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An Objective Function Formulation for Circuit Parameter Extraction Based on the Kullback-Leibler Distance

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***Abstract* — Numerical circuit parameter extraction is widely used for modeling many kinds of high-frequency structures. In particular, parameter extraction can be used to find the optimal parameter values of an equivalent circuit model to match as close as possible a given target response. This paper presents a novel objective function formulation for parameter extraction based on the Kullback-Leibler distance. A rigorous graphical and numerical comparison of the proposed new formulation against classical *l*-th norm formulations is included. One- and two-dimensional synthetic examples are used to illustrate the advantages of the proposed Kullback-Leibler distance formulation. Our results indicate that the proposed new formulation yields similar or better-behaved parameter extraction objective functions than those obtained from *l*-th norm formulations.**

***Index Terms* — Chebyshev, entropy, Euclidean, Kullback-Leibler, Manhattan, parameter extraction, *l*-th norm, Shannon information theory, optimization.**

I. Introduction

The EM response of a high-frequency structure can be locally approximated by using numerical parameter extraction (PE) if there is available an initial parameterized model that approximates the structure performance. PE formulations can be used to minimize the error between the approximating model response and the target EM response. To measure the error between these two responses, formulations with *l*-th norms are widely used. Parameter extraction is also the most critical process in several space mapping (SM) optimization algorithms [1]-[3]; using better formulations for parameter extraction can positively impact on the convergence of SM algorithms that require performing PE at each iteration. This paper also aims towards addressing that opportunity in a future work.

To the best of our knowledge, this paper presents for the first time a formulation of the objective function for parameter extraction exploiting concepts related to Shannon entropy theory, more specifically, by using the Kullback-Leibler (K-L) distance [4] to represent the error function to be minimized during numerical PE. A rigorous graphical and numerical comparison between the proposed K-L formulation and classical Manhattan, Euclidean, and Chebyshev formulations is presented, exploiting one- and two-dimensional synthetic examples where it is feasible to plot the PE objective function versus the model parameters. The results obtained considering these synthetic cases indicate that the proposed formulation based on the Kullback-Leibler distance yields similar or better behaved PE objective functions than those obtained from *l*-th norm formulations, which might be useful for more robust SM algorithms.

II. Circuit Parameter Extraction (PE)

Numerical circuit parameter extraction (PE) can be formulated as the following optimization problem:

$$\boldsymbol{x}^{\mathrm{PE}} = \arg\min_{\boldsymbol{x}} U_{\mathrm{PE}}(\boldsymbol{R}(\boldsymbol{x}), \boldsymbol{R}^{\mathrm{t}}) \tag{1}$$

where vector $\boldsymbol{x} \in \Re^n$ contains the model parameters, vector function $\boldsymbol{R}(\boldsymbol{x}): \Re^n \to \Re^r$ generates the model responses, and vector $\boldsymbol{R}^{\mathrm{t}} \in \Re^r$ contains the target or desired response. The scalar parameter extraction objective function, $U_{\mathrm{PE}}: \Re^n \to \Re^1$, measures the difference between the model response $\boldsymbol{R}(\boldsymbol{x})$ and the target response $\boldsymbol{R}^{\mathrm{t}}$. When performing PE, we aim at finding the optimal model parameters $\boldsymbol{x}^{\mathrm{PE}} \in \Re^n$ that make the model response $\boldsymbol{R}(\boldsymbol{x})$ as close as possible to the target response $\boldsymbol{R}^{\mathrm{t}}$. Ideally, $\boldsymbol{R}(\boldsymbol{x}^{\mathrm{PE}}) = \boldsymbol{R}^{\mathrm{t}}$.

As mentioned before, PE is also used as an important sub-process in several SM optimization algorithms [1]. SM techniques employ at least two models for the device to be optimized: a highly accurate but computationally expensive model, named as fine model, and a fast but insufficiently accurate representation, named as coarse model. In the context of SM, the target response $\boldsymbol{R}^{\mathrm{t}}$ is usually obtained from the fine model response $\boldsymbol{R}_{\mathrm{f}}$ at some fine model design parameters $\boldsymbol{x}_{\mathrm{f}}$, i.e., $\boldsymbol{R}^{\mathrm{t}} = \boldsymbol{R}_{\mathrm{f}}(\boldsymbol{x}_{\mathrm{f}})$, and PE is used to find the coarse model parameters $\boldsymbol{x}_{\mathrm{c}}$ that makes the coarse model response $\boldsymbol{R}_{\mathrm{c}}$ as close as possible to the fine model response. Ideally, $\boldsymbol{R}_{\mathrm{c}}(\boldsymbol{x}_{\mathrm{c}}^{\mathrm{PE}}) = \boldsymbol{R}_{\mathrm{f}}(\boldsymbol{x}_{\mathrm{f}})$, where $\boldsymbol{x}_{\mathrm{c}}^{\mathrm{PE}}$ is the solution of the corresponding PE optimization sub-problem, which is normally inexpensive since $\boldsymbol{R}_{\mathrm{f}}(\boldsymbol{x}_{\mathrm{f}})$ remains fixed during PE. However, PE can be very problematic for SM if U_{PE} has multiple local minima [3],[5].

Several formulations for the PE objective function U_{PE} can be considered. In Section III, the PE objective function U_{PE} using generalized *l*-th norms is revisited. In Section IV, the proposed new formulation based on the Kullback-Leibler distance is used to formulate U_{PE}.

III. *l*-th Norm Formulations for PE

The PE objective function can be formulated as an *l*-th norm,

$$U_{\mathrm{PE}}(\boldsymbol{x}) = \|\boldsymbol{e}(\boldsymbol{x})\|_l \tag{2}$$

where $\boldsymbol{e}(\boldsymbol{x}) \in \Re^r$ is the error of the model response with respect to the target response, $\boldsymbol{e}(\boldsymbol{x}) = \boldsymbol{R}(\boldsymbol{x}) - \boldsymbol{R}^{\mathrm{t}}$. The most widely used norms are as follows.

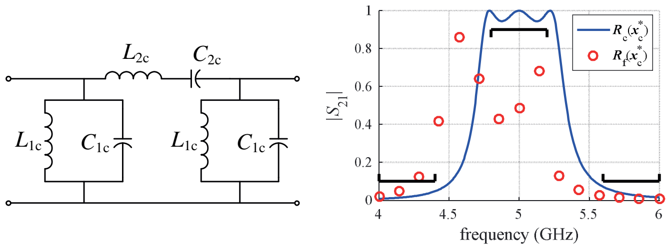

Fig. 1. Coarse model of a sixth-order bandpass filter, its optimal response (continued blue trace) and a fine model target response sampled at $p = 15$ frequency points from 4 to 6 GHz (circled red trace).

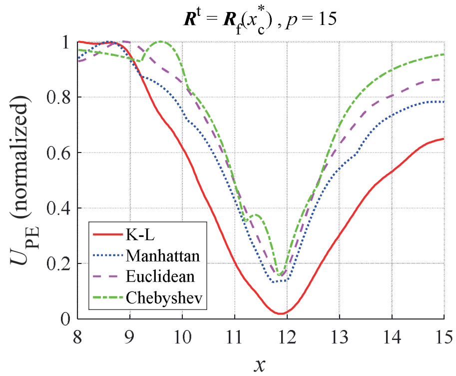

Fig. 2. PE objective functions for sixth-order bandpass filter in Fig. 2 varying $x_c = C_{1c}$ from 8 to 15 pF.

A. Manhattan

The l_1 norm formulation, also known as Manhattan, uses

$$U_{PE}(\boldsymbol{R}(\boldsymbol{x}), \boldsymbol{R}^{t}) = \sum_{i=1}^{r} |e_i(\boldsymbol{x})| \tag{3}$$

and it emphasizes the average absolute error.

B. Euclidean

The l_2 norm formulation, also known as Euclidean, uses

$$U_{PE}(\boldsymbol{R}(\boldsymbol{x}), \boldsymbol{R}^{t}) = \sum_{i=1}^{r} e_i^2(\boldsymbol{x}) \tag{4}$$

and it emphasizes the mean square error.

C. Chebyshev

The infinite norm or Chebyshev formulation uses

$$U_{PE}(\boldsymbol{R}(\boldsymbol{x}), \boldsymbol{R}^{t}) = \max_{i}\{\cdots |e_i(\boldsymbol{x})| \cdots\} \tag{5}$$

and it emphasizes the maximum absolute error.

IV. Kullback-Leibler (K-L) Formulation for PE

According to Shannon information theory [6], the degree of difference between the states of two systems A and B can be represented by relative entropy [7]. This difference can be measured using the Kullback-Leibler distance [8], [9], which can be defined as

$$C = \sum_{i=1}^{N} \left\{ A_i \ln \frac{A_i}{B_i} + (1 - A_i) \ln \frac{1-A_i}{1-B_i} \right\} \tag{6}$$

where C is the relative entropy between the systems A and B. The i-th states of these systems are represented by A_i and B_i ($i = 1, 2, \ldots, N$).

Extending (6) to the PE problem, the following alternative objective function is proposed:

TABLE I. One-Dimensional Example Solved by Numerical PE using the Nelder-Mead Method

$\boldsymbol{x}^{(0)}$	Obj. Func.	i	$\boldsymbol{x}^{PE}$	ε_{avg}	ε_{max}	*NMSE*
8.5	Manhattan	33	11.7251	0.0504	0.2447	0.1937
	Euclidean	28	6.4742	0.2873	0.9800	0.9105
	Chebyshev	31	9.2177	0.3342	0.9187	1.0506
	K-L	28	11.8747	0.0523	0.1599	0.1663
10.1624	Manhattan	30	11.7251	0.0504	0.2447	0.1937
	Euclidean	26	11.8623	0.0522	0.1573	0.1660
	Chebyshev	30	11.8365	0.0520	0.1518	0.1671
	K-L	26	11.8747	0.0523	0.1599	0.1663
14.5	Manhattan	32	11.7251	0.0504	0.2447	0.1937
	Euclidean	27	11.8623	0.0522	0.1573	0.1660
	Chebyshev	31	11.8365	0.0520	0.1518	0.1671
	K-L	27	11.8747	0.0523	0.1599	0.1663

$$U_{PE}(\boldsymbol{R}(\boldsymbol{x}), \boldsymbol{R}^{t}) = \sum_{i=1}^{r} \left\{ R_i \ln \frac{R_i}{R_i^{t}} + (1 - R_i) \ln \frac{1-R_i}{1-R_i^{t}} \right\} \tag{7}$$

where R_i is the i-th model response at model parameters $\boldsymbol{x}$, and R_i^{t} is the i-th target response.

V. PE Comparison using L-th Norms and Kullback-Leibler (K-L) Formulations

Graphical and numerical comparisons between K-L and l-th norm formulations are presented by synthetic examples. The following figures of merit to measure the corresponding matching errors are used: average absolute relative error (ε_{avg}), normalized mean square error (*NMSE*), and maximum absolute relative error (ε_{max}), defined as

$$\varepsilon_{avg} = \frac{\frac{1}{r}\|\boldsymbol{e}_{PE}\|_1}{\|\boldsymbol{R}^{t}\|_\infty} \tag{8}$$

$$NMSE = \frac{\|\boldsymbol{e}_{PE}\|_2}{\|\boldsymbol{R}^{t}\|_2} \tag{9}$$

$$\varepsilon_{max} = \frac{\|\boldsymbol{e}_{PE}\|_\infty}{\|\boldsymbol{R}^{t}\|_\infty} \tag{10}$$

where $\boldsymbol{e}_{PE}$ is the error vector after PE, $\boldsymbol{e}_{PE} = \boldsymbol{R}(\boldsymbol{x}^{PE}) - \boldsymbol{R}^{t}$.

A. One-Dimensional PE Example

The ideal sixth-order π bandpass symmetrical lumped filter shown in Fig. 1 is taken as a coarse model [10]. It uses $L_{1c} = 0.0997$ nH, $L_{2c} = 17.455$ nH, and $C_{2c} = 0.058048$ pF. To treat it as a one-dimensional problem, we use $x_c = C_{1c}$ as the only coarse model parameter.

The target response $\boldsymbol{R}^{t}$ is some fine model response $\boldsymbol{R}_f$ at the optimal coarse model design $\boldsymbol{x}_c^{*} = 10.1624$ pF, using $p = 15$ frequency points from 4 to 6 GHz, as shown in Fig. 1. The PE objective function plots for the four corresponding formulations are in Fig. 2, where the K-L formulation exhibits the best-behaved objective (unimodal, smoother and with larger dynamic range). This is confirmed by the numerical results shown in Table I, where i is the number of iterations.

B. Two-Dimensional PE Example

The ideal 10:1 two-section impedance transformer shown in Fig. 3 is used as a coarse model. Reference impedance is $Z_0 = 50$ Ω. Load impedance is $R_L = 500$ Ω. The transmission lines

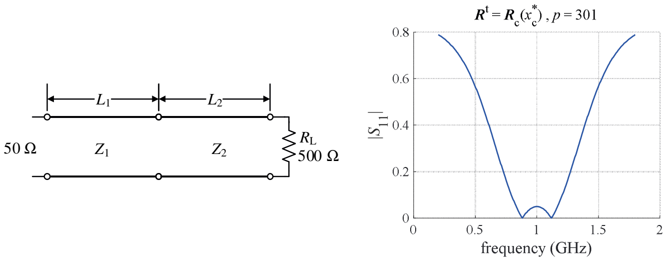

Fig. 3. Ideal 10:1 two-section impedance transformer used as a coarse model and its optimal response taken as target, sampled at p = 301 frequency points from 0.2 to 1.8 GHz.

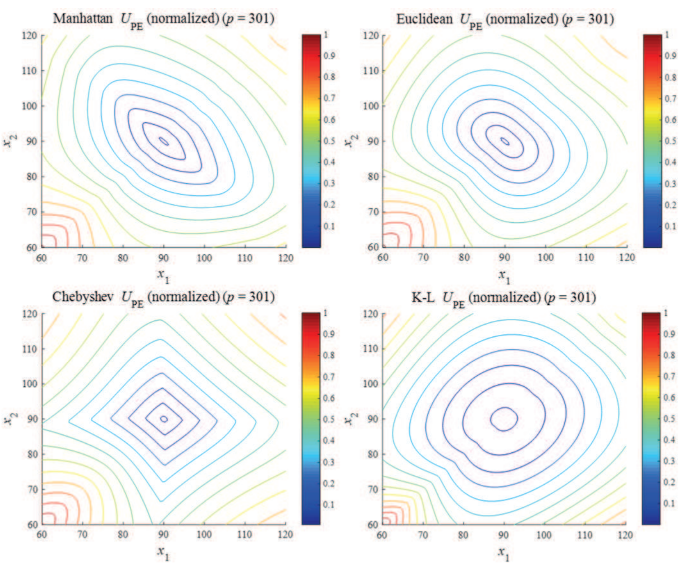

Fig. 4. Normalized PE objective functions for impedance transformer in Fig. 3 varying L_1 and L_2 from 60° to 120°.

characteristic impedances are Z_1 = 1.8233Z_0 Ω, and Z_2 = 5.4845Z_0 Ω, using coefficients for a Chebyshev profile with a 10:1 transformation ratio and a 0.05 ripple. Model parameters are $\boldsymbol{x} = [L_1 \;\; L_2]^T$ (degrees). To test a case with an exact match, the target response is the optimal coarse model response from 0.2 to 1.8 GHz using p = 301 frequency points (see Fig. 3).

The contour plots for the four corresponding PE objective functions are shown in Fig. 4, where the K-L formulation shows the best-behaved surface. This is re-confirmed by numerical results in Tables II and III, where i is again the number of iterations.

VI. Conclusion

This paper presented a formulation for parameter extraction (PE) in terms of the Kullback-Leibler (K-L) distance and a comparison with the classical l-th norm formulations. Our results indicate that the K-L distance formulation yields similar or better PE performance than the corresponding l-th norm formulations. This was confirmed by two findings: 1) smoother objective functions were observed; and 2) similar or smaller number of iterations were needed to obtain satisfactory response matching after numerical optimization.

Table II. Two-Dimensional Example Solved by Numerical PE using the Nelder-Mead Method

$\boldsymbol{x}^{(0)}$	Obj. Func.	i	$\boldsymbol{x}^{PE}$
$[70 \;\; 80]^T$	Manhattan	86	$[90.0000 \;\; 90.0000]^T$
	Euclidean	80	$[90.0000 \;\; 90.0000]^T$
	Chebyshev	80	$[90.0000 \;\; 90.0000]^T$
	K-L	75	$[90.0000 \;\; 90.0000]^T$
$[110 \;\; 80]^T$	Manhattan	90	$[90.0000 \;\; 90.0000]^T$
	Euclidean	77	$[90.0000 \;\; 90.0000]^T$
	Chebyshev	78	$[90.0000 \;\; 90.0000]^T$
	K-L	67	$[90.0000 \;\; 90.0000]^T$
$[90 \;\; 115]^T$	Manhattan	84	$[90.0000 \;\; 90.0000]^T$
	Euclidean	78	$[90.0000 \;\; 90.0000]^T$
	Chebyshev	76	$[90.0000 \;\; 90.0000]^T$
	K-L	72	$[90.0000 \;\; 90.0000]^T$

Table III. Two-Dimensional Example Solved by Numerical PE using the Trust Region Interior-Reflective Newton Method

$\boldsymbol{x}^{(0)}$	OF	i	$\boldsymbol{x}^{PE}$	ε_{avg}	ε_{max}	$NMSE$
$[70 \;\; 80]^T$	M	12	$[89.9999 \;\; 90.0001]^T$	0.0023×10^{-7}	0.1774×10^{-7}	0.0179×10^{-7}
	E	16	$[90.0001 \;\; 89.9999]^T$	0.0029×10^{-7}	0.2080×10^{-7}	0.0209×10^{-7}
	C	14	$[90.0002 \;\; 89.9998]^T$	0.1460×10^{-7}	0.3664×10^{-7}	0.2829×10^{-7}
	K-L	13	$[90.0033 \;\; 89.9968]^T$	0.0878×10^{-5}	0.5782×10^{-5}	0.1777×10^{-5}
$[110 \;\; 80]^T$	M	16	$[90.0002 \;\; 89.9998]^T$	0.0041×10^{-7}	0.4442×10^{-7}	0.0429×10^{-7}
	E	12	$[89.9997 \;\; 90.0003]^T$	0.0051×10^{-7}	0.5286×10^{-7}	0.0519×10^{-7}
	C	17	$[90.0038 \;\; 89.9965]^T$	0.2279×10^{-5}	0.5910×10^{-5}	0.4419×10^{-5}
	K-L	12	$[89.9972 \;\; 90.0029]^T$	0.0341×10^{-5}	0.4006×10^{-5}	0.0788×10^{-5}
$[90 \;\; 115]^T$	M	20	$[89.9999 \;\; 90.0001]^T$	0.0333×10^{-7}	0.1122×10^{-7}	0.0646×10^{-7}
	E	16	$[89.9999 \;\; 90.0001]^T$	0.1856×10^{-8}	0.9690×10^{-8}	0.3660×10^{-8}
	C	3	$[83.0220 \;\; 90.1578]^T$	0.0567	0.1106	0.1047
	K-L	11	$[89.9977 \;\; 90.0023]^T$	0.0450×10^{-5}	0.2442×10^{-5}	0.0905×10^{-5}

(M: Manhattan; E: Euclidean; C: Chebyshev; K-L: Kullback-Leibler)

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