Education for the twenty-first century: assessment to support the development of problem solving goals in mathematics curricula 5 to 16.

Current mathematics education policy perspectives in England, as well as in many other developed nations, privilege problem solving as a key 21st century skill (e.g. DfE, 2014; Kaur, 2014). Importantly, there is among the mathematics, mathematics education and end-user communities widespread embrace of problem solving as a centrally valued mathematical activity (e.g. ACME, 2011). However, English teachers and assessors often have limited experience of teaching for/assessing genuine problem solving, and performance in mathematics in England is high-stakes (Ofsted, 2012). Problem solving is therefore unlikely to be widely developed in classrooms unless summatively assessed at key points including GCSE, the standard English external assessment at age 16. A coherent curriculum system (Schmidt and Prawat, 2006) whereby intended curriculum, assessments, resources and teacher development are aligned and consistent, is key to supporting principled enactment (Golding, 2017).

We report on a set of longitudinal efficacy studies which, among other intentions, evaluate the impact on teachers and students of a leading curriculum and assessment provider's support for, and assessment of, problem solving for the 2014 Mathematics National Curriculum for 5-16 year olds (DfE, 2014). These are highly influential in England because they are widely adopted: the provider's GCSE assessments at 16, for example, accounted for about two-thirds of all cohort entries in 2017. Key theoretical constructs used are those of performativity (Ball, 1994) and curriculum coherence (Schmidt and Prawat, 2006).

Teacher interviews (n=452), student focus groups (n=172), classroom observations (n=101) and student survey responses (n~3300) over two early years of curriculum enactment show teachers and students perceive the approaches adopted in related curriculum materials and in the first set of provider GCSE examination papers not only employ highly valid assessment of, and approaches to, mathematical problem solving, but support that with provision of free surround materials specifically designed to build up students' ability to demonstrate related skills in summative timed assessments.

However, we also evidence early and emerging constraints on both assessment and classroom enactment of problem solving in the curriculum: teacher skills and knowledge for

related teaching across the range of students, teacher time and opportunity to harness the (additional or included) professional development opportunities provided with the resources, perceptions of superficial interpretations of 'problem solving' in national assessments at age 11, and pressures on schools and GCSE assessors to adopt enactments of 'problem solving' that are of limited validity, or for a subset of students only. Teachers attribute this to a) the challenges associated with defining an agreed meaning for mathematical problem solving and b) perceived in-school tensions between validly enacting that and meeting high-stakes performance measures. We discuss some implications.

References:

ACME. 2011b. *Mathematical Needs: The Mathematical Needs of Learners*. Advisory Committee for Mathematics Education. At http://www.acme-uk.org/media/7627/acme_theme_b_final.pdf.

Ball, S. J. 1994. *Education reform: A Critical and Post-structural Approach*. Buckingham: Open University Press.

DfE (Department for Education). 2014. *The 2014 National Curriculum for Mathematics in England* London: HMSO. At https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study

Golding, J. 2017. Mathematics teacher capacity for change *Oxford Review of Education 43/4*, 502-517

Kaur, B. 2014. Evolution of Singapore's school mathematics curriculum. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice* Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia, 1–13). Sydney: MERGA. At https://files.eric.ed.gov/fulltext/ED572633.pdf

Ofsted (Office for Standards in Education) 2012. *Mathematics made to measure.* London: HMSO.

Schmidt, W. & Prawat, R. 2006. Curriculum coherence and national control of education: Issue or non-issue? *Journal of curriculum studies 38*(6), 641-658*.*

Reviews:

An interesting areas that, although seen from a UK context, is more generally relevant and should give some interesting insights.

The authors of this paper address a long standing issue of importance and relevance. They outline clearly and helpfully the educational policy context within which the research was carried out. It would be of interest to all involved in research if the authors could extend their clear summary of their research methodology to give some account of the means by which evidence from four different types of source was triangulated. The final section outlines some important issues arising from the evidence gained through the research; while some of these are of a practical nature (e.g. teacher time) which could be addressed through adequate resourcing, others (e.g. superficial models of problem solving) are more fundamental and raise questions about the validity of the assessments; it is hoped that these can be explored in discussion.

This is a very interesting paper regarding problem solving. It would be great if the authors could provide to the presenters a review of how extensive problem-solving currently is in GCSE or A Level exams. Also, it would be great of it was possible to offer some hints as per whether Awarding Bodies are planning to increase the footprint of problem-solving tasks in exams

Sub-questions included:

- *How is mathematical problem-solving being enacted in classrooms, by whom?*

- *What are the affordances and constraints for teaching problem-solving, of teacher capacity, of resources, and of assessments, and how do these interact?*

Assessment Objective tested over the range of content, specified separately		GCSE Foundation tier (age 16, grades 1-5)	GCSE Higher tier (age 16, grades 4-9)	AS Mathematics (age 17, one-year course)	A Level Mathematics (age 18, two-year course)
AO1	Use and apply standard mathematical techniques*	50%	40%	60%	50%
AO2	Reason, interpret and communicate mathematically	25%	30%	20%	25%
AO3	Solve problems within	25%	30%	20%	25%

	mathematics and in other contexts				

- What is a 'standard technique* depends on one 's experience and learning: I varies between students.
- Mathematics 'problem solving' is a contested term in the literature: I take 'problems' to be those tasks to which there is no familiar approach or algorithm that the student knows almost certainly to result in a solution. Teaching for problem-solving is complex since solving semi-structured or unstructured problems is likely to draw on deep conceptual understanding, mathematical reasoning and well-developed communication (e.g. Schoenfeld, 2007). It therefore makes substantial demands on the capacity for change (Golding, 2017) of teachers coming new to it.
- Reference: Ofqual (2017) An evaluation of the difficulty of the assessments and the characteristic of the problem-solving (AO3) items (in Summer 2017 GCSE Mathematics examinations) Ofqual/17/6329
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An evaluation of the difficulty of the assessments and the characteristics of the problem-solving (AO3) items

AO3: Solve problems within mathematics and in other contexts

Students should be able to:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
- make and use connections between different parts of mathematics
- interpret results in the context of the given problem
- evaluate methods used and results obtained
- evaluate solutions to identify how they may have been affected by assumptions made

Last appears not significantly associated with PSQ

| Open-ended written answer |
| High level of language demand (unusual words used) |
| General knowledge needed |
| High quantity of text to be read |
| Requires selection of parameters to do the calculation |
| No obvious standard method |
| Non-obvious first step |
| Multiple possible approaches |

Dimensions associated with higher quality problem-solving quality (high PSQ) in Ofqual exercise analysing item features

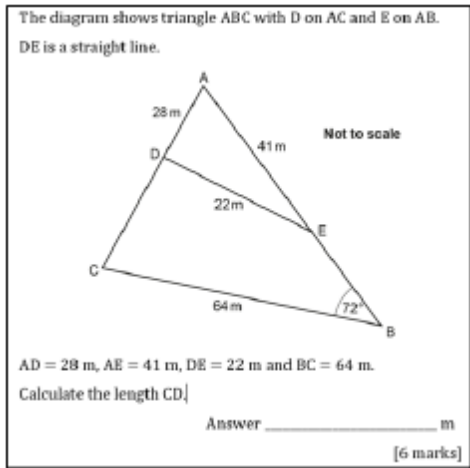| Basic foundation content	Additional foundation content	Higher content only
	know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$ and the trigonometric ratios, $\sin\theta = \frac{opposite}{hypotenuse}$, $\cos\theta = \frac{adjacent}{hypotenuse}$ and $\tan\theta = \frac{opposite}{adjacent}$	
	apply them to find angles and lengths in right-angled triangles in two dimensional figures	apply them to find angles and lengths in right-angled triangles and, where possible, general triangles in two and three dimensional figures

There was a weak relationship between PSQ and item difficulty

On Saturday, some adults and some children were in a theatre.

The ratio of the number of adults to the number of children was 5 : 2

Each person had a seat in the Circle or had a seat in the Stalls.

$\frac{3}{4}$ of the children had seats in the Stalls.

117 children had seats in the Circle.

There are exactly 2600 seats in the theatre.

On this Saturday, were there people on more than 60% of the seats?

You must show how you get your answer. [5 marks]

Figure 19. *The item that was rated second highest on overall problem solving elicited by the question (mean = 3.71). Adapted from Pearson GCSE Mathematics paper 2 (foundation and higher tier) 2017.*

Figure 19 illustrates the item rated second highest for 'overall rating of problem-solving elicited by the question'. This item also scored highly on seven dimensions:

- the requirement for general knowledge
- the quantity of text to read
- the requirement to select parameters to do the calculation
- a non-obvious standard method
- non-obvious first step
- multiple possible approaches
- connections between different parts of maths.

The diagram shows triangle ABC with D on AC and E on AB.

DE is a straight line.

AD = 28 m, AE = 41 m, DE = 22 m and BC = 64 m.

Calculate the length CD.

Answer _____ m

[6 marks]

Strong correlation with PSQ:

Pole with rating of 5	$r(43) =$	$p =$ (one-tailed)
No obvious standard method	0.833	<.001
Requires selection of parameters to do the calculation	0.807	<.001
Non-obvious first step	0.776	<.001
Intermediate steps not obvious	0.608	<.001
Multiple possible approaches	0.565	<.001

Moderate correlation with PSQ:

Requires connections between different parts of maths	0.447	.001
High level of language demand (unusual words used)	0.425	.002
High quantity of text to be read	0.419	.002
General knowledge needed	0.412	.002

19 Here are two right-angled triangles.
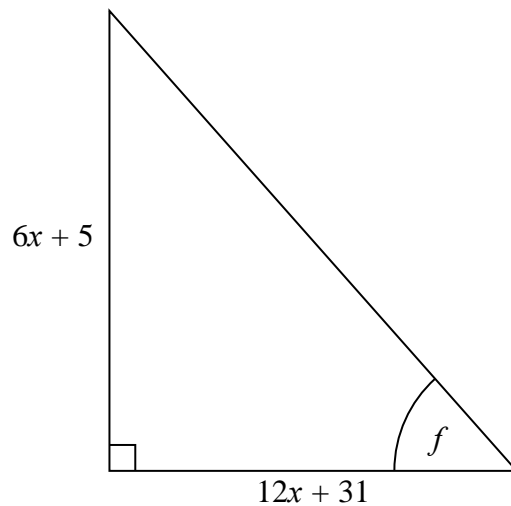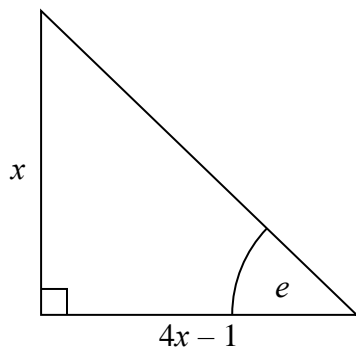
Given that

$$\tan e = \tan f$$

find the value of x.

You must show all your working.

(5 marks, 4 of them for PS)! H demand

4   Raya buys a van for £8500 plus VAT at 20%

Raya pays a deposit for the van.
She then pays the rest of the cost in 12 equal payments of £531.25 each month.

Find the ratio of the deposit Raya pays to the total of the 12 equal payments.
Give your answer in its simplest form.
 M demand 2+0+3. OK.

9   Jean invests £12 000 in an account paying compound interest for 2 years.

In the first year the rate of interest is x%
At the end of the first year the value of Jean's investment is £12 336

In the second year the rate of interest is $\dfrac{x}{2}$%

What is the value of Jean's investment at the end of 2 years?
 M diff demand, 1+0+3 but credibility: is it authentic?

3. Renee buys 5 kg of sweets to sell.
    She pays £10 for the sweets.

Renee puts all the sweets into bags.
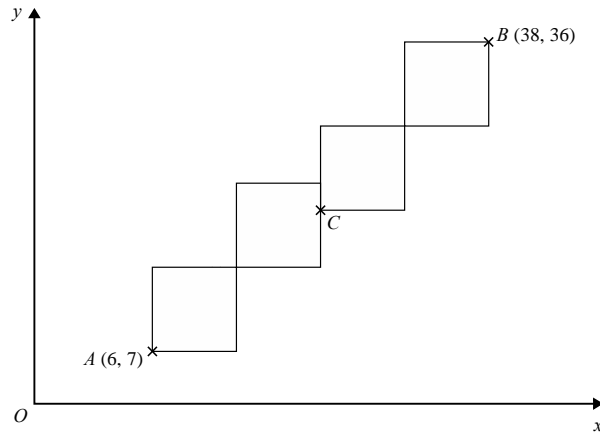She puts 250 g of sweets into each bag.
She sells each bag of sweets for 65p.

Renee sells all the bags of sweets.

Work out her percentage profit. 1+0+3 M demand. OK

6   A pattern is made from four identical squares.

The sides of the squares are parallel to the axes.

Point *A* has coordinates (6, 7)
Point *B* has coordinates (38, 36)
Point *C* is marked on the diagram.

Work out the coordinates of *C*.   m demand 1+0+4 Nice.

19   The point *P* has coordinates (3, 4)
The point *Q* has coordinates (*a*, *b*)

A line perpendicular to *PQ* is given by the equation $3x + 2y = 7$

Find an expression for *b* in terms of *a*.

OK H demand, 1+0+4

12   The diagram shows a scale drawing of a tennis court.

The scale of the drawing is 1 : 200

Work out the perimeter of the real tennis court.
Give your answer in metres.

L demand, 1,0,4

9 This is part of a bus timetable between Bury and Manchester.

Bury	08 25	08 55	09 15	09 30	09 45	10 05
Whitefield	08 34	09 04	09 24	09 39	09 54	10 14
Heaton Park	08 46	09 16	09 36	09 51	10 06	10 27
Cheetham	08 56	09 26	09 46	10 01	10 16	10 37
Manchester	09 05	09 35	09 55	10 10	10 25	10 48

Daniel goes from Whitefield to Manchester by bus.

Daniel takes 17 minutes to get from his house to the bus stop in Whitefield.
He takes 15 minutes to get from the bus stop in Manchester to work.

Daniel has to get to work by 10 am.
He leaves his house at 8.45 am.

(*b*) Does Daniel get to work by 10 am?
You must show all your working.   L demand, 1+0+4

6 Margaret is thinking of a number.
She says,

"My number is odd. It is a factor of 36 and a multiple of 3"

There are two possible numbers Margaret can be thinking of.

Write down these two numbers. 1+0+2, L demand

8 Neil buys 30 pens, 30 pencils, 30 rulers and 30 pencil cases.

Price list	
pens	6 for 82p
pencils	15 for 45p
rulers	10 for £1.25
pencil cases	37p each

What is the total amount of money Neil spends?1+0+4, L demand

19 A farmer has a field in the shape of a semicircle of diameter 50 m.

←——— 50 m ———→
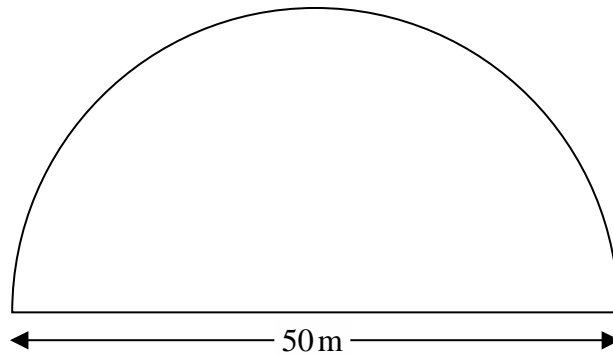
The farmer asks Jim to build a fence around the edge of the field.
Jim tells him how much it will cost.

Total cost = £29.86 per metre of fence plus £180 for each day's work

Jim takes three days to build the fence.

Work out the total cost. 1+0+4, M demand

14 Gavin, Harry and Isabel each earn the same monthly salary.

Each month,

Gavin **saves** 28% of his salary and spends the rest of his salary

Harry spends $\dfrac{3}{4}$ of his salary and **saves** the rest of his salary

the amount of salary Isabel saves : the amount of salary she spends = 3 : 7

Work out who saves the most of their salary each month.
You must show how you get your answer. L demand, 0,0,4 ?? authenticity

13 A piece of wire is 240 cm long.

Peter cuts two 45 cm lengths off the wire.
He then cuts the rest of the wire into as many 40 cm lengths as possible.

Work out how many 40 cm lengths of wire Peter cuts. 0,0,3 L demand

10 Tim and three friends go on holiday together for a week.

The 4 friends will share the costs of the holiday equally.

Here are the costs of the holiday.

£1280 for 4 return plane tickets
£640 for the villa
£220 for hire of a car for the week

Work out how much Tim has to pay for his share of the costs. 1,0,2 L demand