: (a) initial emotions when observing the tasks; (b) beliefs about mathematics and the students' personal relationship to mathematics; and (c) experiences with mathematics in the past. The interviews were recorded in audio (transcription as first written record).

Phase 2. The students were organised in pairs and solved the different tasks collaboratively. We sought to detect evidence of the interaction between the students' affective processes and their understanding as they use the mathematical knowledge. All the mathematical activity was recorded in audio and video (second written record).

Phase 3. The researcher shared her findings based on the results obtained in the previous phases and presented an interpretation of the student's performance in order to reach an agreement with him/her regarding the uses given to mathematical knowledge and their relationship with the affect experienced. Each interview was recorded in audio (third written record).

Data analysis and interpretation

The hermeneutic circle follows the various semiotic, phenomenon-epistemological and dialogical planes included in their interpretative trajectory.

We sought to identify the uses given to the mathematical knowledge and traces of understanding on the semiotic and phenomenon-epistemological planes. The following

analyses served as a reference: (a) the *phenomenon-epistemological analysis* of the problem raised, in which we clarified the essential knowledge that could help to solve the problem; and (b) the *phenomenological analysis* of the student's emerging affective components during mathematical practice. We characterised these components using different representation systems that informed us of what was being communicated and how it was communicated: (i) the *verbal* system (tone of voice and locutions) and (ii) the *kinesthetic* system (facial and body expressions).

On the circle's dialogical plane, we compared the student's mathematical activity during the episode, we established relationships with the uses given to mathematical knowledge, and then structured the conclusions regarding the student's understanding. The search for a consensus also allowed us to contrast information relating to the different affective components displayed by the student during the episode's previous phases. The appropriation that occurred during the agreement-building with the other was expected to generate a transformative effect on the protagonists.

RESULTS

In Phase 1 of the study, students' beliefs and personal mathematical history began to emerge, and we identified a number of initial emotions.

Antonio: (*Low voice, nervous laughter*) I don't know how to solve them (*belief about what I should know*). I should have worked on many of these things at school when I was a child (*belief about school teaching and learning*).

Interviewer: If you had to find a word for that feeling.

Antonio: Being unsure. Hesitations, quite a lot of doubts (*uncertainty*). I think that the lack of usage ... makes me forget about it (*causal attribution*). I don't understand why they don't explain this to me (*belief about teaching*). I am very eager to understand certain things (*belief about oneself*). I would say: I'm going to look for the answer, let's see if I find it (*perseverance*).

Antonio shows shame and, as he describes having doubts, he recognises feeling uncertainty. These emotions originate from the discrepancy between a belief about himself (he should know more about mathematics) and the recognition of not being able to immediately solve the tasks posed. He uses causal attributions as a justification (a characteristic component of the belief system) probably as a way of attempting to minimise the negative effects of his emotions. When describing his mathematical past, he is also showing beliefs about his own preferences and he is shaping a recognised attitude of perseverance.

During Phase 2, Antonio perceives the task as easy and becomes suspicious (belief). In addition, he manifests a limited understanding of the fraction as a measure. The fact of not regarding the triangle as a unit generates uncertainty (emotion):

Antonio: Maybe you can't, let's see... There is always a catch in these exercises. (*In the first figure*) 1, 2, 3, 4, 5, 6, 7, 8... Exactly half is left over (*half a square or a right-angled triangle*). I don't know if it is linked to...

He modifies his strategy and uses knowledge of formulas, without paying much attention to its adequacy. New beliefs are involved that may favour this change (mathematics requires the use of sophisticated calculations):

Antonio: In Primary school, you don't know the Pythagorean theorem, but if I know this (*the cathetus*) and I know this (*the cathetus*), I know how much the diagonal measures (*hypotenuse*). Taking the area of this triangle, I add it to the area of the square and we obtain the surface. And if not, then you would have to calculate the surface of one of the triangles and since all triangles are the same, you simply add the surface. That would be another option.

His beliefs (numbers are needed to calculate measurements) and his understanding influence his emotions. He shows uncertainty and distress that are reflected through tensions (emotional responses in Figure 2). He abandons the first flat figure:

Antonio: With the Pythagorean theorem, you have to know the measurement to get an answer. We don't have the numbers here, so we can't give numbers. The quadrilateral in the first figure is formed with...

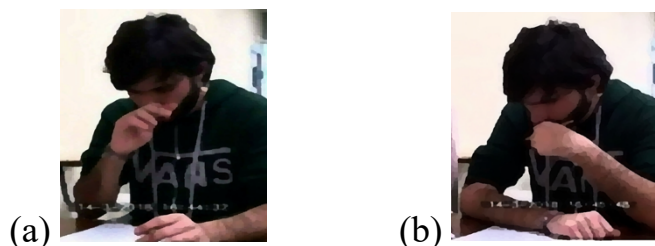


Figure 2: (a) Tense body and face, continuously touches his nose; (b) Covers mouth with hand, hunched shoulders, clenched hands.

Finally, he manages to measure the second figure and, therefore, decides to that the task is solved with an expression of relief (emotion):

Antonio: In this case (*second flat figure*), you can make the square with the additional pieces, those that are left over, and yes, it is possible to calculate the surface. Here we could form six squares. It would be the same. That's it!

In Phase 3, we obtained complementary information which allowed us to confirm the relationships existing between Antonio's affective domain and his understanding.

- 1 I: Did you have doubts while you were doing the exercise?
- 2 A: Yes (*firm voice, sounds very certain*). Uncertainty and frustration, partly. Because I should be able to solve it and... why can't I? I find it hard to believe that things can be simple (*origin of a belief*).
- 3 I: You started talking about the Pythagorean theorem.
- 4 A: I wanted to apply the Pythagorean theorem if a measurement was given. We would multiply the triangle by two and we would obtain the surface of the square. Here the surface would be given in terms of numbers of squares.
- 5 I: How much would it be?
- 6 A: Here, six (*second flat figure*). And for this one (*first flat figure*), let's see: One, two, three, these would form two, four, five. No! Five and a half.

- 7 I: Five and a half. That was the answer. What happened do you think?
- 8 A: I like it when things are complicated (*muffled laugh*). Since there was no tangible measure, I didn't give an answer. Numbers were missing. I'm used to having to give a precise result... but it's not like that (*timid laughs*). But I've still got it there... I don't know if one day I'll lose it...
- 9 I: But you told me that your perspective was changing...
- 10 A: I have to change the way I see things. I have the mathematical knowledge. What I need to understand is how I can really teach that knowledge.

We found evidence of the relationship between the uncertainty and frustration that Antonio felt and his beliefs about himself and the social context in which he was immersed (his own expectations and that of others about what he should be able to do) (1-2, 8-10). Moreover, he again displayed the emotional responses of tension and nervousness (8). The strategy he planned based on the use of the Pythagorean theorem was conditioned by his beliefs (mathematics consists of rules, formulas and complex procedures) (3-4). Subsequently, because he could not apply the theorem due to missing numerical data, he came back to resorting to his initial knowledge, and managed to solve the task (5-7). Finally, as he became aware of his mathematical performance during the task, Antonio encountered fresh motivation regarding his future teaching practice: He opened up to a possible modification of his own previous beliefs manifested in Phase 1 (the learning of mathematics depends on the teacher's actions). Above all, he was aware of the influence of the affective domain on his understanding (I have the knowledge, but I must change the way I see things) (9-10).

DISCUSSION AND CONCLUSION

A mathematical situation was evaluated by Antonio's cognitive system, based on his belief system about mathematics, about his teaching and learning, about himself and about the context. These beliefs were incompatible with the reality of the context of the task, thus generating different emotions. His facial and body expressions during the episode provided emotional evidence of his uncertainty, frustration, distress and relief. These emotions, in turn, generated emotional responses (tension and blockage), and the latter determined his decisions of action. Antonio's affective system thus intervened in his mathematical practice, conditioning the uses given to mathematical knowledge and providing reasons for his understanding of measurement.

The configuration of theoretical frameworks that help to understand the role of affect in mathematical learning is an ongoing objective in the field of Mathematics Education (Goldin, 2004; Hannula, 2012). Another goal is to define procedures that allow to interpret the acknowledged relationship between affective domain and cognition in mathematics (Schlöglmann, 2002). The specific contribution of our study is an approach that enables exploring the understanding of mathematical knowledge through different components of affect. Identifying the relationship between affective domain and understanding allows to obtain a more accurate assessment of the students' actual

mathematical understanding. Such an assessment will help us in the future to guide students' affective responses towards learning with understanding.

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