Algorithmic Approaches to Energy Market Price Prediction

Catherine Mc Hugh (BSc, MSc)

School of Computing, Engineering and Intelligent Systems

Faculty of Computing, Engineering and the Built Environment

Ulster University in collaboration with Click Energy

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I confirm that the word count of this thesis is less than 100,000 words.

"You are braver than you believe, stronger than you seem, and smarter than you think."

— Winnie the Pooh by A.A. Milne

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List of Abbreviations

ACF	AutoCorrelation Function
ADF	Augmented Dickey-Fuller
ADX	Average Directional Movement Index
AIC	Akaike Information Criterion
ANN	Artificial Neural Network
AR	AutoRegressive
ARMA	AutoRegressive Moving Average
ARMAX	AutoRegressive Moving Average with eXogenous inputs
ARIMA	AutoRegressive Integrated Moving Average
ARIMAX	AutoRegressive Integrated Moving Average with eXogenous inputs
ATM	Automatic Teller Machines
ATR	Average True Range
BETTA	British Electricity Trading and Transmission Arrangements
BSC	Balancing and Settlement Code
CART	Classification and Regression Tree
CER	Commission for Energy Regulation
CCGT	Combined Cycle Gas Turbine
EPEXSPOT	European Power Exchange
ERR	Error Reduction Ratio
EVS	Explained Variance Score
ISEM	Integrated Single Electricity Market
KDE	Kernel Density Estimate
KNN	k-Nearest Neighbours
KPSS	Kwiatkowski Phillips Schmidt and Shin
MA	Moving Average
MACD	Moving Average Convergence/Divergence
MAD	Moving Average Deviation
MAPE	Mean Absolute Percentage Error
MCO	Market Coupling Operator
MedAE	Median Absolute Error

MISO	Multiple Input Single Output
MSE	Mean Square Error
NARMAX	Nonlinear AutoRegressive Moving Average with eXogenous inputs
NEMO	Nominated Electricity Market Operators
NORDPOOL	Nordic Power Exchange
OCGT	Open Cycle Gas Turbine
OFGEM	Office of Gas and Electricity Markets
PACF	Partial AutoCorrelation Function
PMOM	Price Momentum
PPCMA	Percentage Price Change Moving Average
PR	Percentage Range
RMSE	Root Mean Squared Error
RMSLE	Root Mean Squared Log Error
RSI	Relative Strength Index
SARIMA	Seasonal AutoRegressive Integrated Moving Average
SARIMAX	Seasonal AutoRegressive Integrated Moving Average with eXogenous inputs
SEM	Single Electricity Market
SEMO	Single Electricity Market Operator
SISO	Single Input Single Output
SMA	Simple Moving Average
SMP	System Marginal Price
SONI	System Operator for Northern Ireland Limited
SVM	Support Vector Machine
XGBoost	Extreme Gradient Boosting

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Abstract

A new energy market, the **I**ntegrated **S**ingle **E**lectricity **M**arket (ISEM), went live in Ireland during October 2018 providing more flexibility and competition to energy traders. This recent development requires energy traders to purchase energy in advance. Therefore, if traders could accurately predict usage and the correct time at which to purchase energy, they could optimise their costs. Price prediction through statistical and computational approaches would be a valuable commercial tool when forecasting electricity prices to capture market trends with the aim of reducing market costs to increase profits.

This thesis explores day-ahead electricity price forecasting within the ISEM and **B**ritish **E**lectricity **T**rading and **T**ransmission **A**rrangements (BETTA) energy markets. Traditional statistical approaches were first considered by utilising time-series prediction models with historical data to observe energy market trends. Appropriate stationarity, integration, and seasonal checks were included in the traditional statistical models for estimation and diagnostic testing. Next, non-linear regression models were applied to model input-output relationships and find key energy-related factors that influence current electricity price. The inclusion of energy-related model inputs were shown to influence price prediction. Refining the statistical models to only include the identified significant factors from the non-linear regression models improved overall accuracy. Technical indicators have shown great promise within the stock market, thus building on this knowledge eight novel energy price technical indicators consisting of trend, oscillator, and momentum types were developed. These technical indicators were used as inputs into machine learning algorithms and demonstrated highly accurate prediction performance. Therefore computational approaches would be advantageous for energy trading prediction modelling.

Overall this thesis demonstrates that many approaches may be used to predict energy prices. However, the combination of novel technical indicators and machine learning provided compact models that accurately represent the variability and dynamics within the energy market.

Chapter 1

Introduction

1.1 Research Area and Motivation

Computational modelling has been used extensively in a number of areas for price prediction such as financial trading [1] and the stock market [2], [3] often applying hybrid models [4] and machine learning [5]. A new unique cross-border energy market, the **I**ntegrated **S**ingle **E**lectricity **M**arket (ISEM), went live in October 2018 in Northern Ireland and Republic of Ireland, increasing transparency and competition in the market [6]. Before this new ISEM market came into place energy companies, like Click Energy, had no control over purchase price in the previous **S**ingle **E**lectricity **M**arket (SEM) and it was the Market Operator who set the final market price four days after consumption. The ISEM consists of multiple markets allowing traders to purchase electricity units beforehand in the Day-Ahead or Intra-Day markets. If market traders do not purchase electricity units in either of these two markets, they have to pay the balancing market price as well as any financial costs through the imbalance settlement price if the electricity price has increased. The ISEM brings complexity in purchasing and selling electricity units with a need to forecast as it adapts to the European Target Model process. Therefore this thesis aims to address the ISEM requirements and challenges by developing reliable price forecasting models that energy traders, in particular Click Energy, could utilise for future electricity pricing. A prediction model that can function robustly at all times, is adaptable, can spot patterns, and predicts accurate electricity prices is desirable for competitive energy trading.

Forecasting algorithms are valuable mechanisms that have been considered for modelling electricity price prediction, particularly time-series models, and are trained with real electricity price data to analyse patterns and aid as a commercial tool reducing market costs for energy traders [7]. However, the energy market is difficult to forecast due to the complex behaviour and unpredictable nature of energy data [1]. Another obstacle in any commodity market is the constant battle between supply and demand with fluctuating prices due to economic and technical factors. Short term forecasting models with a window size of one day or one week display a strong relationship between past and predicted electricity prices [7], hence a short time period is often considered more reliable for better model accuracy. Forecasting models can receive a range of historical electricity market variables as possible input factors; it is important to consider other significant energy-related factors for inputs in price prediction energy models as they can influence current electricity price and would be worthwhile to consider for exploring input-output relationships. Historical input data will show if a relationship exists between the exogenous variables and the dependent price variable [8].

Computational techniques such as machine learning algorithms make predictions by learning trends from data to capture market behaviour but do not rely on rules [9]. Stock price financial trading has similar data characteristics and market behaviours as energy data. Therefore computational modelling and machine learning algorithms will be investigated for modelling electricity price data. Additionally stock market technical analysts examine price change patterns to summarise and predict price behaviour by building technical indicators; technical analysts do not consider fundamental factors such as expenses or assets as this information is already accumulated into historical prices [10]. Therefore, developing novel energy-related technical indicators would also aid in decision making, market competition, and balance supply/demand within ISEM [11]. The majority of current techniques that focus on electricity price forecasting use opaque black-box models [12], [13], [14] which provide no insight into how predictions are made or the variables that have contributed to the prediction; this thesis will utilise transparent non-linear models for better data understanding and analysis.

This thesis explores algorithmic approaches to develop optimal price forecasting models with the end goal being an accurate robust system that will aim to increase Click Energy's profits within the ISEM market through purchasing electricity units at the most advantageous time. Examining different statistical regression and computational modelling techniques and applying these with historical electricity price data will help discover market trends and overcome forecasting issues by understanding the strengths and weaknesses of an ideal energy trading process. The outcome will be an optimal forecasting model that can detect past market trends and adapt to current market conditions. Time will be spent at Click Energy to test the ISEM market in a live environment and to receive company feedback on model findings to validate the results.

1.2 Research Aims and Questions

This research aims to achieve a successful electricity price forecasting model which is applicable to the ISEM market and that can accurately predict future prices. The following research questions are addressed in this thesis:

1. Are traditional statistical methods or more recent computational models appropriate techniques for day-ahead electricity price forecasting?
2. Do energy-related exogenous variables improve model performance?
3. Can transparent models identify key factors that influence electricity price?

4. Can prediction accuracy be improved by developing representative energy-related technical indicators compared with the use of electricity prices?
5. Can model performance be improved by building on the strengths of statistical models and machine learning models?

1.3 Thesis Outline and Key Contributions

The remainder of the thesis is structured as follows:

- **Chapter 2 – Review of energy trading markets:** A critical analysis of the **B**ritish **E**lectricity **T**rading and **T**ransmission **A**rrangements (BETTA), SEM and ISEM energy markets has been completed, outlining trading processes focusing on new developments and highlighting key energy-related factors that influence day-ahead electricity price for energy traders to consider in prediction models. A review of trading use for the statistical regression and machine learning approaches utilised in the study chapters is also discussed.
- **Chapter 3 – Methods:** Each of the main traditional statistical models, non-linear regression approaches, and computational algorithms are presented to consider the advantages and disadvantages of each technique for suitability in short-term electricity price forecasting.
- **Chapter 4 – Single input single output models:** Time-series prediction models can follow market trends to predict future values. Various time-series models (**A**uto**R**egressive **M**oving **A**verage (ARMA), **A**uto**R**egressive **I**ntegrated **M**oving **A**verage (ARIMA), and **S**easonal **A**uto**R**egressive **I**ntegrated **M**oving **A**verage (SARIMA)) with historical raw electricity price as model input are analysed for data understanding and price prediction for both the BETTA and ISEM energy markets.
- **Chapter 5 – Multiple inputs single output models:** Time-series prediction models (**A**uto**R**egressive **M**oving **A**verage with e**X**ogenous inputs (ARMAX), **A**uto**R**egressive **I**ntegrated **M**oving **A**verage with e**X**ogenous inputs (ARIMAX), **S**easonal **A**uto**R**egressive **I**ntegrated **M**oving **A**verage with e**X**ogenous input (SARIMAX), and **N**onlinear **A**uto**R**egressive **M**oving **A**verage with e**X**ogenous inputs (NARMAX)) with multiple inputs energy-related factors are examined for both the BETTA and ISEM markets to investigate the most significant factors influencing day-ahead electricity price.
- **Chapter 6 – Refined models:** Applying significant ISEM energy-related factors from the non-linear models as inputs in the traditional statistical models will determine if model accuracy improves. Correlated peak lags are also considered for refinement to enhance ISEM model performance.

- **Chapter 7 – Computational models:** Machine learning regression algorithms (Random Forest, Gradient Boosting, and E**x**treme **G**radient (XG)Boost) combined with technical indicator model inputs derived specifically for the ISEM energy market are examined as an alternative to traditional approaches. The aim is to develop a robust optimal computational prediction system consistent with ISEM procedures that can be considered for future electricity price predictions.
- **Chapter 8 – Conclusion and future work:** A summary of each thesis contribution and an outline of the overall research findings from each chapter are provided. A concluding section on electricity price prediction applications is provided alongside possible future work.

The main research contributions focussing on predicting day-ahead electricity price are included in Chapters 4-7 as outlined below.

Chapter 4 addresses the first research question by exploring traditional statistical methods and numerous experiments are performed to examine trend/seasonality and evaluate model accuracy. Each experiment considers the four key modelling stages: identification, estimation, diagnostics, and prediction. This research work is published in the conference proceeding:

- C. McHugh, S. Coleman, D. Kerr, and D. McGlynn, "Forecasting Day-Ahead Electricity Prices with a SARIMAX Model", in Proceedings of the 2019 IEEE Symposium Series on Computational Intelligence, SSCI 2019, pp. 1523–1529.

Chapter 5 continues to explore traditional forecasting methods and undertakes the second research question by including external energy-related model inputs. Both statistical models and transparent non-linear models are examined to determine key contributing exogenous variables through prediction modelling that greatly enhance model performance. This research work is published in the following conference proceedings:

- C. McHugh, S. Coleman, D. Kerr, and D. McGlynn, "Daily Energy Price Forecasting Using a Polynomial NARMAX Model," in Advances in Computational Intelligence Systems, UKCI 2018, pp. 71–82.
- C. McHugh, S. Coleman, D. Kerr, and D. McGlynn, "A Linear Polynomial NARMAX Model with Multiple Factors to Forecast Day-Ahead Electricity Prices," in Proceedings of the 2018 IEEE Symposium Series on Computational Intelligence, SSCI 2018, pp. 2125–2130.

- C. McHugh, S. Coleman, and D. Kerr, “Hourly Electricity Price Forecasting with NARMAX”, submitted to Elsevier Statistical Methods for Machine Learning with Applications (under review).

Chapter 6 addresses the third research question through applying the key exogenous variables identified from the non-linear models as inputs in the traditional statistical models. The aim of the refined original and correlated lags models was to improve day-ahead prediction accuracy. This research work is published in the conference proceeding:

- C. McHugh, S. Coleman, D. Kerr, and D. McGlynn, “Seasonal Models for Forecasting Day-Ahead Electricity Prices”, in Proceedings of International Conference on Time Series and Forecasting, ITISE 2019, pp. 310–320.

Chapter 7 targets both the fourth and fifth research questions by examining regression based machine learning algorithms for electricity price prediction and through developing novel price technical indicators to be used as model inputs. Machine learning prediction models are trained and tested to determine if they are more robust than the standard statistical models. Price technical indicators focussing on trends, oscillations, and momentum are derived specifically for the energy market. This research work is published or under review as follows:

- C. McHugh, S. Coleman, and D. Kerr, “Technical Indicators for Hourly Energy Market Trading”, in Proceedings of The Ninth International Conference on Data Analytics, Data Analytics 2020, pp. 72-77.
- C. McHugh, S. Coleman, and D. Kerr, “Technical Indicators and Prediction for Energy Market Forecasting”, in Proceedings of 19th IEEE International Conference on Machine Learning and Applications, ICMLA 2020, pp. 1241-1246.
- C. McHugh, S. Coleman, and D. Kerr, “Technical Indicators for Energy Market Trading”, in Elsevier Machine Learning with Applications, vol. 6, 2021.

Chapter 2
Review of Energy Trading Markets

2.1 Introduction

Electricity price forecasting is becoming popular due to deregulation within the energy industry. Energy markets differ throughout the world and there is no standard optimal price forecasting technique that can be used, instead the approach depends on the market type [15]. A price forecasting model is a valued mechanism for profitable trading that energy traders can utilise for hedging market volatility and risk [16]. Due to the elevated frequency of trading and spikes in demand, computational forecasting algorithms tend to be more desirable than statistical techniques to predict future prices. Electricity prices may be considered over different temporal scales depending on the trading period. Energy traders will, over time, reduce their costs if they can develop commercial price prediction algorithms which accurately represent the market dynamics [7].

Fundamental economic factors and historical raw data can influence electricity price forecasting; therefore in price modelling the key indicators should be included as inputs in prediction models for profitable forecasting [15]. Electricity price can be impacted by temporal changes in demand (hour/day/week), weather, generation costs, or seasonality. Electricity price data are prone to spikes corresponding to changes in demand making the market clearing price, which is set as the price where demand and supply curves join [17], unpredictable. Demand is a key price contributor as when demand increases so does the price; nonetheless all significant energy factors must be identified and considered to estimate the cheapest bid in the energy market in order to stay competitive [18]. For an accurate prediction tool to spot trends, it is important to have both historical and real-time data in the forecasting model [19] as well as significant key factors [20] and/or representation of them such as technical indicators. A technical indicator is derived from raw data to observe market trends, helping traders decide whether to buy, sell or hold price units.

This chapter describes three energy markets: **S**ingle **E**lectricity **M**arket (SEM) and **I**ntegrated **S**ingle **E**lectricity **M**arket (ISEM) in Ireland and **B**ritish **E**lectricity **T**rading and **T**ransmission **A**rrangements (BETTA) in Great Britain. The trading processes and format for each market are explained in Sections 2.2 to 2.4, focussing in particular on the developments of the new ISEM and the changes from the old Irish market, SEM. Section 2.5 explores literature on statistical and computational price techniques focussing on day-ahead forecasting and observes which key energy-related factors influence electricity price. Section 2.6 reviews the financial stock markets as this type of market has similar characteristics to energy trading markets and thus considering prediction algorithms which have been used to successfully forecast stock prices would underpin the creation of accurate electricity price forecasting models.

2.2 Single Electricity Market

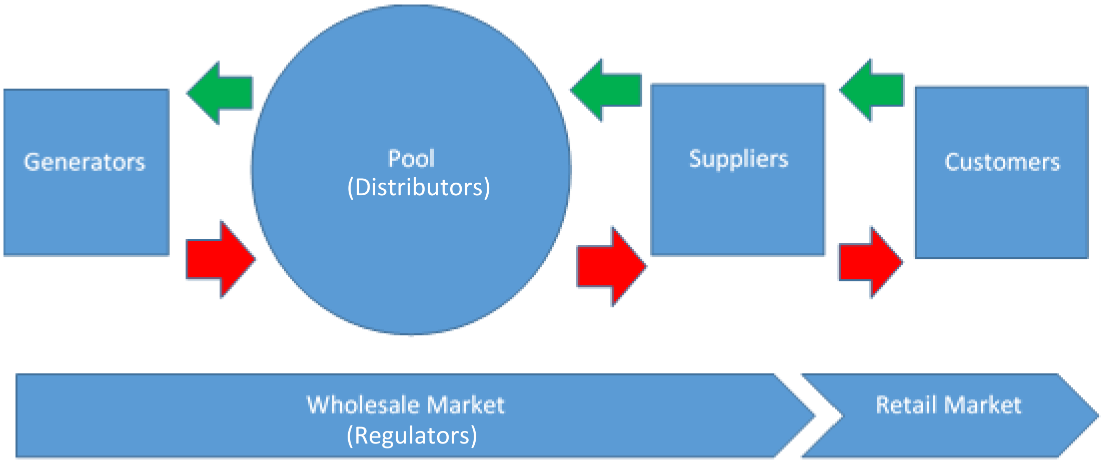

Figure 2.1: Single Electricity Market network [21]

The collaborative **S**ingle **E**lectricity **M**arket (SEM) for both Republic of Ireland and Northern Ireland, formed by the Irish and UK governments, began operating in November 2007 consisting of a network of five stakeholders: (i) *Generators*, who produce and supply the required energy bidding into a single pool setting the price; (ii) *Distributors*, who transmit and network the produced energy; (iii) *Regulators*, who safeguard the pricing and efficiently operate the wholesale market to meet demand; (iv) *Suppliers*, who buy electricity from the wholesale market and sell in the retail market; and (v) *Consumers*, who buy the electricity from the suppliers through the retail market [22], [23]. The network, illustrated in Figure 2.1, allows free trade through one efficient cross-border market, with all generators delivering and all suppliers buying at the **S**ystem **M**arginal **P**rice (SMP), which is the final cheapest price in the wholesale market to meet customer demand in the retail market. This cross-border network is unique as it is the first example of such a market in Europe [24]. With one mandatory pool for bidding and selling, there is no price control and hence more competition within the market.

The **S**ingle **E**lectricity **M**arket **O**perator (SEMO) is a joint operator providing security of supply through one single pool involving two transmission operators: the **S**ystem **O**perator for **N**orthern **I**reland Limited (SONI) and EirGrid, based in the Republic of Ireland [25]. The bidding occurs over five pricing and scheduling cycles: *Ex Ante1* (-1 trade day published by 11am), *Ex Ante2* (-1 trade day published by 1pm), *Within Day* (trading day), *Ex Post Indicative* (+1 trade day), and *Ex Post Initial* (+4 trade day) [26]. The SMP is a combination of the shadow price, which is the cost needed to match demand, and the uplift, which is the required operating cost for a generator to recover [26] at 30-minute trading periods.

There are various types of generation units currently in use: thermal generation such as gas, coal, and biomass; hydroelectric generation including pumped storage; interconnectors consisting of imported and exported energy transferred across borders; and wind generation [25]. The SMP can be influenced by demand, fuel costs, wind distribution, and interconnection flows [25]. Total production costs from generators consist of operating, no-load, and start-up costs. Interconnector generation is fixed for the five pricing and scheduling cycles. Pumped storage generation submits costs with no value and therefore has no actual bids but instead adopts the SMP which has been set for each 30-minute trading period [26]. There is a code of practice to be followed for generator bids in such that marginal costs have to be declared truthfully to ensure the correct price is set by the generators [27]. Capacity payments, paid for by suppliers, are given to generators to help contribute to a share of their fixed generation costs and to deliver stability in meeting demand requirements [18]. A continuous balance between energy generation and consumption is necessary for a stable energy market as the wholesale price is decided by this connection [28].

The SEMO controls the trading process trying to sustain minimum production costs and keep electricity prices low by setting the final cheapest price for each 30-minute period in the trading day after the last cycle has occurred, four days after trading [26]. This final price is determined through post-processing for the market schedule during *Ex Ante1* cycle to get the forecast SMP values, however real-time fluctuations have to be accounted for so two further runs are completed on the *Ex Post Indicative* and *Ex Post Initial* cycles, with this last run being considered the final SMP [18]. The SEMO is also accountable for administration duties in terms of making payments to generators and sending invoices to suppliers [29]. The market trading design and process is governed by the SEM committee and regulated jointly by the Utility Regulator in Northern Ireland and the **C**ommission for **E**nergy **R**egulation (CER) in the Republic of Ireland to avoid exploitation of control by any key market player [25].

The procedure of how electricity price is determined daily was outlined in [22] as follows: bidding prices are placed by generators the day before trading day; for each 30-minute interval SEMO first accepts renewable power generation and then begins accepting bids of lowest prices through the market scheduling software cycles and continues until demand matches electricity supply; the last generator bid accepted that meets demand requirement sets the final price for all units at that 30-minute interval for everyone; suppliers then buy from the wholesale market at this price and compete with one another by selling to consumers in the retail market at tariff expected prices. It is hoped the tariff expected price is similar to the final wholesale price in order for the supplier to make a profit but there is the possible risk of loss. The retail market can be competitive as generators and suppliers still have a relationship until the bids are paid and thus retail prices for consumers are influenced by competition level [30]. Increased competition also has benefits allowing more choice to consumers and enhanced guarantee of supply [24].

2.3 Integrated Single Electricity Market

In October 2018, the SEM changed to a new development, the **I**ntegrated **S**ingle **E**lectricity **M**arket (ISEM), moving from single to multiple markets to follow and integrate the European Target Model standards allowing suppliers greater control and flexibility. The main features of the European Target Model are: a standard price algorithm for market scheduling, open cross-border trading, hedging services, an increase in renewable energy, and balancing between regions [31]. The new ISEM has significant differences from SEM; mainly focusing on balancing, the option to trade in multiple markets, and economical interconnector usage [31].

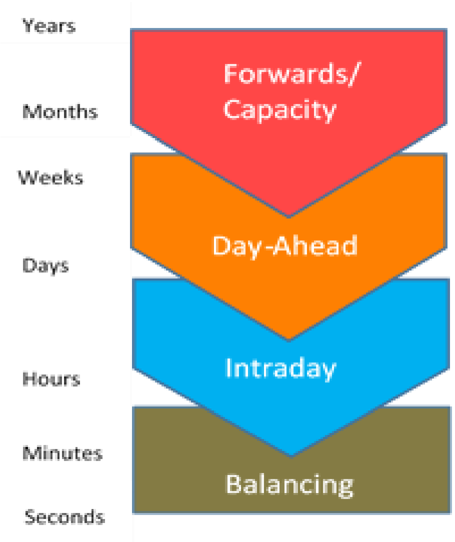

Figure 2.2: Integrated Single Electricity Market outline [31]

In the ISEM markets, illustrated in Figure 2.2, each market operates independently: *Forwards* which occur months or years in advance to provide hedging for trading participants against prices rising and falling; *Capacity* which involves generators or interconnectors effectively delivering the required energy and thus is a compulsory market for both to participate in; *Day-Ahead* which sets initial electricity price linking to other European market regions through cross-border trade via interconnectors and closes one day before, and the **M**arket **C**oupling **O**perator (MCO) operates this *ex ante* market via interaction from both SONI and EirGrid who are the **N**ominated **E**lectricity **M**arket **O**perators (NEMOs); *Intraday* which opens following Day-Ahead and closes one hour before release allowing room for adjustments as this *ex ante* market fluctuates near real-time; and *Balancing*, which keeps stability into real-time by equalising demand and supply through the SEMO providing imbalance settlement prices (market participants are required to pay the difference if prices increase and they are paid the surplus if prices decrease). Nonetheless, it is a compulsory market for the majority of generators to participate in. Generators and suppliers prefer less contact in this market due to the risk of additional financial costs [31].

The Day-Ahead market occurs over 24 hourly periods, the Intraday market occurs over 48 30-minute periods, and the Balancing market is split in half-hourly imbalance settlements with six 5-minute periods for each settlement interval [6]. A price coupling algorithm, known formally as EUPHEMIA, calculates price and market position to assign cross-border volume and join various energy markets into one coupling [6]. The imbalance price is decided through a flagging and tagging method that aims to keep prices low by considering only marginal costs from energy functions accumulated during the balancing market [32]. The complexity of the multiple trading markets brought about challenges and the requirement for market traders to forecast electricity prices in advance of buying and selling to secure market positions and stay balanced. To limit interaction in the Balancing market and reduce financial costs, it is beneficial for market participants to forecast accurately with realistic optimal bids in the Day-Ahead market [6]. In order to achieve accurate day-ahead bids and reduce balancing risk, price forecasting is a vital requirement for companies to maintain stability in the market [33].

The ISEM brought about various changes from operation and competitive trading in multiple markets over large timeframes with both generators and suppliers bidding simultaneously, risk management and forecasting techniques being required, and generators having more responsibility when contributing in the day-ahead bidding stage. Therefore market traders had to adapt quickly [34]. Competitive prices from SEM remained in the new ISEM trading process which was considered positive [30]. Unlike the SEM, generation costs are no longer announced separately arising to greater obligations for generators to have capable production dispatch schedules that allow for all running costs to be met [27]. Bids are submitted during the day-ahead market following the structure of the EUPHEMIA algorithm [30]. The available generation units are: wind, pumped storage, battery storage, demand, assetless, and dual rated [6]. The ISEM is more transparent for commercial investment when trading as market signals provide decisions on whether to raise or reduce assets [35], but market traders first had to learn how to work the new trading systems and train their staff on business processes and strategies to become market-ready [34]. The ISEM has limited access to European markets due to geographical restrictions, only connected via Great Britain, and therefore it is harder to promote competition through the forwards market; however regulation would guarantee that consumers still receive a competitive market price [27]. The first month of ISEM showed promising results with a stable market highly correlated with both EUPHEMIA traders and efficient interconnectors [36]. ElectroRoute, an independent energy trading and service company, compared the first year of ISEM prices with the old SEM market and noted minimal differences in baseload prices and a slight decrease in ISEM day-ahead prices compared with the SEM SMP prices, highlighting a positive outlook that the ISEM market continues to keep prices low [37].

The main goal of the ISEM market is to create a more equal trading process for market participants increasing trading competition with multiple markets and promoting renewable energy generation [33]. It focusses on many different energy generation units and is less reliant on large power plants with the possible chance of plant closures as ISEM progresses [35]. Another key feature of the ISEM is cross border trading through interconnectors. The ISEM has two interconnectors with Great Britain (GB): the *Moyle* linking Northern Ireland to Scotland which has operated since 2001 and the *East-West* linking the Republic of Ireland to Wales which has operated since 2012. Both these links offer competitive trading opportunities through access to the rest of Europe via the GB market [31]. However, ISEM is interconnected only to Great Britain and the GB market is not fully integrated with other markets in Europe due to exchange rates, hence it is doubtful if there will be much competition between ISEM and other EU traders [27]. Free flowing energy allows for one integrated price among coupled markets; however network congestion leads to conflicting prices [6] that has been noted in cross-border studies [38], [39].

Since the ISEM market did not go live until October 2018, historical data were unavailable at the beginning of this research work. The **B**ritish **E**lectricity **T**rading and **T**ransmission **A**rrangements (BETTA) market in Great Britain is designed to allow trading in multiple markets and the requirement to propose electricity prices before the start of the balancing market period. Until ISEM market data became publicly available for data analysis, BETTA market data were collected and examined for time-series modelling to observe energy trends. As ISEM is a recent development there has been limited current research work in this area; one research work focussed on energy storage developing a smart grid for the ISEM retail market [40], another examined volatility connectedness between ISEM and BETTA markets [41], and another analysed real-time ISEM imbalance pricing [32]. Literature prior to 2018 discussed ISEM development, regulations and challenges [27], [30] with forecasting scenarios performed using SEM data [18], [29], [33], but to date I am not aware of any current research on ISEM electricity price forecasting since the market went live. Technical indicators considered for energy market prediction have also been limited with only one research work currently available that created a set of hourly technical indicators for the Day-Ahead Belgian market [42].

2.4 British Electricity Trading and Transmission Arrangements Market

The BETTA market has been in operation since 2005 and has created a fully-competitive British-wide market for trading electricity generation [43]. This market is similar to ISEM in terms of bilateral trading in multiple markets (Day-Ahead, Intraday, and Balancing), the requirement of hedging tactics in the Forwards market to limit risk, and the need for generators and suppliers to propose predicted electricity prices in the *ex ante* periods to reduce exposure in the Balancing market [44]. Since 2005 coal and gas electricity generation has decreased with the reduction of generation power plants, however renewable based electricity generation (biomass, wind, solar) has increased [41]. The increase in renewable source electricity generation brings additional overheads to the market due to renewable support costs per electricity consumption [45]. There were initially four interconnectors when the BETTA market was first formed: *England-Scotland* which links between Scotland and England/Wales; *Moyle* which links Scotland and Northern Ireland, *Manx* which links England/Wales and the Isle of Man, and *Anglo-French* which links England/Wales and France [46]. Since 2012 an additional interconnector, the *East-West* has been established, which links Wales and the Republic of Ireland.

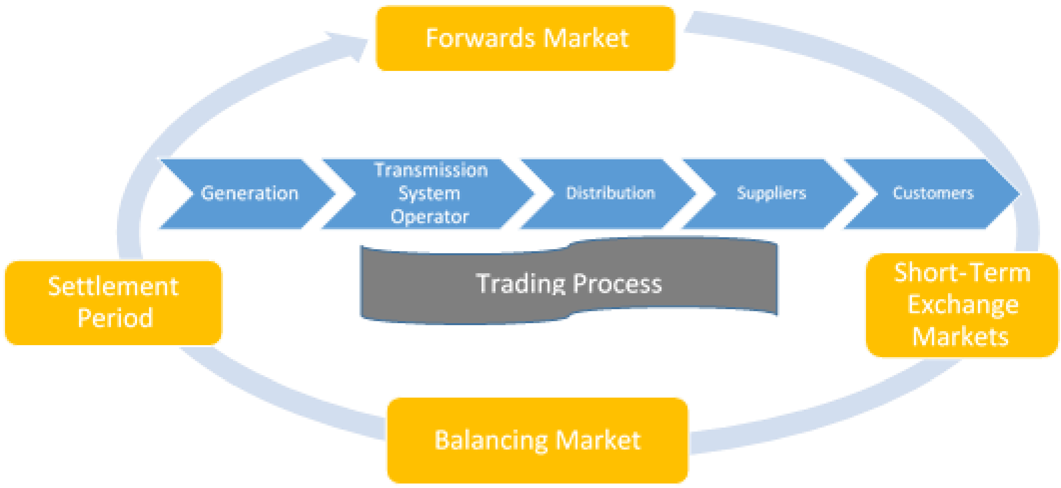

Figure 2.3: British Electricity Trading and Transmission Arrangements Market [47]

The BETTA market, illustrated in Figure 2.3, focusses around predictable generation trading through bilateral contracts [47]. The trading measures comprise of a Forwards market with the option of contracts starting in several years; two power exchange Short-term markets in which participant contracts can be amended; a Balancing market in which the system operator ensures stability between demand and supply; and a Settlement period in which participants are charged or paid if their contracts do not equal the final electricity price [46]. National Grid is the transmission system operator who is required to be informed of the trade volume during the bidding stage [43] and the **O**ff**ice** of **G**as and **E**lectricity **M**arkets (Ofgem) is the government regulatory body. ELEXON is a private company overseeing the Balancing market guaranteeing the **B**alancing and **S**ettlement **C**ode (BSC) is implemented by comparing actual observed energy against what was specified by generators for production and by suppliers for consumption, with ELEXON ensuring price stability and system security through imbalance settlements provided by the system operator [48]. The BETTA Day-Ahead wholesale market contains 24 hourly settlement periods in which energy suppliers enter into a contracted bid with generators before the start of each settlement period. This market has two trading platforms: the **E**uropean **P**ower **Ex**change (EpexSpot) and Nordic Power Exchange (Nordpool), both settling all sales and purchases for electricity price trading.

2.5 Approaches to Forecasting

There have been numerous statistical and computational price forecasting methods applied in the trading area. A common approach to price forecasting involves numerous steps outlined in [15] as: (i) gather and analyse historical data, (ii) data preparation, (iii) data fitting for model selection, and (iv) model refinement. Pandey and Upadhyay [15] outline the three different categories of price forecasting: short-term which covers a period of days or weeks, making this type suitable for market traders; medium-term which covers a period of weeks to months thereby improving discussions between provider and consumers; and long-term which emerges over months or years swaying long term decisions and planning. In energy forecasting, short-term is considered advantageous to smooth fluctuations as balancing demand and supply with shorter timeframes helps identify peaks and trends [49]. If demand and supply are imbalanced, price fluctuations (which are normal due to commercial and technical factors [15]) tend to arise making accurate forecasting difficult [50]. Hence a steady balance between consumption and production will help to prevent this.

Techniques for price forecasting are becoming more progressive within the energy industry as electricity is expensive to source over longer periods. Time-series analysis is a traditional statistical forecasting approach that involves historical data [15]. A **S**imple **M**oving **A**verage (SMA) model is a statistical time-series forecasting model that calculates the average price over a set period to smooth fluctuations and identify trends [51]. For example, a SMA model calculates average daily price over 24 hours to identify trends [51]. Another simple technique is linear regression which models linear relationships between one dependent variable and one or multiple independent variables building into multivariate regression [8], however multiple variables can lead to multicollinearity and errors. To overcome this Nogales et al. [12] utilised dynamic regression when predicting day-ahead electricity prices by linking current price at time t with historical prices at time $t-1$, etc. to develop a model with uncorrelated errors.

Regression analysis using lagged independent variables indicates if variable relationships exist [8].

Basic statistical models like **A**uto**R**egressive **I**ntegrated **M**oving **A**verage (ARIMA) have been applied to predict electricity prices demonstrating promising forecasts [15]. An ARIMA model was applied by García-Martos et al. [52] to forecast the Spanish energy market and concluded that separating an entire time-series into smaller time periods (24-hours) lowered the prediction errors improving the parameter estimates. ARIMA models work best with a stable market to predict trends but have difficulty with complex modelling [53], requiring more historical data for accurate forecasts [50]. To account for seasonality, a **S**easonal **A**uto**R**egressive **I**ntegrated **M**oving **A**verage (SARIMA) is a common forecasting technique. Due to the stochastic trend, before creating a SARIMA model, the time-series must be made stationary (no trend) with a constant mean and variance over time [4]. This can be applied through seasonal differencing [8] which is required in electricity price forecasting to remove the non-stationary trend [7]. A **S**easonal **A**uto**R**egressive **I**ntegrated **M**oving **A**verage with e**X**ogenous variables (SARIMAX) has been shown to outperform SARIMA with improved performance accuracy forecasting energy load [54], especially for short-term forecasting [55]. A SARIMAX model can be used to determine significant external factors by examining the statistical summary output that describes the price dynamics of each variable [56]. An example of such an external factor is temperature which could be included as a model input in energy forecasting models, and it was a key input when included in a SARIMAX model to predict energy load [57]. Khashei et al. [4] enhanced the limitations of a SARIMA model by combining it with computational techniques to improve prediction accuracy.

Computational techniques, like machine learning, may be useful in price forecasting as they learn from training data without depending on programming rules and the algorithms try to replicate market trends from patterns [9] to generate optimal prediction models [7]. There are various types of machine learning algorithms for example: **S**upport **V**ector **M**achines (SVM) which involve a supervised learning algorithm and labelled data for either classification or regression methods [8]; K-means clustering which is an unsupervised learning algorithm using unlabelled data and involves pattern recognition among similar groups [1]; Naïve bayes which is supervised classification technique with independent class inputs [58]; and **k**-**N**earest **N**eighbours (kNN) which is a simple supervised method assigning new labels depending on the common class of their nearest neighbours [59]. A Weighted kNN approach was applied for Spanish electricity price forecasting and it was noted that extreme weather variables caused inflated forecasting errors [60].

Statistical time-series techniques work best when a small forecasting window is used [16] as the relationship between historical and predicted values is stronger [7] making them a starting point for price forecasting. However, problems can develop from weakly correlated relationships between the inputs and output and therefore forecasting techniques that can accommodate non-linearities would be beneficial. To effectively model non-linear relationships between input and output variables, a **N**onlinear **A**uto**R**egressive **M**oving **A**verage with e**X**ogenous inputs (NARMAX) model can be considered [61]. Polynomial NARMAX models in particular are very popular due to the simple model representation [62]. There are numerous studies involving NARMAX models, for example, modelling the relationship between air pressure and turbines of diesel engines [62]; identifying the key features influencing house prices in China [63]; modelling West Africa monthly rainfall [64]; forecasting demand of **A**utomatic **T**eller **M**achines (ATMs) using seasonal input factors [65]; and modelling solar wind to determine the parameters that provide the optimal magnetosphere function [66]. Multiple energy-related factors utilised with a transparent polynomial NARMAX model would be advantageous to remove redundant exogenous variables and identify statistically significant variables that influence electricity price. Non-linear data can lead to prediction issues but a forecasting model that can handle volatility, provide accurate predictions, and thus reduce market costs is desirable.

Time-series models are popular for price forecasting but it has been shown that short-term forecasting results in improved precision and model accuracy. Mosbah and El-Hawary [67] trained a multilayer neural network with gas, load, and temperature hourly non-linear data and Vijayalakshmi and Girish [68] examined short-term forecasting and observed that time-series models performed better than an **A**rtificial **N**eural **N**etwork (ANN). Gao et al. [7] also noted a similar finding when they compared ANN and ARIMA forecasting models with respect to **R**oot **M**ean **S**quared **E**rror (RMSE). The ARIMA model had the lowest RMSE but it was found that as the forecasting period increased both models were less accurate. Severiano et al. [69] utilised short-term fuzzy logic time-series models for solar energy forecasting highlighting that prediction accuracy improves with a short-term period. Therefore short-term forecasting would be a valuable tool for Day-Ahead energy market trading.

2.6 Day-Ahead Forecasting

The Day-Ahead market is a key component of the ISEM and thus both energy consumers and suppliers would gain from accurate day-ahead price predictions before the trading day for stability [49]. Kavanagh [70] performed a time-series day-ahead forecasting scenario before the ISEM went live using actual historical SEM load data from 2013 to 2014 to observe market volatility and noted the appearance of daily and weekly patterns, with daily load on week days moving in peaks and troughs between 8am to 5pm. For the ISEM Day-Ahead market, in the first year of going live the average prices followed average demand with peaks occurring between 8am to 10am and between 4pm to 6pm [71]. Analysing price spread between both day-ahead and intraday with econometric models is beneficial when historical prices, exogenous variables, lag structure, and forecasting window length are considered [72].

Feature selection and general data preparation improves model accuracy and model precision achieving successful day-ahead trading [1]. It can be worthwhile to also include energy-related factors as model inputs and examine how each factor influences day-ahead electricity price and which factors are most significant. Due to changing market behaviour it may be necessary to consider price spikes in forecasting models [73]. Huurman et al. [74] focussed on the impact of weather variables for predicting day-ahead Nordpool prices and found weather forecasts provide information that help to anticipate possible price spikes. Nogales et al. [12] applied time-series methods to forecast day-ahead electricity producing accurate results, however during peak hours the prediction accuracy was reduced when spikes occurred.

A study comparing various models over different time periods with the inclusion/exclusion of external factors highlighted that **M**ean **A**bsolute **P**ercentage **E**rror (MAPE) in forecasting models, with the inclusion of wind, improved significantly in day-ahead forecasting (MAPE=7.53%) compared to week-ahead forecasting (MAPE=14.93%) [75]. Another study examined electricity price behaviour with multiple external factors that describe market behaviour through a **L**oss of **L**oad **P**robability (LOLP) methodology. In this study Random Forest, Gradient Boosting, and XGBoost regression algorithms were applied to test performance with the highest performance algorithm, XGBoost, selected for real-time forecasting implementation [48]. Ziel [76] applied regression models using hourly European data with Lasso estimation techniques to allow for variation over the day and stop overfitting when predicting day-ahead electricity prices.

Both statistical and computational models have been identified for modelling the energy market. It is clear from the vast amount of literature on day-ahead electricity price forecasting that it is essential for energy traders to develop prediction machine learning algorithms which are robust to enable to be competitive in the balancing market and not incur excessive pricing costs. With accurate forecasting knowledge available before placing bids in the day-ahead market, ISEM participants would be market-aware and able to plan in advance for purchasing and trading using an innovative price prediction system with their end goal of reducing costs.

2.7 Relevance with Trading in the Financial Stock Market

Financial trading is split into two types: fundamental analysis which explores economic features and technical analysis which examines historical behaviour. In price forecasting, technical analysis is more suitable for short-term forecasting and fundamental is generally more appropriate for long-term forecasting [77]. However an integrated system merging both has also been considered with promising results [77], [78]. Early financial trading relied on qualitative data such as financial reports, but in more recent years computational intelligence has been applied which relies mostly on quantitative data such as technical indicators [1].

The use of computational intelligence is receiving increased attention among financial traders, especially in stock market trading due to its ability to analyse vast amounts of trading data [1], [2]. Computational techniques have been extensively used to model historical financial data offline but it is uncertain how well the methods would function online [51]. Machine learning models have been used with technical indicators [3] and trading data to determine relationships and achieve profitable returns [5]. A trading forecasting approach using technical indicators as inputs to the k-nearest neighbour algorithm [51] was shown to predict well for short-term forecasting. More recently, deep learning techniques have been applied to analyse stock market patterns and predict intraday stock prices [78].

Forecasting techniques applied to financial data, in particular stock price, could be considered for energy trading price algorithms as both markets have similar characteristics: historical energy trends aid in future price prediction (technical) and energy-related variables influence electricity price (fundamental). Fundamental input variables, which can be internal or external [1], have been applied to energy forecasting models and findings showed robust correlation for same hour data [18]. Often machine learning algorithms and technical indicators are combined in financial stock market trading to extract key features; the indicators are built from raw stock prices capturing trends over time by following price movements to predict future prices [79]. To date, only one previous research work [42] has created and examined technical indicators using Belgian market data, therefore research in this area is limited. Developing novel energy technical indicators with ISEM market data would be beneficial and can be included as inputs for optimal day-ahead price prediction algorithms.

2.8 Conclusion

This chapter has reviewed a range of topics relating to price prediction and energy markets. Three energy markets were reviewed: SEM, ISEM and BETTA. The review considers the market in terms of overview, trading standards, market schedules, and operating changes. The presented literature highlights numerous statistical and computational intelligence techniques in terms of price forecasting and has explained the key areas (day-ahead, time-series, non-linear regression, machine learning, technical indicators) to consider for developing the optimal price prediction model to use in energy trading.

Algorithmic approaches for trading in financial markets were also reviewed to consider the similarities with energy market trading and hence determine appropriate algorithms to use in this research. In particular, computational intelligence techniques have become increasingly popular in the financial market for price forecasting. These techniques could be considered in energy market trading as historical trends aid future prices (technical) and energy-related factors influence prices (fundamental). Machine learning and technical indicators are often combined to forecast stock market prices. Therefore creating novel technical indicators with ISEM data would be worthwhile exploring in a price forecasting model.

Similar to any real-world data, electricity demand data display dynamic behaviours and short-term forecasting can help to smooth these price fluctuations. Therefore a day-ahead price prediction model should be considered to deal with market trends. The literature has also identified that including energy-related factors as model inputs would be beneficial to improve forecasting prediction.

The next chapter will explore the theory and methodology behind the key statistical and machine learning techniques that will be used in this thesis. Statistical time-series models, such as ARIMA and SARIMAX, will be explained in terms of trend, seasonality, and multiple model inputs. Non-linear regression models, such as NARMAX, help to identify key energy-related factors and therefore these types of models will be explained. Machine learning techniques, such as Gradient Boosting that find patterns in data will also be introduced.

Chapter 3

Methods

3.1 Introduction

Energy data display non-linear and non-stationary characteristics [67] thus an understanding of statistical and computational approaches in terms of forecasting is crucial for analysts to create viable price prediction models. Time-series analysis often uses popular traditional statistical methods to create predictive models for forecasting. As these mainly rely on statistical properties, they perform best when the prediction window is small since there is often a stronger relationship between historical and predicted values over a short time period [7]. Time-series analysis is applied (and often preferred) in commercial industries such as business, finance and health to build simple prediction models that capture behaviour and help in forecasting. Even if forecasting models only reduce prediction error by a small percentage, the long-term reduction in the financial costs is still worthwhile [14]. More recently, time-series techniques have made use of machine learning approaches for time-series forecasting [60]. This research will consider a hybrid method for day-ahead prediction that applies a statistical model framework with a **N**onlinear **A**uto**R**egressive **M**oving **A**verage model with e**X**ogenous inputs (NARMAX) to improve forecasting accuracy.

This chapter explores statistical regression and machine learning techniques in terms of price forecasting for time-series, highlighting the strengths and weaknesses to be considered when developing an energy price forecasting model. Section 3.2 examines **S**ingle **I**nput **S**ingle **O**utput (SISO) time-series price forecasting models in detail. Section 3.3 looks at **M**ultiple **I**nput **S**ingle **O**utput (MISO) models that can be used to observe exogenous factors and investigate how transparent non-linear models can help determine key model input factors. Section 3.4 outlines the more recent machine learning approaches that can be used for price forecasting. This overview provides an understanding of statistical and computational techniques necessary for the research undertaken and presented in the remaining chapters of this thesis.

3.2 SISO Models

Time-series methods are well-established models that apply a simple framework for model identification and evaluation. Time-series models predict future values by collecting historical market data as input and by observing past market trends to analyse patterns and make accurate predictions. The notation of a time-series, X, is:

$$X = \{X_t : t \, \varepsilon \, T\} \tag{3.1}$$

where values are recorded at an exact time t in the complete series T [56]. The stages in modelling a time-series involve (i) model identification, (ii) model estimation, and (iii) evaluation of the identified model [75]. As time-series models break down patterns over time to determine relationships, they are often considered top-down models [8]. Moving averages are common representation of time-series data that can identify trends by smoothing variations in the underlying data. Traditional time-series approaches such as statistical models combine autoregressive and moving average methods. SISO models consider single input (e.g., historical price values) to predict a single output (e.g., future price).

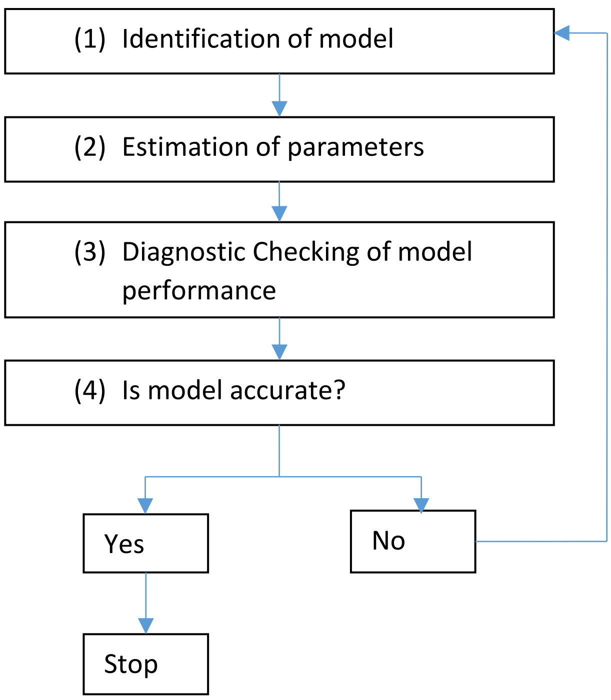

Figure 3.1: Procedure stages for Box-Jenkins model selection [80]

The Box-Jenkins methodology, developed in the early 1970s, is a simple framework applied to statistical time-series models (such as ARMA or ARIMA) to understand the data, identify, fit, and forecast future values through an iterative stochastic process outlined by Hall [80] in Figure 3.1. The four stages are: (1) model identification to determine parameter order terms *p*, *q* and *d*, where *p* is the autoregressive term, *q* is the moving average term and *d* is the difference order; (2) parameter estimation using maximum likelihood to fit the selected model; (3) diagnostic checking to validate the model accuracy; and (4) forecasting. If the model accuracy is poor or autocorrelation is detected in the residuals, stages (1) to (3) are repeated until model optimization is achieved [80]. The Box-Jenkins methodology is an iterative forecasting technique in which the model is only fitted once, with the new predicted values being fed step-wise into the model to forecast over a desired period. At stage (1) the error term is assumed to have a random normal distribution with zero mean and constant variance [81]. The model identification stage is of key importance as this determines how the final model is classified and is wholly dependent on the initial data identified and the parameters selected. Identifying the correct model terms involves trial and error of the Box-Jenkins stages; two key diagnostic tools to help with identification are the correlogram and partial autocorrelation function [80].

Traditional linear statistical time-series models assume strict stationarity such that the process $(x_{t_1}, \dots, x_{t_k})$ is time invariant and its statistical properties do not change over time. Stationarity is required for a model to function correctly and this may not always be the case with real-world data applications [82]. If a time-series is strictly stationary, has no trend or seasonal patterns and stays around its average, then it can be modelled using an **A**uto**R**egressive **M**oving **A**verage (ARMA) model. These assumptions are not reasonable in real-world applications, instead the data tend to be weakly stationary (a less limiting condition to strictly stationary) where values fluctuate around a fixed mean level with constant variance, and therefore weakly stationary is assumed in practice [56].

Determining if a time-series is stationary can be achieved through various techniques: visually observing a plot of the data, checking the mean and variance randomly throughout the data to see if each set of summary statistics remains similar, or from stationary unit root tests in which the null hypothesis is that the time-series is stationary. Well known stationary tests include **K**wiatkowski **P**hillips **S**chmidt and **S**hin (KPSS) or **A**ugmented **D**ickey-**F**uller (ADF) [56]. Data which have a stochastic trend have to become stationary first (constant mean and variance) by removing the seasonal cycle or trend before creating a forecasting model [4].

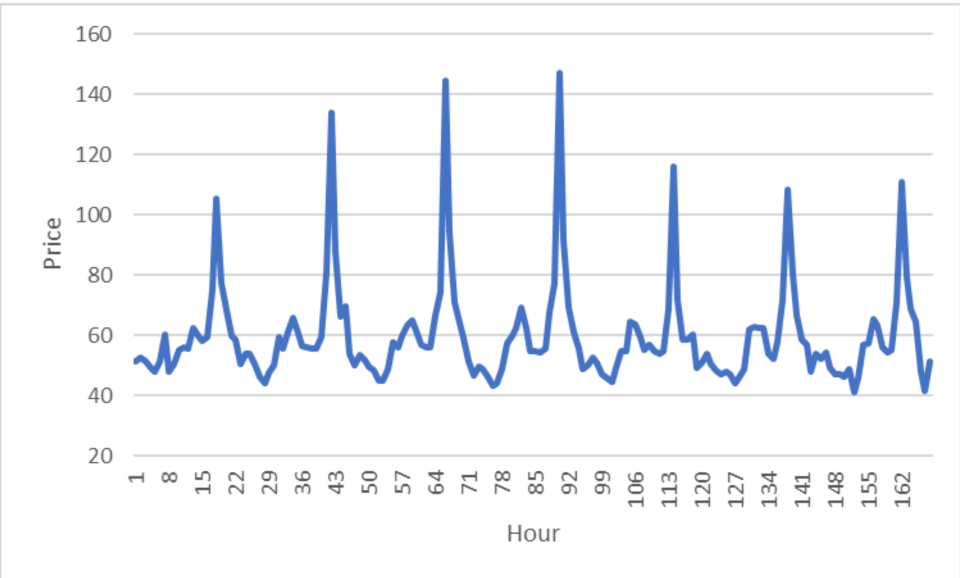

Figure 3.2: Weekly electricity price data from 02 Jan to 08 Jan 2017

Electricity prices over time start to resemble a stationary process (Figure 3.2) staying around the average with a fixed mean and constant variance, however there are still signs of seasonal cycles with peaks appearing throughout the weekly data. In Figure 3.2, the data are sampled from 2017 for illustrative purposes. In day-ahead forecasting it is important to examine daily time-series and from observing Figure 3.3 it is clear that hourly electricity prices have a noticeable trend showing signs of non-stationarity.

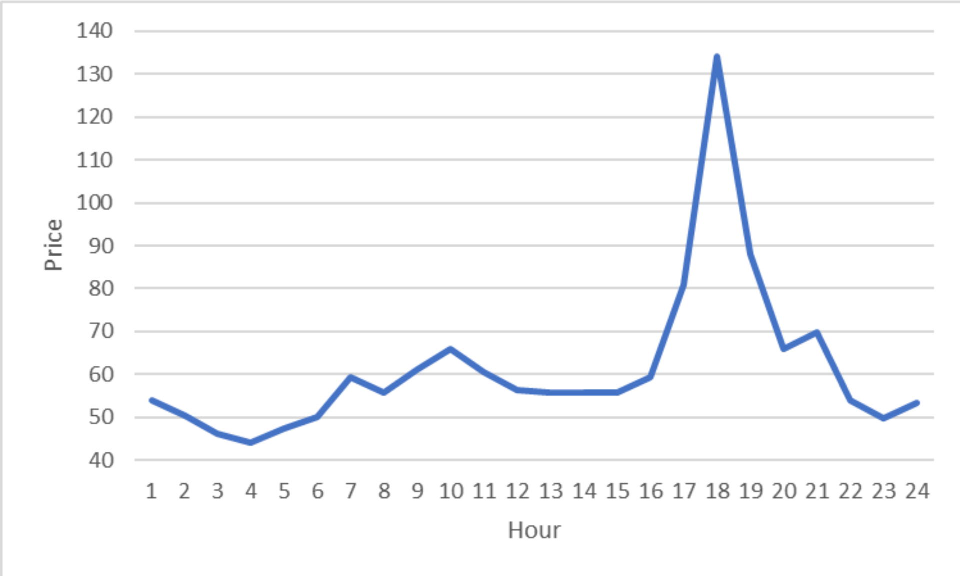

Figure 3.3: Hourly electricity price data from 02 Jan 2017

Model terms are selected to fit the model appropriately and determine the most suitable model. An important step is to prepare the data to be stationary and a popular method to do this transformation is differencing. The number of differencing steps required to transform the series into a stationary series is denoted as (d) [14]. Determining if differencing is required is the first aspect of the identification stage of Box-Jenkins modelling. If a time-series needs to be made stationary then an order of integration (I) is also necessary, which changes the ARMA model to an **A**uto**R**egressive **I**ntegrated **M**oving **A**verage (ARIMA) model by applying a differencing rate of change transformation between current and previous values to remove non-stationarity. First order differencing is calculated as the difference between the current value at time t and the previous value at time $t-1$ such that:

$$\nabla x_t = x_t - x_{t-1} \tag{3.2}$$

and if this is sufficient to make the data stationary then the order of differencing is set to 1, if not the order of differencing transformations is increased to 2, 3, etc., until stationarity is met [7]. To determine if differencing has achieved stationarity it is possible to plot the data and check if the differenced series displays a symmetrical pattern showing consistency over time and therefore stationarity. Figure 3.4 displays first order differencing which appears to achieve stationarity as the majority of values fall around 0 with a consistent pattern.

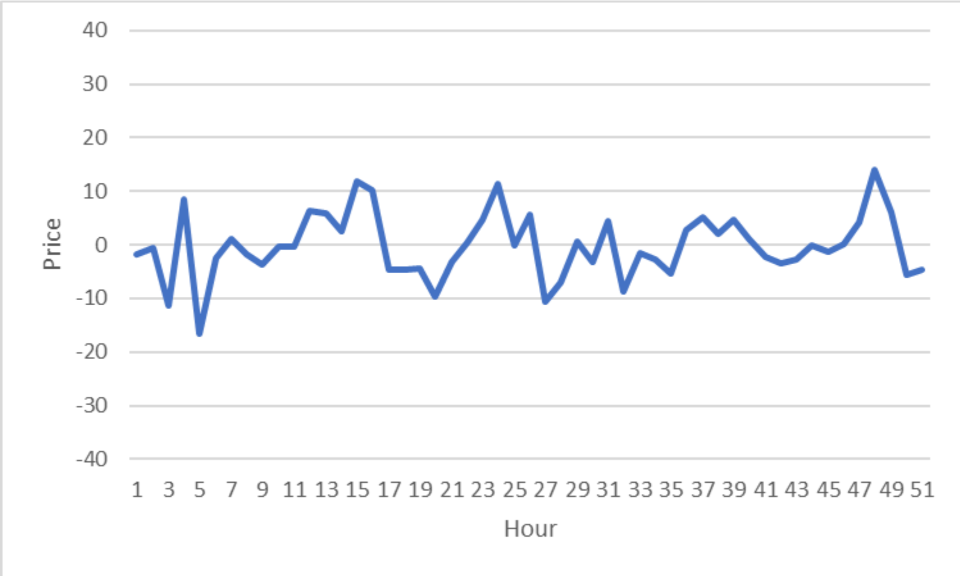

Figure 3.4: First order difference of hourly 2017 electricity price data

Once d has been determined correctly, both the p and q order terms need to be manually set for the AR and MA terms, respectively. These terms can be chosen visually from observing the **P**artial **A**uto**C**orrelation **F**unction (PACF) and **A**uto**C**orrelation **F**unction (ACF) plots which display data trends, illustrated in Figures 3.5 and 3.6 respectively. The terms are identified as the last lag to be significant (maximum lag) in the PACF plot for p , which from Figure 3.5 is 7, and the ACF plot for q , which from Figure 3.6 is 2.

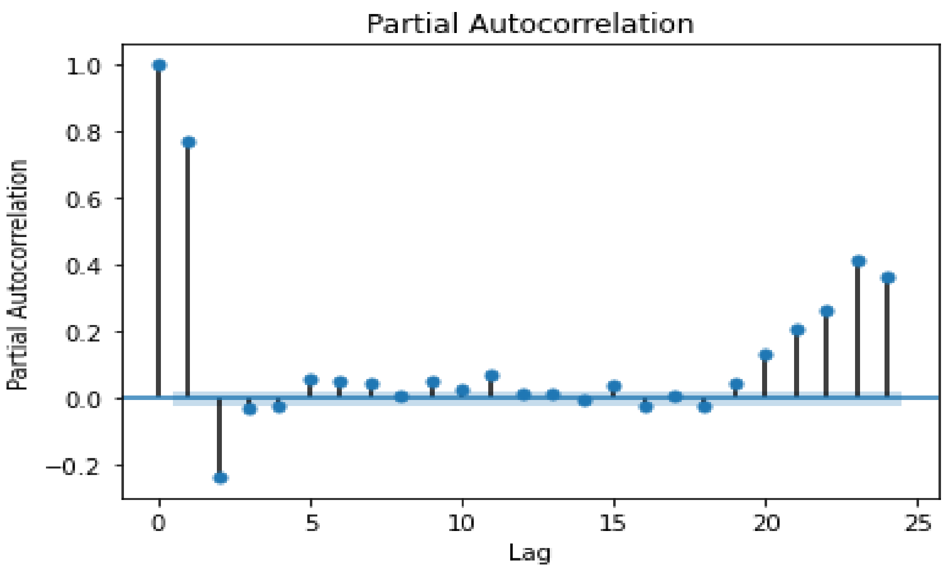

Figure 3.5: Partial autocorrelation (PACF) plot of hourly 2017 electricity price data

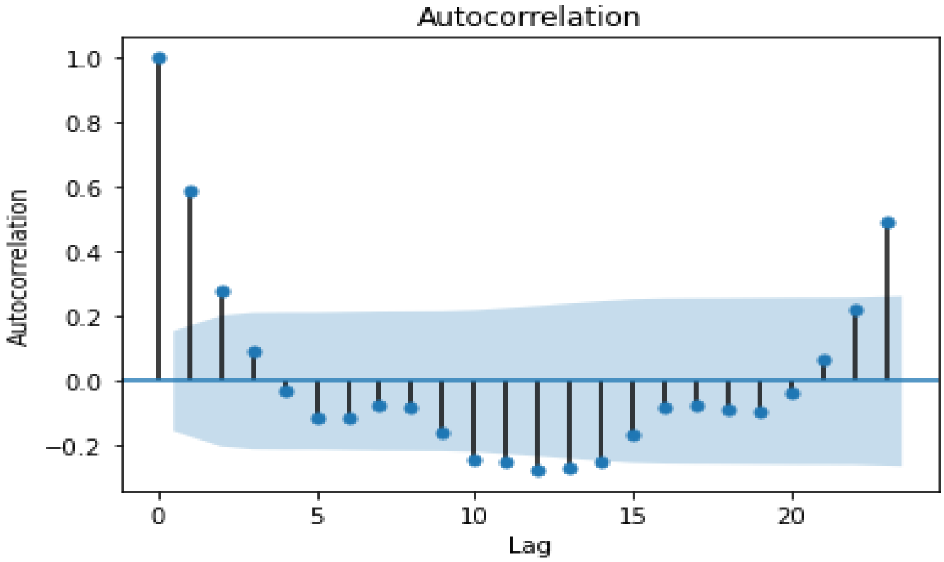

Figure 3.6: Autocorrelation (ACF) plot of hourly 2017 electricity price data

Another technique for selecting the order terms is the **A**kaike **I**nformation **C**riterion (AIC) method, an error criterion, which chooses the optimal model as the one with the lowest AIC value and is calculated in [56] as:

$$AIC = -2\ ln\ (L)\ + 2k \tag{3.3}$$

where L is the maximum likelihood and k is the number of model parameters. AIC measures the quality of a statistical model fit and is utilized for model selection. The AIC method can also be applied to non-linear data which makes this technique advantageous for electricity price forecasting [83]. The AIC values are compared for different order term combinations for models with the same input data. The order terms required for the model with the lowest AIC value are selected as they generate a less complex model. Only AIC values produced from similar types of models can be directly compared, i.e. all models should be differenced or all models should not be differenced before selecting the optimal terms. Identification determines the order terms (p, d, q) and defining the correct model order is a fundamental step to fitting an ARMA or ARIMA model [82].

3.2.1 ARMA

The **A**uto**R**egressive (AR) model is the time-series regression of the variable onto itself and is described as “regression of self” such that its current values depend strongly on the pattern of previous values and lags from preceding periods. An AR model is considered one of the simplest and is represented as:

$$Y_t = \theta_0 + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \cdots + \varphi_p Y_{t-p} + \varepsilon_t \quad (3.4)$$

where Y_t is the prediction output, θ_0 is the intercept, p is the quantity of autoregressive terms (order term), φ_p is the set of autoregressive parameters, Y_{t-p} is the value at time $t-p$, and ε_t is the error term [84]. PACF plots help to decide what lag period to set for p, through correctly identifying the AR order term by assigning it as the last significant lag identified from the PACF plot. For instance in Figure 3.5, p=2 represents using two previous periods from the time-series. If the data follow a stationary process, the PACF plot will quickly decline to zero.

The **M**oving **A**verage (MA) model is the time-series historical error and again current values depend on previous lags and noise. The MA model is another basic model and is represented as:

$$Y_t = \mu - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q} + \varepsilon_t \quad (3.5)$$

where Y_t is the prediction output, μ is the mean of the series, q is the quantity of moving average terms (order term), θ_q is the set of moving average parameters, and ε_t is the error term [84]. To correctly identify the MA order term, ACF plots are used to determine the appropriate error lag for q by assigning it as the last significant lag from the plot.

A combination of these two separate time-series models becomes an ARMA model [84]:

$$Y_t = \theta_0 + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \cdots + \varphi_p Y_{t-p} + \mu - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q} + \varepsilon_t \quad (3.6)$$

This model can be further simplified [54] by merging the AR terms and the MA terms separately:

$$\varphi_p(B) Y_t = \theta_q(B) \varepsilon_t \quad (3.7)$$

where

$$\varphi_p(B) = 1 - \sum_{i=1}^{p} \varphi_i B^i \quad (3.8)$$

$$\theta_q(B) = 1 - \sum_{i=1}^{q} \theta_i B^i \quad (3.9)$$

in which the backward shift operator (B) shifts Y_t and ε_t over time: $(B^p)Y_t = Y_{t-p}$ and $(B^q)\varepsilon_t = \varepsilon_{t-q}$ [54]. As the model combines autoregressive and moving average techniques there are less parameters to identify. The ARMA (p, q) model can be re-written in the order terms of p and q:

$$pY_t = q\varepsilon_t \quad (3.10)$$

3.2.2 ARIMA

An ARIMA model, developed by Box-Jenkins in early 1970s, is another combination of AR and MA models that includes differencing and is presented mathematically in [7] as:

$$\varphi_p(B)(1-B)^d Y_t = \theta_q(B) \varepsilon_t \quad (3.11)$$

where $(1-B)^d$ is the differencing term and all other models are the same as those defined for the ARMA model. Hence, the ARIMA (p, d, q) model can be re-written in the order terms of p, d, and q:

$$pdY_t = q\varepsilon_t \tag{3.12}$$

ARIMA models are widely-used in time-series analysis and integrate the data through differencing until the series becomes stationary [80].

ARIMA models display relationships between current and historical prices interpreting correlation among data [17]. ARIMA models are stochastic processes that can spot trends particularly when observing 24-hour periods, but generally for an ARIMA model to reach maximum forecasting potential a large dataset of historical records is required [50]. A large volume of historical data allows for better parameter estimations which improves the prediction errors and resulting forecasting accuracy [52]. Energy traders have considered the use of ARIMA models for electricity price forecasting and have seen promising results, especially when predicting daily or weekly electricity prices in energy commodities markets [15]. Gao et al. [7] compared an ARIMA model and an **A**rtificial **N**eural **N**etwork (ANN) model in terms of short-term electricity price forecasting and found the ARIMA model to have better forecasting accuracy. In a steady market, ARIMA models can accurately predict electricity prices; nonetheless these types of models struggle with compound price forecasting [53].

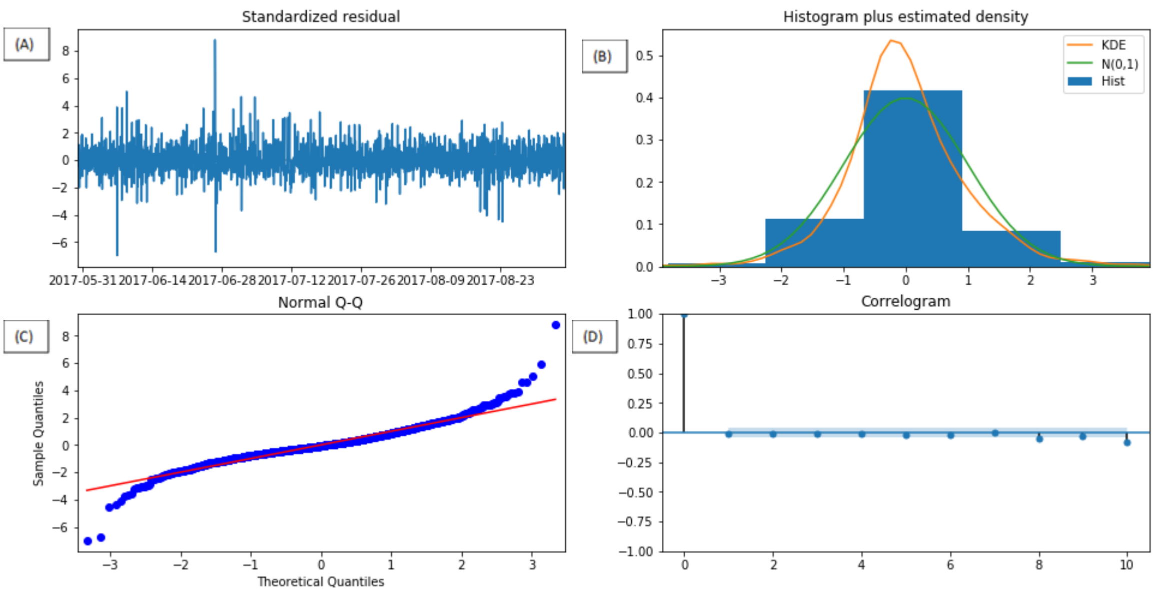

Figure 3.7: A diagnostic plot with simulated electricity price data

Errors arise from autocorrelation and it is helpful that ARIMA models can manage these errors by examining the residuals in the model and, if autocorrelation exists, choose appropriate test statistics to handle the errors [17]. Diagnostic checking on fitted residuals requires a robust statistical approach and is an important step to examine goodness-of-fit and to confirm that a model is appropriate as outliers can affect order term identification and the estimation of parameters [85]. Diagnostic plots observe if the fitted residuals are uncorrelated and help determine if the residuals follow a stationary pattern. Stationary residuals resemble a normal distribution and do not display white noise. A diagnostic plot, illustrated in Figure 3.7, is split into four sections: Figure 3.7(A) displays a residual error plot which should fluctuate around 0 with standardised variance if the residuals are uncorrelated, Figure 3.7(B) is a **K**ernel **D**ensity **E**stimate (KDE) plot which should be normally distributed with mean zero, Figure 3.7(C) shows a quantile plot which should align close to the red line, and Figure 3.7(D) is an ACF plot which should display the residual errors close to zero if they are uncorrelated.

3.2.3 SARIMA

Seasonality is defined as a recurrent periodic pattern influenced by external factors frequently accompanied by a stochastic trend [4]. Trend and seasonality cause spikes, often making the market unpredictable. With energy data having the same hour/day patterns (time patterns) or spring/summer patterns (seasonal patterns), seasonality and trend need to be accounted for. An extension of ARIMA is a **S**easonal **A**uto**R**egressive **I**ntegrated **M**oving **A**verage (SARIMA) model which considers non-seasonal as well as seasonal behaviours/trends and is outlined in [54] as:

$$\varphi_p(B)\Phi_P(B^S)(1-B)^d(1-B^S)^D Y_t = \theta_q(B)\Theta_Q(B^S)\varepsilon_t \tag{3.13}$$

where $\varphi_p(B)$ is the standard autoregressive term, $(1-B)^d$ is the standard differencing term, and $\theta_q(B)$ is the standard moving average term. $\Phi_P(B^S)$ is the seasonal autoregressive term, $(1-B^S)^D$ is the seasonal differencing term, $\Theta_Q(B^S)$ is the seasonal moving average term, Y_t is the prediction output at time t, ε_t is the error term and (B) is the backward lag shift. Therefore, the SARIMA (p, d, q) (P, D, Q) model can be re-written in the non-seasonal order terms of p, d, q, and the seasonal order terms of P, D, Q:

$$pPdDY_t = qQ\varepsilon_t \tag{3.14}$$

A SARIMA model is used if the data display periodic behaviour, for instance monthly patterns. A SARIMA model follows the Box-Jenkins method illustrated in Figure 3.1 as this process can function with seasonal factors [86].

SARIMA models assume a linear relationship exists in the time-series between past and current values, therefore input data must first be made stationary to remove the stochastic trend that arises from seasonality before fitting the model [4]. Removing seasonal variations from the model adjusts seasonality and then the model is scaled back before applying techniques for forecasting [4]. The seasonal difference order terms (P, D, Q) show relationships between current and historical values within consecutive seasons [4]. Typically, the number of significant lags for P and Q depend on multiples of the length of the season period S. Integration orders (non-seasonal d and seasonal D) are the number of times differencing is needed for both standard and seasonal ARIMA to have fixed mean and constant variance. D is applying differencing with lags or seasons and is set to 0 if the seasonal time-series pattern is unstable. The number of periods between seasons (S) is also required which involves repetition of the seasonal pattern: for instance a periodic 24-hour lag is a seasonal daily recurrence [56]. Updating the parameters periodically allows the model to keep in line with price data trends [56].

3.3 MISO Models

Electricity prices are controlled by multiple energy input factors and vary depending on seasonal patterns. Commercial and technical features such as fuel markets, power systems, and weather lead to price fluctuations and therefore need to be considered as exogenous variables in prediction modelling. MISO models consider multiple inputs (historical exogenous variables) to predict a single output (actual price). Adding exogenous variables can help to explain energy price movements as external factors can contribute strongly to generation costs. Determination of the most significant factors is vital to include as model inputs in order to improve day-ahead prediction accuracy. As well as the current exogenous variables, lag values can also be included as model inputs [56].

3.3.1 ARMAX

ARMA models can be further extended to include eXogenous variables and become multivariate models, known as ARMAX models, which contain multiple explanatory variables as inputs as well as the historical time-series data. An ARMAX model is defined as:

$$\varphi_p(B)Y_t = \beta_k x_{k,t}{}' + \theta_q(B)\varepsilon_t \tag{3.15}$$

where $\varphi_p(B)$ is the autoregressive term, Y_t is the prediction output, $\beta_k x_{k,t}{}'$ is the exogenous variables of the k^{th} input at time t, $\theta_q(B)$ is the moving average term, and ε_t is the error term [56]. The ARMAX (p, q) model can be written with respect to the order terms p and q:

$$pY_t = \beta_k x'_{k,t} + q\varepsilon_t \tag{3.16}$$

Including the key contributing exogenous variables as model inputs and selecting the most significant order terms is essential for an accurate forecasting performance [87]. This research will consider energy-related variables from **B**ritish **E**lectricity **T**rading and **T**ransmission **A**rrangements (BETTA) and **I**ntegrated **S**ingle **E**lectricity **M**arket (ISEM) energy data.

3.3.2 ARIMAX

If integration is required, the ARMAX model changes to ARIMAX denoted in [56] as:

$$\varphi_p(B)(1-B)^d Y_t = \beta_k x_{k,t}{}' + \theta_q(B)\varepsilon_t \tag{3.17}$$

where $(1-B)^d$ is the differencing term to make the data stationary, which is not required for an ARMAX model [56]. Thus, the ARIMAX (p, d, q) model can be written with respect to the order terms *p*, *d*, and *q*:

$$pdY_t = \beta_k x'_{k,t} + q\varepsilon_t \tag{3.18}$$

3.3.3 SARIMAX

A further extension to SARIMA to improve model performance with the inclusion of explanatory variables is a **S**easonal **A**uto**R**egressive **I**ntegrated **M**oving **A**verage model with e**X**ogenous inputs (SARIMAX) computed in [54] as:

$$\varphi_p(B)\Phi_P(B^S)(1-B)^d(1-B^S)^D Y_t = \beta_k x_{k,t}{}' + \theta_q(B)\Theta_Q(B^S)\varepsilon_t \tag{3.19}$$

where $\varphi_p(B)$ is the non-seasonal AR term, $(1-B)^d$ is the non-seasonal differencing term, and $\theta_q(B)$ is the non-seasonal MA term. $\Phi_P(B^S)$ is the seasonal AR term, $(1-B^S)^D$ is the seasonal differencing term, $\Theta_Q(B^S)$ is the seasonal MA term, Y_t is the prediction output, $\beta_k x_{k,t}{}'$ is the exogenous variable of the *k*[th] input at time t and ε_t is the error term. Therefore, similar to the other models, the SARIMAX (p, d, q) (P, D, Q) model can be re-written in the order terms of *p*, *d*, *q*, *P*, *D*, and *Q*:

$$pPdDY_t = \beta_k x'_{k,t} + qQ\varepsilon_t \tag{3.20}$$

With the inclusion of exogenous variables, the performance accuracy of SARIMAX tends to produce better results than a SARIMA model. The transparency of a SARIMAX model easily identifies which exogenous variables are significant through outputting model terms and summary statistics for each variable [56].

3.3.4 NARMAX

Non-linear regression models may be more suitable for electricity price forecasting as the data are often non-linear. Non-linear regression models can calculate the dependent price variable by using a non-linear combination of one or more additional independent variables [8]. A NARMAX model detects the structure of a non-linear difference equation displaying a relationship between current output and previous inputs/outputs [88]. Parameters are estimated in a transparent NARMAX model through simple regression algorithms which model the relationship between input and output variables [61] and their values can be determined with simple linear identification methods [89]. A NARMAX model identifies the most significant input variables and therefore it creates a compact price forecasting model. NARMAX models also consider seasonality and can detect strong periodic series [65]. To establish an optimal forecasting tool for predicting electricity price it is important to include all significant energy-related factors. Transparent learning models are fast to compute and can be graphically analysed from the input/output relationships or interpreted from the model term information [90]. In particular polynomial models have a simple transparent model structure, making them the most attractive type of NARMAX [88].

A polynomial NARMAX model (Leontaritis and Billings, 1985 [91]) is represented as:

$$y(t) = F^l[y(t-1), \ldots, y(t-N_y), u(t), \ldots, u(t-N_u), \mathcal{E}(t-1), \ldots\ \mathcal{E}(t-N_\mathcal{E})] + \mathcal{E}(t) \quad (3.21)$$

where $y(t)$ is the output time-series, F^l is an unknown non-linear function generally taken as a polynomial, N_y is the output regression lag order, $u(t)$ is the input time-series, N_u is the input regression lag order, $N_\mathcal{E}$ is the prediction error regression lag order, and $\mathcal{E}(t)$ is the prediction error [91]. NARMAX is a popular technique for estimating the unknown parameters by controlling inputs and outputs [62]. There are various polynomial model structures (linear-, quadratic-, and cubic-polynomial) to consider. The model first attempts to estimate F^l which removes unnecessary terms to create a single polynomial [89]. With a polynomial model the difficult part of the set-up is deciding on the interaction terms and which polynomial degree to use, but this can be determined through different trial and error combinations of inputs, degrees, and interaction terms [89]. Once F^l is identified, the unknown parameters can be estimated. F^l is limited to multivariable polynomials which prevents step-wise regression occurring during model identification [92]. To ensure model reliability, the polynomial model is validated using unseen data and verified against a predetermined error threshold [93].

The NARMAX methodology estimates and identifies the appropriate model terms needed for an accurate model through five stages: (1) structure selection, (2) parameter estimation, (3) model validation, (4) prediction, and (5) analysis. NARMAX aims to determine the best model by using the first 50% of the data for estimation and the remaining 50% of the data are used as testing to validate goodness-of-fit [90]. Structure selection and estimating parameters using experimental data are important as the learning process prunes insignificant coefficients [94]. For structure selection, the polynomial and regression orders are required to determine the initial model structure [90]. All possible combinations of historical inputs and outputs are analysed to provide unbiased estimates and to determine the most significant model terms [66]. This is determined through applying simple orthogonal estimation algorithms from Korenberg et al. [91] for structure detection to find model terms and prune parameter coefficients. The orthogonal algorithm initially estimates linear independent parameters without $\mathcal{E}(t)$ allowing the addition of extra terms to be included without re-estimating, then it estimates prediction errors, and finally estimates the $\mathcal{E}(t)$ in each iteration until all model coefficients have been estimated [91]. Since terms are orthogonal for any input, each coefficient is independent of the other terms and can be independently estimated [91]. This method keeps the model simple and avoids over-fitting or under-fitting by adding each new term separately and checking the significance of the new coefficient against the output's variance after each step [88].

Since the NARMAX model can include numerous inputs, this increases the difficulty in the model reaching substantial accuracy. The orthogonal estimation algorithm outputs the ERR which is the reduction percentage from the total **M**ean **S**quared **E**rror (MSE) and signifies the contribution of each model term with respect to the output [83]. The ERR is outlined by Zito & Landau [62] as:

$$ERR_i = \frac{g_i^2 \sum_{k=1}^{N} w_i^2(t)}{\sum_{k=1}^{N} y_i^2(t)} \tag{3.22}$$

where g_i is the parameter (energy-related factors) and w_i is the regressor (day-ahead electricity price) and y_i is the output regressor. ERR helps develop a parsimonious model to select significant model terms by ranking regressors in a forward regression from high to low in terms of the MSE reduction [64].

While the model structure is identified from the orthogonal algorithm, validation tests are required to confirm an adequate fit before acceptance [91]. The need to identify the structure of the model accurately is a limitation of the NARMAX model [89]. The last part of the identification process is model validation and verification using unseen data [95] to make predictions and check the accuracy. The final NARMAX model provides the model terms of the statistically significant input variables which have been determined and ranked in order of significance [63]. Throughout the methodology the outcome is dependent on the selected input data therefore it is important to choose the input correctly to achieve accurate model fitness

[94]. Due to the volatile nature of the energy market with many significant factors exhibiting non-linear relationships, NARMAX could be useful to determine the model structure.

3.4 Machine Learning

Machine learning algorithms are built around input knowledge, learning from training data to find meaningful patterns and predict outputs [9]. Machine learning models can handle noise and respond to changing trends; thus a machine learning forecasting model that can avoid overfitting, provide user transparency, and adapt to new inputs is desirable [96]. Popular machine learning techniques are described in detail below.

3.4.1 Random Forest

Random Forest is an ensemble model built with multiple decision trees. Bagging is one technique to grow the forest and this method consists of the following steps for a training set $R = \{y, x\}$ to predict y for an observation x [97]:

(1) Select n random observations from R to create smaller sample S datasets.
(2) For each S dataset, grow a decision tree.
(3) Each tree outputs a prediction for every observation; the one with the most votes is selected as the final prediction.

The Random Forest algorithm combines the **C**lassification **a**nd **R**egression **T**ree (CART) technique with the Bagging (bootstrap aggregation) method to stop overfitting and reduce variance by outputting a confidence interval that can be linked to prediction to improve accuracy [98]. The algorithm contains a tuning parameter which recursively decides the nature of split at each node for classifying the input data [99]. The best split at each node is chosen by the highest information gain [100] which is calculated as:

$$IG = I(N) - P_L * I(N_L) - P_R * I(N_R) \tag{3.23}$$

where $I(N)$ is the impurity measure (either Gini or Shannon Entropy) of node N, N_L is the left child of the node, N_R is the right child of the node, P_L is the proportion that goes to N_L after the split and P_R is the proportion that goes to N_R after the split. Splitting at nodes means no individual tree sees the full set of training data [100]. Once the Random Forest is completely built, with a prediction from each tree, the final value is calculated as the average of all the trees' predictions [101].

A Random Forest model is very adaptable as it can adjust to the latest seasonal and market trends. As well as adaptability a Random Forest has many advantageous prediction features: simple to tune, robust, accurate estimates, and expandable for data growth [98]. Random Forest can be considered for feature selection as during the training an overall ranking of the importance of each feature is calculated [102]. Since correlation between trees is reduced, a Random Forest algorithm would be suitable for prediction whenever there is little knowledge on the relationship of the input variables [97]. Random Forest can be applied with any modelling approach to improve model accuracy but is most frequently used with decision trees or regression trees to increase efficiency [103].

3.4.2 Gradient Boosting

Boosting algorithms have become quite popular over the last few decades. Using sequential learning to train the model [102], a gradient boosting algorithm combines weak learner models and through loss function optimisation builds one strong learner prediction model [104]. A loss function is optimised from the combination of weak learners minimising the residual errors in the strong learner model [97]. The algorithm outlined in [105] for input data x, loss function G and iteration number M is as follows:

(1) Firstly initialise a model $\widehat{f_0}$.

For $t = 1$ to M do:

(2) Calculate the negative gradient, $g_t(x)$.

(3) Fit a new basic learner, $h(x, \theta_t)$.

(4) Find the optimal gradient step-size, p_t.

(5) Update function estimate: $\widehat{f_t} \leftarrow \widehat{f_{t-1}} + p_t h(x, \theta_t)$

end

Gradient boosting algorithms are highly flexible with the learning process continually fitting new models to generate an accurate final estimate, unlike the Random Forest which takes the average of all trees. However, gradient boosting techniques do have some computational complexities in terms of memory consumption, which are dependent on the number of iterations required for boosting [105].

3.4.3 Extreme Gradient Boosting

A popular advancement to gradient boosting is the Extreme Gradient Boosting (XGBoost) regression algorithm. XGBoost has additional features with weighted predictors which make it easier to differentiate the model's performance compared with the performance of a simple decision tree [104]. The algorithm outlined in [104] for training data x is as follows:

(1) Firstly initialise a model $F_0(x)$.

For $t = 1$ to M do:

(2) Calculate the pseudo-residuals.

(3) Fit basic learner, $h_t(x)$ to pseudo-residuals.

(4) Calculate multiplier, γ_t.

(5) Update model: $F_t(x) \leftarrow F_{t-1}(x) + \gamma_t h_t(x)$

end

The recursive steps of the algorithm include learning the regression predictor, computing the residual errors, and then ensemble learning to predict the residual through estimating the loss function's gradient [104].

XGBoost has been successful over the last few years due to its scalability; the speed of the system is much improved because of algorithmic optimizations and justified weights allowing the model to learn quicker and function well with large datasets [106].

3.5 Conclusion

This chapter has discussed in detail statistical regression and machine learning techniques and their potential suitability for accurately predicting day-ahead electricity prices. First the theory of statistical models was explained, discussing why stationarity and integration are important and how to define them. If the necessary stationarity and seasonal checks are complete, then ARIMA or SARIMA may be suitable transparent statistical models for electricity price forecasting. As electricity prices are often impacted by exogenous factors then the use of ARIMAX or SARIMAX can also be considered. Statistical forecasting models, in particular short-term models, are very popular. However, these models assume linearity and often energy related data are non-linear. Thus a non-linear regression model, like NARMAX, that uses a non-linear combination of inputs and outputs may be suitable for forecasting electricity prices. The transparency of NARMAX can help identify key factors to refine statistical models and hence improve accuracy of day-ahead price forecasting models. Similarly, machine learning techniques can be used to model complex data, learning from training data to find patterns and predict accurate output values. Table 3.1 provides a critical evaluation of each method stating the assumptions, the pros/cons and the situation when the method is best suited for use. The contribution chapters following this chapter, will utilise this range of statistical regression and machine learning techniques.

Table 3.1: Review table of all methods

Method	Assumptions	Pros/Cons	Best Suited When
ARMA	Weakly stationary	*Pro:* Simple to tune *Con:* Need to determine *p* and *q*	Single input with no trend or seasonality
ARIMA	Weakly stationary	*Pro:* Performs well short-term *Con:* Requires differencing	Single input with trend and no seasonality
SARIMA	Weakly stationary	*Pro:* Performs well short-term *Con:* Requires differencing and seasonality	Single input with both trend and seasonality
ARMAX	Weakly stationary	*Pro:* Simple to tune *Con:* Need to determine *p* and *q*	Multiple inputs with no trend or seasonality
ARIMAX	Weakly stationary	*Pro:* Performs well short-term *Con:* Requires differencing	Multiple inputs with trend and no seasonality
SARIMAX	Weakly stationary	*Pro:* Performs well short-term *Con:* Requires differencing and seasonality	Multiple inputs with both trend and seasonality
NARMAX	Non-linear	*Pro:* Transparency *Con:* Trial and error combinations of inputs	Removing redundant exogenous variables
Random Forest	Tuning parameter	*Pro:* Avoids overfitting, easy tuning, robust to outliers *Con:* Slow to train	Trying to find patterns and learn trends
Gradient Boosting	Loss function	*Pro:* Highly flexible during sequential learning *Con:* Can overfit	Trying to find patterns and learn trends
Extreme Gradient Boosting	Weighted predictors	*Pro:* Fast to train, performs well with large datasets *Con:* Difficult to tune	Trying to find patterns and learn trends

The next chapter will investigate various SISO models for electricity price forecasting using only historical raw price values as input. Day-ahead price prediction models with actual electricity prices from both the BETTA and ISEM markets will be analysed to follow trends and forecast electricity price values. Three SISO time-series models (ARMA, ARIMA and SARIMA) will be examined through experiments focussing on the four key modelling steps: model identification, parameter estimation, diagnostic checking, and forecasting performance.

Chapter 4

Single Input Single Output Models

4.1 Introduction

Single **I**nput **S**ingle **O**utput (SISO) models focus on one input/one output relationships to predict values. In this chapter, the first research question listed in Chapter 1 is addressed ("Are traditional statistical methods or more recent computational models appropriate techniques for day-ahead electricity price forecasting?"). This research contribution aims to understand and interpret time-series based statistical methods and apply them to real electricity price data to establish if such techniques are suitable for day-ahead electricity price forecasting. The research work is published in the following:

- C. McHugh, S. Coleman, D. Kerr, and D. McGlynn, "Forecasting Day-Ahead Electricity Prices With A SARIMAX Model", in Proceedings of the 2019 IEEE Symposium Series on Computational Intelligence, SSCI 2019, pp. 1523–1529.

This chapter explores different statistical SISO time-series models and analyses each separately in terms of data understanding and predicting day-ahead electricity prices. To evaluate the performance of each forecasting technique, historical electricity price data were used to obtain prediction models. Data retrieved from the **B**ritish **E**lectricity **T**rading and **T**ransmission **A**rrangements (BETTA) market examines the period from May 2017 until April 2018. Data collected from the Irish **I**ntegrated **S**ingle **E**lectricity **M**arket (ISEM) explores the period from May 2019 until April 2020.

The research work first analyses an **A**uto**R**egressive **M**oving **A**verage (ARMA) model by following the steps of the Box-Jenkins model [80], as outlined in Section 3.2. Historical values are influential in time-series models as past observations influence future values resulting in autocorrelation as the time-series is used twice in terms of original and lagged values [107]. The data will be checked through **A**ugmented **D**ickey-**F**uller (ADF) unit root testing to confirm if stationarity (constant mean and variance) is present. **P**artial **A**uto**C**orrelation **F**unction (PACF) and **A**uto**C**orrelation **F**unction (ACF) plots will be displayed and observed to measure the range for order terms *p* and *q*, however these terms will be verified and selected from the **A**kaike **I**nformation **C**riterion (AIC) statistical method. Next, differencing will be applied and plotted to observe how an **A**uto**R**egressive **I**ntegrated **M**oving **A**verage (ARIMA) model handles electricity price forecasting. Various models built from statistical techniques are extended to enable the inclusion of seasonal variation and have been shown to improve prediction accuracy [13]. Since electricity prices display a seasonal trend, mostly due to daily and weekly patterns [81], a

Seasonal **A**uto**R**egressive **I**ntegrated **M**oving **A**verage (SARIMA) model will be examined as well as diagnostic plots to check residuals and model fit. In this thesis, robustness is defined as the accuracy between actual and predicted electricity price and is measured by the **R**oot **M**ean **S**quared **E**rror (RMSE) which calculates the distance between actual and predicted values. For each of the statistical models, the RMSE was chosen as the performance metric as it checks how accurately the predicted electricity price compares with the actual market price; a low RMSE value indicates a robust model fit. Throughout the sections of this chapter, the four key modelling stages are outlined for each technique: identification, estimation, diagnostic testing, and forecasting. The software utilised for each of the modelling stages was Python through the NumPy, Pandas and Statsmodels libraries.

4.2 ARMA Experiment

ARMA models were described in Section 3.2.1. Hourly BETTA market electricity prices are available from the exchange traded day-ahead market (N2EX) on the Nordpool website [108]. To build the ARMA experiment and evaluate model performance, data were obtained resulting in a total of 8736 hourly electricity prices. The task is to predict the price in the same one hour period during the next day (day ahead price). Price data ranges from 02nd May 2017 to 30th April 2018 and is used as the target day-ahead price with all previous hour prices, ranging from 01st May 2017 to 29th April 2018 used as the input data. The data records were split 50/50 for model estimation (02nd May Hour 0 to 30th October Hour 23) and model validation (31st October Hour 0 to 30th April Hour 23). The traditional 50/50 split was applied for the statistical models to follow the standard approach whereas the computational models in Chapter 7 applied a split of 85/15 as they require more data for training.

The behaviour of electricity prices is generally quite volatile therefore the prices need to be plotted over time to observe the pattern. Figure 4.1 displays the BETTA market electricity prices for the overall time period. In Figure 4.1 there are some peaks and troughs with a slight trend, but generally overall it can be observed that the data are stationary. This is further confirmed using the ADF stationarity test which determines that the data follow a stationary pattern with constant mean and variance. The ADF test checks for stationarity by assuming non-stationarity for the null hypothesis and stationarity following an ARMA structure for the alternative hypothesis [56]. For the data presented, the ADF statistic is -5.77 and the probability is extremely low (0.000001), rejecting the null hypothesis and confirming stationarity. In this case, d = 0 since the data are already stationary and therefore suitable for an ARMA model.

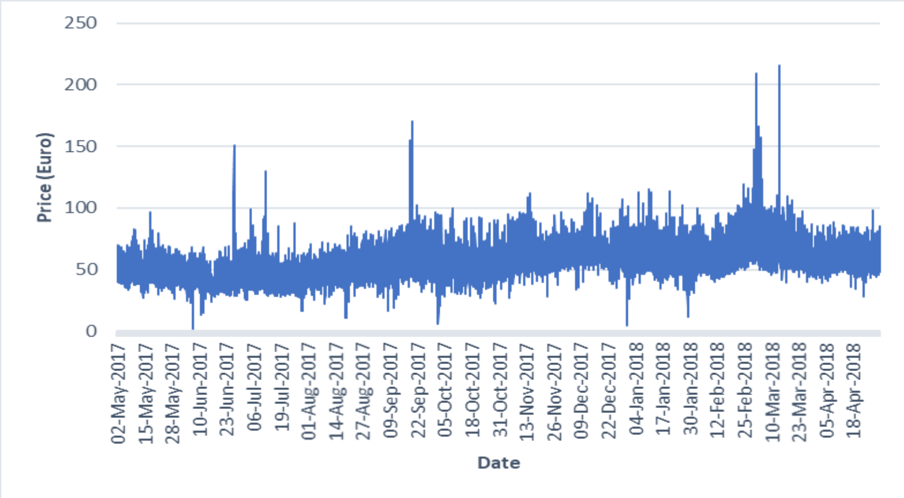

Figure 4.1: BETTA market electricity prices from May 2017 until April 2018

Model identification, discussed in Section 3.2, is required to find the parameter order terms to determine which type of time-series model fits the data. Examining a PACF plot identifies *p* as the number of past values up until the last significant lag and an ACF plot identifies *q* as the number of past deviations from the mean up until the last significant lag [109]. The order terms can either be set manually from observation or the plots can be used to determine the ranges required for the AIC method. To determine *p* and *q*, PACF and ACF values were plotted with the target electricity prices up to Lag 24. The confidence interval for the correlations is represented by the blue shaded area. Examining the autoregressive lags in Figure 4.2, it is difficult to identify exactly which is the last significant lag for *p*, but it appears to range from 0 to 10. Examining the moving average lags in Figure 4.3, the last significant lag for *q* is 7.

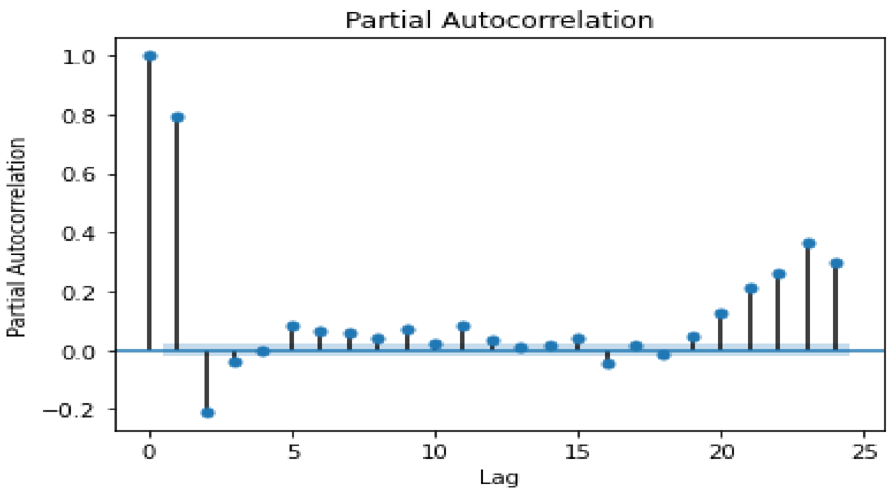

Figure 4.2: Partial autocorrelation (PACF) plot to determine p for BETTA market

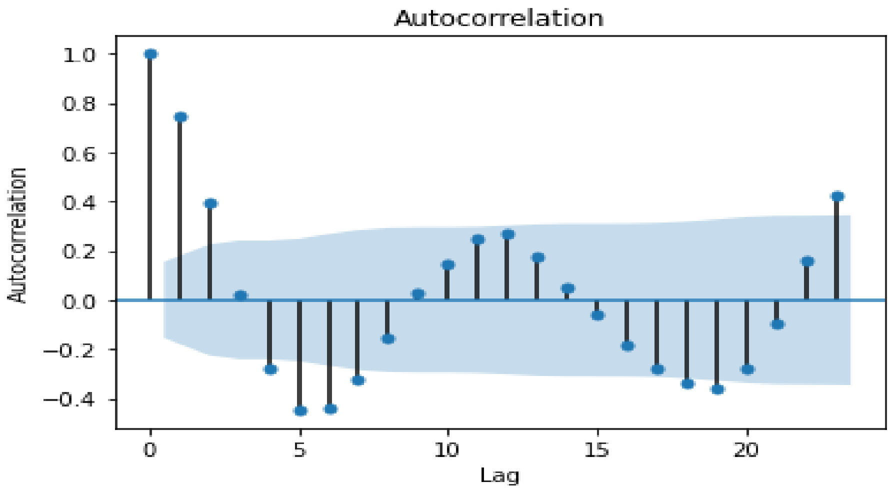

Figure 4.3: Autocorrelation (ACF) plot to determine q for BETTA market

The order terms p and q are identified using the AIC technique which is applied to verify the chosen order terms from the selected ranges by measuring the quality of the model fit [56]. To verify the order terms chosen, the AIC technique follows the process described in [56] using a brute force search of all the combinations within the set range. The ranges for the order terms are set as p = (0,10) and q = (0,10) for the number of permutations to rank the AIC values which converge from lowest to highest. The lowest AIC value is selected (28673.03) and the best ARMA order terms estimating parameter values using maximum likelihood are p = 9, q = 7. The final ARMA(9, 7) model function for predicted Y_t is taken from the model coefficients and given as:

$$Y_t = 0.090Y_{t-1} - 0.059Y_{t-2} + 0.70Y_{t-3} - 0.067Y_{t-4} - 0.68Y_{t-5} - 0.35Y_{t-6} - 0.35Y_{t-7} + 0.65Y_{t-8} - 0.097Y_{t-9} + 0.71\varepsilon_{t-1} + 0.50\varepsilon_{t-2} - 0.37\varepsilon_{t-3} - 0.32\varepsilon_{t-4} + 0.52\varepsilon_{t-5} + 0.76\varepsilon_{t-6} + 0.92\varepsilon_{t-7} + 21.89 \quad (4.1)$$

consisting of weighted terms in a linear combination of 9 autoregressive lags for Y and 7 moving average lags for prediction error ε with a RMSE value of 10.91.

To confirm if ARMA(9, 7) is an appropriate model fit and matches the ideal criteria presented in Figure 3.7, diagnostic checking is performed on the standardised residuals. Figure 4.4(A) plots the standardised residuals which fluctuate around 0, however there is a pattern of peaks and troughs from outliers; Figure 4.4(B) is a histogram with a density plot that is normally distributed with a narrow bell-shaped pattern symmetrical around 0; Figure 4.4(C) is a normal quantile-quantile plot with the quantiles mainly on, or close to, the red line suggesting a normal distribution but the sharp curves at the ends highlight extreme data values that the model is unable to fit; Figure 4.4(D) is a correlogram plot with slight autocorrelation present as a few lag errors fall outside the blue boundary. Therefore the ARMA(9, 7) model is confirmed as an appropriate fit as the standardised residuals display a weakly stationary pattern.

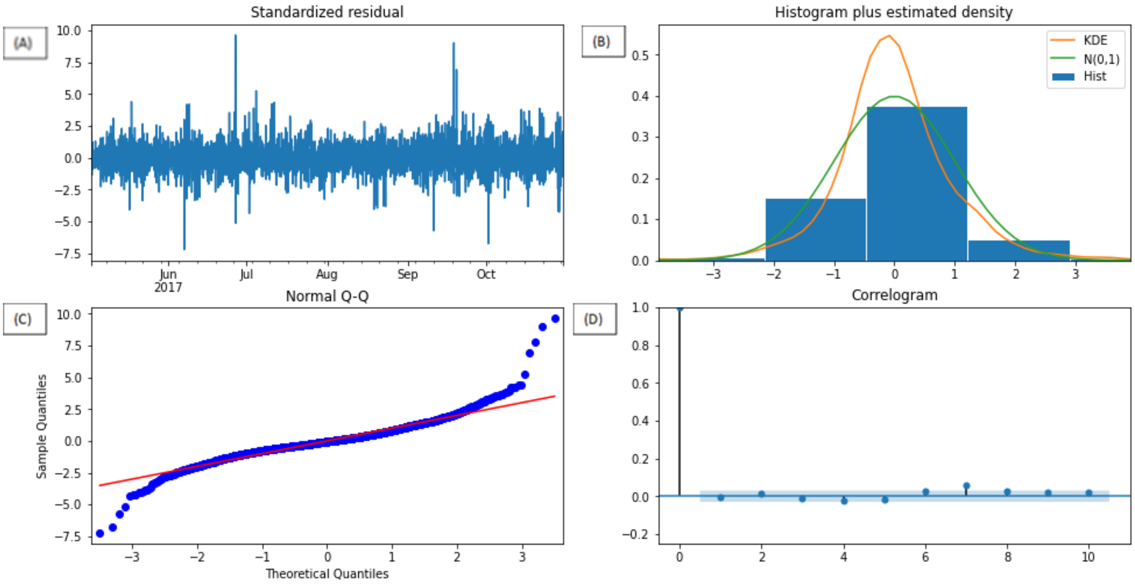

Figure 4.4: Residual diagnostic checks for BETTA market ARMA(9,7)

The ARMA(9, 7) model validation results for hourly BETTA electricity prices are presented in Figure 4.5 to illustrate the model fit and it can be seen that the predicted price values closely match the actual price values around the centre but fail to reach the peaks.

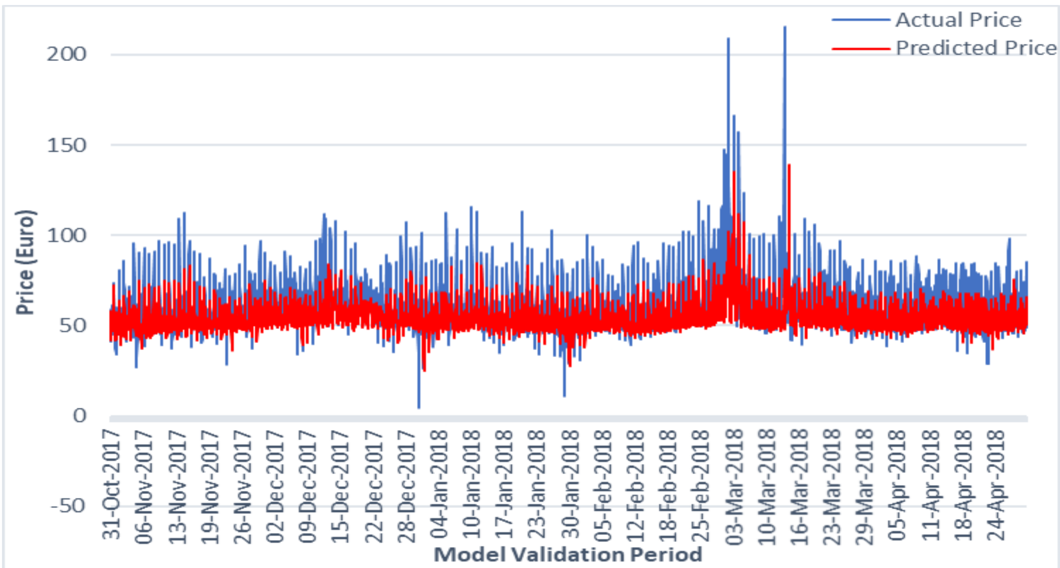

Figure 4.5: BETTA market ARMA(9,7) model

The same ARMA approach was used with the ISEM market data with hourly day-ahead electricity price available from the **S**ingle **E**lectricity **M**arket **O**perator (SEMOpx) website [110]. For this experiment, data were obtained resulting in a total of 8760 hourly electricity prices. The task is to predict the price in the same one hour period for the next day. The price being predicted, ranges from 02nd May 2019 to 30th April 2020 and is used as the target day-ahead price with all previous hour prices, ranging from 01st May 2019 to 29th April 2020 used as the input data. The data records were split 50/50 for model estimation (02nd May Hour 0 to 31st October Hour 11) and model validation (31st October Hour 12 to 30th April Hour 23). Figure 4.6 displays the electricity ISEM market prices from the time period May 2019 until April 2020. Over time the data appears to be consistent, nonetheless there is some trend with periods of high and low peaks. Applying the ADF stationarity test, the ADF statistic is -10.16 and the null hypothesis is rejected with probability 0.000000. Since the data displays stationarity, *d* is set to 0.

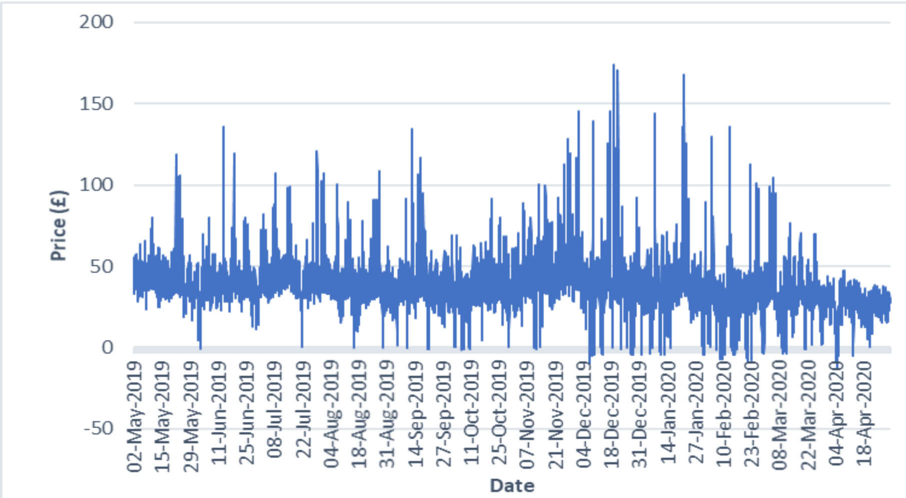

Figure 4.6: ISEM market electricity prices from May 2019 until April 2020

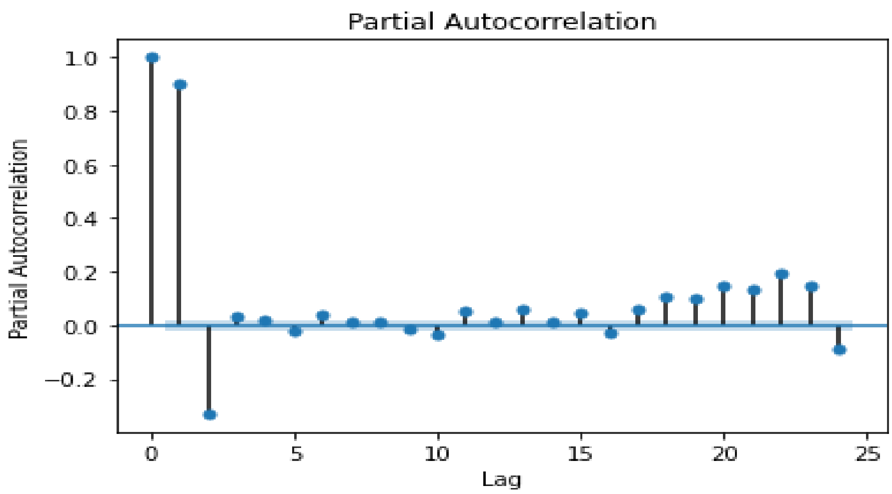

Figure 4.7: Partial autocorrelation (PACF) plot to determine p for ISEM market

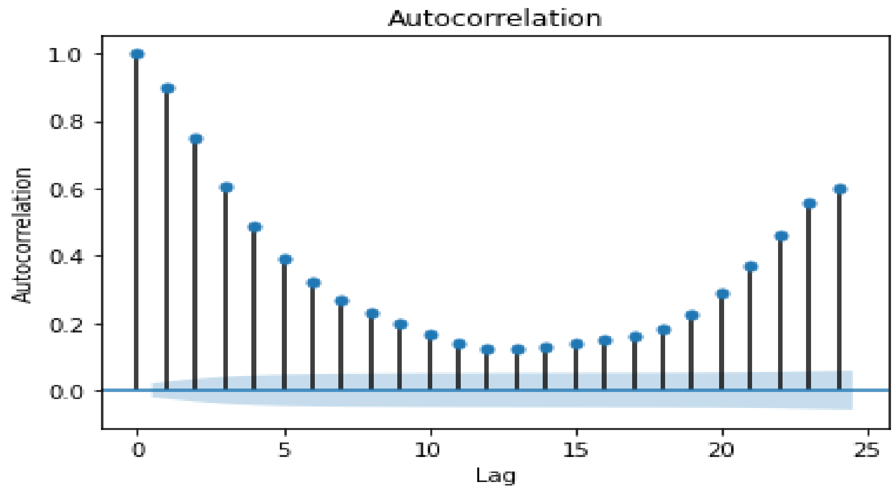

Figure 4.8: Autocorrelation (ACF) plot to determine q for ISEM market

The parameter order terms are found through model identification. First, the ranges for the order terms are identified by manually observing the PACF and ACF plots. Manually observing the PACF plot (Figure 4.7), which displays the past value lags up to 24, the range of the significant lag value for p goes from 0 to 10. Manually observing the ACF plot (Figure 4.8), which displays the past deviation from the mean lags up to 24, the significant lag value for q is likely to be either 8 or 9 before the lags drop therefore the range goes from 0 to 10.

The order terms *p* and *q* are calculated using the AIC technique following the process described in [56]. The ranges for the order terms are set as *p* = (0,10) and *q* = (0, 10) and the best ARMA parameter order terms are *p* = 9, *q* = 8 with an AIC value of 27681.54. The model function for ARMA(9, 8) for predicted Y_t is given as:

$$Y_t = 0.19Y_{t-1} + 0.075Y_{t-2} + 0.46Y_{t-3} + 0.10Y_{t-4} - 0.041Y_{t-5} + 0.43Y_{t-6} + 0.081Y_{t-7} + 0.21Y_{t-8} - 0.57Y_{t-9} + 0.83\varepsilon_{t-1} + 0.68\varepsilon_{t-2} + 0.089\varepsilon_{t-3} - 0.076\varepsilon_{t-4} - 0.13\varepsilon_{t-5} - 0.59\varepsilon_{t-6} - 0.64\varepsilon_{t-7} - 0.79\varepsilon_{t-8} + 18.66 \quad (4.2)$$

with 9 autoregressive lags for Y_t , 8 moving average lags for prediction error ε and a RMSE value of 14.99.

Figure 4.9 displays the residual diagnostic plots to check how well ARMA(9, 8) has performed. In Figure 4.9(A) the standardised residuals fluctuate around 0 with many peaks and troughs displaying a trend, in Figure 4.9(B) the density is normally distributed with a narrow bell-shaped pattern (orange line), in Figure 4.9(C) the majority of the quantiles are close to the red line but are curved at both ends due to outliers, and in Figure 4.9(D) the residual errors are close to zero with no autocorrelation present. Therefore the ARMA(9, 8) model is confirmed as a reasonable approximation fit to forecast electricity prices as the fitted residuals are generally uncorrelated and mostly follow a normal pattern.

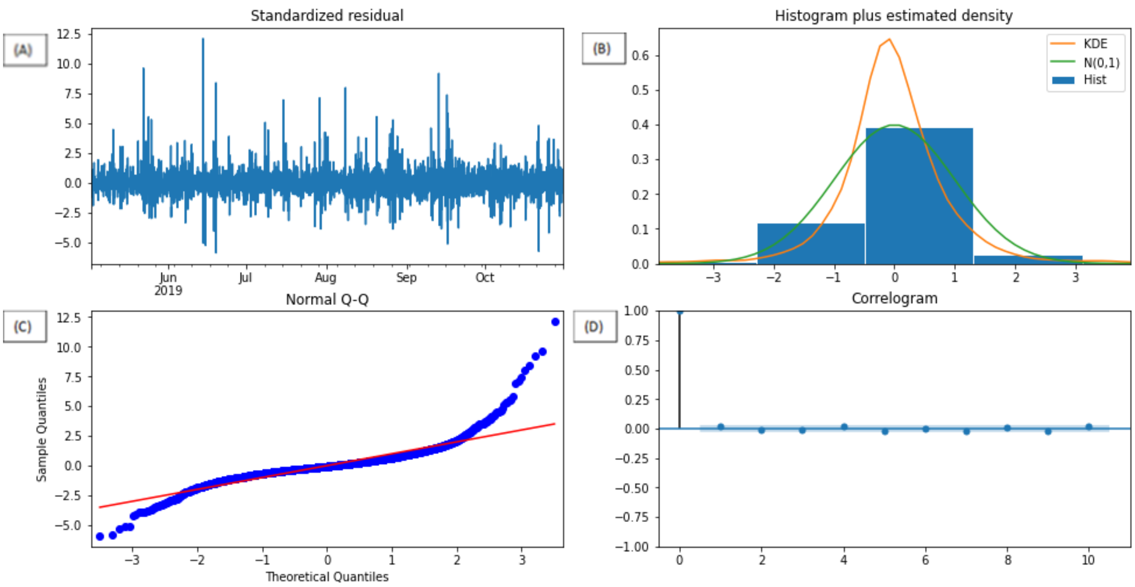

Figure 4.9: Residual diagnostic checks for ISEM market ARMA(9,8)

Figure 4.10 displays the ARMA(9, 8) model validation results for hourly ISEM electricity prices and illustrates that the predicted prices generally match the actual prices around the centre but fail to reach the high or low actual price values. Table 4.1 presents the RMSE values of the best ARMA model for both the BETTA and ISEM markets. Comparing the two markets, BETTA electricity prices provide a lower RMSE and better model accuracy than when using the ISEM electricity prices.

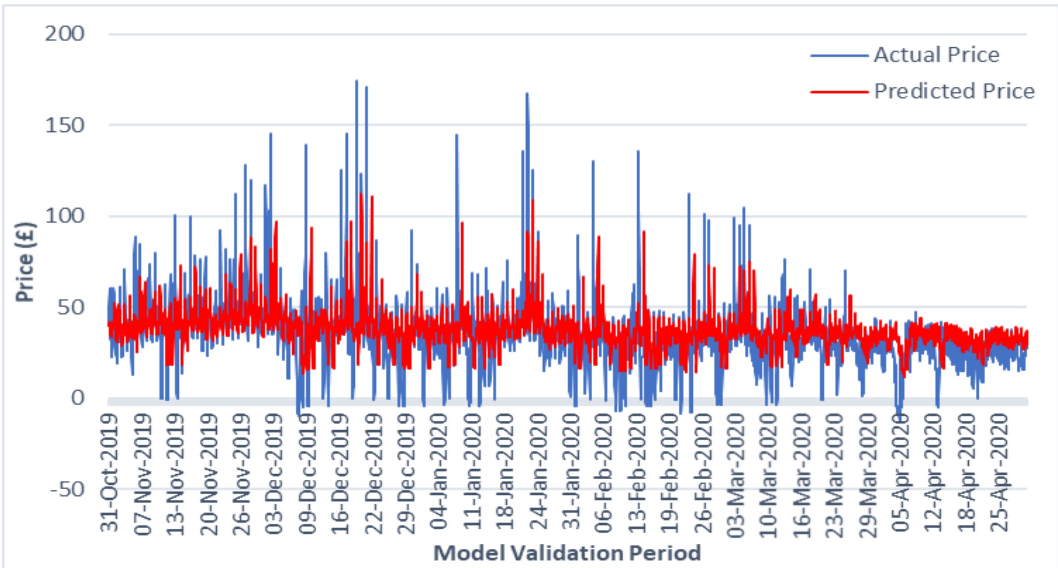

Figure 4.10: ISEM market ARMA(9,8) model

Table 4.1: RMSE values (ARMA models)

Market	Model	RMSE
BETTA	ARMA(9, 7)	10.91
ISEM	ARMA(9, 8)	14.99

4.3 ARIMA Experiment

As seen in the ARMA experiments, electricity prices have a long term trend as prices tend to fluctuate up and down. Over time electricity prices follow a similar pattern, however they also display a daily trend. This section investigates the use of ARIMA models, discussed in Section 3.2.2, to determine if applying differencing to remove trends between the current and previous prices will improve model performance and the accuracy of the model fit. Generally first-order differencing (differencing once) is sufficient to remove trends [14]. Figure 4.11 shows that after first-difference the data displays consistency over time with a smoother, straight horizontal pattern compared with Figure 4.1 which had more peaks and troughs. To include first-order differencing in the model fit, d is set to 1.

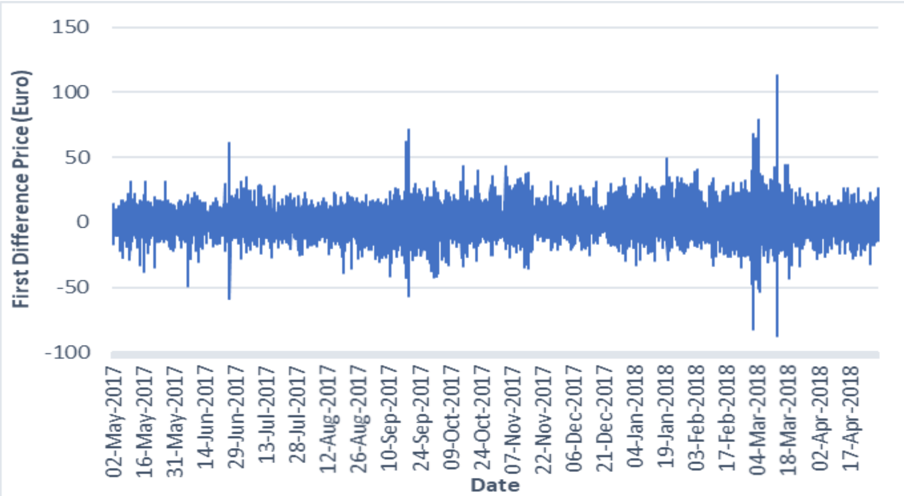

Figure 4.11: BETTA market first-difference price from May 2017 until April 2018

In this experiment, model identification is applied to the first-difference lags up to Lag 24 when examining the PACF and ACF plots. Observing Figure 4.12, the last significant autoregressive lag for p ranges between 0 and 10 and observing Figure 4.13 the last significant moving average lag for q is approximately 6.

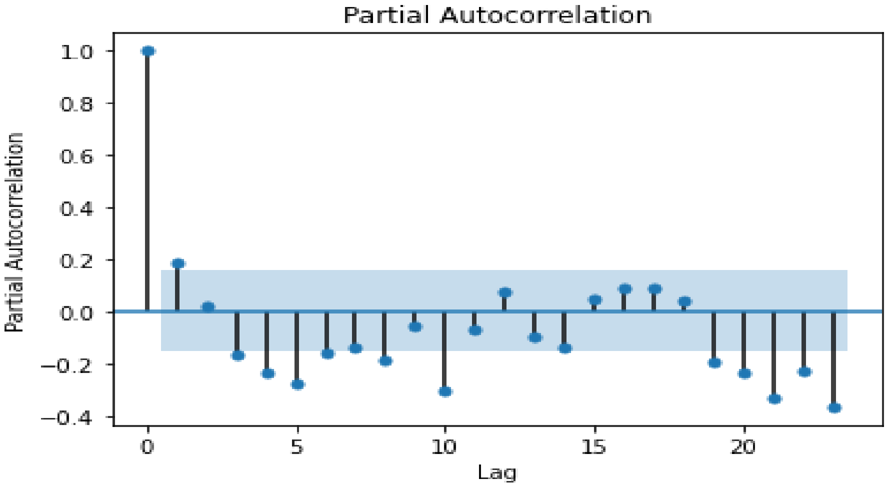

Figure 4.12: First difference partial autocorrelation (PACF) plot to determine p for BETTA market

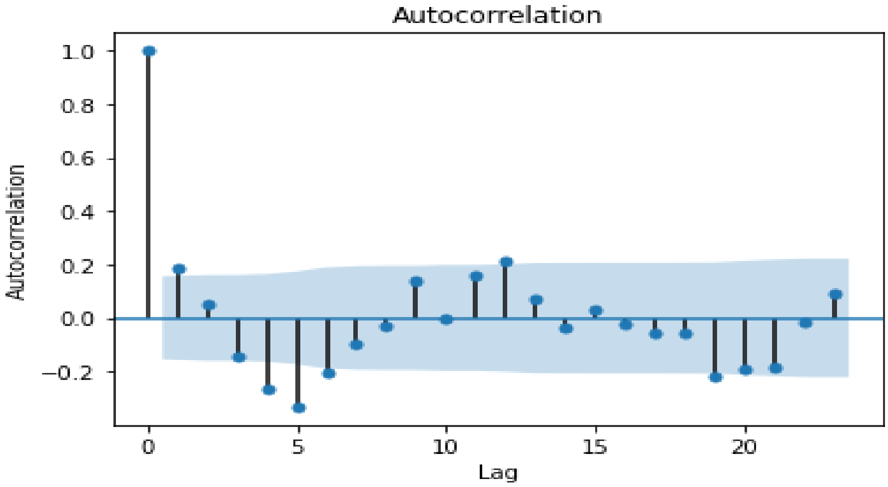

Figure 4.13: First difference autocorrelation (ACF) plot to determine q for BETTA market

The order terms *p* and *q* are determined using the AIC technique following the process outlined in [56]. The parameter ranges are set as *p* = (0,10) and *q* = (0,10). The best ARIMA parameter order terms are *p* = 9, *d* = 1, *q* = 7 and AIC value is 29677.70. The ARIMA(9, 1, 7) model function for predicted Y_t with a RMSE value of 9.94 is given as:

$$Y_t = -0.95\nabla Y_{t-1} - 0.89\nabla Y_{t-2} - 0.58\nabla Y_{t-3} - 0.20\nabla Y_{t-4} + 0.13\nabla Y_{t-5} + 0.42\nabla Y_{t-6} - 0.18\nabla Y_{t-7} - 0.033\nabla Y_{t-8} + 0.011\nabla Y_{t-9} + 0.76\nabla \varepsilon_{t-1} + 0.45\nabla \varepsilon_{t-2} - 0.026\nabla \varepsilon_{t-3} - 0.53\nabla \varepsilon_{t-4} - 0.75\nabla \varepsilon_{t-5} - 0.88\nabla \varepsilon_{t-6} - 0.0096\nabla \varepsilon_{t-7} + 43.45 \quad (4.3)$$

Diagnostic checking on the fitted residuals is performed to determine if ARIMA(9, 1, 7) is a suitable model fit. Figure 4.14(A) shows the standardised residuals fluctuate around 0 with a trend pattern. Figure 4.14(B) presents a histogram with a density plot that resembles a narrow bell-shaped pattern which indicates that the data are normally distributed and symmetrical around 0. Figure 4.14(C) displays a quantile-quantile plot where the majority of the quantiles are on or near to the red line, however the shape curves on both ends highlighting extreme outliers among the data. Figure 4.14(D) presents a correlogram plot showing that slight autocorrelation is present in Lag 10 but the majority of the lag errors are within the blue boundary.

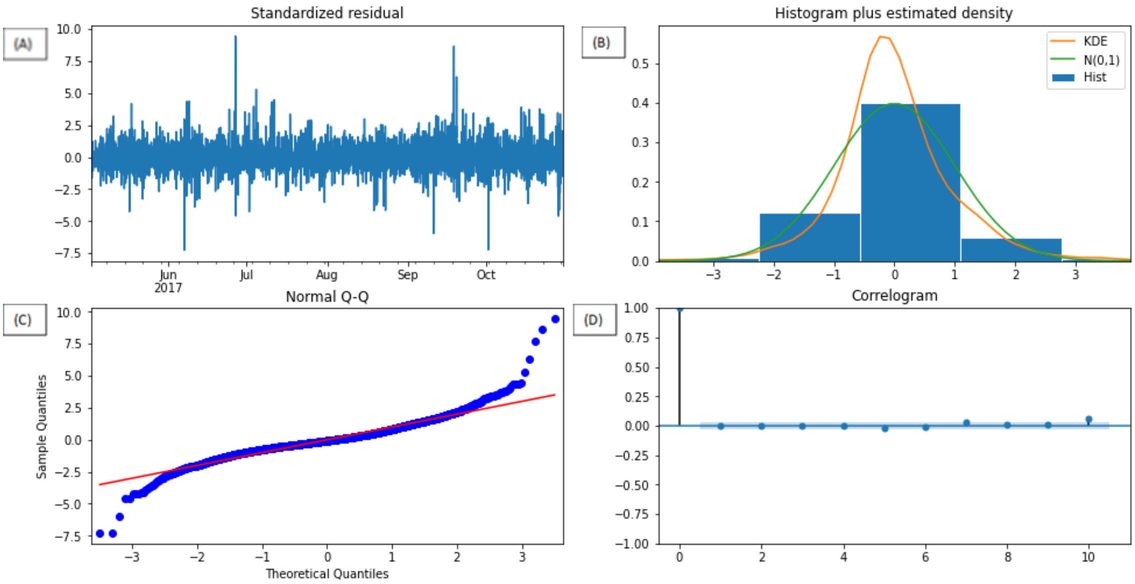

Figure 4.14: Residual diagnostic checks for BETTA market ARIMA(9,1,7)

The ARIMA(9, 1, 7) model validation results for hourly BETTA electricity prices are displayed in Figure 4.15. The majority of the predicted prices follow the same trend as the actual prices displaying an accurate representation suggesting that day-ahead electricity prices can be predicted using an ARIMA model.

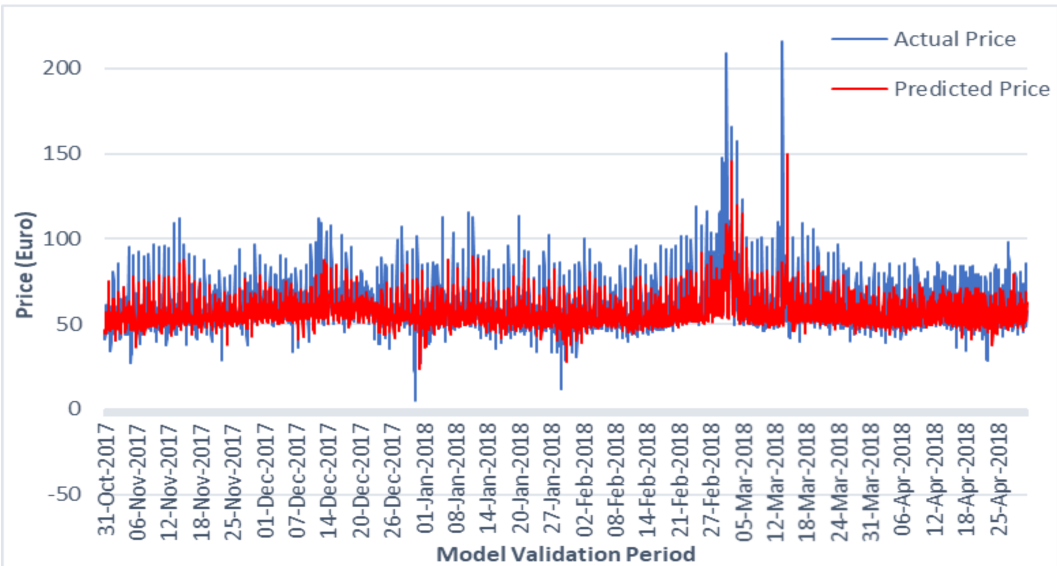

Figure 4.15: BETTA market ARIMA(9,1,7) model

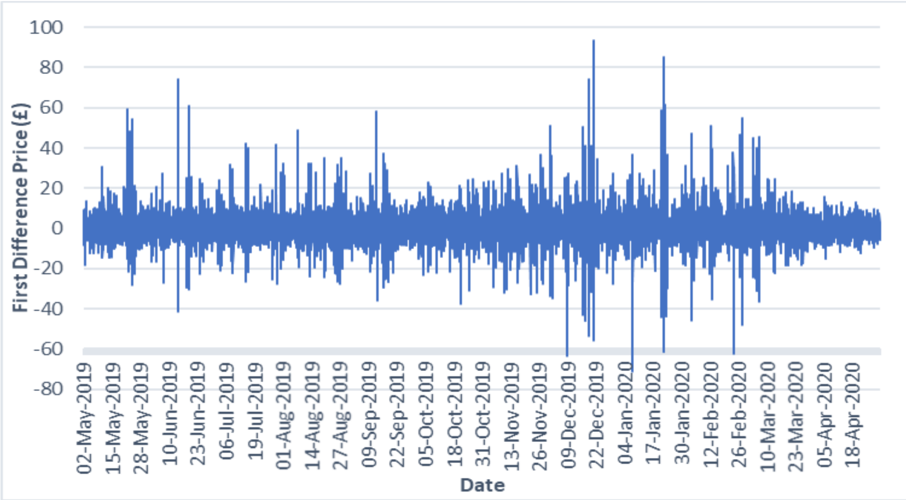

Figure 4.16: ISEM market first-difference price from May 2019 until April 2020

The same ARIMA approach was applied to the ISEM market data and *d* is set to 1. Figure 4.16 shows that the first-order differencing prices follow a straight horizontal pattern with fluctuations compared with the sporadic curve-like pattern of Figure 4.6. The parameter order terms for the first-difference lags up to Lag 24 are found through model identification. Manually observing the PACF plot in Figure 4.17, the last significant autoregressive lag for *p* ranges between 0 and 10. Manually observing the ACF plot in Figure 4.18, the last significant moving average lag for *q* is approximately 7.

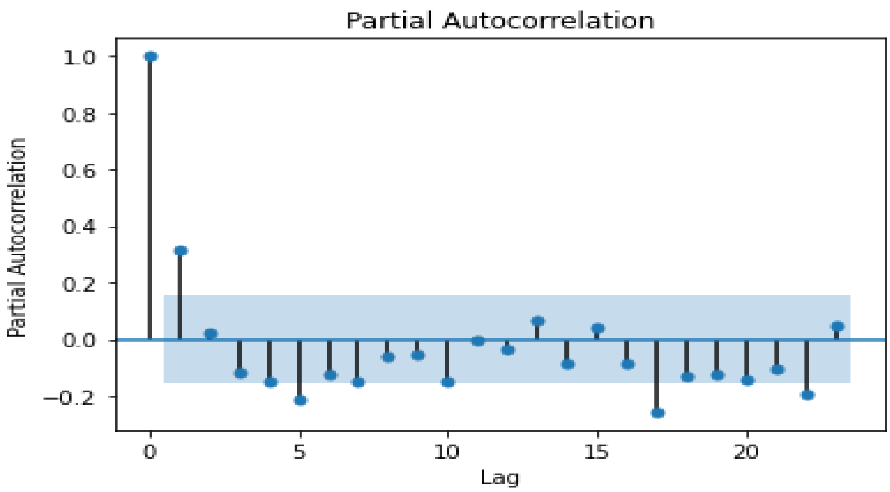

Figure 4.17: First difference partial autocorrelation (PACF) plot to determine p for ISEM market

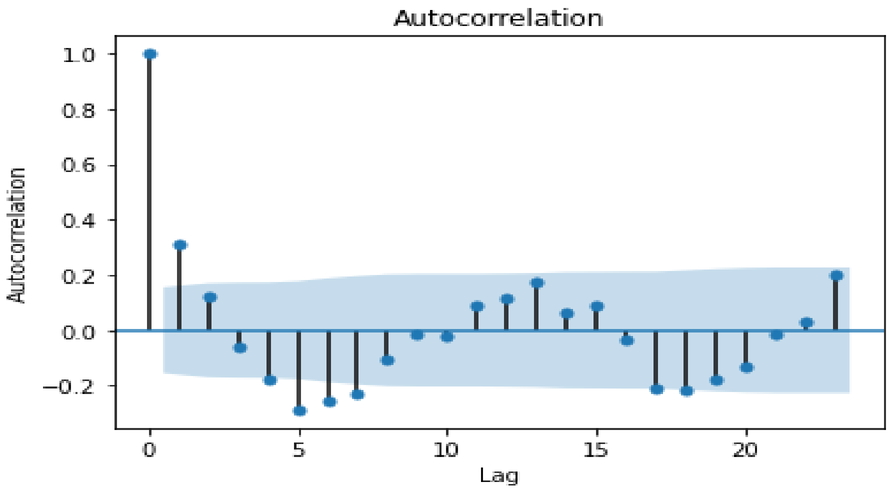

Figure 4.18: First difference autocorrelation (ACF) plot to determine q for ISEM market

The AIC technique [56] identifies the parameter order terms and the parameter ranges are *p* = (0,10) and *q* = (0,10). The best order terms with the lowest AIC value (28090.44) are *p* = 8, *d* = 1, *q* = 8. The ARIMA(8, 1, 8) model function for the predicted Y_t is taken from the weighted terms and, with a RMSE value of 14.86, is given as:

$$Y_t = -0.61\nabla Y_{t-1} - 0.51\nabla Y_{t-2} - 0.29\nabla Y_{t-3} + 0.056\nabla Y_{t-4} - 0.18\nabla Y_{t-5} + 0.33\nabla Y_{t-6} + 0.51\nabla Y_{t-7} + 0.19\nabla Y_{t-8} + 0.66\nabla \varepsilon_{t-1} + 0.46\nabla \varepsilon_{t-2} + 0.13\nabla \varepsilon_{t-3} - 0.24\nabla \varepsilon_{t-4} - 0.13\nabla \varepsilon_{t-5} - 0.63\nabla \varepsilon_{t-6} - 0.80\nabla \varepsilon_{t-7} - 0.43\nabla \varepsilon_{t-8} + 32.32 \quad (4.4)$$

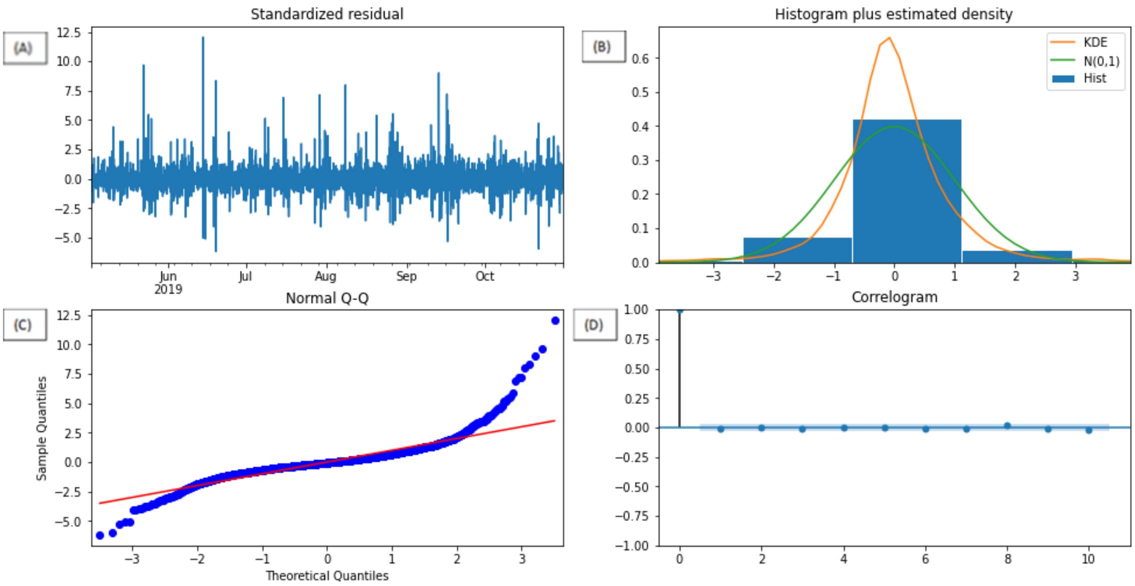

Figure 4.19: Residual diagnostic checks for ISEM market ARIMA(8,1,8)

Figure 4.19 displays the residual diagnostic plots to confirm if ARIMA(8, 1, 8) is a suitable model fit. In Figure 4.19(A) the residuals fluctuate around 0 displaying a trend, in Figure 4.19(B) the density is normally distributed with a narrow bell-shaped pattern, in Figure 4.19(C) the majority of the quantiles fall on or around the red line but are curved at both ends due to outliers, and in Figure 4.19(D) the residual errors are close to zero with no autocorrelation present. Therefore the ARIMA(8, 1, 8) model is a reasonable approximation fit as the residuals are generally uncorrelated and mostly follow a normal distribution.

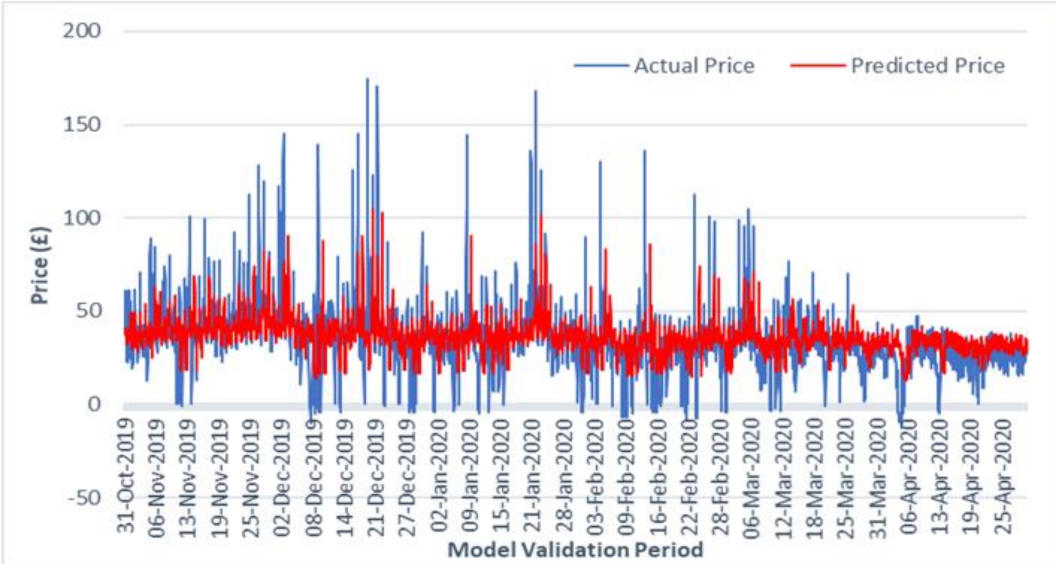

Figure 4.20: ISEM market ARIMA(8,1,8) model

Table 4.2 presents the RMSE values corresponding to the best ARMA and ARIMA models for both the BETTA and ISEM markets. The ARIMA models for both markets had slightly lower RMSE values and better model accuracies compared with the ARMA models. Since the peaks for electricity prices occur at regular intervals this could be indicative of seasonality and therefore it would be worthwhile to account for this in the statistical model.

Table 4.2: RMSE values (ARMA & ARIMA models)

Market	Model	RMSE
BETTA	ARMA(9, 7)	10.91
	ARIMA(9, 1, 7)	9.94
ISEM	ARMA(9, 8)	14.99
	ARIMA(8, 1, 8)	14.86

4.4 SARIMA Experiment

Seasonal variations in model performance are considered with standard time-series models to determine if model accuracy can be improved. Following on from Section 3.2.3, a SARIMA model was utilised to investigate the impact of trend and seasonality. Figure 4.21 shows a large amount of seasonality during May 2017 and this short-term cycle is repeated over and over throughout the entire time period. In this experiment, the first stage of the modelling process (identification) includes the order terms: *p*, *d*, *q*, *P*, *D*, *Q*, and *S*, to determine an appropriate SARIMA model. To manage the non-stationarity of the electricity prices, differencing was applied to the data between current and previous prices to make the series trend stationary; *d* is set to 1. Due to heterogeneity in the data it was decided not to include seasonal differencing in the SARIMA model; *D* is set to 0. For data to be considered to have seasonality, the seasonal pattern must repeat itself over a time span *S* [56]. As electricity prices are recorded hourly, the seasonality length for each day is a daily 24-hour recurring cycle; *S* is set to 24.

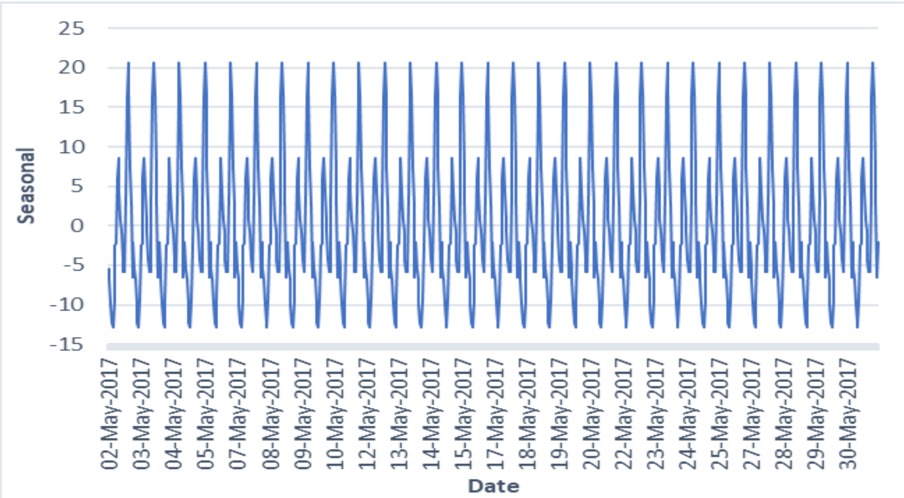

Figure 4.21: BETTA market seasonality for May 2017

Model identification determines the optimal *p*, *q*, *P*, and *Q* order terms required for estimation. Examining the PACF plot (Figure 4.22), the last significant autoregressive lag for *p* ranges between 2 and 5. By observing the trend in Figure 4.22, the significant lag for seasonal order *P* can also be determined. The PACF plot indicates a slight seasonal pattern, therefore *P* ranges between 1 and 2. Examining the ACF plot (Figure 4.23), the last significant moving average lag for *q* ranges between 2 and 7. Figure 4.23 also displays a seasonal pattern, therefore *Q* ranges between 1 and 2.

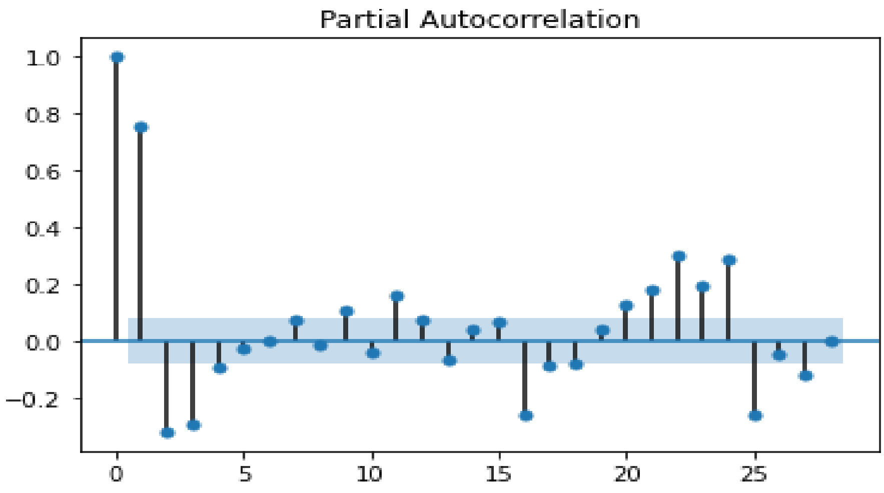

Figure 4.22: Seasonal partial autocorrelation (PACF) plot to determine p for BETTA market

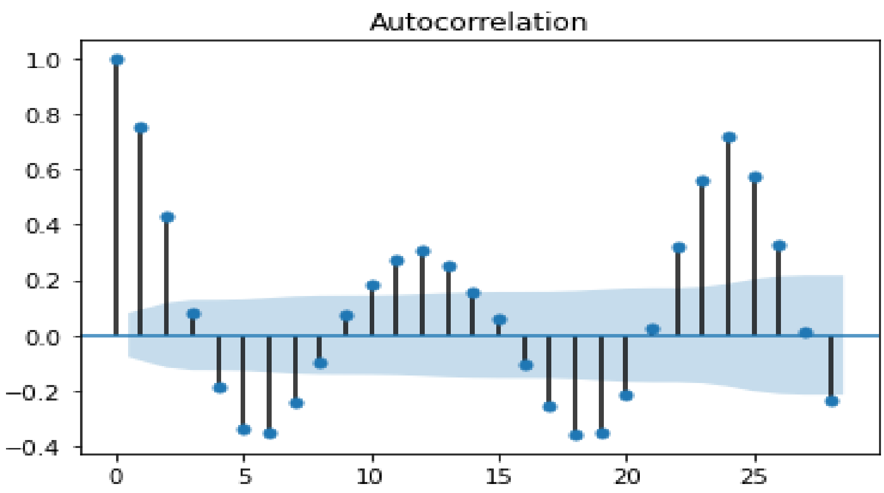

Figure 4.23: Seasonal autocorrelation (ACF) plot to determine q for BETTA market

The AIC method [56] selects the best SARIMA model by determining the optimal p, q, P, and Q order terms required for estimation. For the SARIMA models the parameter ranges are p = (2, 5), q = (2, 7), P = (1, 3), and Q = (1, 3) and the AIC measure is used to determine the best overall SARIMA model. The final model is SARIMA p = 3, d = 1, q = 2, P = 2, D = 0, Q = 2, S = 24 with the lowest AIC value of 27885.83 and RMSE of 9.67 given as:

$$Y_t = -0.17\nabla Y_{t-1} + 0.66\nabla Y_{t-2} - 0.098\nabla Y_{t-3} - 0.029\nabla\varepsilon_{t-1} - 0.97\nabla\varepsilon_{t-2} + 0.88S^{24}Y_{t-24} - 0.18S^{48}Y_{t-48} - 1.62S^{24}\varepsilon_{t-24} + 0.69S^{48}\varepsilon_{t-48} + 34.27 \quad (4.5)$$

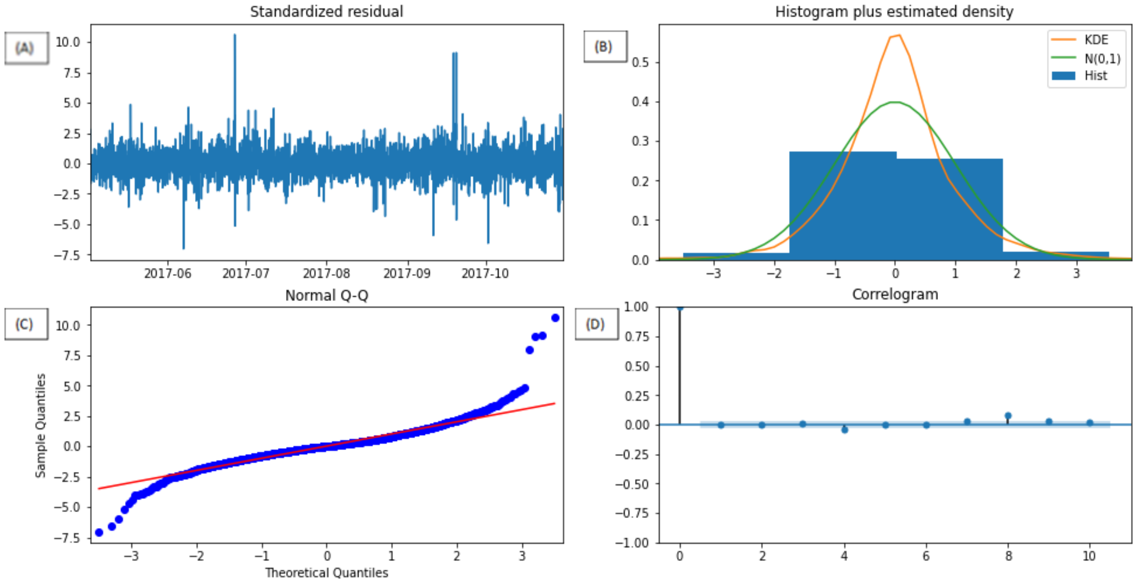

Figure 4.24: Residual diagnostic checks for BETTA market SARIMA(3, 1, 2)(2, 0, 2, 24)

To confirm if this is an appropriate SARIMA model, diagnostic checking on the fitted residuals is performed. Figure 4.24 displays the respective diagnostic plot, and it was noted that the residuals are generally uncorrelated and follow a normality plot pattern. In Figure 4.24(A) the residuals fluctuate around 0 however there is a trend pattern, in Figure 4.24(B) the density is normally distributed with a narrow bell-shaped pattern symmetrical around 0, in Figure 4.24(C) the quantiles mainly fall on or around the red line suggesting a normal distribution but are slightly curved at the ends highlighting extreme data values, and in Figure 4.24(D) the residual errors are mostly close to zero but there is some autocorrelation present as Lag 4 and Lag 8 fall outside the blue boundary. Therefore the SARIMA(3, 1, 2) (2, 0, 2, 24) model is an accurate prediction approximation to forecast electricity prices as the fitted residuals resemble a normality plot. Figure 4.25 illustrates the model fit during the validation stage and the majority of the predicted prices very closely match the actual prices. Therefore, the SARIMA(3, 1, 2)(2, 0, 2, 24) model could be considered an option for forecasting day-ahead electricity price in the BETTA market.

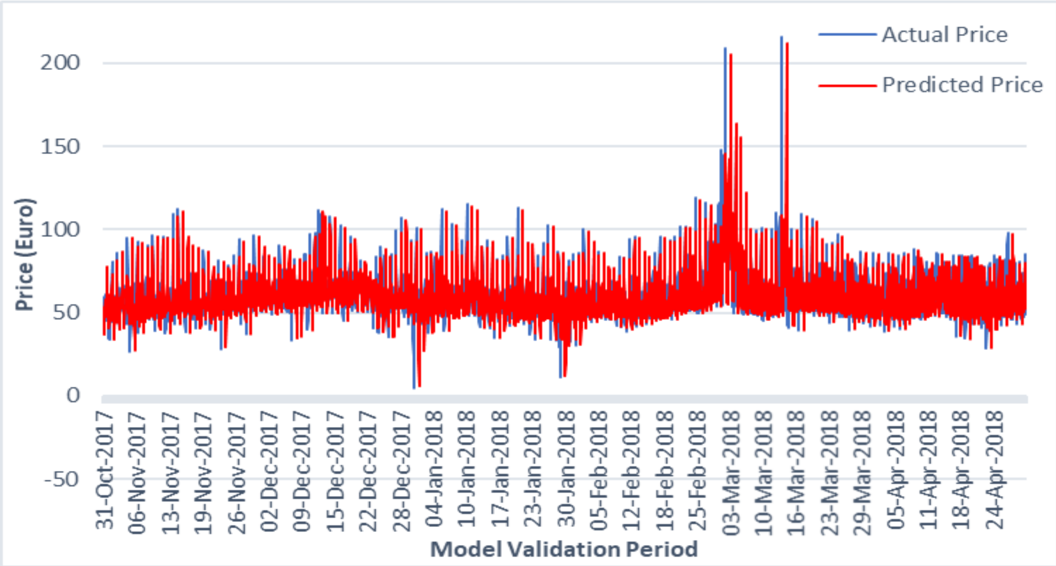

Figure 4.25: BETTA market SARIMA(3, 1, 2)(2, 0, 2, 24) model

The same SARIMA approach was applied to the ISEM market data and Figure 4.26 displays seasonality repeating the seasonal cycle during May 2019 and this continues throughout the entire time period. In this experiment, *d* is set to 1, *D* is set to 0, and to factor seasonality length for each day, *S* is set to 24.

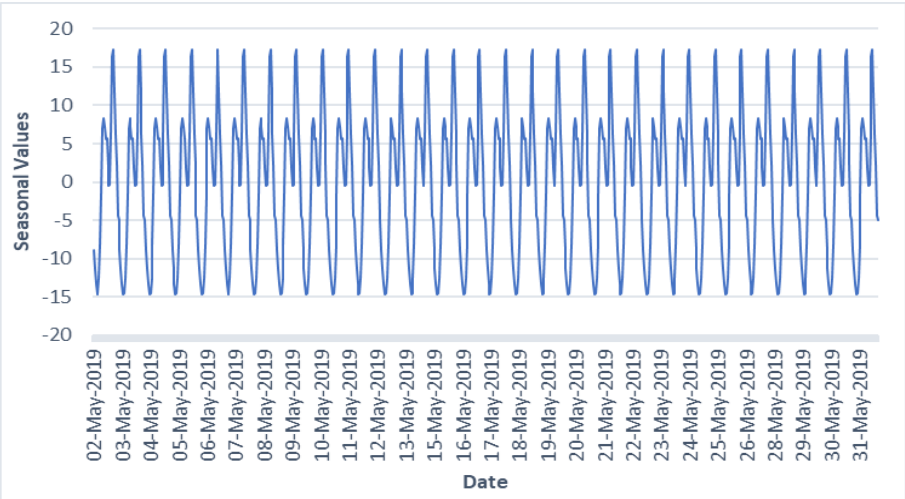

Figure 4.26: ISEM market seasonality for May 2019

Model identification confirms the parameter order terms by observing both Figure 4.27 and Figure 4.28 to find the ranges of the significant lags. Observing Figure 4.27 (PACF plot), the last significant autoregressive lag for *p* ranges between 1 and 4 and the significant lag for seasonal order *P* ranges between 1 and 2. Observing Figure 4.28 (ACF plot), the last significant moving average lag for *q* ranges between 1 and 4 and the significant lag for seasonal order *Q* ranges between 1 and 2.

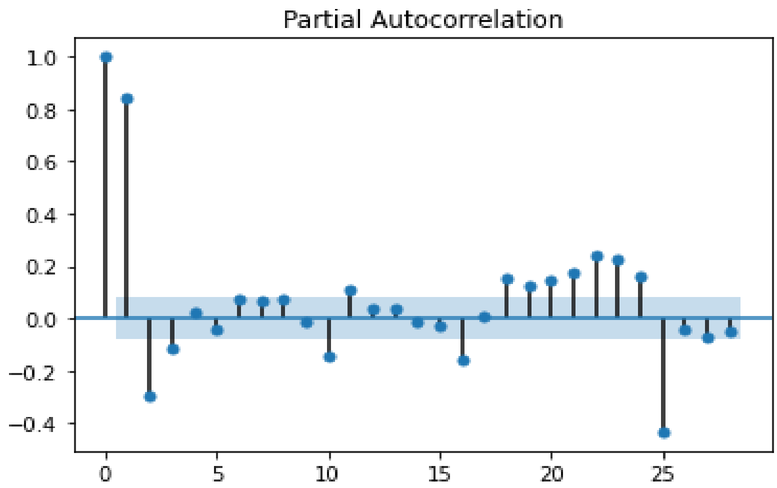

Figure 4.27: Seasonal partial autocorrelation (PACF) plot to determine p for ISEM market

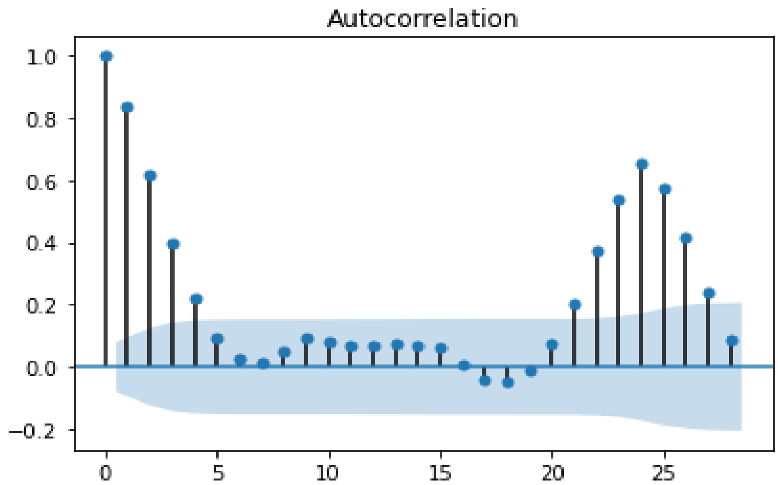

Figure 4.28: Seasonal autocorrelation (ACF) plot to determine q for ISEM market

The AIC technique [56] selects the best SARIMA model by determining the optimal *p*, *q*, *P*, and *Q* order terms. The parameter ranges are *p* = (1, 4), *q* = (1, 4), *P* = (1, 3), and *Q* = (1, 3) and the final SARIMA model is determined to be SARIMA *p* = 3, *d* = 1, *q* = 3, *P* = 2, *D* = 0, *Q* = 2, *S* = 24. The SARIMA model function for predicted Y_t with the lowest AIC value of 27002.82 and RMSE of 14.12 is:

$$Y_t = -0.20\nabla Y_{t-1} + 0.32\nabla Y_{t-2} + 0.58\nabla Y_{t-3} + 0.19\nabla \varepsilon_{t-1} - 0.45\nabla \varepsilon_{t-2} - 0.74\nabla \varepsilon_{t-3} + 0.11S^{24}Y_{t-24} + 0.89S^{48}Y_{t-48} - 0.085S^{24}\varepsilon_{t-24} - 0.85S^{48}\varepsilon_{t-48} + 27.38 \quad (4.6)$$

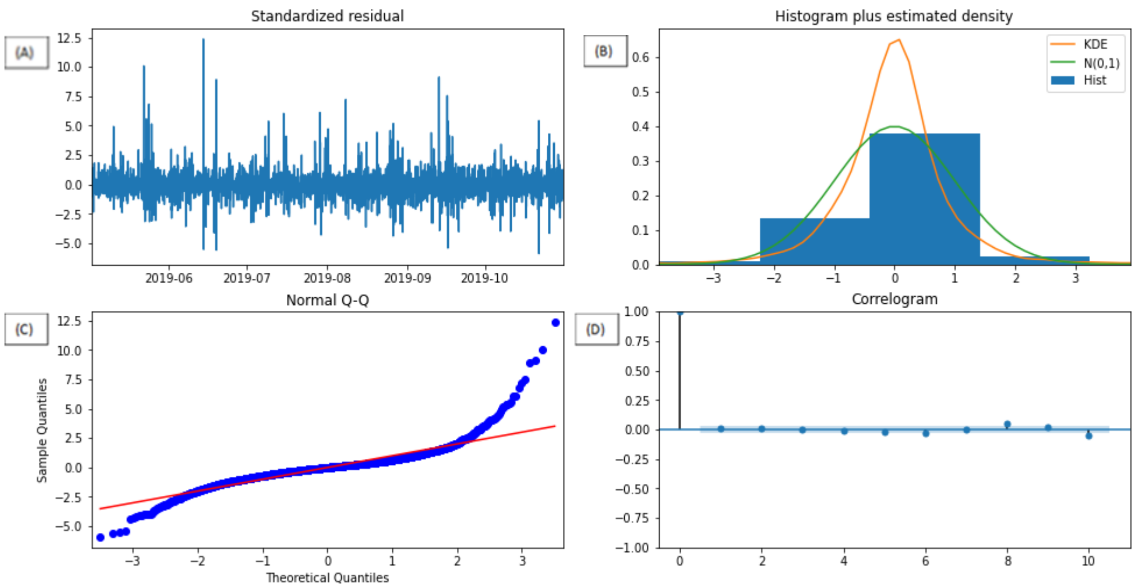

Figure 4.29: Residual diagnostic checks for ISEM market SARIMA(3, 1, 3)(2, 0, 2, 24)

Figure 4.29 displays the residual diagnostic plots. In Figure 4.29(A) the residuals fluctuate around 0 with many peaks displaying a trend, in Figure 4.29(B) the density is normally distributed with a narrow bell-shaped pattern (orange line), in Figure 4.29(C) the quantiles mostly fall on or around the red line but are slightly curved at both ends due to outliers highlighting weak stationarity, and in Figure 4.29(D) the residual errors are close to zero but some autocorrelation is present as Lag 8 and Lag 10 appear outside the blue boundary. Therefore the SARIMA(3, 1, 3)(2, 0, 2, 24) model is a reasonable prediction approximation to forecast electricity prices as the fitted residuals are generally uncorrelated and mostly follow a normality plot pattern. Figure 4.30 displays the model validation period from the selected model and the majority of the predicted prices generally match the mid actual prices, but still struggle to reach the high or low actual price values.

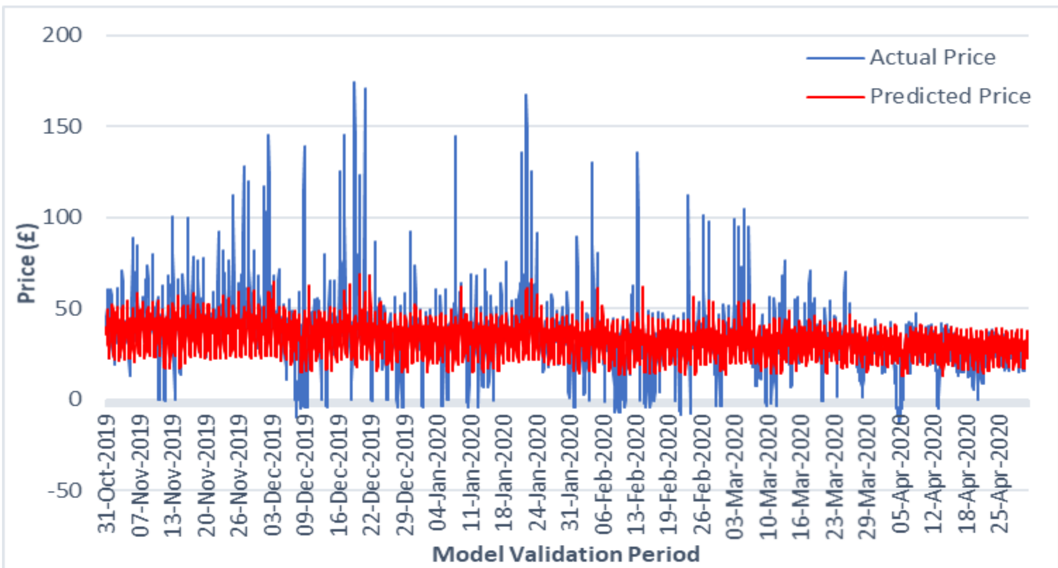

Figure 4.30: ISEM market SARIMA(3, 1, 3)(2, 0, 2, 24) model

Table 4.3 presents the RMSE values of the best ARMA, ARIMA, and SARIMA models for both the BETTA and ISEM markets. The SARIMA models for both markets had the lowest RMSE values improving model performance and accurately predicting day-ahead electricity price. Comparing the two markets, the BETTA market is slightly easier to predict. The experiments were conducted using an Intel Pentium Quad Core Processor N4200, and the run-times for the ISEM models to output the AIC values for each of the listed ranges are included in Table 4.4. The simple ARMA model is the quickest to compute all the permutations with the more advanced models taking a longer run time.

Table 4.3: RMSE values (ARMA, ARIMA, & SARIMA models)

Market	Model	RMSE
BETTA	ARMA(9, 7)	10.91
	ARIMA(9, 1, 7)	9.94
	SARIMA(3, 1, 2)(2, 0, 2, 24)	**9.67**
ISEM	ARMA(9, 8)	14.99
	ARIMA(8, 1, 8)	14.86
	SARIMA(3, 1, 3)(2, 0, 2, 24)	**14.12**

Table 4.4: Run times to fit the optimal ISEM SISO models

Model	Range	Run Time
ARMA(9, 8)	p = (0, 10), q = (0, 10)	3,119 seconds
ARIMA(8, 1, 8)	p = (0, 10), q = (0, 10)	11,388 seconds
SARIMA(3, 1, 3)(2, 0, 2, 24)	p = (1, 4), q = (1, 4), P = (1, 3), Q = (1, 3)	24,714 seconds

4.5 SISO Modelling with Click Energy Data

Electricity price data were provided by Click Energy for the ISEM market 2020/2021. The target day-ahead price being predicted ranges from 02nd December 2020 to 30th June 2021, with all previous hour prices used as the input ranging from 01st December 2020 to 29th June 2021. The data records were split for model estimation (02nd December to 31st May) and model validation (01st June to 30th June). For each of the statistical models, the parameter order terms are found through model identification.

Table 4.5 presents the RMSE values for the ARMA, ARIMA and SARIMA models for the ISEM market 2020/2021. Comparing these findings to Table 4.3, the results are robust as they remain consistent for the two different time periods with SARIMA performing the best. It is also noted that 2020/2021 data performs better with a higher model accuracy (lower RMSE values). Figures 4.31, 4.32 and 4.33 display the ARMA(8, 5) model, ARIMA(9, 1, 9) model and SARIMA(2, 1, 5)(1, 0, 1, 24) model validation results respectively. These figures illustrate that the predicted prices, especially for SARIMA, generally match the actual prices for the month of June.

Table 4.5: RMSE values (ISEM market 2020/2021 SISO models)

Model	RMSE
ARMA(8, 5)	14.89
ARIMA(9, 1, 9)	14.74
SARIMA(2, 1, 5)(1, 0, 1, 24)	**13.76**

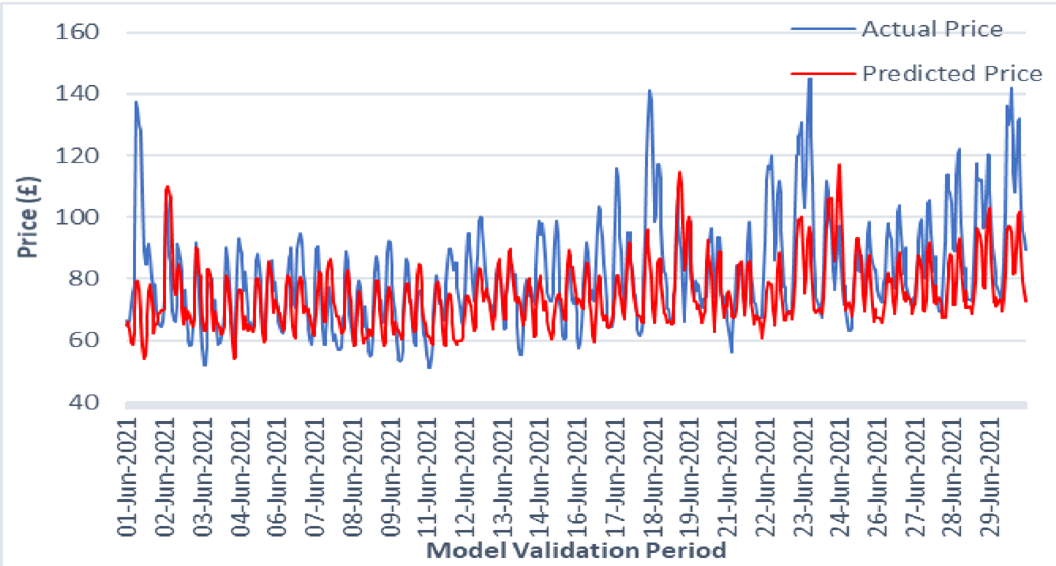

Figure 4.31: ISEM market 2020/2021 ARMA(8, 5) model

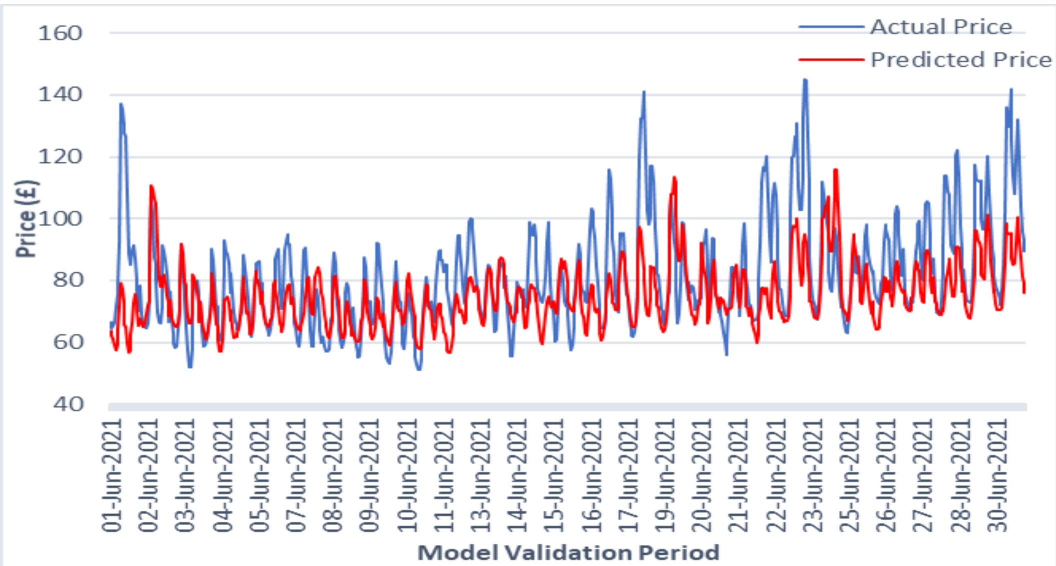

Figure 4.32: ISEM market 2020/2021 ARIMA(9, 1, 9) model

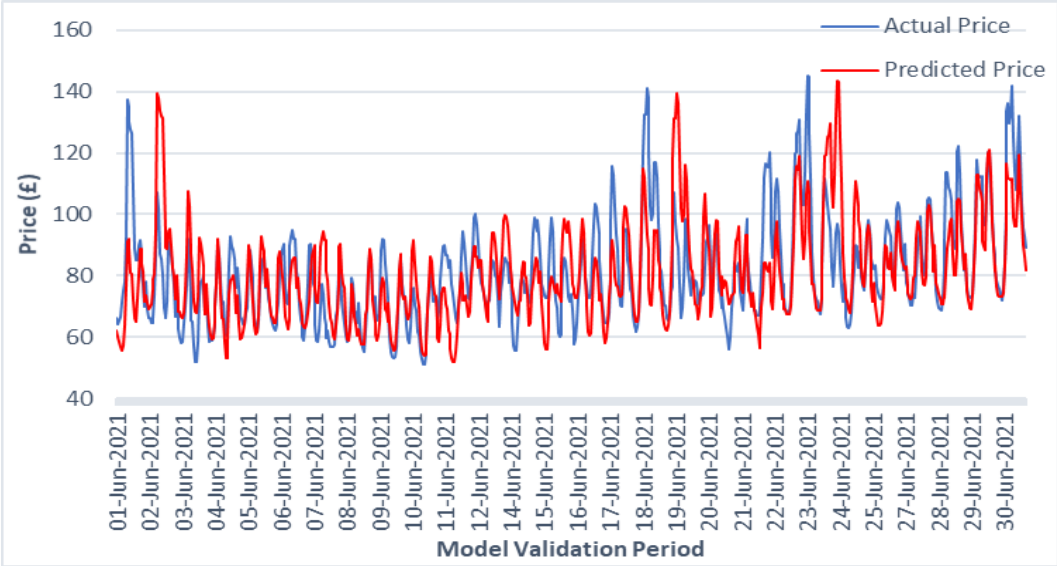

Figure 4.33: ISEM market 2020/2021 SARIMA(2, 1, 5)(1, 0, 1, 24) model

4.6 Conclusion

SISO statistical prediction models follow historical values and price market trends to predict future values. In this chapter, various time-series prediction models using only historical prices as inputs were interpreted and analysed using both BETTA and ISEM market data to determine if they could accurately forecast day-ahead electricity prices. During the investigation of the SISO models, stationarity, trend, and seasonality were all considered when identifying order terms, estimating parameters, and model validation. This type of analysis and comparison for each model helps to determine which trend, order term, or seasonal input improves prediction accuracy the most.

Firstly the ARMA model was chosen and the experimental results show that when using BETTA market data, the best model is ARMA(9, 7) and when using ISEM market data, the best model is ARMA(9, 8). The corresponding RMSE values were promising for the ARMA models, however since electricity prices have long term trends which fluctuate up and down it was decided to try ARIMA to determine if model accuracy could be improved. The experimental results show that with BETTA market data, the best model is ARIMA(9, 1, 7) and with ISEM market data, the best model is ARIMA(8, 1, 8). ARIMA performed generally well with the corresponding RMSE values slightly lower than the ARMA models. To account for seasonality, a SARIMA model was utilised in future experiments to try to further improve model performance. The model generated a RMSE value of 9.67 for SARIMA(3, 1, 2)(2, 0, 2, 24) when using the BETTA market data and a RMSE value of 14.12 for SARIMA(3, 1, 3)(2, 0, 2, 24) when using the ISEM market data. These experimental results found SARIMA models to be the best for predicting day-ahead electricity price and that the BETTA market was the easiest to forecast for each individual SISO model.

Model performance remained consistent for both 2019/2020 and 2020/2021 data highlighting the robustness of the statistical models.

This chapter addressed the four key statistical modelling stages: identification, estimation, diagnostic testing, and forecasting. Although performance was sufficient, there is still room for improvements and hence the next chapter will focus on **M**ultiple **I**nput **S**ingle **O**utput (MISO) models to investigate if external energy-related factors improve model performance further and to determine, in particular, which exogenous factors have the most impact on day-ahead electricity price.

Chapter 5

Multiple Input Single Output Models

5.1 Introduction

Multiple **I**nput **S**ingle **O**utput (MISO) forecasting models explore the relationship between multiple inputs and one output in order to predict values. This research follows on from Chapter 4 by continuing to understand and analyse real electricity price data with traditional time-series statistical models, however this chapter includes external energy-related input data. As well as statistical multivariate models, non-linear learning models are also examined to discover key energy input factors that influence day-ahead electricity price prediction. These factors are identified through prediction modelling using short-term regression techniques and by examining and developing **N**onlinear **A**uto**R**egressive **M**oving **A**verage models with e**X**ogenous inputs (NARMAX). This chapter addresses the second research question listed in Chapter 1: "Do energy-related exogenous variables improve model performance?". The aim of this chapter is to investigate MISO models and establish whether model performance is enhanced with the inclusion of factors which are known to influence energy price. This research work resulted in the following publications:

- C. McHugh, S. Coleman, D. Kerr, and D. McGlynn, "Daily Energy Price Forecasting Using a Polynomial NARMAX Model," in Advances in Computational Intelligence Systems, UKCI 2018, pp. 71–82.
- C. McHugh, S. Coleman, D. Kerr, and D. McGlynn, "A Linear Polynomial NARMAX Model with Multiple Factors to Forecast Day-Ahead Electricity Prices," in Proceedings of the 2018 IEEE Symposium Series on Computational Intelligence, SSCI 2018, pp. 2125–2130.
- C. McHugh, S. Coleman, and D. Kerr, "Hourly Electricity Price Forecasting with NARMAX", submitted to Elsevier Statistical Methods for Machine Learning with Applications (under review).

Throughout this chapter, various MISO time-series models are separately analysed for two energy markets to evaluate model performance and forecast day-ahead prices. The two energy markets remain the same as previously explored in the **S**ingle **I**nput **S**ingle **O**utput (SISO) models: **B**ritish **E**lectricity **T**rading and **T**ransmission **A**rrangements (BETTA) and the Irish **I**ntegrated **S**ingle **E**lectricity **M**arket (ISEM). To compare performance results between the SISO and MISO models, the forecasting periods examined remain as May 2017 until April 2018 for the BETTA market and May 2019 until April 2020 for the ISEM market.

This research begins by analysing an **A**uto**R**egressive **M**oving **A**verage with e**X**ogenous inputs (ARMAX) model, which includes multiple energy-related variables as inputs as well as the historical price data. Adding exogenous variables can help to explain energy price movements as external factors can contribute strongly to price changes. The same range for the order terms *p* and *q* from the ARMA model in Section 4.2 are applied here. Next, an **A**uto**R**egressive **I**ntegrated **M**oving **A**verage with e**X**ogenous inputs (ARIMAX) model is examined to include differencing. The previous chapter demonstrated that seasonal techniques provided the best performance for SISO models improving overall prediction accuracy, therefore a **S**easonal **A**uto**R**egressive **I**ntegrated **M**oving **A**verage with e**X**ogenous input (SARIMAX) model is also considered here. A NARMAX model is analysed to determine which external factors significantly influence electricity price and which are necessary to include in a forecasting model for energy market traders to predict day-ahead prices. Determination of the most significant contributory factors is vital in order to improve day-ahead prediction accuracy. The performance of each of the MISO models in this chapter is evaluated by observing their **R**oot **M**ean **S**quared **E**rror (RMSE) values. The software utilised for the statistical models was Python through the NumPy, Pandas and Statsmodels libraries and for the NARMAX models was ScicosLab.

5.2 Energy-Related Data

It is important in price prediction models to consider external input factors as these can have a significant impact on the accuracy of a forecasting model. As discussed in Chapter 2, electricity generation prices fluctuate when supply and demand vary, making it important to consider these influential factors for electricity price forecasting to establish an accurate prediction tool [50]. The system marginal price (which is the cheapest bid set in the market) is controlled by external factors and therefore it is important to consider such external factors in a predictive model [18]. Energy-related factors which have previously been shown to be significant on influencing electricity prices are: system demand [8]; system load, interconnection contributions (from other markets), and supply generation from power plants [18]; wind generation [111]; solar generation [69]; fuel market prices such as gas and coal [15]; and environmental temperature, which can influence additional consumer demand through heating or hot water usage [75]. Appropriate selection of which of these contributing factors can be used as inputs is necessary for energy traders to know when to buy or sell in the market and over time develop a successful trading system. If energy traders train forecasting models using external factors as input variables, the resulting models, whilst more complex, should become more adaptable to factors that can cause price fluctuations in the energy market.

5.2.1 British Electricity Trading and Transmission Arrangements (BETTA)

For this research, BETTA 2017-2018 market data were downloaded from multiple sources. Historical hourly electricity price records were collected from the Nordpool day-ahead exchange traded auction market [108]. Hourly transmission system average gas prices were retrieved via the data item explorer available through National Grid [112]. The hourly fuel-type generation data taken from Gridwatch [113] which displays data in five-minute period intervals were as follows: demand (overall total not including exports), wind (total contribution from wind farms), solar (estimated power), coal (coal plants), Moyle interconnector (connected from Scotland to Northern Ireland), nuclear (power stations), pumped storage (small hydroelectric storage stations), hydroelectric power (combination of small stations mainly located in Scotland), biomass (renewable power stations), **C**ombined **C**ycle **G**as **T**urbine [CCGT] (boiler and steam turbines), and **O**pen **C**ycle **G**as **T**urbine [OCGT] (gas without steam). Hourly temperature records were gathered from Speedwell [114] for five UK weather stations (Birmingham, Glasgow, London, St. Athan, and Yeovilton) and averaged for each hour to achieve an accurate representation of the hourly temperature over the whole UK energy market. These energy-related factors are displayed alongside their unit of measure in Table 5.1.

Table 5.1: Energy-related factors from BETTA market

Energy-Related Factors	Model Input Terms	Unit
Historical Electricity Price	u1	Euro per Megawatt Hour
Demand	u2	Megawatt
Gas	u3	Pence per Kilowatt Hour
Wind	u4	Megawatt
Solar	u5	Megawatt
Coal	u6	Megawatt
Moyle Interconnector	u7	Megawatt
Nuclear	u8	Megawatt
Pumped Storage	u9	Megawatt
Hydroelectric Power	u10	Megawatt
Biomass	u11	Megawatt
Combined Cycle Gas Turbine [CCGT]	u12	Megawatt
Open Cycle Gas Turbine [OCGT]	u13	Megawatt
Temperature	u14	Celsius

5.2.2 Integrated Single Electricity Market (ISEM)

For this research, ISEM 2019-2020 market data were downloaded from multiple sources. Historical hourly electricity price records were collected from the **S**ingle **E**lectricity **M**arket **O**perator (SEMOpx) [110] day-ahead trading auction market. Half-hourly load forecast generation was retrieved from SEMO [115]. The 15-minute intervals of energy-related data taken from EirGrid smart grid dashboard [116] were as follows: actual demand (predicted electricity production), system generation (total electricity production), forecast wind (total all island wind farms), East-West interconnector (connected from Ireland to Wales), Moyle interconnector (connected from Scotland to Northern Ireland), CO2 intensity (average of CO2 emissions), and CO2 emissions (estimated total of all power stations). Hourly temperature records were gathered (both Northern Ireland [117] and Republic of Ireland [118]) from five all island weather Met Office stations (Belfast, Cork, Donegal, Dublin, and Galway) and averaged for each hour to achieve an accurate representation of the hourly temperature over the whole ISEM energy market. Each of these energy-related factors are displayed alongside their unit of measure in Table 5.2.

Table 5.2: Energy-related factors from ISEM market

Energy-Related Factors	Model Input Terms	Unit
Historical Electricity Price	u1	GBP per Megawatt Hour
System Generation	u2	Megawatt
Demand	u3	Megawatt
Wind	u4	Megawatt
East-West Interconnector	u5	Megawatt
Moyle Interconnector	u6	Megawatt
CO2 intensity	u7	Kilowatt Hour
CO2 emissions	u8	CO2 intensity per Hour
Load	u9	Megawatt
Temperature	u10	Celsius

5.3 ARMAX Experiment

The definition and background of ARMAX models were outlined in Section 3.3.1. The ARMAX experiment using BETTA market data was set up identically to the ARMA BETTA experiment in Section 4.2 with the same forecasting period and same aim of predicting the day-ahead price. A total of 8736 samples for the electricity price and for each of the energy-related exogenous variables was used. All input factors ranged from 01st May 2017 to 29th April 2018 and the target day-ahead prices (+24hours) ranged from 02nd May 2017 to 30th April 2018. For the experiment, the data records were split for model estimation and model validation where 02nd May Hour 0 to 30th October Hour 23 was used for model estimation and 31st October Hour 0 to 30th April Hour 23 was used for model validation.

Using the information from the **A**uto**C**orrelation **F**unction (ACF) and **P**artial **A**uto**C**orrelation **F**unction (PACF) plots in Section 4.2, the ranges for order terms *p* and *q* are set from 0 to 10. The **A**kaike **I**nformation **C**riterion (AIC) verifies the optimal order terms by using a brute force search of all given combinations ranking the AIC values from lowest to highest, following the process outlined in [56]. The lowest AIC value was 28765.10 when order terms were *p*=8 and *q*=0 and therefore these were selected as the best ARMAX order terms. The ARMAX(8, 0) model function for predicted Y_t is given as:

$$Y_t = 0.78Y_{t-1} - 0.12Y_{t-2} + 0.0073Y_{t-3} - 0.056Y_{t-4} + 0.041Y_{t-5} - 0.025Y_{t-6} + 0.042Y_{t-7} + 0.028Y_{t-8} + 0.55U_1 + 2.23^{e-05}U_2 + 3.33U_3 + 4.71^{e-05}U_4 - 1.23^{e-06}U_5 -$$

$$1.79^{e-05}U_6 - 4.19^{e-05}U_7 - 0.0001U_8 + 0.0002U_9 + 0.0003U_{10} + 3.09^{e-05}U_{11} + 4.86^{e-06}U_{12} - 0.0032U_{13} + 0.11U_{14} + 12.78 \quad (5.1)$$

consisting of weighted terms in a linear combination of autoregressive lags for Y and coefficient values for each of the exogenous input variables (U). Gas was the most weighted external variable (3.33), followed by historical electricity price (0.55), and then temperature (0.11).

Table 5.3 contains the exogenous variables corresponding p-value for the z-statistic significance (P>|z|) for the ARMAX(8, 0) model. The level of significance (α=0.05) was chosen based on the 95% confidence interval and is rejected if it falls in the 5% critical region. The p-value significance for each variable was compared against α: if the p-value was ≤ α, the exogenous variable was significant. Observing the p-values for each of the variables in Table 5.3, it can be seen that historical electricity price, gas, pumped storage, hydroelectric power, and OCGT were deemed significant out of the 14 possible inputs.

Table 5.3: BETTA market ARMAX(8,0) model summary statistics

Variable (Model Term)	**p-value**
Historical Electricity Price (**U_1**)	**<0.001**
Demand (**U_2**)	0.423
Gas (**U_3**)	**<0.001**
Wind (**U_4**)	0.110
Solar (**U_5**)	0.489
Coal (**U_6**)	0.650
Moyle Interconnector (**U_7**)	0.972
Nuclear (**U_8**)	0.070
Pumped Storage (**U_9**)	**0.006**
Hydroelectric Power (**U_{10}**)	**0.013**
Biomass (**U_{11}**)	0.612
CCGT (**U_{12}**)	0.865
OCGT (**U_{13}**)	**0.012**
Temperature (**U_{14}**)	0.212

Diagnostic checking is performed on the standardised model residuals to confirm if ARMAX(8, 0) is an appropriate model fit. Figure 5.1(A) plots the standardised residuals which fluctuate around 0, however there is a pattern of peaks and troughs highlighting the varying trend of electricity prices. Figure 5.1(B) is a histogram with a density plot which generally resembles a bell-shaped pattern and comparison of the orange line against the green line suggests the data are normally distributed. Figure 5.1(C) is a normal quantile-quantile plot with the majority of the quantiles falling on or near to the red line suggesting a normal distribution, but the shape is curved on both ends suggesting extreme values among the data. Figure 5.1(D) is a correlogram plot showing that no autocorrelation is present as the residual errors are close to zero. All these plots confirm that the residuals are uncorrelated mostly following a normal pattern and that the model fits reasonably well.

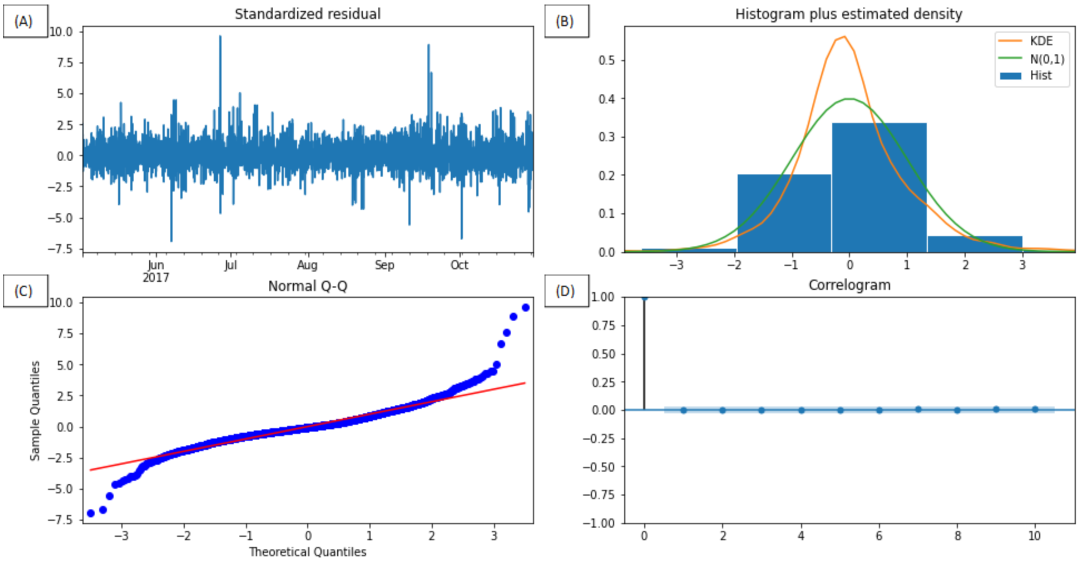

Figure 5.1: Residual diagnostic checks for BETTA market ARMAX(8,0)

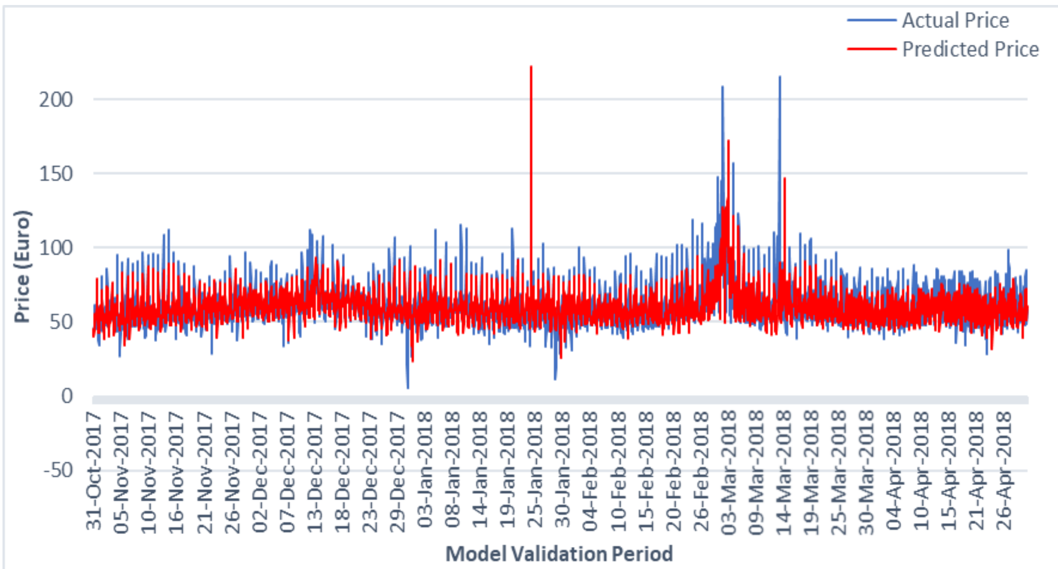

Figure 5.2: BETTA market ARMAX(8,0) model

The selected model fit is used to predict electricity price and compare the model prediction against the historical actual electricity price. The RMSE value measures the accuracy of the model validation (testing stage) with the accuracy improving as RMSE tends towards zero. The RMSE value for ARMAX(8, 0) including all exogenous variables was 9.59 indicating that the model is reasonably accurate. Comparing this value to the RMSE value from the ARMA(9, 7) model presented in Section 4.2, which was 10.91, this demonstrates that the inclusion of exogenous input variables for day-ahead BETTA electricity price forecasting improves the overall model accuracy. The ARMAX(8, 0) model validation results for hourly BETTA electricity prices are presented in Figure 5.2 and illustrate that the predicted price values closely match the actual historical price values but fail to reach the peaks. Given the overall accuracy the ARMAX(8, 0) model could be considered an option for forecasting day-ahead electricity price.

For day-ahead forecasting, since ARMAX could model the BETTA market accurately, the same approach was applied to the ISEM market to determine if ARMAX is robust to the dynamics of different markets. The ARMAX experiment included 8760 records of electricity price and each of the energy-related exogenous variables. All input factors ranged from 01st May 2019 to 29th April 2020 and the target day-ahead price (+24h) ranged from 02nd May 2019 to 30th April 2020. For the experiment, the data records were split 50/50 for model estimation and model validation: 02nd May Hour 0 to 31st October Hour 11 for model estimation and 31st October Hour 12 to 30th April Hour 23 for model validation. The ranges for order terms *p* and *q* were set from 0 to 10 to match the ISEM experiment in Section 4.2. In this experiment, the lowest AIC value (27406.75) for the forecasting period occurred when the order terms were *p*=3 and *q*=9. The model function for ARMAX(3, 9) for predicted Y_t consists of autoregressive lags for Y, moving average lags for prediction error ε, exogenous input values for U and is given as:

$$Y_t = 0.71Y_{t-1} - 0.27Y_{t-2} + 0.41Y_{t-3} + 0.30\varepsilon_{t-1} + 0.47\varepsilon_{t-2} - 0.018\varepsilon_{t-3} + 0.0029\varepsilon_{t-4} - 0.11\varepsilon_{t-5} - 0.057\varepsilon_{t-6} - 0.049\varepsilon_{t-7} - 0.0016\varepsilon_{t-8} + 0.087\varepsilon_{t-9} + 0.34U_1 - 0.0006U_2 + 0.0022U_3 + 0.0002U_4 + 0.0004U_5 - 3.62^{e-05}U_6 - 0.0032U_7 + 0.0006U_8 + 0.0008U_9 - 0.46U_{10} + 3.17 \quad (5.2)$$

Table 5.4: ISEM market ARMAX(3,9) model summary statistics

Variable (Model Term)	**p-value**
Historical Electricity Price (**U_1**)	**<0.001**
System Generation (**U_2**)	**0.004**
Demand (**U_3**)	**<0.001**
Wind (**U_4**)	0.452
East-West Interconnector (**U_5**)	0.117
Moyle Interconnector (**U_6**)	0.916
CO2 Intensity (**U_7**)	**0.044**
CO2 Emissions (**U_8**)	0.179
Load (**U_9**)	**0.027**
Temperature (**U_{10}**)	**0.006**

Temperature was the most weighted external variable (-0.46) followed by historical electricity price (0.34). The exogenous variables p-values for the ARMAX(3, 9) model are presented in Table 5.4. Observing these it can be seen that historical electricity price, system generation, demand, CO2 intensity, load, and temperature were all highly significant. Similar to the BETTA market results, historical electricity price is extremely dominant. The significant factors for the BETTA market were mainly from generating power and hydroelectric stations as well as gas prices and gas turbines. The ISEM market takes advantage of the full range of generation types rather than only focussing on power plants [35]. Therefore the ISEM ARMAX summary statistics are more varied with multiple significant factors identified. These factors are inherently linked together through supply and demand in electricity production (system generation, demand, and load). These findings are consistent with previous literature as Li et al. [18] discussed that energy production and demand are among the key energy forecasting factors.

Figure 5.3 displays the fitted residual diagnostic plots to check how well ARMAX(3, 9) has performed. In Figure 5.3(A) the standardised residuals fluctuate around 0, however there are many peaks and troughs throughout the period. In Figure 5.3(B) the density plot (orange line) has a narrow bell-shaped pattern indicating that the data are normally distributed. In Figure 5.3(C) the majority of the quantiles are close to the red line suggesting a normal distribution, but due to extreme outliers the blue line has sharp curves at both ends. Figure 5.3(D) shows that the residual errors are close to zero and thus no autocorrelation is present. All these plots confirm that the residuals are uncorrelated and that the ISEM ARMAX(3, 9) model is a reasonable approximation fit.

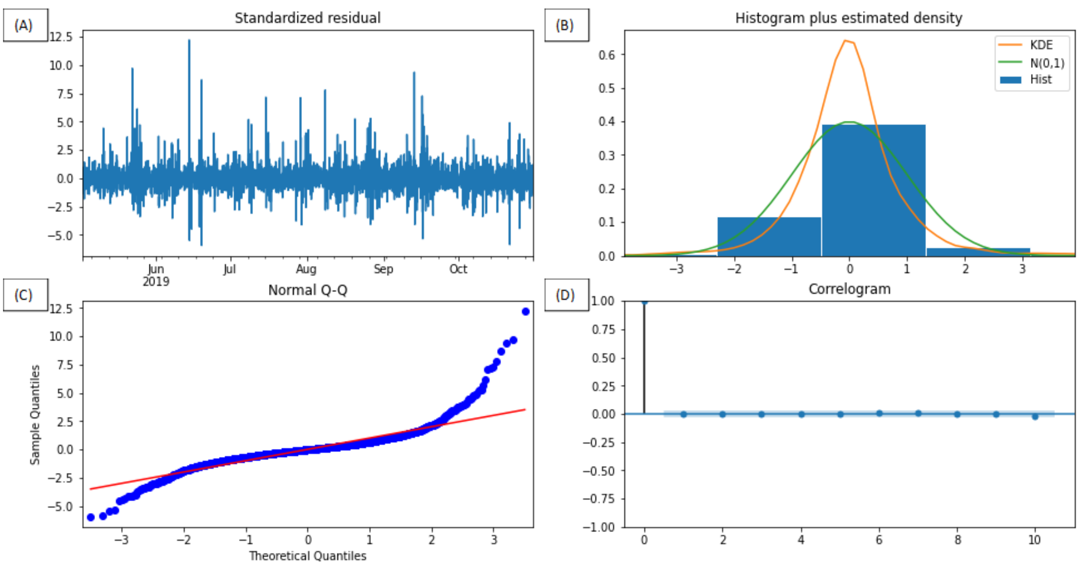

Figure 5.3: Residual diagnostic checks for ISEM market ARMAX(3,9)

To check the performance of the ARMAX(3, 9) model during the model validation stage, the predicted price values were plotted against the historical actual values, shown in Figure 5.4. It is clear from Figure 5.4 that for the majority of records the predicted values closely matched the actual values. The RMSE value for ARMAX(3, 9) was 18.73 and this highlighted that the overall model performance of ISEM day-ahead forecasting was less accurate than for the BETTA market. Comparing this value to the RMSE value from the ARMA(9, 8) model displayed in Section 4.2, which was 14.99, this indicates that the ISEM day-ahead market is slightly more difficult to forecast with the inclusion of exogenous input variables.

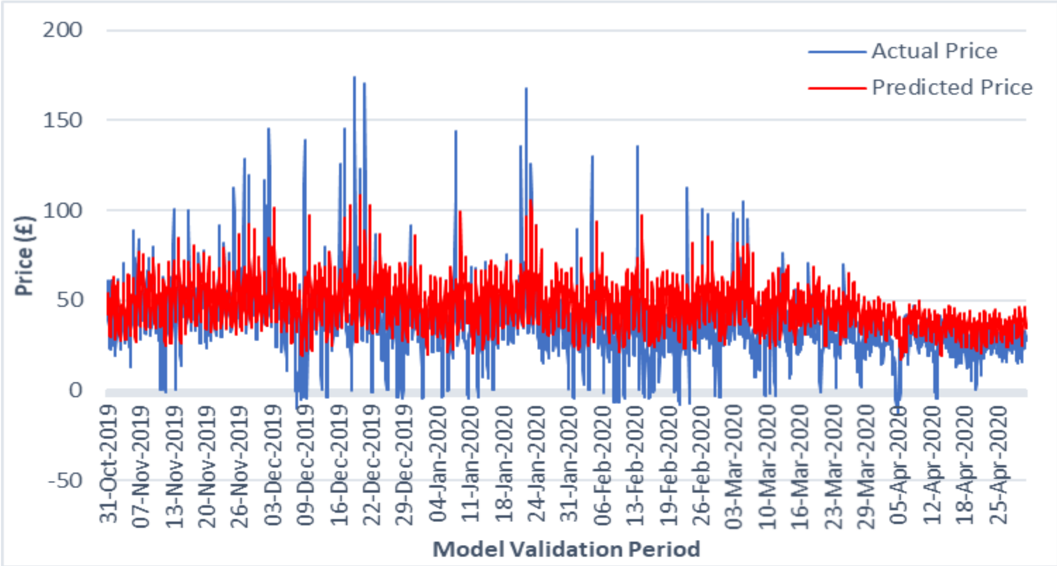

Figure 5.4: ISEM market ARMAX(3,9) model

Table 5.5 presents the RMSE values of the best ARMAX model for both the BETTA and ISEM markets. Overall, comparing the two markets with the inclusion of exogenous input variables, the BETTA market results in a lower RMSE value and better model accuracy than the ISEM market, hence the need to further investigate possible models for the ISEM market in particular.

Table 5.5: RMSE values (ARMAX models)

Market	**Model**	**RMSE**
BETTA	ARMAX(8, 0)	9.59
ISEM	ARMAX(3, 9)	18.73

5.4 ARIMAX Experiment

ARIMAX models were previously discussed in Section 3.3.2 and this experiment is conducted to determine if removing trends through first-order differencing improves model performance for both the BETTA and ISEM markets. The parameter ranges for both order terms *p* and *q* were 0 to 10 and *d* was set to 1. The order terms *p*=8, *d*=1, and *q*=2 provided the lowest AIC value (30705.84) and the ARIMAX(8, 1, 2) model function for predicted Y_t is given as:

$$Y_t = -0.030\nabla Y_{t-1} - 0.072\nabla Y_{t-2} - 0.18\nabla Y_{t-3} - 0.20\nabla Y_{t-4} - 0.13\nabla Y_{t-5} - 0.14\nabla Y_{t-6} - 0.10\nabla Y_{t-7} - 0.071\nabla Y_{t-8} - 0.12\nabla \varepsilon_{t-1} - 0.17\nabla \varepsilon_{t-2} + 0.51U_1 + 4.17^{e-05}U_2 + 0.81U_3 + 7.68^{e-05}U_4 - 1.32^{e-06}U_5 - 4.87^{e-06}U_6 + 4.94^{e-06}U_7 - 0.0003U_8 + 7.81^{e-06}U_9 + 0.0006U_{10} + 0.0001U_{11} - 2.00^{e-06}U_{12} - 0.0019U_{13} - 0.078U_{14} + 44.20 \quad (5.3)$$

Table 5.6: BETTA market ARIMAX(8,1,2) model summary statistics

Variable (Model Term)	**p-value**
Historical Electricity Price (**U_1**)	**<0.001**
Demand (**U_2**)	0.144
Gas (**U_3**)	0.277
Wind (**U_4**)	**0.029**
Solar (**U_5**)	0.610
Coal (**U_6**)	0.911
Moyle Interconnector (**U_7**)	0.966
Nuclear (**U_8**)	**0.023**
Pumped Storage (**U_9**)	0.182
Hydroelectric Power (**U_{10}**)	**<0.001**
Biomass (**U_{11}**)	0.172
CCGT (**U_{12}**)	0.947
OCGT (**U_{13}**)	0.241
Temperature (**U_{14}**)	0.515

The most weighted variable was gas (0.81) and the second most weighted was historical electricity price (0.51), similar to the findings for ARMAX(8, 0) model (Section 5.3). The individual p-values are displayed in Table 5.6 for the ARIMAX(8, 1, 2) model. Observing the p-values, 4 of the 14 inputs were significant: historical electricity price, wind, nuclear, and hydroelectric power. Comparing these findings to the ARMAX(8, 0) model, two input factors were consistently significant in both model findings: historical electricity price and hydroelectric power.

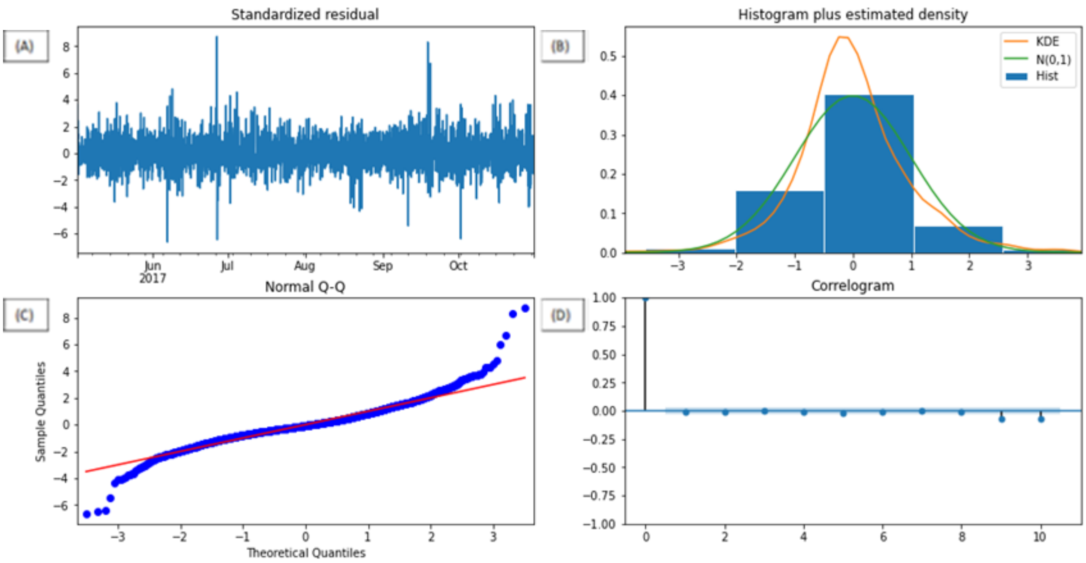

Figure 5.5: Residual diagnostic checks for BETTA market ARIMAX(8,1,2)

Figure 5.5(A) shows the standardised residuals fluctuate around 0, however the width of the pattern highlights a trend in the data. Figure 5.5(B) is a histogram with a density plot that resembles a narrow bell-shaped pattern, observing the orange line, which indicates that the data are normally distributed and symmetrical around mean 0. Figure 5.5(C) is a normal quantile-quantile plot and illustrated that the majority of the quantiles are on or close to the red line, but the shape is curved on both ends highlighting extreme values among the data. Figure

5.5(D), the correlogram plot, shows slight autocorrelation is present in Lags 9 and 10 but the majority of the residual errors are close to zero and within the blue boundary. Figure 5.5 confirms that the residuals generally follow a normal pattern displaying a reasonable model fit.

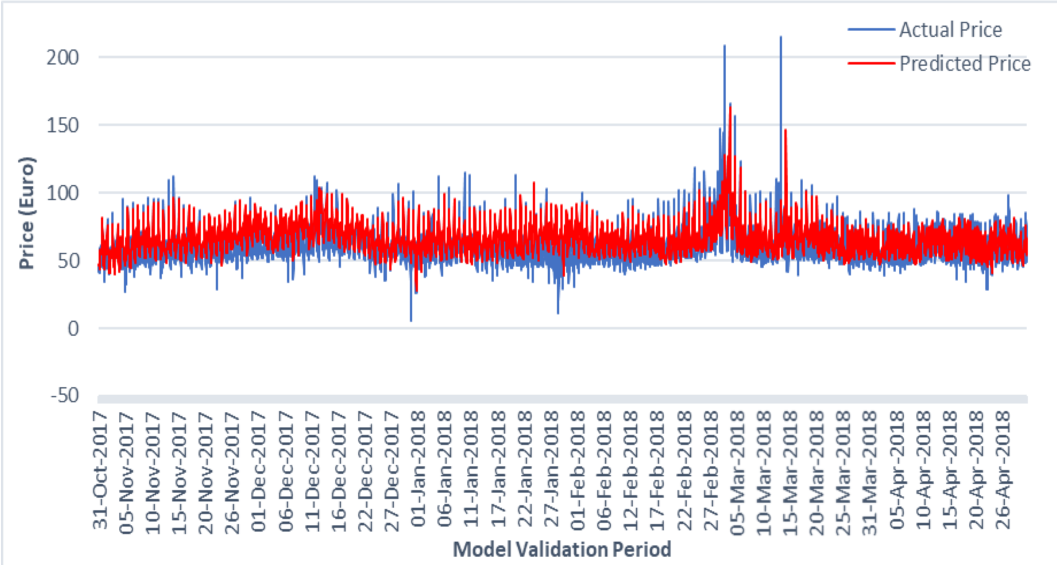

Figure 5.6: BETTA market ARIMAX(8,1,2) model

The RMSE value for ARIMAX(8, 1, 2) was 11.09. The RMSE is slightly higher than the ARIMA(9, 1, 7) model in Section 4.3, which produced a RMSE value of 9.94, suggesting that the ARIMAX model is not as accurate at predicting BETTA day-ahead electricity prices compared to the ARIMA model. The BETTA ARIMAX model prediction against actual fit is illustrated in Figure 5.6 which shows that the predicted prices are similar to the actual prices.

The same ARIMAX technique was used with the ISEM market data, where the *p* and *q* order terms ranged from 0 to 10 and *d* was set to 1. The best ARIMAX order terms are *p*=1, *d*=1, and *q*=9 outputting the lowest AIC value of 28412.94. The ARIMAX(1, 1, 9) model function for predicted Y_t is given as:

$$Y_t = 0.14\nabla Y_{t-1} - 0.12\nabla\varepsilon_{t-1} - 0.071\nabla\varepsilon_{t-2} - 0.12\nabla\varepsilon_{t-3} - 0.020\nabla\varepsilon_{t-4} - 0.17\nabla\varepsilon_{t-5} - 0.084\nabla\varepsilon_{t-6} - 0.032\nabla\varepsilon_{t-7} - 0.024\nabla\varepsilon_{t-8} + 0.023\nabla\varepsilon_{t-9} + 0.32U_1 - 0.0005U_2 + 0.0022U_3 + 0.0009U_4 + 0.0004U_5 - 9.75^{e-06}U_6 - 0.0025U_7 + 0.0005U_8 + 0.008U_9 - 0.84U_{10} + 31.38 \quad (5.4)$$

The most weighted variable was temperature (-0.84) followed by historical electricity price as the second most weighted variable (0.32), which is the same as the ARMAX(3, 9) model findings. The ARIMAX(1, 1, 9) model summary p-values are presented in Table 5.7. Observing the p-values, 5 of the 10 energy-related inputs were significant: historical electricity price, demand, wind, load, and temperature. Similar to the ARMAX(3, 9) model, the factors linked to supply and demand electricity production are highly significant within the ARIMAX(1, 1, 9) model.

Table 5.7: ISEM market ARIMAX(1,1,9) model summary statistics

Variable (Model Term)	**p-value**
Historical Electricity Price (**U_1**)	**<0.001**
System Generation (**U_2**)	0.051
Demand (**U_3**)	**<0.001**
Wind (**U_4**)	**<0.001**
East-West Interconnector (**U_5**)	0.096
Moyle Interconnector (**U_6**)	0.978
CO2 Intensity (**U_7**)	0.183
CO2 Emissions (**U_8**)	0.329
Load (**U_9**)	**0.004**
Temperature (**U_{10}**)	**<0.001**

Figure 5.7(A) shows the standardised residuals fluctuate around 0, however there are peaks and troughs throughout the period suggesting a trend. Figure 5.7(B) is a histogram with density plot resembling a narrow bell-shaped normal distribution pattern from the orange line, symmetrical around mean 0. Figure 5.7(C) is a normal quantile-quantile plot illustrating the majority of the quantiles are on or close to the red line, but there are extreme values highlighted from the curves on both ends of the line. Figure 5.7(D), the correlogram plot, shows slight autocorrelation is present with some lag errors outside the blue boundary, nonetheless the majority are close to zero. These plots confirm that the model residuals are generally uncorrelated and follow a normal distribution.

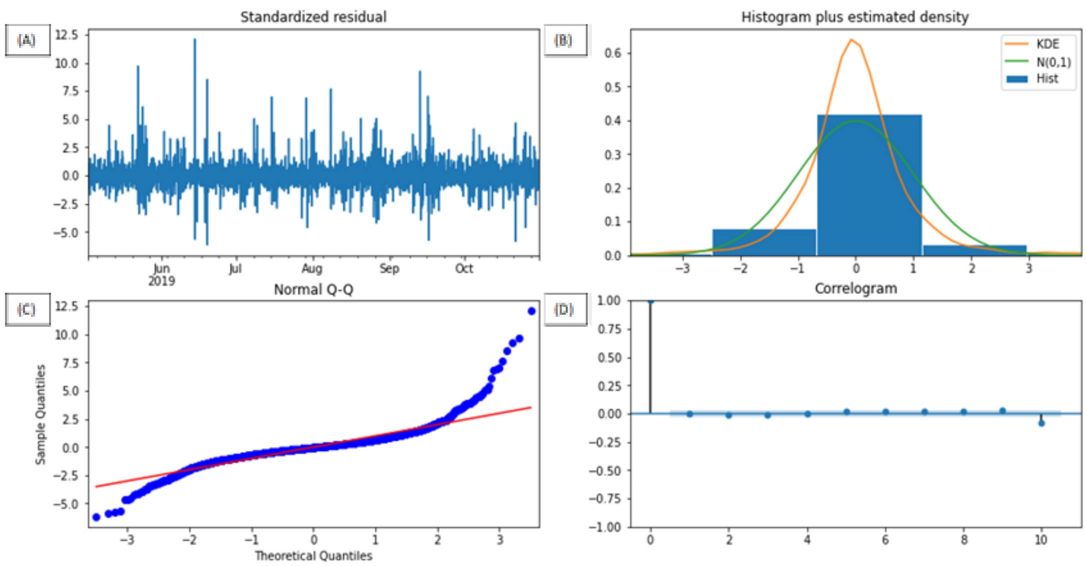

Figure 5.7: Residual diagnostic checks for ISEM market ARIMAX(1,1,9)

The RMSE value for the model validation period was 14.87. Comparing model performance with the ARIMA(8, 1, 8) model from Section 4.3, which outputted a RMSE value of 14.86, it is clear that the ARIMAX model can accurately predict day-ahead ISEM electricity prices just like the ARIMA model. This is further highlighted in Figure 5.8 which illustrates that the predicted electricity prices generally match the pattern of the actual prices, however, have difficulty reaching the peaks and troughs.

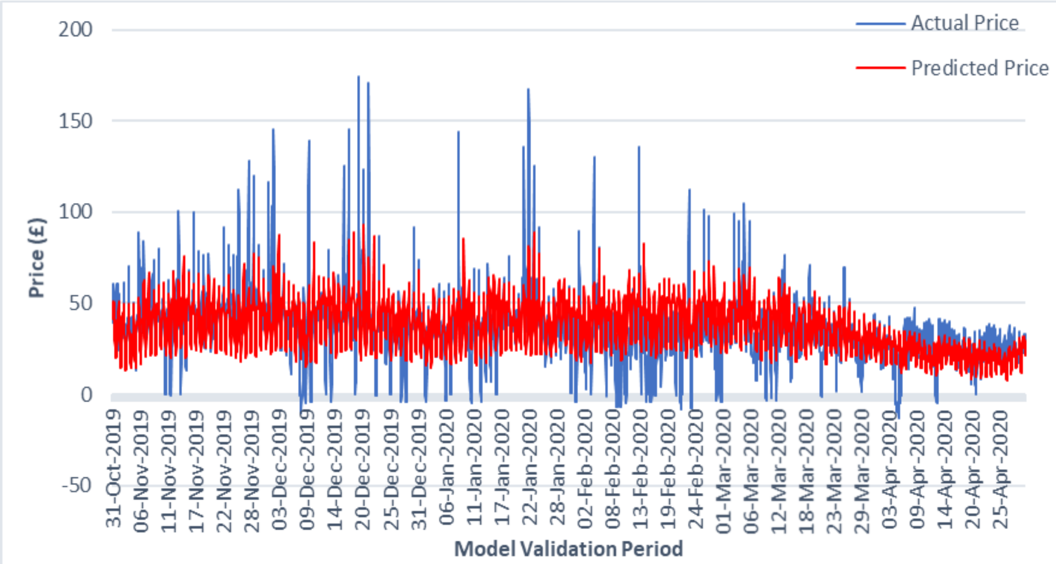

Figure 5.8: ISEM market ARIMAX(1,1,9) model

Table 5.8: RMSE values (ARMAX & ARIMAX models)

Market	Model	RMSE
BETTA	ARMAX(8, 0)	9.59
	ARIMAX(8, 1, 2)	11.09
ISEM	ARMAX(3, 9)	18.73
	ARIMAX(1, 1, 9)	14.87

Table 5.8 displays the best ARMAX and ARIMAX models alongside the corresponding RMSE value for both the BETTA and ISEM markets. The ARMAX model performed the best for the BETTA market, while the ARIMAX model performed the best for the ISEM market. There are regular daily trends for both markets which could indicate seasonality; therefore the energy-related inputs will be utilised in a seasonal statistical model to check if model accuracy can be improved. When modelling trends, there are advantages to starting with a simple model: it is less complex with faster computational time. If a simple model outputs a similar accuracy to a model adjusted for seasonality, then it would be chosen to save time during modelling.

5.5 SARIMAX Experiment

A SARIMAX model is considered to handle trend and seasonal variations with the aim of improving model accuracy. The BETTA SARIMAX model used the same data period and hourly records as the SARIMA model in Section 4.4. The following order terms are included: p ranged between 2 and 5, q ranged between 2 and 7, both P and Q ranged between 1 and 3, d was set to 1 to make the trend stationary, *D* was empirically set to 0 as when D was varied it made the seasonal pattern unstable, and *S* was set to 24 to capture the daily seasonality recurring cycle. The best SARIMAX model for the BETTA market is when *p*=2, *d*=1, *q*=3, *P*=2, *D*=0, *Q*=1, and *S*=24 corresponding to an AIC value of 28726.62. The SARIMAX(2, 1, 3)(2, 0, 1, 24) model function for predicted Y_t is given as:

$$Y_t = 0.26\nabla Y_{t-1} + 0.32\nabla Y_{t-2} - 0.41\nabla\varepsilon_{t-1} - 0.52\nabla\varepsilon_{t-2} - 0.071\nabla\varepsilon_{t-3} - 0.050S^{24}Y_{t-24} + 0.096S^{48}Y_{t-48} - 0.094S^{24}\varepsilon_{t-24} + 0.51U_1 + 3.15^{e-05}U_2 + 0.81U_3 + 1.06^{e-05}U_4 - 4.75^{e-07}U_5 + 1.64^{e-05}U_6 - 1.80^{e-06}U_7 - 0.0002U_8 + 0.0001U_9 + 0.0006U_{10} + 9.38^{e-05}U_{11} - 2.41^{e-06}U_{12} - 0.0019U_{13} - 0.078U_{14} + 44.39 \quad (5.5)$$

Gas was the most weighted exogenous variable (0.81) with historical electricity price the second most weighted (0.51), similar to the previous models using BETTA market data. The summary statistics are displayed in Table 5.9 and out of the 14 input factors, 4 were significant: historical electricity price, nuclear, pumped storage, and hydroelectric power. This is similar to the ARMAX(8, 0) model with three factors (historical electricity price, pumped storage, and hydroelectric power) being significant for both models.

Table 5.9: BETTA market SARIMAX(2,1,3)(2,0,1,24) model summary statistics

Variable (Model Term)	**p-value**
Historical Electricity Price (**U_1**)	**<0.001**
Demand (**U_2**)	0.314
Gas (**U_3**)	0.295
Wind (**U_4**)	0.754
Solar (**U_5**)	0.866
Coal (**U_6**)	0.721
Moyle Interconnector (**U_7**)	0.989
Nuclear (**U_8**)	**0.005**
Pumped Storage (**U_9**)	**0.041**
Hydroelectric Power (**U_{10}**)	**<0.001**
Biomass (**U_{11}**)	0.153
CCGT (**U_{12}**)	0.940
OCGT (**U_{13}**)	0.339
Temperature (**U_{14}**)	0.489

Figure 5.9(A) illustrates that the residuals fluctuate around 0, however there is a pattern of peaks and troughs highlighting a trend in electricity prices. Figure 5.9(B) generally resembles a bell-shaped pattern from comparing the orange line against the green line and therefore the data are normally distributed. Figure 5.9(C) is a normal quantile-quantile plot with the majority of the quantiles falling on or near to the red line indicating a normal distribution, but the shape is curved on both ends suggesting extreme values among the data. Figure 5.9(D) shows that there is autocorrelation present at Lag 4, but otherwise the residual errors are close to zero. All these plots confirm that the residuals are uncorrelated and that the data mostly follows a normal

pattern. Therefore the BETTA SARIMAX model (2, 1, 3) (2, 0, 1, 24) is an accurate prediction approximation to forecast electricity prices.

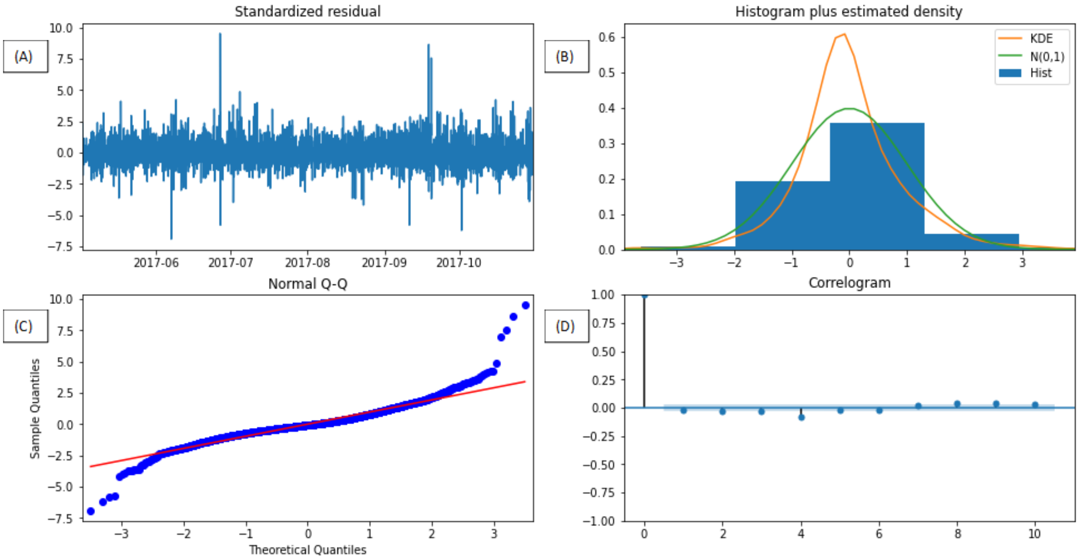

Figure 5.9: Residual diagnostic checks for BETTA market SARIMAX(2,1,3)(2,0,1,24)

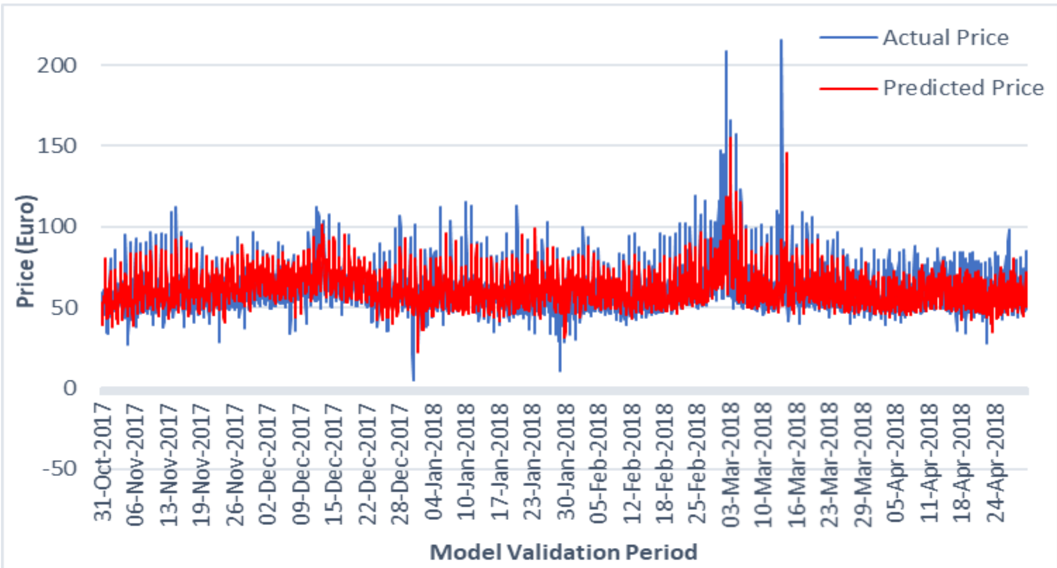

Figure 5.10: BETTA market SARIMAX(2,1,3)(2,0,1,24) model

The model validation performance was quite accurate with a RMSE value of 9.31. This was a slight improvement over the RMSE value of 9.67 for the SARIMA(3, 1, 2)(2, 0, 2, 24) model in Section 4.4 highlighting that the inclusion of exogenous variables in the seasonal model helps predict BETTA market data. Figure 5.10 illustrates the model validation results and the predicted values generally match the historical electricity price values throughout the modelling period. The promising results means a SARIMAX(2, 1, 3)(2, 0, 1, 24) model would be an option for day-ahead prediction in the BETTA market.

The ISEM market data were used with the SARIMAX model to predict day-ahead electricity prices in the Irish energy market. The ISEM SARIMAX model has the same study setup as the ISEM SARIMA experiment outlined in Section 4.4. The order terms are as follows: both *p* and *q* ranged between 1 and 4, both *P* and *Q* ranged between 1 and 3, *d* was set to 1 for stationary trend, *D* was set to 0, and *S* was set to 24 to factor daily seasonality. The lowest AIC value of 27497.02 was outputted from an ISEM SARIMAX(2, 1, 2) (2, 0, 2, 24) model whose function is given as:

$$Y_t = 0.59\nabla Y_{t-1} + 0.27\nabla Y_{t-2} - 0.57\nabla \varepsilon_{t-1} - 0.38\nabla \varepsilon_{t-2} + 0.32S^{24}Y_{t-24} + 0.35S^{48}Y_{t-48} - 0.29S^{24}\varepsilon_{t-24} - 0.19S^{48}\varepsilon_{t-48} + 0.15U_1 - 0.0004U_2 + 0.0028U_3 + 0.0002U_4 + 0.0005U_5 - 1.59^{e-05}U_6 - 0.0012U_7 + 0.0004U_8 + 0.0004U_9 - 0.83U_{10} + 31.72 \quad (5.6)$$

Temperature was the most weighted variable (-0.83) and historical electricity price was the second most weighted variable (0.15). This is similar to the model findings for ARMAX(3, 9). Historical electricity price, demand, East-West interconnector, and temperature had very low p-values as seen from Table 5.10. Three of these four factors were also highly significant for the previous two models using ISEM data, but this is the first time that the East-West interconnector is a dominant factor.

Table 5.10: ISEM market SARIMAX(2,1,2)(2,0,2,24) model summary statistics

Variable (Model Term)	**p-value**
Historical Electricity Price (**U_1**)	**0.011**
System Generation (**U_2**)	0.142
Demand (**U_3**)	**<0.001**
Wind (**U_4**)	0.353
East-West Interconnector (**U_5**)	**0.015**
Moyle Interconnector (**U_6**)	0.965
CO2 Intensity (**U_7**)	0.535
CO2 Emissions (**U_8**)	0.439
Load (**U_9**)	0.272
Temperature (**U_{10}**)	**<0.001**

Figure 5.11(A) illustrates that the standardised residuals fluctuate around 0, however there are many peaks and troughs throughout the period. In Figure 5.11(B) the histogram is symmetrical and the density plot (orange line) has a narrow bell-shaped pattern indicating the data are normally distributed. In Figure 5.11(C) the majority of the quantiles are close to the red line suggesting a normal distribution, but due to extreme outliers the blue line has sharp curves at both ends. Figure 5.11(D) shows that the residual errors are close to zero but some autocorrelation is present. All these plots confirm that the ISEM SARIMAX(2, 1, 2) (2, 0, 2, 24) model is a reasonable prediction approximation.

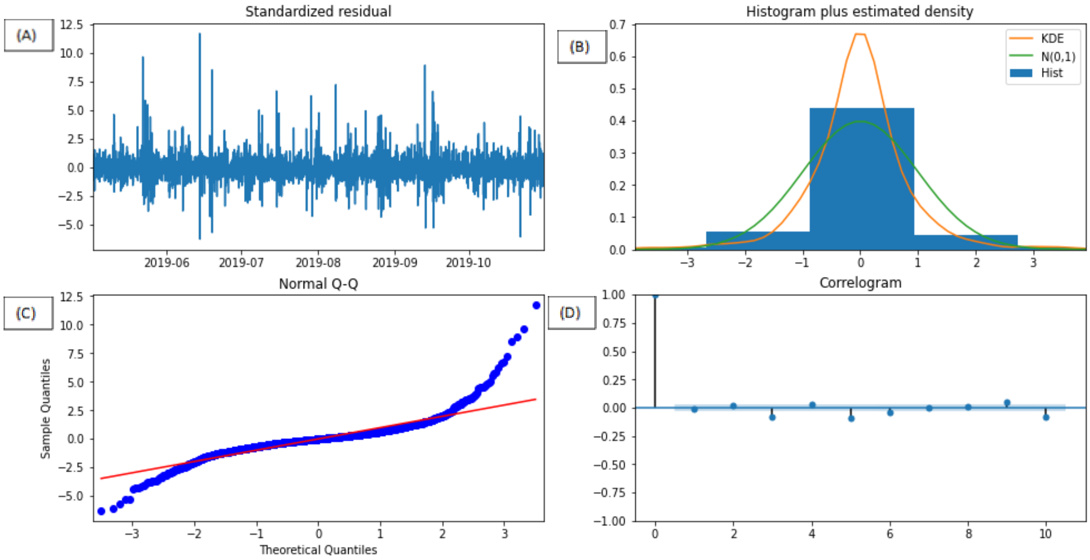

Figure 5.11: Residual diagnostic checks for ISEM market SARIMAX(2,1,2)(2,0,2,24)

The RMSE value of 14.32 for the model validation period improved greatly from the ISEM ARMAX(3, 9) model where the RMSE was 18.73, highlighting that including seasonality in a model improves forecasting. However the ISEM SARIMA(3, 1, 3) (2, 0, 2, 24) from Section 4.4 had a slightly better model fit, with a RMSE of 14.12, suggesting that the inclusion of exogenous factors makes it harder to predict day-ahead electricity price in the ISEM market.

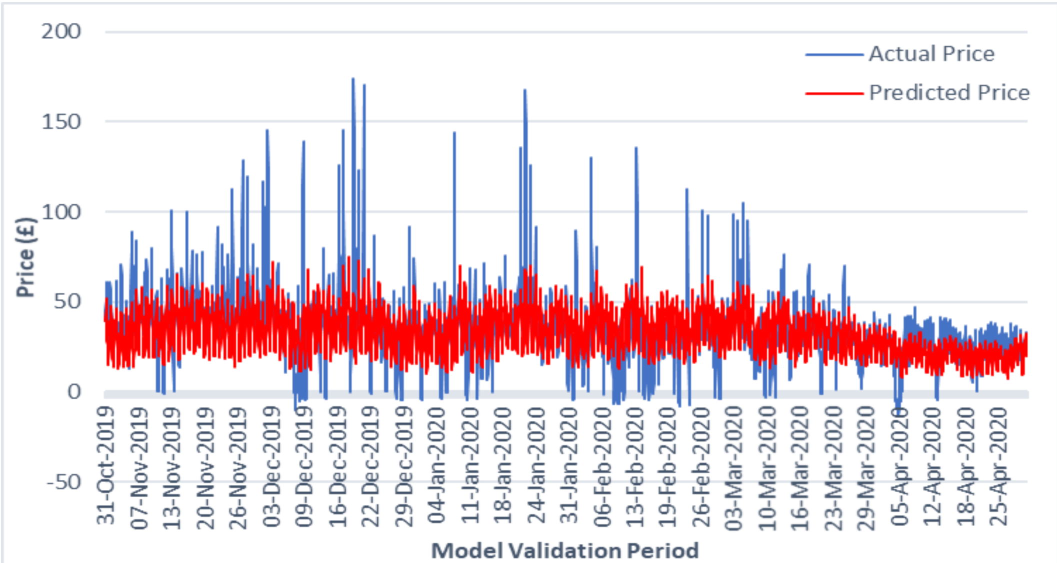

Figure 5.12: ISEM market SARIMAX(2,1,2)(2,0,2,24) model

In Figure 5.12, the predicted values follow the same trend around the middle of the plot as the actual historical values, however the model struggles with some fluctuations of the actual price values. Table 5.11 displays the RMSE values of the optimal ARMAX, ARIMAX, and SARIMAX models for both the BETTA and ISEM markets. Observing both markets, the SARIMAX models provided the lowest RMSE values improving overall model performance and therefore could be considered for predicting day-ahead electricity price. Observing the two markets individually, the BETTA market is much easier to accurately predict compared with the ISEM market when including exogenous variables.

Table 5.11: RMSE values (ARMAX, ARIMAX, & SARIMAX models)

Market	Model	RMSE
BETTA	ARMAX(8, 0)	9.59
	ARIMAX(8, 1, 2)	11.09
	SARIMAX(2, 1, 3)(2, 0, 1, 24)	**9.31**
ISEM	ARMAX(3, 9)	18.73
	ARIMAX(1, 1, 9)	14.87
	SARIMAX(2, 1, 2)(2, 0, 2, 24)	**14.32**

5.6 NARMAX Experiment

A NARMAX model, for which the theory was discussed in Section 3.3.4, is a forecasting technique that only keeps significant model terms for prediction. This research work analyses linear polynomial NARMAX models as simpler polynomial models perform better with unseen data (see polynomial model findings in Appendix).

The first NARMAX experiment used BETTA market data from May 2017 until April 2018; 8736 records in total. The energy-related input data ranged from 01st May 2017 until 29th April 2018 and electricity price was the target output to predict day-ahead and ranged from 02nd May 2017 until 30th April 2018. Initially, all energy-related inputs were included and the model structuring and estimation process removed unnecessary energy factors, keeping only significant model terms. For this experiment, a linear polynomial was chosen and the **E**rror **R**eduction **R**atio (ERR) threshold was set to 0.05. The ERR develops an accurate, parsimonious final model by ranking the **M**ean **S**quared **E**rror (MSE) from largest to smallest reduction to select the key model terms above the ERR cut-off threshold [64]. As the model is constructed it changes iteratively, eliminating variables, one at a time, that have no influence on the prediction and retaining the influential variables throughout the iterative process until all remaining variables are significant. The final NARMAX model showed a strong relationship between the input exogenous variables and electricity price and is given as:

$$Y_t = 0.56U_1 + 0.000016U_2 + 0.000035U_4 - 0.000067U_8 + 0.00034U_9 + 0.24U_{10} - 0.0036U_{13} + 17.77 \tag{5.7}$$

Both the modelling estimation and validation stages resulted in reasonable predictions with RMSE values of 9.23 and 9.46, respectively. The seven significant factors retained during the iteration process were historical electricity price, demand, wind, nuclear, pumped storage, hydroelectric power and OCGT. This suggests that including significant energy-related factors does influence the forecasting prediction, in particular historical electricity price as it was the most dominant factor.

Table 5.12 ranks the ERR terms from largest to smallest for the BETTA market energy-related factors. Each ERR value represents the measure of contribution of each variable against the model output (day-ahead price). If any variable was to be removed from the final model, the ERR value represents the ratio accuracy of how much the model loses in relation to day-ahead electricity price. The three largest ERR values approximate to a 49.40 proportion of the variance of the day-ahead price and these three values were made up of the model terms from historical electricity price (47.10), then demand (1.69), and finally wind (0.61). Historical electricity price was both the most weighted factor (0.56) and had the largest ERR ranking (47.10).

Table 5.12: Error Reduction Ratio for BETTA market NARMAX model

ERR	Energy-Related Factors (Model Term)
47.098855	Historical electricity price (**U_1**)
1.686628	Demand (**U_2**)
0.609266	Wind (**U_4**)
0.470711	Pumped storage (**U_9**)
0.244902	Hydroelectric power (**U_{10}**)
0.115161	OCGT (**U_{13}**)
0.077452	Nuclear (**U_8**)

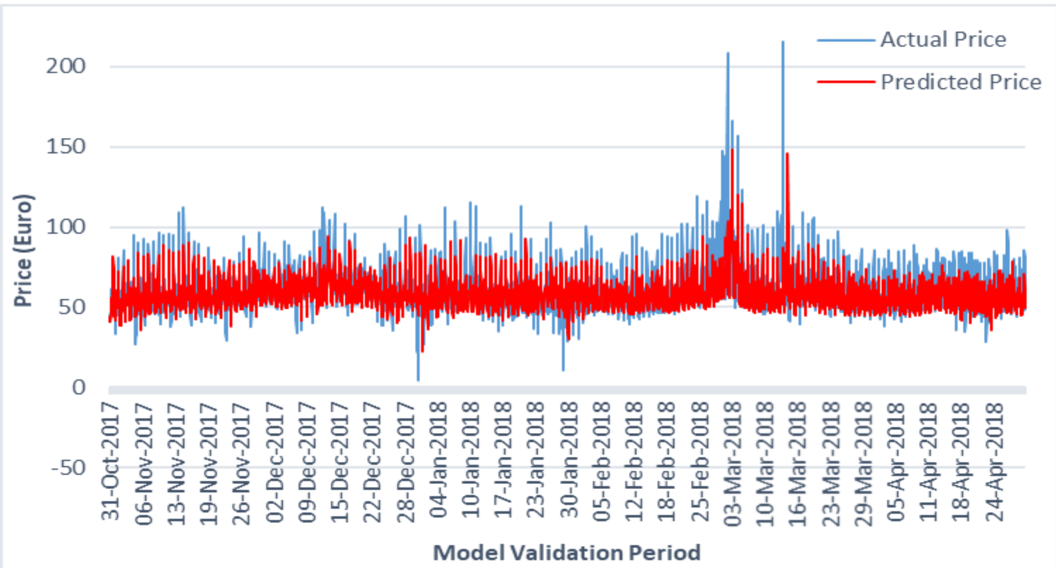

Figure 5.13: Best model validation for BETTA market NARMAX model

Model validation uses the unseen data (historical energy-related data from 31st October 2017 Hour 0 until 30th April 2018 Hour 23) and therefore is used to verify how well the model fits the data. Figure 5.13 illustrates the BETTA NARMAX model outputs with only the significant energy-related factors included as inputs. It can be seen from Figure 5.13 that the predicted price output matched the actual price output constantly around the middle data points and troughs, but the predicted values sometimes struggled to span all actual data points and reach the peaks.

The second NARMAX experiment used ISEM market data from May 2019 until April 2020 resulting in 8760 records in total. The energy-related input data ranged from 01st May 2019 until 29th April 2020 and electricity price, which was the target output data to predict day-ahead, ranged from 02nd May 2019 until 30th April 2020. In this experiment a linear polynomial was selected and the ERR was set to 0.05. At the beginning, all energy-related inputs were included in the model but after the required iterations to reach the optimal final model, the unnecessary factors were removed. The final NARMAX model contained the significant model terms required for an optimal forecasting model to predict the day-ahead ISEM market.

The model function of the linear NARMAX model for the ISEM market is given as:

$$Y_t = 0.38U_1 - 0.00053U_2 + 0.00086U_3 + 0.00082U_7 - 0.00080U_8 + 0.0021U_9 + 0.26U_{10} + 31.72 \tag{5.8}$$

The model estimation had a RMSE value of 12.58 and the model validation had a RMSE value of 15.15. The seven significant factors retained during the iterative process were historical electricity price, system generation, demand, CO2 intensity, CO2 emissions, load, and temperature.

Table 5.13: Error Reduction Ratio for ISEM market NARMAX model

ERR	Energy-Related Factors (Model Term)
30.653281	Historical electricity price (**U_1**)
2.847335	Demand (**U_3**)
1.511024	System generation (**U_2**)
0.355038	Load (**U_9**)
0.284880	CO2 intensity (**U_7**)
0.179546	Temperature (**U_{10}**)
0.059359	CO2 emissions (**U_8**)

Table 5.13 displays the percentage variance of each significant variable for the ISEM market, ranking the ERR terms from largest to smallest. The three largest ERR values approximated to a 35.01 proportion of the variance of the day-ahead price and these three values were made up of the model terms from historical electricity price (30.65), demand (2.85), and system generation (1.51). Similar to the BETTA market results, historical electricity price and demand were the two most dominant factors. Historical electricity price was both the most weighted factor (0.38) and had the largest ERR ranking (30.65); however it does not have as much influence in the ISEM market as both the weighted and ERR values are lower. Comparing both the BETTA and ISEM results, it is clear that historical electricity price and demand have a large influence on forecasting electricity prices.

Historical energy-related data from 31st October 2019 Hour 12 until 30th April 2020 Hour 23 were considered for the model validation period to verify how well the model fits the data. Figure

5.14 illustrates the ISEM NARMAX model with only the significant energy-related factors included as inputs. It can be seen from Figure 5.15 that the predicted price output matched the actual price output consistently around the middle data points, but the predicted values failed to reach the peaks and troughs.

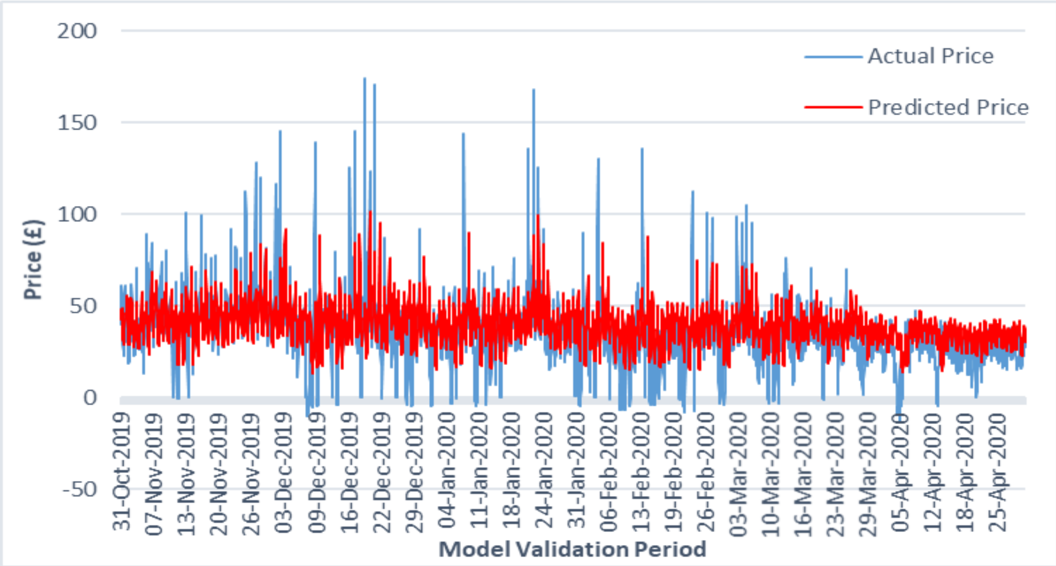

Figure 5.14: Best model validation for ISEM market NARMAX model

Table 5.14 displays the RMSE values of the optimal ARMAX, ARIMAX, SARIMAX, and NARMAX models for both the BETTA and ISEM markets. The BETTA market prediction remained quite stable with the inclusion of exogenous inputs. The ISEM market prediction was not as accurate as the BETTA, however model performance did improve for ARIMAX, SARIMAX, and NARMAX forecasting models. It is clear form Table 5.14 that the SARIMAX model provided the lowest RMSE value for both the BETTA and ISEM markets.

Table 5.14: RMSE values (ARMAX, ARIMAX, SARIMAX & NARMAX models)

Market	Model	RMSE
BETTA	ARMAX(8, 0)	9.59
	ARIMAX(8, 1, 2)	11.09
	SARIMAX(2, 1, 3)(2, 0, 1, 24)	**9.31**
	NARMAX	9.46
ISEM	ARMAX(3, 9)	18.73
	ARIMAX(1, 1, 9)	14.87
	SARIMAX(2, 1, 2)(2, 0, 2, 24)	**14.32**
	NARMAX	15.15

The experiments were conducted using an Intel Pentium Quad Core Processor N4200, and the run times taken for the ISEM MISO models to output all the combinations to find the lowest AIC values are included in Table 5.15. Compared to Table 4.4, the computation time for each model to compute all the combinations of order terms is twice as long for the MISO models.

Table 5.15: Run times to fit the optimal ISEM MISO models

Model	Range	Run Time
ARMAX(3, 9)	p = (0, 10), q = (0, 10)	6,881 seconds
ARIMAX(1, 1, 9)	p = (0, 10), q = (0, 10)	25,124 seconds
SARIMAX(2, 1, 2)(2, 0, 2, 24)	p = (1, 4), q = (1, 4), P = (1, 3), Q = (1, 3)	54,523 seconds

5.7 Conclusion

This chapter examined and compared MISO time-series prediction models with energy-related input data and explored their suitability for accurately forecasting day-ahead electricity price in the BETTA and ISEM markets. For each of the statistical MISO models stationarity, trend, and seasonality were considered during model identification and estimation to help find optimal order terms and improve overall model performance. RMSE values were compared for each of the standard and seasonal model experiments to determine if model validation accuracy can improve depending on modelling inputs, trend, and seasonality.

The ARMAX experimental results determined ARMAX(8, 0) to be the best model with BETTA market data and ARMAX(3, 9) to be the best model with ISEM market data. Due to fluctuating trends in energy data, ARIMAX was considered to determine if model performance could be further enhanced. The ARIMAX experimental results noted ARIMAX(8, 1, 2) to be the best model with BETTA market data and ARIMAX(1,1,9) to be the best model with ISEM market data. To capture daily trends but to try to improve model accuracy further, seasonality was considered in future prediction models. The SARIMAX experimental results displayed reasonable model accuracy with SARIMAX(2, 1, 3) (2, 0, 1, 24) being the best model when using the BETTA market data and SARIMAX(2, 1, 2) (2, 0, 2, 24) being the best model when using the ISEM market data. The NARMAX experimental results emphasized that exogenous variables do help to improve model accuracy, in particular with ISEM market data, and determined that the most dominant energy-related factors were historical electricity price, demand, and wind in the BETTA market and were historical electricity price, demand, and system generation in the ISEM market. The experiments observed that SARIMAX models were the best at forecasting day-ahead electricity price. Comparing the two markets, the BETTA market resulted in a better overall model accuracy for each of the individual MISO models.

Throughout Chapter 4 and Chapter 5, SISO and MISO models were examined with real energy market data and the corresponding RMSE values for each of these models are presented in Table 5.16. The findings are reasonable as the results are similar between the two chapters: SARIMAX is the most accurate model and BETTA is the easier market to predict. Seasonal models performed the best overall, outputting the lowest RMSE values: when all exogenous variables are included as model inputs for the BETTA market and when only historical price is included for the ISEM market. This indicates that including all energy-related factors as model inputs might reduce prediction accuracy due to multicollinearity in the data when forecasting in the ISEM market and thus it may help improve model performance if only significant energy-related factors identified from NARMAX were included as model inputs.

Table 5.16: RMSE values of all SISO and MISO models

Market	**Model**	**RMSE**
BETTA	ARMA(9, 7)	10.91
	ARIMA(9, 1, 7)	9.94
	SARIMA(3, 1, 2)(2, 0, 2, 24)	9.67
	ARMAX(8, 0)	9.59
	ARIMAX(8, 1, 2)	11.09
	SARIMAX(2, 1, 3)(2, 0, 1, 24)	**9.31**
	NARMAX	9.46
ISEM	ARMA(9, 8)	14.99
	ARIMA(8, 1, 8)	14.86
	SARIMA(3, 1, 3)(2, 0, 2, 24)	**14.12**
	ARMAX(3, 9)	18.73
	ARIMAX(1, 1, 9)	14.87
	SARIMAX(2, 1, 2)(2, 0, 2, 24)	14.32
	NARMAX	15.15

This chapter investigated model performance of MISO models to determine if energy-related factors as inputs improve model accuracy and to determine which exogenous variables had the most influence on day-ahead electricity price. With the inclusion of exogenous inputs, model performance does improve therefore it is important to consider energy-related factors in electricity price forecasting. Since traditional statistical time-series models assume stationarity but it is normal for energy data to display non-stationary traits, it would be beneficial to predict day-ahead electricity price by first utilizing the statistical modelling frameworks and combining them with a transparent linear polynomial NARMAX model to enhance the accuracy of the statistical models. In the next chapter, the identified key energy-related factors from the NARMAX summary results will be applied as inputs to refined statistical models (parsimonious) with the aim of further improving model performance and electricity price prediction accuracy.

Chapter 6

Refined Models

6.1 Introduction

This chapter will first examine correlated peak lags for each individual energy-related factor. Li et al. [18] showed, through autocorrelation testing, that every 24-hour lag peaked suggesting that same hour energy data provides strong correlations and therefore would be best to use as input in a forecasting model. Lagged input variables in a regression analysis can be used to determine if a relationship exists with the dependent price variable [8]. Autocorrelation reveals the similarity between data observations in terms of the time lag function and autocorrelation testing identifies correlations when applied to time-series lag input. Dividing a time-series into fixed daily time periods of 24 hours enables the observation of trends to create a homogeneous process and improve parameter estimates compared with a model which considers the whole time-series [52].

This research follows on from Chapter 5 by developing parsimonious models through utilising the identified key energy-related factors from the transparent **N**onlinear **A**uto**R**egressive **M**oving **A**verage models with e**X**ogenous variables (NARMAX) model as inputs to the refined statistical **A**uto**R**egressive **M**oving **A**verage with e**X**ogenous variables (ARMAX), **A**uto**R**egressive **I**ntegrated **M**oving **A**verage with e**X**ogenous variables (ARIMAX), and **S**easonal **A**uto**R**egressive **I**ntegrated **M**oving **A**verage with e**X**ogenous variables (SARIMAX) original and correlated lags models. This chapter focusses on the third research question listed in Chapter 1: “Can transparent models identify key factors that influence electricity price?” with the aim of improving model performance and accuracy of day-ahead electricity price forecasting models. Research work in this chapter is published in the following:

- C. McHugh, S. Coleman, D. Kerr, and D. McGlynn, “Seasonal Models for Forecasting Day-Ahead Electricity Prices”, in Proceedings of International Conference on Time Series and Forecasting, ITISE 2019, pp. 310–320.

All the models presented in this chapter were tested separately with the Irish **I**ntegrated **S**ingle **E**lectricity **M**arket (ISEM) market data, analysed from May 2019 until April 2020.

First, each of the correlated lags models are displayed and examined to determine the key energy-related lag factors for the ISEM market. Next, the refined original statistical models are tested including only the following significant energy-related factors from the NARMAX model findings in Section 5.6 as inputs: historical electricity price, system generation, demand, CO2 intensity, CO2 emissions, load, and temperature. Finally, the refined correlated lags models are tested with only the significant input factors from the NARMAX model examined in Section 6.6: historical electricity price, historical electricity price Lag 2, historical electricity price Lag 23, historical electricity price Lag 24, system generation, system generation Lag 1, system generation Lag 2, system generation Lag 3, system generation Lag 22, system generation Lag 23, demand, demand Lag 1, demand Lag 2, demand Lag 3, demand Lag 22, East-West interconnector, CO2 intensity, CO2 emissions, CO2 emissions Lag 1, CO2 emissions Lag 2, load, load Lag 1, temperature , temperature Lag 1, and temperature Lag 22. Throughout the chapter, the **R**oot **M**ean **S**quared **E**rror (RMSE) values for each refined model are compared against the RMSE values from the NARMAX modelling results. The experimental set-up and presented run-times in Chapter 5 are the same in this chapter.

6.2 Correlated Lags

The previous statistical models displayed in Chapter 5 provided favourable results but there was still room for improvement to lower the RMSE values and improve model accuracy. Lagging energy-related inputs will determine the peak lags when an exogenous variable exhibits strong correlation [18]. Considering lagged exogenous variables that influence electricity prices as model inputs should improve prediction accuracy, which was noted as a limitation in a study focussed on Sweden’s energy market which only considered a one-day influence and no time lags [56]. Autocorrelation testing was performed on each of the energy-related factors individually with the initial lags selected from ACF plots as any positive lag which falls outside the 95% confidence interval. The ACF plots are displayed in Figure 6.1 – Figure 6.10 for each energy-related factor.

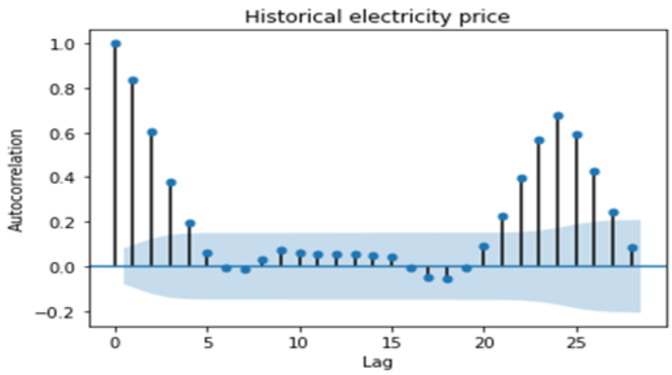

Figure 6.1: ISEM market autocorrelation testing of lagged historical electricity price

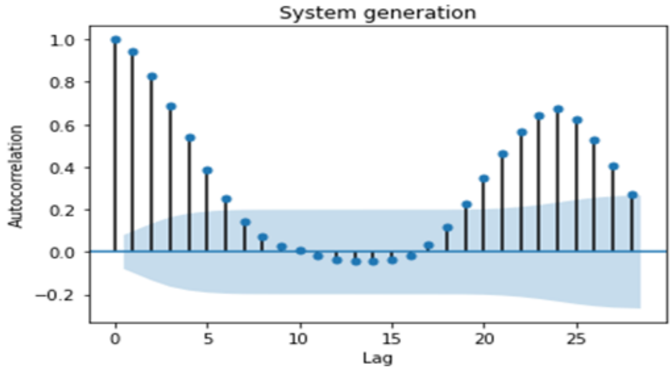

Figure 6.2: ISEM market autocorrelation testing of lagged system generation

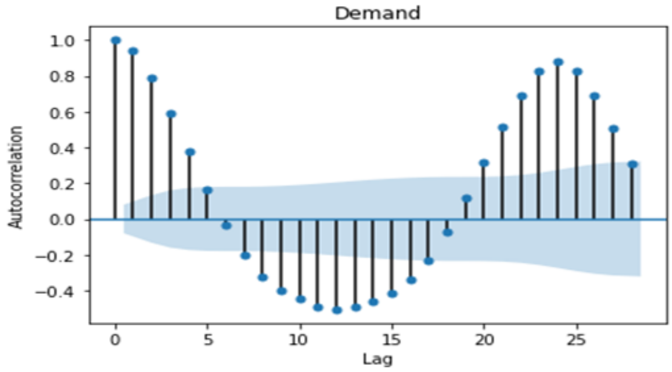

Figure 6.3: ISEM market autocorrelation testing of lagged demand

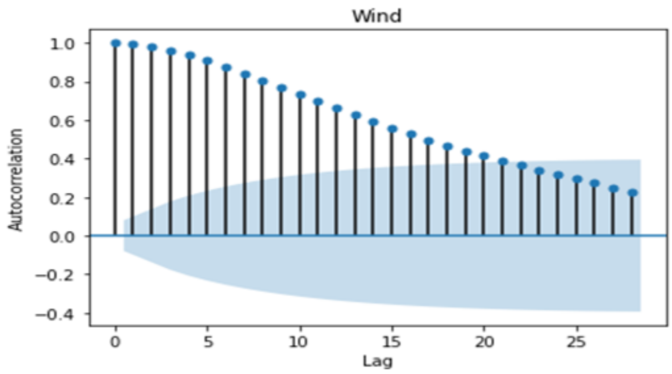

Figure 6.4: ISEM market autocorrelation testing of lagged wind

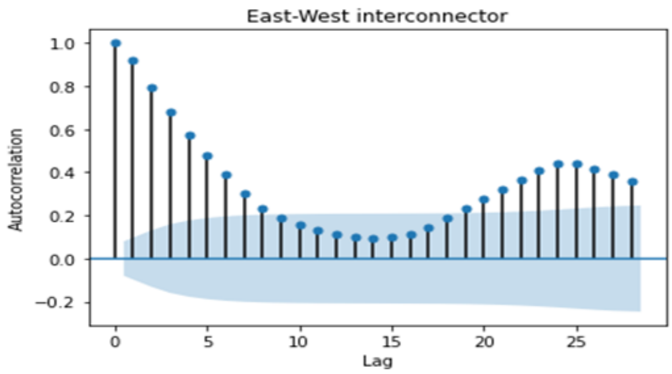

Figure 6.5: ISEM market autocorrelation testing of lagged East-West interconnector

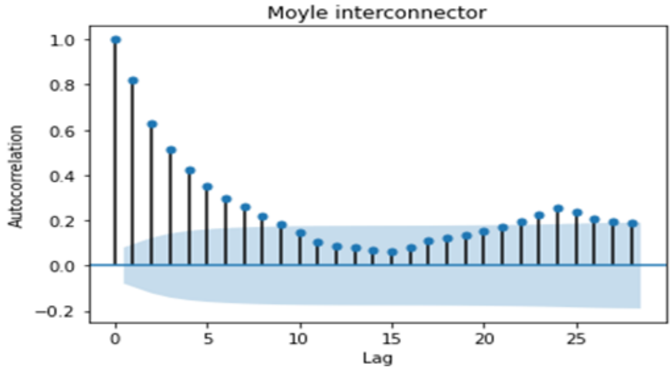

Figure 6.6: ISEM market autocorrelation testing of lagged Moyle interconnector

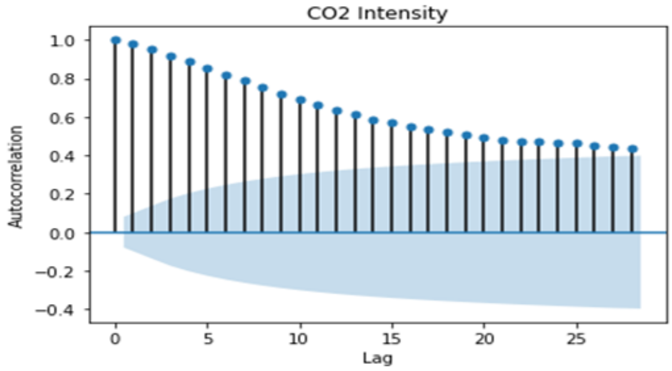

Figure 6.7: ISEM market autocorrelation testing of lagged CO2 intensity

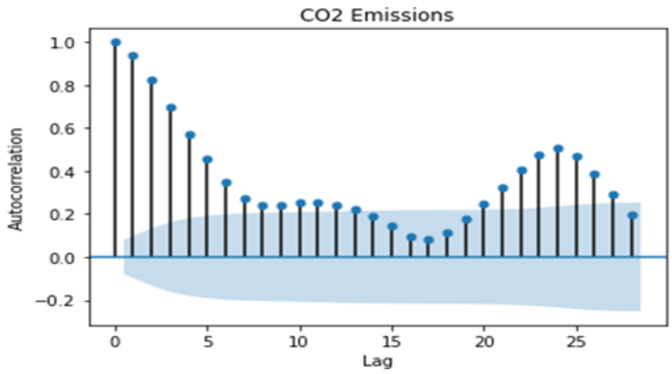

Figure 6.8: ISEM market autocorrelation testing of lagged CO2 emissions

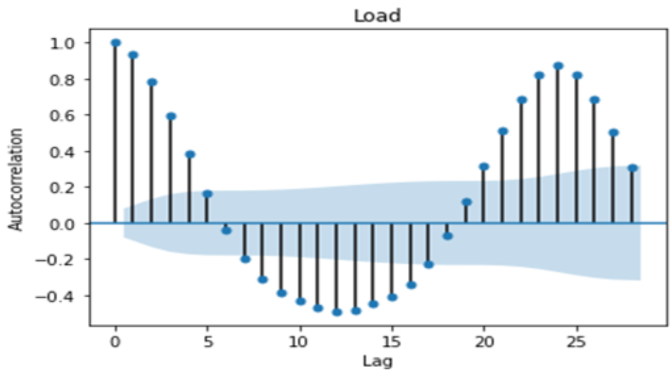

Figure 6.9: ISEM market autocorrelation testing of lagged load

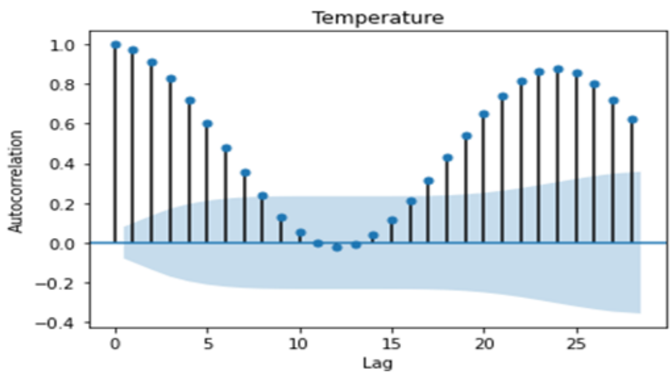

Figure 6.10: ISEM market autocorrelation testing of lagged temperature

As expected the majority of exogenous variables displayed strong correlation every 24 hours. For any ACF plot displaying a downward slope (e.g. Figure 6.4), only the first six lags were selected as initial lags. The significant peak lags were identified as a subset from the initial lags and subsequently included as model inputs in the statistical and non-linear models to examine whether correlated lags influence model accuracy. The peak lags from each factor's autocorrelation plots are presented in Table 6.1 for the ISEM market. These peak lags will be used in Sections 6.3 to 6.6 as model inputs in the forecasting models to observe if significant lags can improve prediction accuracy.

Table 6.1: ISEM energy-related factors peak lags

Energy-Related Factors	Unit	Peak Lags from Autocorrelation Plots
Historical Electricity Price	GBP per Megawatt Hour	Lags 1, 2, 23, 24
System Generation	Megawatt	Lags 1, 2, 3, 22, 23, 24
Demand	Megawatt	Lags 1, 2, 3, 22, 23, 24
Wind	Megawatt	Lag 1
East-West Interconnector	Megawatt	Lags 1, 2, 3, 24
Moyle Interconnector	Megawatt	Lags 1, 2, 3, 24
CO2 intensity	Kilowatt Hour	Lag 1
CO2 emissions	CO2 intensity per Hour	Lags 1, 2, 3, 22, 23, 24
Load	Megawatt	Lags 1, 2, 3, 22, 23, 24
Temperature	Celsius	Lags 1, 2, 3, 22, 23, 24

6.3 Correlated Lags ARMAX Experiment

Including the additional ISEM market lag data, the ranges for order terms *p* and *q* were set from 0 to 10. From the brute force search the lowest AIC value was 27374.90 with optimal order terms being *p*=1 and *q*=9. The RMSE for ARMAX(1, 9) model with all initial lags included was 16.00 whereas the RMSE for the ARMAX(1, 9) model with peak lags was 14.66, therefore peak lags were selected for the experiment. The model function for correlated lags ARMAX(1, 9) for predicted Y_t is given as:

$$Y_t = 0.80Y_{t-1} + 0.16\varepsilon_{t-1} + 0.060\varepsilon_{t-2} + 0.032\varepsilon_{t-3} + 0.10\varepsilon_{t-4} - 0.012\varepsilon_{t-5} +$$
$$0.047\varepsilon_{t-6} + 0.071\varepsilon_{t-7} + 0.042\varepsilon_{t-8} + 0.065\varepsilon_{t-9} + 0.28U_1 - 0.014U_{1(t-1)} +$$

$$0.0023U_{1(t-2)} + 0.023U_{1(t-23)} + 0.093U_{1(t-24)} - 0.0008U_2 - 0.0003U_{2(t-1)} + 2.87^{e-05}U_{2(t-2)} + 0.0004U_{2(t-3)} - 0.0002U_{2(t-22)} + 0.0005U_{2(t-23)} - 0.0002U_{2(t-24)} + 0.0018U_3 + 0.0001U_{3(t-1)} - 3.83^{e-05}U_{3(t-2)} - 0.0012U_{3(t-3)} + 0.0009U_{3(t-22)} + 0.0007U_{3(t-23)} - 0.0008U_{3(t-24)} - 0.0002U_4 - 0.0002U_{4(t-1)} + 0.0004U_5 + 8.53^{e-05}U_{5(t-1)} + 0.0001U_{5(t-2)} + 0.0003U_{5(t-3)} - 0.0001U_{5(t-24)} - 0.0004U_6 + 4.58^{e-05}U_{6(t-1)} - 0.0008U_{6(t-2)} - 0.0005U_{6(t-3)} + 0.0001U_{6(t-24)} - 0.0047U_7 - 0.0027U_{7(t-1)} + 0.0011U_8 + 0.0007U_{8(t-1)} - 1.15^{e-05}U_{8(t-2)} - 0.0004U_{8(t-3)} - 0.0004U_{8(t-22)} - 1.13^{e-07}U_{8(t-23)} + 5.71^{e-05}U_{8(t-24)} + 0.0001U_9 + 0.0014U_{9(t-1)} + 0.0009U_{9(t-2)} + 0.0001U_{9(t-3)} - 0.0010U_{9(t-22)} - 0.0014U_{9(t-23)} - 0.0002U_{9(t-24)} - 1.96U_{10} + 1.45U_{10(t-1)} + 1.35U_{10(t-2)} - 0.95U_{10(t-3)} + 0.087U_{10(t-22)} + 0.049U_{10(t-23)} + 0.38U_{10(t-24)} + 16.44 \quad (6.1)$$

Table 6.2: ISEM market correlated lags ARMAX(1,9) model summary statistics

Variable (Model Term)	**Current p-value**	**Lag 1 p-value**	**Lag 2 p-value**	**Lag 3 p-value**	**Lag 22 p-value**	**Lag 23 p-value**	**Lag 24 p-value**
Historical Electricity Price (**U_1**)	**<0.001**	0.379	0.881			0.149	**<0.001**
System Generation (**U_2**)	**<0.001**	0.200	0.894	0.078	0.237	**0.014**	0.307
Demand (**U_3**)	**<0.001**	0.751	0.926	**<0.001**	0.032	0.131	**0.014**
Wind (**U_4**)	0.389	0.483					
East-West Interconnector (**U_5**)	0.136	0.747	0.660	0.180			0.615
Moyle Interconnector (**U_6**)	0.245	0.897	**0.027**	0.189			0.682
CO2 Intensity (**U_7**)	**0.006**	0.121					
CO2 Emissions (**U_8**)	**0.014**	0.118	0.975	0.204	0.199	1.000	0.867
Load (**U_9**)	0.780	**0.045**	0.193	0.869	**0.004**	**0.041**	0.754
Temperature (**U_{10}**)	**<0.001**	**<0.001**	**<0.001**	**<0.001**	0.692	0.836	0.118

The most weighted exogenous variable was temperature (-1.96) followed by temperature Lag 1 (1.45). Table 6.2 presents the summary statistics for the ISEM ARMAX(1, 9) correlated model. Observing the p-values, the following is noted: historical electricity price, system generation, demand, CO2 intensity, CO2 emissions, and temperature were significant for current inputs; load and temperature were significant for Lag 1; Moyle interconnector and temperature were significant for Lag 2; demand and temperature were significant for Lag 3; demand and load were significant for Lag 22; system generation and load were significant for Lag 23, and historical electricity price and demand were significant for Lag 24. Wind and East-West interconnector were the only two variables not significant in the correlated model. The original ISEM ARMAX model had similar results without the Moyle interconnector and CO2 emissions.

Figure 6.11 illustrates the actual and predicted prices for the ARMAX(1, 9) model validation period. The figure shows an accurate model fit with a RMSE value of 14.66, which is a great improvement from the original ISEM ARMAX model performance (RMSE=18.73) in Section 5.3. Again the predicted values struggle to reach all the peaks and troughs but overall the model performs consistently with promising results. The ISEM ARMAX(1, 9) model could be considered to forecast day-ahead electricity prices.

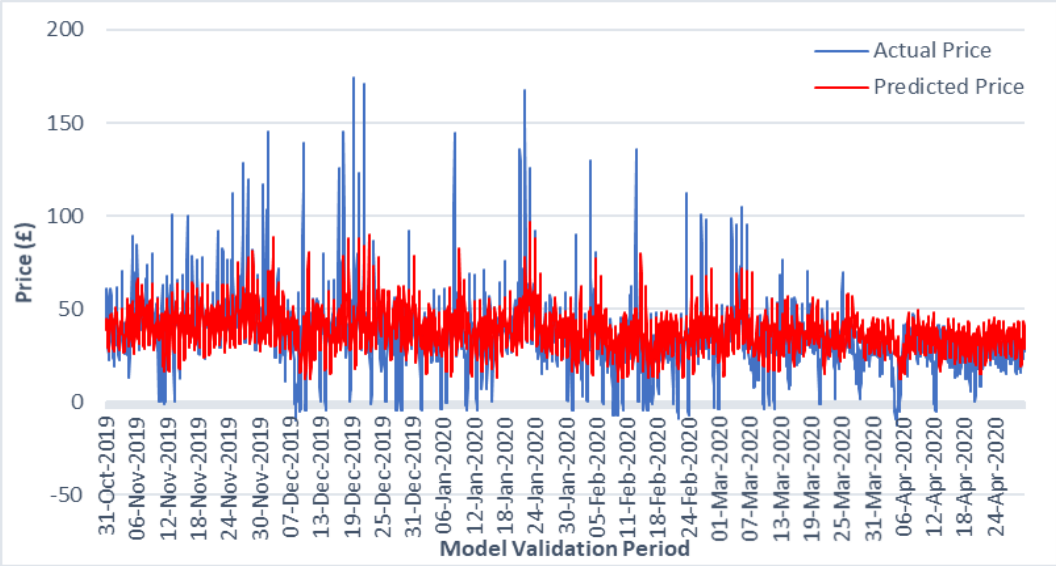

Figure 6.11: ISEM market correlated lags ARMAX(1,9) model

6.4 Correlated Lags ARIMAX Experiment

For the ISEM ARIMAX correlated lags model, the ranges for order terms *p* and *q* were set from 0 to 10. From the brute force search the lowest AIC value was 27379.15 with optimal order terms being *p*=7, *d*=1, and *q*=6. The model function for correlated lags ARIMAX(7, 1, 6) for predicted Y_t is given as:

$$Y_t = 0.18Y_{t-1} + 0.10Y_{t-2} - 0.022Y_{t-3} + 0.21Y_{t-4} + 0.17Y_{t-5} + 0.021Y_{t-6} + 0.087Y_{t-7} - 0.049\varepsilon_{t-1} - 0.096\varepsilon_{t-2} + 0.086\varepsilon_{t-3} - 0.046\varepsilon_{t-4} - 0.13\varepsilon_{t-5} + 0.022\varepsilon_{t-6} + 0.27U_1 - 0.023U_{1(t-1)} + 0.0051U_{1(t-2)} + 0.013U_{1(t-23)} + 0.10U_{1(t-24)} - 0.0012U_2 - 0.0002U_{2(t-1)} + 8.67^{e-06}U_{2(t-2)} + 0.0003U_{2(t-3)} + 9.70^{e-05}U_{2(t-22)} + 0.0004U_{2(t-23)} - 0.0001U_{2(t-24)} + 0.0024U_3 + 0.0003U_{3(t-1)} - 0.0002U_{3(t-2)} - 0.0008U_{3(t-3)} + 0.0004U_{3(t-22)} + 0.0011U_{3(t-23)} - 0.0006U_{3(t-24)} + 3.04^{e-05}U_4 - 5.87^{e-05}U_{4(t-1)} + 0.0002U_5 + 0.0001U_{5(t-1)} + 0.0001U_{5(t-2)} + 0.0004U_{5(t-3)} - 0.0001U_{5(t-24)} - 0.0002U_6 + 0.0002U_{6(t-1)} - 0.0007U_{6(t-2)} - 0.0004U_{6(t-3)} + 0.0002U_{6(t-24)} - 0.012U_7 + 6.07^{e-06}U_{7(t-1)} + 0.0023U_8 + 0.0010U_{8(t-1)} + 3.16^{e-05}U_{8(t-2)} - 0.0004U_{8(t-3)} - 0.0003U_{8(t-22)} + 9.69^{e-05}U_{8(t-23)} - 0.0002U_{8(t-24)} - 0.0002U_9 + 0.0004U_{9(t-1)} + 0.0009U_{9(t-2)} + 0.0002U_{9(t-3)} - 0.0004U_{9(t-22)} - 0.0011U_{9(t-23)} - 0.0009U_{9(t-24)} - 0.38U_{10} + 0.48U_{10(t-1)} + 0.45U_{10(t-2)} - 0.58U_{10(t-3)} - 0.16U_{10(t-22)} - 0.44U_{10(t-23)} - 0.38U_{10(t-24)} + 30.34 \quad (6.2)$$

Table 6.3: ISEM market correlated lags ARIMAX(7,1,6) model summary statistics

Variable (Model Term)	**Current p-value**	**Lag 1 p-value**	**Lag 2 p-value**	**Lag 3 p-value**	**Lag 22 p-value**	**Lag 23 p-value**	**Lag 24 p-value**
Historical Electricity Price ($\mathbf{U_1}$)	**<0.001**	**0.048**	0.708			0.332	**<0.001**
System Generation ($\mathbf{U_2}$)	**<0.001**	0.365	0.967	0.167	0.616	**0.023**	0.587
Demand ($\mathbf{U_3}$)	**<0.001**	0.326	0.630	**<0.001**	0.296	**<0.001**	**0.029**
Wind ($\mathbf{U_4}$)	0.926	0.853					
East-West Interconnector ($\mathbf{U_5}$)	0.376	0.567	0.575	0.120			0.643
Moyle Interconnector ($\mathbf{U_6}$)	0.486	0.616	**0.043**	0.231			0.492
CO2 Intensity ($\mathbf{U_7}$)	**<0.001**	0.997					
CO2 Emissions ($\mathbf{U_8}$)	**<0.001**	**0.041**	0.937	0.335	0.418	0.783	0.605
Load ($\mathbf{U_9}$)	0.691	0.567	0.192	0.807	**0.402**	0.135	0.180
Temperature ($\mathbf{U_{10}}$)	0.107	**0.056**	**0.027**	**0.007**	0.478	0.083	0.130

The most weighted exogenous variable was temperature Lag 3 (-0.58) followed by temperature Lag 1 (-0.48). Table 6.3 presents the summary statistics for the ISEM ARIMAX(7, 1, 6) correlated model. Observing the p-values, the following is noted: historical electricity price, system generation, demand, CO2 intensity, and CO2 emissions were significant for current inputs; historical electricity price, CO2 emissions, and temperature were significant for Lag 1; Moyle interconnector and temperature were significant for Lag 2; demand and temperature were significant for Lag 3; load was significant for Lag 22; system generation and demand were significant for Lag 23, and historical electricity price and demand were significant for Lag 24. Wind and East-West interconnector were the only two variables not significant in the correlated model. The original ISEM ARIMAX model had wind as a significant variable and five variables not considered significant: system generation, East-West interconnector, Moyle interconnector, CO2 intensity, and CO2 emissions.

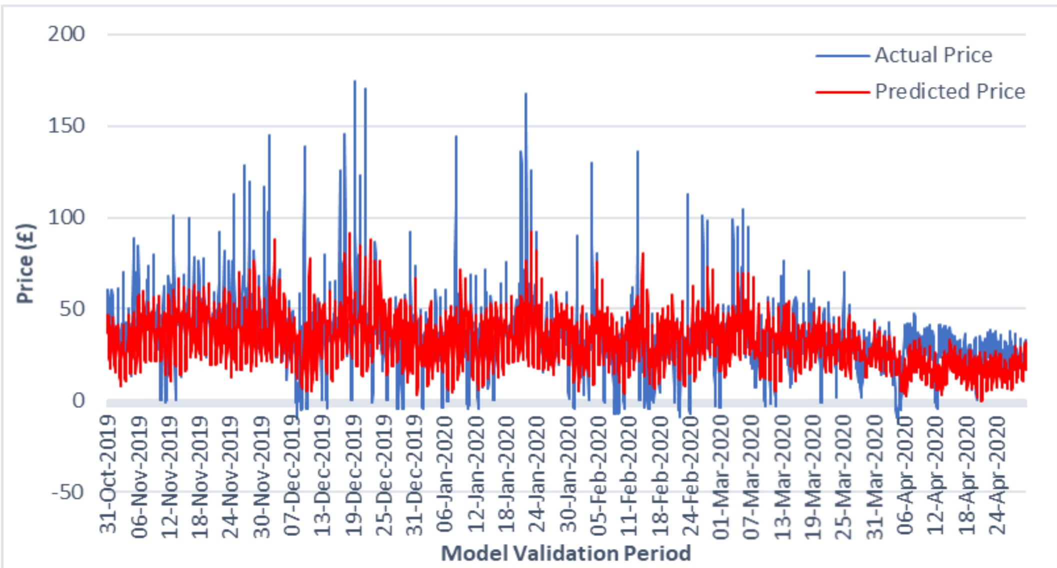

Figure 6.12: ISEM market correlated lags ARIMAX(7,1,6) model

Figure 6.12 illustrates the actual and predicted price values for the ARIMAX(7, 1, 6) model validation period. The figure shows an accurate model fit with a RMSE value of 14.47, which is a slight improvement from the original ISEM ARIMAX model performance (RMSE=14.87) in Section 5.4. Overall, the predicted electricity prices generally match the pattern of the actual prices but have some difficulty reaching all the peaks and troughs.

6.5 Correlated Lags SARIMAX Experiment

The ISEM SARIMAX model ranges are identical to the ranges set in Section 4.4 for the ISEM SARIMA experiment: *p* and *q* ranged between 1 and 4, *P* and *Q* ranged between 1 and 3, *d* was set to 1, *D* was set to 0, and *S* was set to 24. The lowest AIC value was 27478.86 with optimal order terms *p*=3, *q*=3, *P*=2, and *Q*=1. The SARIMAX(3, 1, 3) (2, 0, 1, 24) correlated lags model function is given as:

$$Y_t = 0.023\nabla Y_{t-1} + 0.28\nabla Y_{t-2} + 0.26\nabla Y_{t-3} - 0.0019\nabla \varepsilon_{t-1} - 0.36\nabla \varepsilon_{t-2} - 0.37\nabla \varepsilon_{t-3} + 0.19S^{24}Y_{t-24} - 0.053S^{48}Y_{t-48} - 0.22S^{24}\varepsilon_{t-24} + 0.28U_1 - 0.023U_{1(t-1)} + 0.0008U_{1(t-2)} + 0.012U_{1(t-23)} + 0.10U_{1(t-24)} - 0.0004U_2 - 6.71^{e-06}U_{2(t-1)} + 0.0001U_{2(t-2)} + 0.0003U_{2(t-3)} - 6.61^{e-05}U_{2(t-22)} + 0.0006U_{2(t-23)} - 0.0002U_{2(t-24)} + 0.0014U_3 + 5.54^{e-05}U_{3(t-1)} - 2.03^{e-05}U_{3(t-2)} - 0.0009U_{3(t-3)} + 9.79^{e-05}U_{3(t-22)} + 0.0004U_{3(t-23)} - 0.0004U_{3(t-24)} + 0.0002U_4 + 2.17^{e-05}U_{4(t-1)} + 0.0004U_5 + 0.0002U_{5(t-1)} + 0.0003U_{5(t-2)} + 0.0004U_{5(t-3)} + 2.38^{e-05}U_{5(t-24)} - 0.0002U_6 + 5.92^{e-05}U_{6(t-1)} - 0.0007U_{6(t-2)} - 0.0004U_{6(t-3)} + 0.0001U_{6(t-24)} - 0.0013U_7 + 0.0006U_{7(t-1)} + 0.0006U_8 + 0.0001U_{8(t-1)} - 0.0003U_{8(t-2)} - 0.0004U_{8(t-3)} - 0.0002U_{8(t-22)} + 0.0001U_{8(t-23)} - 5.54^{e-05}U_{8(t-24)} + 0.0002U_9 + 0.0016U_{9(t-1)} + 0.0011U_{9(t-2)} + 0.0001U_{9(t-3)} - 0.0005U_{9(t-22)} - 0.0003U_{9(t-23)} + 0.0002U_{9(t-24)} - 0.38U_{10} + 0.48U_{10(t-1)} + 0.45U_{10(t-2)} - 0.58U_{10(t-3)} - 0.16U_{10(t-22)} - 0.44U_{10(t-23)} - 0.38U_{10(t-24)} + 30.77 \tag{6.3}$$

Temperature Lag 3 is the most weighted exogenous variable (-0.58) and temperature Lag 1 is the second most weighted variable (0.48). The summary statistics for SARIMAX(3, 1, 3)(2, 0, 1, 24) model are displayed in Table 6.4. From the p-values the following can be determined: historical electricity price and demand were significant for current inputs; load was significant for Lag 1; demand and temperature were significant for Lag 3; and system generation was significant for Lag 23. Overall, 5 variables were significant in the correlated model results (historical electricity price, system generation, demand, load, and temperature). These are similar findings to the original SARIMAX model in Section 5.5 with the addition of system generation and load.

Table 6.4: ISEM market correlated lags SARIMAX(3,1,3)(2,0,1,24) model summary statistics

Variable (Model Term)	**Current p-value**	**Lag 1 p-value**	**Lag 2 p-value**	**Lag 3 p-value**	**Lag 22 p-value**	**Lag 23 p-value**	**Lag 24 p-value**
Historical Electricity Price (**U_1**)	**0.012**	0.083	0.959			0.424	0.070
System Generation (**U_2**)	0.194	0.981	0.680	0.202	0.759	**0.006**	0.426
Demand (**U_3**)	**<0.001**	0.889	0.961	**0.002**	0.830	0.287	0.252
Wind (**U_4**)	0.527	0.953					
East-West Interconnector (**U_5**)	0.121	0.529	0.368	0.126			0.926
Moyle Interconnector (**U_6**)	0.575	0.875	0.080	0.270			0.717
CO2 Intensity (**U_7**)	0.523	0.785					
CO2 Emissions (**U_8**)	0.322	0.816	0.538	0.366	0.669	0.770	0.887
Load (**U_9**)	0.729	**0.031**	0.172	0.873	0.317	0.693	0.778
Temperature (**U_{10}**)	0.152	0.094	0.058	**0.016**	0.530	0.121	0.193

The model is an accurate fit with a RMSE value of 14.36, which is slightly higher from the original ISEM SARIMAX model performance (RMSE=14.32). Figure 6.13 shows the actual and predicted electricity prices plotted for the SARIMAX(3, 1, 3)(2, 0, 1, 24) model. The model fit is reasonably accurate with no extreme declines in the consistency between actual and predicted values.

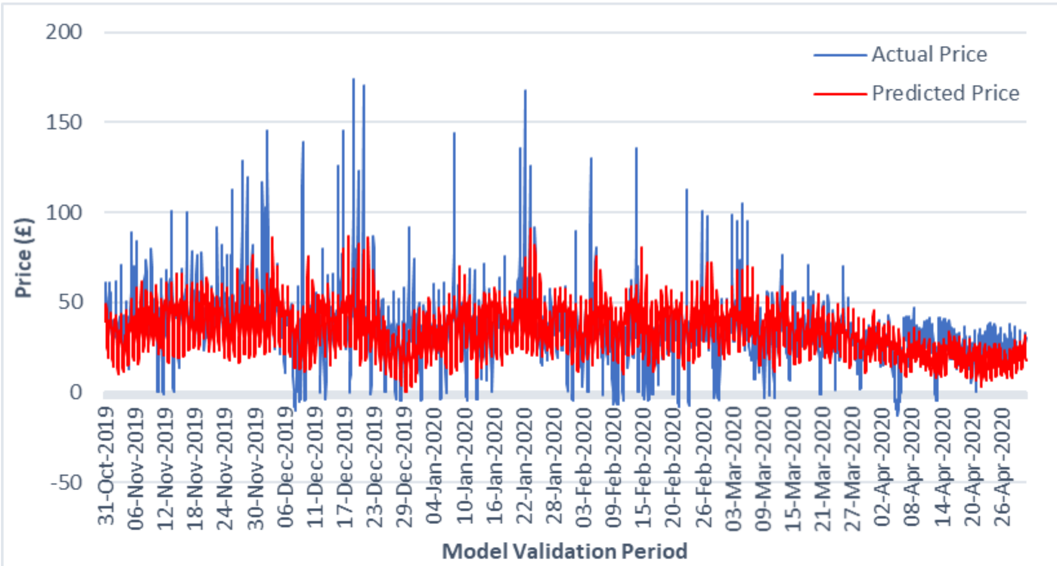

Figure 6.13: ISEM market correlated lags SARIMAX(3,1,3)(2,0,1,24) model

6.6 Correlated Lags NARMAX Experiment

A linear polynomial NARMAX model including correlated lags with **E**rror **R**eduction **R**atio (ERR) set to 0.05 was considered in this experiment. All energy-related inputs and correlated peak lags were included and, once the required iterations were reached, the final optimal model contained only the significant key exogenous variables. The model function of the linear NARMAX correlated model is given as:

$$Y_t = 0.32U_1 - 0.074U_{1(t-2)} + 0.032U_{1(t-23)} + 0.029U_{1(t-24)} - 0.0020U_2 + 0.0010U_{2(t-1)} + 0.0002U_{2(t-2)} + 0.0006U_{2(t-3)} - 0.0008U_{2(t-22)} + 0.0007U_{2(t-23)} +$$

$$0.0016U_3 - 0.0015U_{3(t-1)} + 0.0033U_{3(t-2)} - 0.0036U_{3(t-3)} - 0.0012U_{3(t-22)} + 0.0005U_5 + 0.0038U_7 + 0.0015U_8 - 0.0005U_{8(t-1)} - 0.0021U_{8(t-2)} + 0.0028U_9 + 0.0021U_{9(t-1)} - 2.52U_{10} + 2.54U_{10(t-1)} + 0.37U_{10(t-22)} + 12.34 \quad (6.4)$$

The model estimation RMSE value was 12.10 which is an improvement from the ISEM original NARMAX model (RMSE=12.58) in Section 5.6. The model validation RMSE value was 15.02 which is a slight improvement over the original RMSE value (15.15) in Section 5.6. Temperature Lag 1 was the most weighted factor (2.54) and historical electricity price had the largest ERR ranking (30.65). The final model had 8 significant factors and 17 lagged versions consisting of historical electricity price, system generation, demand, East-West interconnector, CO2 intensity, CO2 emissions, load, and temperature. Like the original NARMAX results, wind and Moyle interconnector were found to be insignificant and were completely removed from the final NARMAX model. However the East-West interconnector was now significant for current value only.

Table 6.5: Error Reduction Ratio for ISEM market correlated lags NARMAX model

ERR	Energy-Related Factors (Model Term)
30.653281	Historical electricity price (**U_1**)
2.673473	Demand (**U_3**)
0.879478	System generation (**U_2**)
0.670583	Historical electricity price lag 23 (**U_1**)
0.668477	Demand lag 3 (**U_3**)
0.591910	Historical electricity price lag 24 (**U_1**)
0.484539	System generation lag 23 (**U_2**)
0.476113	Demand lag 22 (**U_3**)
0.453832	Demand lag 2 (**U_3**)
0.451905	System generation lag 3 (**U_2**)
0.443696	Temperature lag 1 (**U_{10}**)
0.353654	Demand lag 1 (**U_3**)
0.298665	Load (**U_9**)
0.262550	Historical electricity price lag 2 (**U_1**)
0.236631	CO2 emissions lag 1 (**U_8**)
0.214814	Temperature (**U_{10}**)
0.199812	CO2 emissions lag 2 (**U_8**)
0.166720	System generation lag 2 (**U_2**)
0.110188	Temperature lag 22 (**U_{10}**)
0.098272	CO2 intensity (**U_7**)
0.091632	System generation lag 1 (**U_2**)
0.081114	East-West interconnector (**U_5**)
0.068008	System generation lag 22 (**U_2**)
0.065651	Load lag1 (**U_9**)
0.050782	CO2 emissions (**U_8**)

Table 6.5 displays the percentage variance of each significant variable, ranking the ERR terms from largest to smallest. The three largest ERR values approximated to a 34.20 proportion of the day-ahead price variance and these three values were made up of the model terms from historical electricity price (30.65), demand (2.67), and system generation (0.88). Considering historical electricity price appeared in the three largest ERR for both the original and correlated

NARMAX model suggests that historical electricity price has a significant influence on predicting electricity prices in the ISEM market. Figure 6.14 displays the model validation period (31st October 2019 Hour 12 until 30th April 2020 Hour 23) for the optimal ISEM NARMAX model with correlated lags. The predicted price values consistently followed the pattern of the actual price values but failed to reach the peaks and troughs.

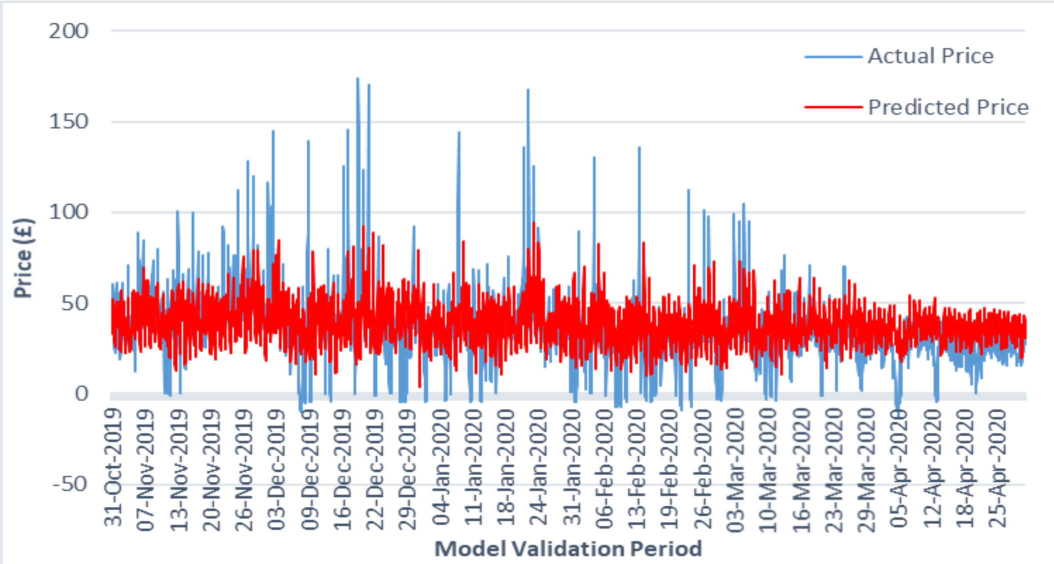

Figure 6.14: ISEM market correlated lags NARMAX model

Table 6.6 displays the RMSE values of the optimal ARMAX, SARIMAX, and NARMAX original and correlated lags models for the ISEM market. The ARMAX prediction model accuracy greatly improved and the NARMAX slightly improved with the inclusion of correlated lags. The SARIMAX correlated lags model was not as accurate as the original SARIMAX model, however the RMSE values are similar.

Table 6.6: ISEM original and correlated lags models RMSE values

Type	Model	RMSE
Original	ARMAX(3, 9)	18.73
	ARIMAX(1, 1, 9)	14.87
	SARIMAX(2, 1, 2)(2, 0, 2, 24)	14.32
	NARMAX	15.15
Correlated	ARMAX(1, 9)	14.66
	ARIMAX(1, 1, 9)	14.47
	SARIMAX(3, 1, 3)(2, 0, 1, 24)	14.36
	NARMAX	15.02

6.7 Refined ARMAX Experiment

An approach was applied to determine if the significant NARMAX factors could refined the statistical ARMAX model and improve model accuracy. The model function for a refined ARMAX(3, 9) consists of autoregressive lags for Y, moving average lags for prediction error ε, exogenous input values for U and is given as:

$$Y_t = 0.70Y_{t-1} - 0.32Y_{t-2} + 0.44Y_{t-3} + 0.30\varepsilon_{t-1} + 0.52\varepsilon_{t-2} + 0.0070\varepsilon_{t-3} + 0.023\varepsilon_{t-4} - 0.092\varepsilon_{t-5} - 0.039\varepsilon_{t-6} - 0.035\varepsilon_{t-7} + 0.0038\varepsilon_{t-8} + 0.090\varepsilon_{t-9} + 0.33U_1 - 0.0006U_2 + 0.0022U_3 - 0.0031U_7 + 0.0005U_8 + 0.0008U_9 - 0.44U_{10} + 2.71 \quad (6.5)$$

Table 6.7 displays the summary results for the refined ISEM ARMAX(3, 9) model. Temperature was the most weighted variable (0.44) and historical electricity price was the second most weighted variable (0.33). This is the same outcome as the original ISEM ARMAX(3, 9) model from Section 5.3. All variables have significant p-values apart from CO2 intensity and CO2 emissions. Previously CO2 intensity was significant in the original model, however in the refined model it is narrowly insignificant with a p-value of 0.054.

Table 6.7: ISEM market refined ARMAX(3,9) model summary statistics

Variable (Model Term)	**p-value**
Historical Electricity Price (**U_1**)	**<0.001**
System Generation (**U_2**)	**0.006**
Demand (**U_3**)	**<0.001**
CO2 Intensity (**U_7**)	0.054
CO2 Emissions (**U_8**)	0.243
Load (**U_9**)	**0.029**
Temperature (**U_{10}**)	**0.008**

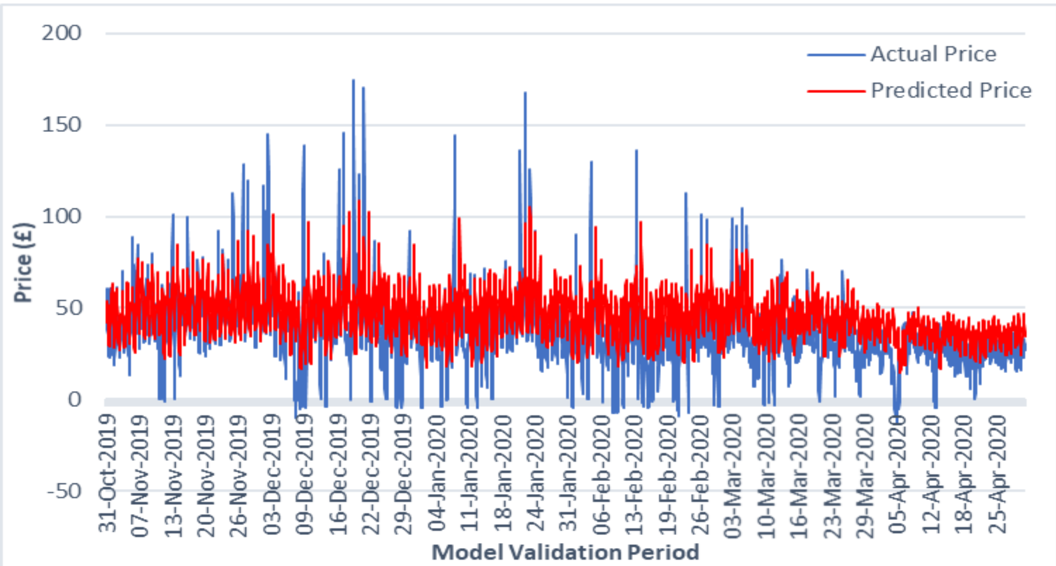

Figure 6.15: ISEM market refined ARMAX(3,9) model

The RMSE value for the refined model was 18.20 which is a slight improvement in model accuracy compared with the RMSE value of 18.73 in the original model. Figure 6.15 illustrates the refined ARMAX(3, 9) model validation period for the ISEM market. The predicted price values match the pattern of the actual price values but struggle to reach the peaks and troughs. This suggests that even though the refined ARMAX model slightly improves model performance, the ISEM market is still difficult to forecast.

To compare model performance of the NARMAX, original ARMAX, and refined ARMAX models, the RMSE values are presented in Table 6.8. Observing the results, when only the significant NARMAX factors were considered in the refined ARMAX model, the RMSE value slightly reduced and therefore model accuracy improved. The refined ISEM ARMAX model does not perform better than the ISEM NARMAX model. Nonetheless these results emphasise that utilising a NARMAX model to find key external factors and then applying these factors as inputs in popular statistical models enhances overall model accuracy.

Table 6.8: RMSE values for original and refined ARMAX models

Model	**RMSE**
NARMAX	15.15
ARMAX(3, 9)	18.73
Refined ARMAX(3, 9)	18.20

6.8 Refined ARIMAX Experiment

The model function for refined ARIMAX(1, 1, 9) is given as:

$$Y_t = 0.16Y_{t-1} - 0.10\varepsilon_{t-1} - 0.070\varepsilon_{t-2} - 0.11\varepsilon_{t-3} - 0.018\varepsilon_{t-4} - 0.14\varepsilon_{t-5} - 0.069\varepsilon_{t-6} - 0.025\varepsilon_{t-7} - 0.015\varepsilon_{t-8} + 0.030\varepsilon_{t-9} + 0.32U_1 - 0.0002U_2 + 0.0021U_3 - 0.0032U_7 + 0.0002U_8 + 0.0008U_9 - 0.77U_{10} + 31.56 \quad (6.6)$$

Table 6.9: ISEM market refined ARIMAX(1,1,9) model summary statistics

Variable (Model Term)	**p-value**
Historical Electricity Price (**U_1**)	**<0.001**
System Generation (**U_2**)	0.294
Demand (**U_3**)	**<0.001**
CO2 Intensity (**U_7**)	0.072
CO2 Emissions (**U_8**)	0.682
Load (**U_9**)	**0.006**
Temperature (**U_{10}**)	**<0.001**

Table 6.9 displays the summary results for the refined ISEM ARIMAX(1, 1, 9) model. Temperature was the most weighted variable (0.77) and historical electricity price was the second most

weighted variable (0.16). This is the same outcome as the original ISEM ARIMAX(1, 1, 9) model from Section 5.4. All variables have significant p-values apart from system generation, CO2 intensity, and CO2 emissions. The RMSE value for the refined model was 14.86 which is a similar finding to the RMSE value of 14.87 in the original model. Figure 6.16 illustrates the refined ARIMAX(1, 1, 9) model validation period. The predicted price values generally match the pattern of the actual price values but struggle to reach the peaks and troughs. Overall the refined ARIMAX model can accurately predict day-ahead ISEM electricity prices.

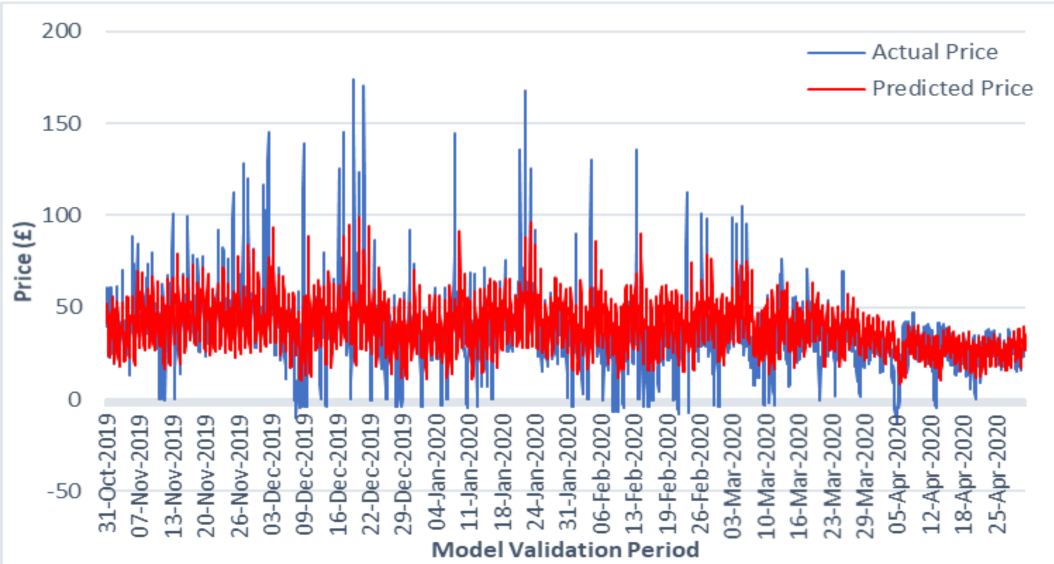

Figure 6.16: ISEM market refined ARMAX(1,1,9) model

The RMSE values of the NARMAX, original ARIMAX, and refined ARIMAX models are presented in Table 6.10. Observing the results all models have a similar accuracy, nonetheless NARMAX performed the worst and the refined ARIMAX performed the best. This highlights that only keeping significant NARMAX factors as model inputs does improve model performance.

Table 6.10: RMSE values for original and refined ARIMAX models

Model	RMSE
NARMAX	15.15
ARIMAX(1, 1, 9)	14.87
Refined ARIMAX(1, 1, 9)	14.86

6.9 Refined SARIMAX Experiment

The refined SARIMAX(2, 1, 2)(2, 0, 2, 24) model function for the ISEM market is given as:

$$Y_t = 0.54\nabla Y_{t-1} + 0.27\nabla Y_{t-2} - 0.49\nabla\varepsilon_{t-1} - 0.42\nabla\varepsilon_{t-2} + 0.29S^{24}Y_{t-24} + 0.30S^{48}Y_{t-48} - 0.31S^{24}\varepsilon_{t-24} - 0.20S^{48}\varepsilon_{t-48} + 0.24U_1 - 9.22^{e-05}U_2 + 0.0026U_3 - 5.39^{e-05}U_7 + 0.0001U_8 + 0.0003U_9 - 0.77U_{10} + 32.14 \quad (6.7)$$

Table 6.11 presents the summary statistics with temperature being the most weighted variable (0.77) followed by historical electricity price (0.24). This was similar to the most weighted variables from the original ISEM SARIMAX(2, 1, 2)(2, 0, 2, 24) model results in Section 5.5. Observing the p-values: historical electricity price, demand, and temperature were highly significant (0.000) which is similar to the original model results apart from the East-West interconnector, which was not included in the refined model.

Table 6.11: ISEM market refined SARIMAX(2,1,2)(2,0,2,24) model summary statistics

Variable (Model Term)	**p-value**
Historical Electricity Price (**U_1**)	**<0.001**
System Generation (**U_2**)	0.691
Demand (**U_3**)	**<0.001**
CO2 Intensity (**U_7**)	0.978
CO2 Emissions (**U_8**)	0.789
Load (**U_9**)	0.341
Temperature (**U_{10}**)	**<0.001**

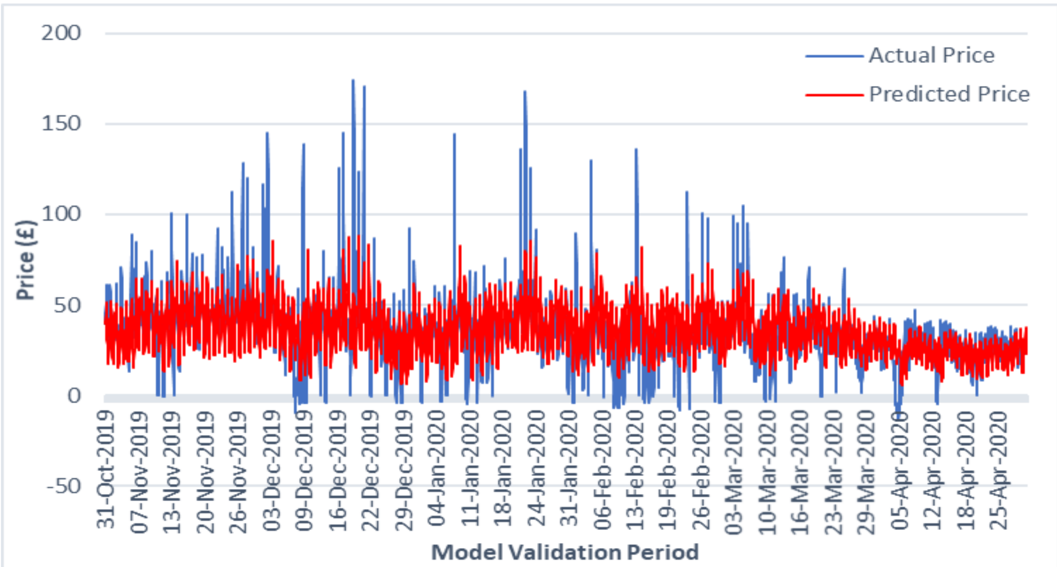

Figure 6.17: ISEM market refined SARIMAX(2,1,2)(2,0,2,24) model

The refined model resulted in a RMSE value of 14.10 which is slightly lower than the original model RMSE value of 14.32. Figure 6.17 displays the model validation period for the refined SARIMAX(2, 1, 2)(2, 0, 2, 24) model and, compared with the original model results (Figure 5.12 in Section 5.5), the predicted price values accurately match more of the actual price values. Therefore the refined ISEM SARIMAX does improve model performance and achieves a reasonable forecast. Table 6.12 presents the RMSE values of the NARMAX, original SARIMAX, and refined SARIMAX models for the ISEM energy market. For this table of results, NARMAX performed the worst with the refined SARIMAX being the optimal model, highlighting that significant NARMAX factors do improve model performance of statistical models.

Table 6.12: RMSE values for original and refined SARIMAX models

Model	**RMSE**
NARMAX	15.15
SARIMAX(2, 1, 2)(2, 0, 2, 24)	14.32
Refined SARIMAX(2, 1, 2)(2, 0, 2, 24)	14.10

6.10 Refined Correlated Lags ARMAX Experiment

Even though the NARMAX correlated lags model performed poorer than ARMAX(1, 9) with correlated lags, the ARMAX correlated model was refined to only include the significant NARMAX factors. The model function for predicted Y_t is given as:

$$Y_t = 0.81Y_{t-1} + 0.15\varepsilon_{t-1} + 0.070\varepsilon_{t-2} + 0.010\varepsilon_{t-3} + 0.84\varepsilon_{t-4} - 0.033\varepsilon_{t-5} + 0.0025\varepsilon_{t-6} + 0.045\varepsilon_{t-7} + 0.046\varepsilon_{t-8} + 0.090\varepsilon_{t-9} + 0.30U_1 + 0.018U_{1(t-2)} + 0.023U_{1(t-23)} + 0.074U_{1(t-24)} - 0.0009U_2 - 0.0001U_{2(t-1)} + 6.17^{e-05}U_{2(t-2)} + 0.0002U_{2(t-3)} - 0.0003U_{2(t-22)} + 0.0004U_{2(t-23)} + 0.0013U_3 + 0.0002U_{3(t-1)} + 0.0002U_{3(t-2)} - 0.0012U_{3(t-3)} - 9.37^{e-05}U_{3(t-22)} + 0.0004U_5 - 0.0057U_7 + 0.0010U_8 + 0.0002U_{8(t-1)} - 0.0002U_{8(t-2)} + 0.0006U_9 + 0.0013U_{9(t-1)} - 2.47U_{10} + 2.33U_{10(t-1)} + 0.30U_{10(t-22)} + 12.35 \quad (6.8)$$

Table 6.13: ISEM market refined correlated lags ARMAX(1,9) model summary statistics

Variable (Model Term)	**Current p-value**	**Lag 1 p-value**	**Lag 2 p-value**	**Lag 3 p-value**	**Lag 22 p-value**	**Lag 23 p-value**	**Lag 24 p-value**
Historical Electricity Price (**U_1**)	**<0.001**		0.246			0.115	**<0.001**
System Generation (**U_2**)	**<0.001**	0.537	0.756	0.278	**0.048**	**0.005**	
Demand (**U_3**)	**0.001**	0.400	0.509	**<0.001**	0.599		
East-West Interconnector (**U_5**)	0.117						
CO2 Intensity (**U_7**)	**0.001**						
CO2 Emissions (**U_8**)	**0.020**	0.530	0.605				
Load (**U_9**)	0.154	**0.045**					
Temperature (**U_{10}**)	**<0.001**	**<0.001**			0.114		

Table 6.13 displays the summary statistics for the refined ARMAX(1, 9) correlated lags model. Temperature remains the most weighted variable (-2.47) from the summary output. Historical electricity price, system generation, demand, CO2 intensity, CO2 emissions, and temperature were significant for current inputs; load and temperature were significant for Lag 1; demand was significant for Lag 3; system generation was significant for Lag 22; system generation was significant for Lag 23, and historical electricity price was significant for Lag 24. Current and Lag

1 inputs had the same significant variables as the correlated lags model and the East-West interconnector still remained not significant. The RMSE value of 15.18 was higher than the RMSE value of 14.66 for the ARMAX(1, 9) correlated lags model. Figure 6.18 illustrates the refined ARMAX(1, 9) correlated lags model fit which is reasonably accurate but compared with Figure 6.11 the model performance has decreased.

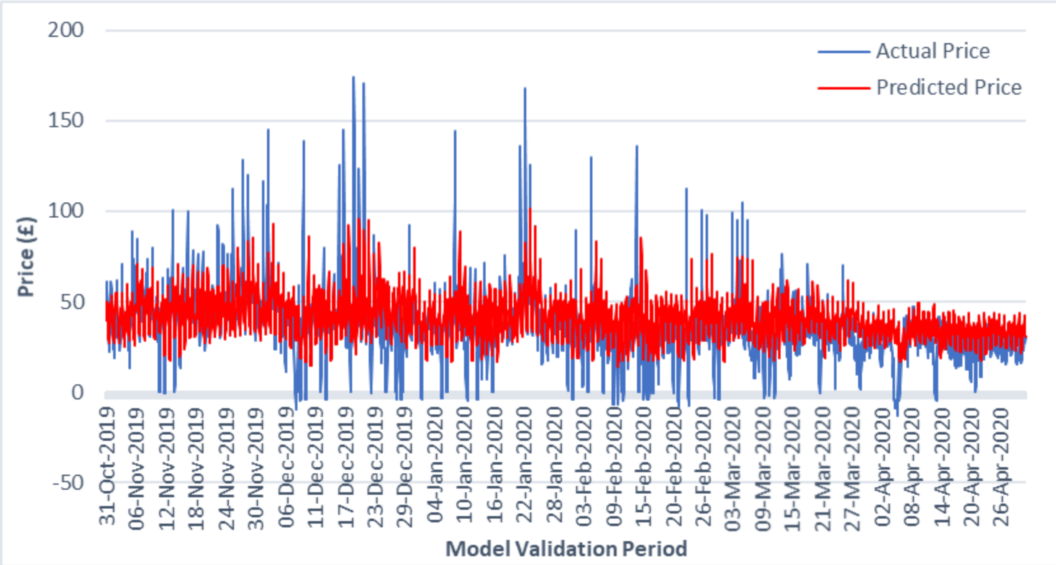

Figure 6.18: ISEM market refined correlated lags ARMAX(1,9) model

6.11 Refined Correlated Lags ARIMAX Experiment

The ARIMAX(7, 1, 6) correlated model was refined to only include the significant NARMAX factors. The model function for predicted Y_t is given as:

$$Y_t = 0.28Y_{t-1} + 0.18Y_{t-2} - 0.21Y_{t-3} + 0.24Y_{t-4} + 0.25Y_{t-5} - 0.078Y_{t-6} + 0.072Y_{t-7} - 0.27\varepsilon_{t-1} - 0.28\varepsilon_{t-2} + 0.10\varepsilon_{t-3} - 0.21\varepsilon_{t-4} - 0.37\varepsilon_{t-5} + 0.038\varepsilon_{t-6} + 0.29U_1 + 0.0040U_{1(t-2)} + 0.014U_{1(t-23)} + 0.094U_{1(t-24)} - 0.0004U_2 - 2.53^{e-05}U_{2(t-1)} + 0.0003U_{2(t-2)} + 0.0003U_{2(t-3)} - 0.0001U_{2(t-22)} + 0.0005U_{2(t-23)} + 0.0011U_3 + 0.0004U_{3(t-1)} - 7.24^{e-08}U_{3(t-2)} - 0.0009U_{3(t-3)} - 2.21^{e-05}U_{3(t-22)} + 0.0004U_5 - 0.0013U_7 + 0.0004U_8 + 0.0002U_{8(t-1)} - 0.0002U_{8(t-2)} + 0.0005U_9 + 0.0014U_{9(t-1)} - 0.48U_{10} + 0.30U_{10(t-1)} - 0.37U_{10(t-22)} + 30.45 \quad (6.9)$$

Table 6.14 displays the summary statistics for the refined ARIMAX(7, 1, 6) correlated lags model. Temperature was the most weighted variable (-0.48) from the summary output. Historical electricity price, system generation, demand, and temperature were significant for current inputs; load was significant for Lag 1; demand was significant for Lag 3; system generation and temperature were significant for Lag 22; system generation was significant for Lag 23, and historical electricity price was significant for Lag 24. Like the correlated lags model findings, the

East-West interconnector remained not significant. However in the refined model CO2 intensity and CO2 emissions are no longer significant.

Table 6.14: ISEM market refined correlated lags ARIMAX(7,1,6) model summary statistics

Variable (Model Term)	**Current p-value**	**Lag 1 p-value**	**Lag 2 p-value**	**Lag 3 p-value**	**Lag 22 p-value**	**Lag 23 p-value**	**Lag 24 p-value**
Historical Electricity Price (**U_1**)	**<0.001**		0.798			0.309	**<0.001**
System Generation (**U_2**)	**0.097**	0.896	0.237	0.130	**0.449**	**0.001**	
Demand (**U_3**)	**0.002**	0.063	1.000	**<0.001**	0.923		
East-West Interconnector (**U_5**)	0.070						
CO2 Intensity (**U_7**)	0.546						
CO2 Emissions (**U_8**)	0.464	0.653	0.574				
Load (**U_9**)	0.217	**0.038**					
Temperature (**U_{10}**)	**0.049**	0.219			**0.090**		

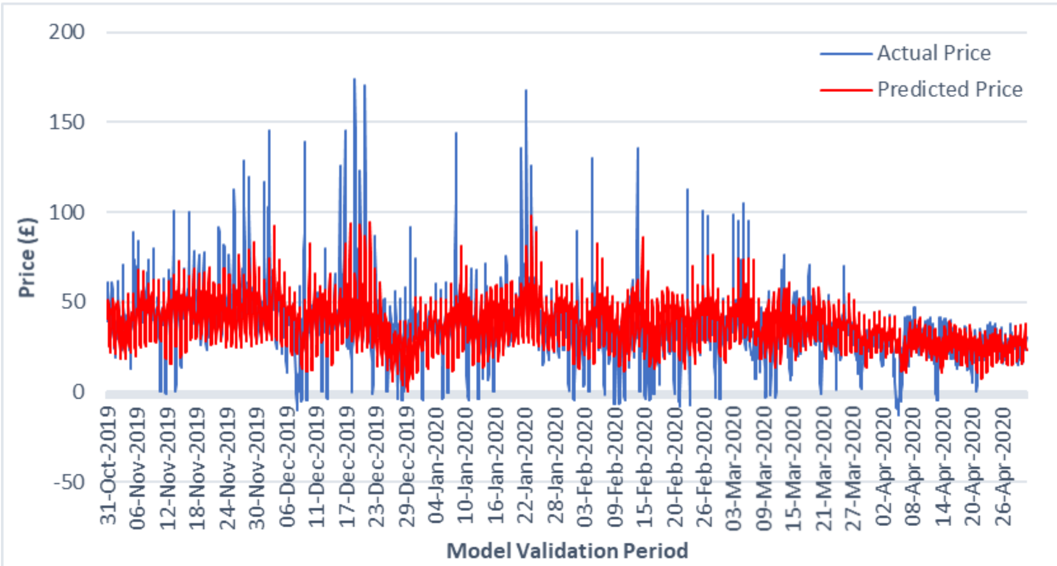

Figure 6.19: ISEM market refined correlated lags ARIMAX(7,1,6) model

The RMSE value of 13.99 was lower than the RMSE value of 14.47 for the ARIMAX(7, 1, 6) correlated lags model. Figure 6.19 illustrates the refined ARIMAX(7, 1, 6) correlated lags model fit which is reasonably accurate and compared with Figure 6.12 the model performance has improved.

6.12 Refined Correlated Lags SARIMAX Experiment

The SARIMAX(3, 1, 3)(2 ,0, 1, 24) correlated lags model was refined and the model function is given as:

$$Y_t = 0.065\nabla Y_{t-1} + 0.27\nabla Y_{t-2} + 0.20\nabla Y_{t-3} - 0.031\nabla\varepsilon_{t-1} - 0.36\nabla\varepsilon_{t-2} - 0.33\nabla\varepsilon_{t-3} + 0.17S^{24}Y_{t-24} - 0.053S^{48}Y_{t-48} - 0.19S^{24}\varepsilon_{t-24} + 0.30U_1 + 0.035U_{1(t-2)} +$$

$0.013U_{1(t-23)} + 0.11U_{1(t-24)} - 0.0004U_2 - 5.36^{e-05}U_{2(t-1)} + 0.0003U_{2(t-2)} + 0.0004U_{2(t-3)} - 0.0002U_{2(t-22)} + 0.0006U_{2(t-23)} + 0.0013U_3 + 0.0003U_{3(t-1)} - 3.04^{e-05}U_{3(t-2)} - 0.0010U_{3(t-3)} - 0.0001U_{3(t-22)} + 0.0004U_5 - 0.0042U_7 + 0.0005U_8 + 0.0003U_{8(t-1)} - 0.0003U_{8(t-2)} + 0.0005U_9 + 0.0008U_{9(t-1)} - 0.48U_{10} + 0.30U_{10(t-1)} - 0.37U_{10(t-22)} + 31.09$ (6.10)

The summary statistics for the refined SARIMAX(3, 1, 3)(2, 0, 1, 24) correlated lags model are presented in Table 6.15. Temperature is the most weighted variable (-0.48) closely followed by historical electricity price (0.30). Historical electricity price, demand, CO2 intensity, and temperature were significant for current inputs; system generation and demand were significant for Lag 3; and system generation was significant for Lag 23. Lag 23 had the same significant variable as the correlated lags model and overall five variables still remained significant, however load was replaced by CO2 intensity.

Table 6.15: ISEM market refined correlated lags SARIMAX(3,1,3)(2,0,1,24) model summary statistics

Variable (Model Term)	**Current p-value**	**Lag 1 p-value**	**Lag 2 p-value**	**Lag 3 p-value**	**Lag 22 p-value**	**Lag 23 p-value**	**Lag 24 p-value**
Historical Electricity Price (**U_1**)	**0.001**		0.819			0.364	0.061
System Generation (**U_2**)	0.116	0.784	0.175	**0.037**	0.348	**<0.001**	
Demand (**U_3**)	**<0.001**	0.099	0.885	**<0.001**	0.533		
East-West Interconnector (**U_5**)	0.065						
CO2 Intensity (**U_7**)	**0.035**						
CO2 Emissions (**U_8**)	0.404	0.403	0.565				
Load (**U_9**)	0.271	0.240					
Temperature (**U_{10}**)	**0.045**	0.216			0.078		

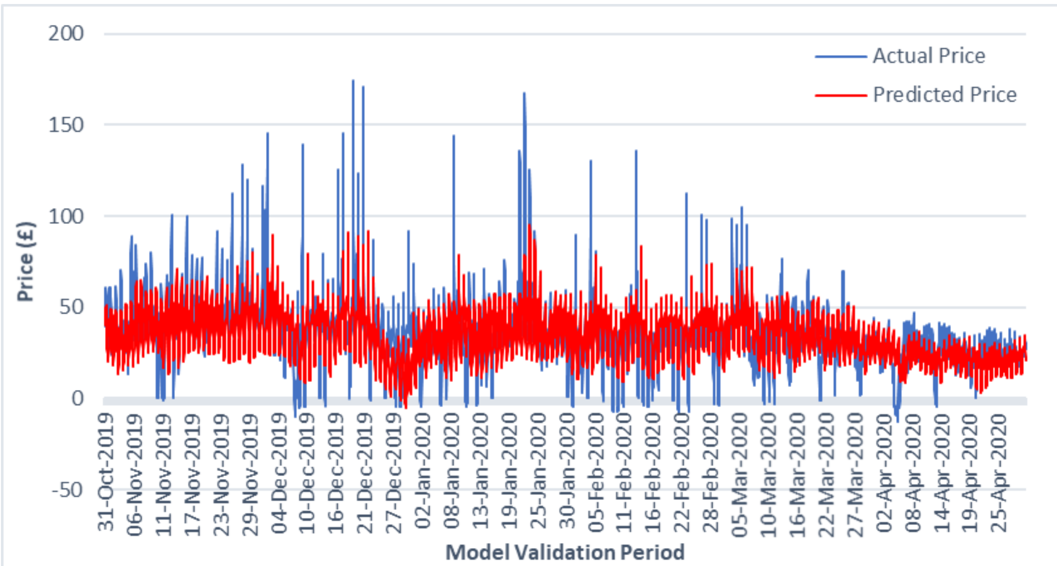

Figure 6.20: ISEM market refined correlated lags SARIMAX(3,1,3)(2,0,1,24) model

The RMSE value of 13.99 was lower than the RMSE value of 14.36 for the SARIMAX(3, 1, 3)(2, 0, 1, 24) correlated lags model. Figure 6.20 displays the refined SARIMAX(3, 1, 3)(2, 0, 1, 24) correlated lags model fit between the actual and predicted price values. Compared with Figure 6.13, the model performance was similar with an accurate model fit displayed between the actual and predicted values.

6.13 Conclusion

This chapter first applied autocorrelation testing to energy-related factors from the ISEM market to determine peak correlated lags. The remainder of this chapter explored model performance of refined statistical time-series prediction models to determine if identified NARMAX significant energy-related factors and their corresponding peak lags, used as model inputs in statistical models, improved performance accuracy. Model parsimony was addressed through utilising the NARMAX model; insignificant variables are pruned as the NARMAX methodology removes non-weighted terms one at a time producing a compact parsimonious model. For all experiments, RMSE values were compared to determine if any of the correlated lags or refined models could further improve model performance and day-ahead electricity price forecasting.

For the correlated lag models, ARMAX greatly improved model performance and ARIMAX slightly improved compared to the original models; however the correlated SARIMAX model was slightly worse than the original. The refined ARMAX(3, 9) model did improve slightly in overall model performance, however the NARMAX model still generated the lowest RMSE value of 15.15. The refined ARIMAX(1, 1, 9) model displayed similar performance to the original model, with a RMSE value of 14.86. The refined SARIMAX(2, 1, 2)(2, 0, 2, 24) model improved overall model performance, with a RMSE value of 14.10. Observing the results from the refined correlated lags models, the refined ARMAX was less accurate than the original correlated model; however the refined ARIMAX and SARIMAX models had lower RMSE values and improved model performance. The four key exogenous variables for the ISEM market, which were significant in most of the statistical models, were historical electricity price, system generation, demand, and temperature. The best models in this chapter are the refined correlated ARIMAX and SARIMAX; both generated a RMSE value of 13.99.

Throughout Chapter 4, Chapter 5, and Chapter 6 various time-series prediction models were examined and applied to real ISEM energy market data. Table 6.16 displays the RMSE values for each of these statistical and regression models. Seasonal models performed the best in each of the model experiment groups. The optimal ISEM prediction model was both the refined correlated ARIMAX and SARIMAX, therefore significant factors and correlated lags together improve model performance for ISEM.

Table 6.16: RMSE values of all ISEM models

Model	RMSE
ARMA(9, 8)	14.99
ARIMA(8, 1, 8)	14.86
SARIMA(3, 1, 3)(2, 0, 2, 24)	14.12
ARMAX(3, 9)	18.73
ARIMAX(1, 1, 9)	14.87
SARIMAX(2, 1, 2)(2, 0, 2, 24)	14.32
NARMAX	15.15
Refined ARMAX(3, 9)	18.20
Refined ARIMAX(1, 1, 9)	14.86
Refined SARIMAX(2, 1, 2)(2, 0, 2, 24)	14.10
Correlated ARMAX(1, 9)	14.66
Correlated ARIMAX(7, 1, 6)	14.47
Correlated SARIMAX(3, 1, 3)(2, 0, 1, 24)	14.36
Correlated NARMAX	15.02
Refined correlated ARMAX(1, 9)	15.18
Refined correlated ARIMAX(7, 1, 6)	**13.99**
Refined correlated SARIMAX(3, 1, 3)(2, 0, 1, 24)	**13.99**

The next chapter will focus on computational modelling and examine machine learning as an alternative to the traditional statistical approaches. ISEM historical electricity price market data will be used as input for three machine learning regression algorithms. This research will also implement technical indicators, specifically derived for ISEM, as model inputs instead of the raw price data to develop day-ahead machine learning forecasting models consistent with the ISEM energy market. Finally, exploring a 24-hour model versus hourly models should distinguish key features for an efficient and accurate prediction model.

Chapter 7

Computational Models

7.1 Introduction

Computational modelling has emerged as a viable alternative to traditional statistical methods in many fields. For details on some of the computational modelling techniques available please see Chapter 3. Ensemble non-linear regression models such as Random Forest or boosting algorithms can be utilised to follow trends and forecast. In this thesis Random Forest, Gradient Boosting, and Extreme Gradient Boosting were considered. These models are commonly used in financial price forecasting: previous financial literature exploring machine learning techniques found that the majority used technical indicators [1]. Within the stock market, machine learning algorithms and technical indicators are often combined to establish relationships and reach profitable stock returns [5]. Technical indicators are popular in short-term financial trading to forecast stock market prices and are more desirable than fundamental techniques [77] due to their suitability for discovering information regarding future prices [79]. The three main types of technical indicators are momentum, oscillator, and trend [3]. Energy market data have comparable characteristics and behaviours to financial data and therefore developing energy market technical indicators and including them as inputs in forecasting models would aid traders in observing market trends to understand when to buy or sell electricity units and over time reduce purchasing costs.

This chapter explores the fourth and fifth research questions listed in Chapter 1: "Can prediction accuracy be improved by developing representative energy-related technical indicators compared with the use of electricity prices?" and "Can model performance be improved by building on the strengths of statistical models and machine learning models?". This chapter is based on the following publications:

- C. McHugh, S. Coleman, and D. Kerr, "Technical Indicators for Hourly Energy Market Trading", in Proceedings of The Ninth International Conference on Data Analytics, Data Analytics 2020, pp. 72-77.
- C. McHugh, S. Coleman, and D. Kerr, "Technical Indicators and Prediction for Energy Market Forecasting", in Proceedings of 19th IEEE International Conference on Machine Learning and Applications, ICMLA 2020, pp. 1241-1246.
- C. McHugh, S. Coleman, and D. Kerr, "Technical Indicators for Energy Market Trading", in Elsevier Machine Learning with Applications, vol. 6, 2021.

Recapping on the machine learning methods outlined in Section 3.4, a successful machine learning process is implemented in this chapter through Jansen's key steps of understanding model inputs, selecting suitable algorithms, and training/testing prediction models [102]. Three regression machine learning algorithms are considered: Random Forest, Gradient Boosting, and E**x**treme **G**radient (XG)Boost. These algorithms are trained and tested with Integrated Single Electricity Market (ISEM) market raw hourly electricity price data ranging from a core window between September 2019 until March 2020 downloaded from the SEMOpx website [110]. The findings are compared in Section 7.2 against results obtained in previous chapters to determine if machine learning algorithms are more robust than statistical techniques. Additionally, a set of novel technical indicators is derived in Section 7.3 for the ISEM using electricity price data from 01st February 2019 until 31st March 2020. The technical indicators are used as model inputs in Section 7.4 to first build one single 24-hour day-ahead price prediction system. Building on this, 24 individual 1-hour models for day-ahead prediction were developed. The technical indicator driven models will be compared against the persistence raw electricity price models to see if model performance improves. The overall objective is to determine the optimal approach for predicting electricity prices using machine learning. The software utilised for the experiments was Python through the SkLearn library.

7.2 Machine Learning Persistence Models

Machine learning algorithms can reflect market trends by developing optimal price forecasting models. Persistence models are those that are trained with raw data to forecast and have the assumption that between actual time and the future, the conditions remain the same [119]. The Random Forest algorithm, described in Section 3.4.1, is an ensemble regression technique that is adaptable, avoids overfitting, provides easy tuning, and is robust to outliers [98]. Boosting algorithms build one strong model through sequential learning by merging weak models [120]; Gradient Boosting methods are described in Section 3.4.2 and XGBoost methods are described in Section 3.4.3.

For this experiment the Random Forest algorithm was implemented on SkLearn with 1000 trees and no pruning, and the algorithm default parameters for minimum sample split of 2 and minimum sample leaf of 1 applied. The Gradient Boosting algorithm was implemented on SkLearn with 1000 trees and the algorithm default parameters were applied with minimum sample split of 2, minimum sample leaf of 1, and a learning rate of 0.1. The XGBoost algorithm was implemented on SkLearn with 1000 trees and parameters for model optimisation were set through a heuristic approach: minimum sample split was set to 500, minimum sample leaf set to 50, maximum depth of tree set to 4, learning rate set to 0.05, alpha set to 10, column fraction set to 0.6, and observation fraction set to 0.8.

Historical hourly electricity price data from the ISEM market were used as model inputs to train the three regression algorithms. Model input was electricity price at Hour T (08th September 2019 to 30th March 2020) and model output was electricity price at Hour T+24 hours (09th September 2019 to 31st March 2020). 85% of the data were used for training (09th September 2019 to 01st March 2020) and 15% of the data were used for testing (01st March 2020 to 31st March 2020). Actual electricity price values for the testing period are displayed in Figure 7.1 (738 hours).

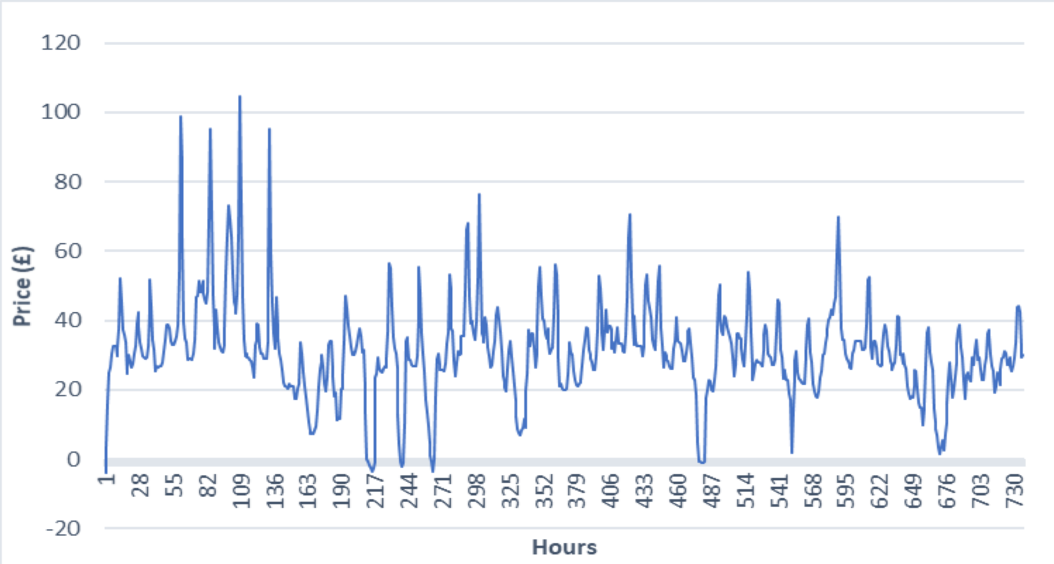

Figure 7.1: Testing period from 1st March 2020 to 31st March 2020

To evaluate model performance four performance metrics were computed with respect to actual and predicted electricity price data:

- **E**xplained **V**ariance **S**core (EVS);
- **Med**ian **A**bsolute **E**rror (MedAE);
- **R**oot **M**ean **S**quared **E**rror (RMSE);
- **R**oot **M**ean **S**quared **L**og **E**rror (RMSLE).

The EVS metric signifies an excellent fit if values are close or equal to 1 and represents overfitting if it results in small or negative values [121]. The formula is given as:

$$EVS = 1 - \left[\frac{Var(Actual - Predicted)}{Var(Actual)}\right] \tag{7.1}$$

The MedAE [121] is insensitive to outliers and a low value indicates good model fit. The metric is given as:

$$MedAE = median(|Actual_1 - Predicted_1|, \dots, |Actual_n - Predicted_n|) \tag{7.2}$$

The RMSE [122] measures the distance between predicted and actual values. When predicted values closely resemble the actual values, the RMSE is small. The formula is given as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Actual_i - Predicted_i)^2} \tag{7.3}$$

The RMSLE [122] represents the difference of two logarithmic functions and a value tending towards zero indicates good model fit and robustness. The formula is given as:

$$RMSLE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(log\ (Actual_i + 1)\ - log\ (Predicted_i + 1)\ )^2} \tag{7.4}$$

Table 7.1 presents the summary results for Random Forest, Gradient Boosting, and XGBoost during both the training and testing periods. EVS ranged from 0.59 to 0.83 for the training stage and ranged from -0.29 to 0.10 for the testing stage. Random Forest had the lowest MedAE (3.45), RMSE (8.07), and RMSLE (0.27) values during training and therefore performed the best for this period. For the testing period, XGBoost performed the best overall with MedAE (7.53), RMSE (12.89), and RMSLE (0.37). Comparing the testing RMSE values to Table 6.16 in Section 6.13, the model accuracy is similar for Random Forest and the time-series model accuracy has improved for both Gradient Boosting and XGBoost as the lowest RMSE value for ISEM market in Table 6.9 was 13.99.

Table 7.1: Machine learning 24-hour models summary results

Period	**Algorithm**	**EVS (↑)**	**MedAE (↓)**	**RMSE (↓)**	**RMSLE (↓)**
Training	Random Forest	**0.83**	**3.45**	**8.07**	**0.27**
	Gradient Boosting	0.66	5.43	11.28	0.32
	XGBoost	0.59	5.94	12.44	0.34
Testing	Random Forest	-0.29	9.85	15.43	0.41
	Gradient Boosting	0.03	7.90	13.39	0.39
	XGBoost	**0.10**	**7.53**	**12.89**	**0.37**

Hourly persistence models were trained and tested with the same data period (09th September 2019 to 31st March 2020) and model split (85%/15%) as the 24-hour single models. For each hour only electricity price data were utilised with model input at Hour T and model output at Hour T+24. Table 7.2 displays the results during the testing period for Random Forest, Gradient Boosting, and XGBoost. Observing Random Forest results, the persistence models' EVS ranged from -2.16 to -0.22. Hour 21 outputted the lowest RMSLE (0.090) while Hour 22 outputted both the lowest MedAE (4.04) and lowest RMSE (6.90) for the persistence models. Examining Gradient Boosting, the persistence models' EVS ranged from -4.20 to -0.30. The lowest MedAE (4.71) was outputted at Hour 21, the lowest RMSE (8.45) at Hour 22, and the lowest RMSLE

(0.042) at Hour 10. Looking through XGBoost the persistence models' EVS ranged from -2.61 to -0.30. Hour 22 outputted the lowest MedAE (4.45) and RMSE (7.08) and the lowest RMSLE (0.095) was computed at Hour 21. It is shown from the overall averages that Random Forest was the best performing out of the three regression algorithms: lowest EVS average (-0.74), lowest RMSE average (14.35), and lowest RMSLE average (0.31).

Table 7.2: Hourly persistence models summary results

Hour	Random Forest				Gradient Boosting				XGBoost			
	EVS (↑)	MedAE (↓)	RMSE (↓)	RMSLE (↓)	EVS (↑)	MedAE (↓)	RMSE (↓)	RMSLE (↓)	EVS (↑)	MedAE (↓)	RMSE (↓)	RMSLE (↓)
0	-0.72	7.33	14.68	0.38	-1.37	9.71	17.35	0.50	-0.82	8.53	15.14	0.35
1	-0.92	9.44	15.97	0.46	-1.20	12.46	17.20	0.49	-0.95	9.30	16.09	0.46
2	-0.32	5.51	13.76	0.47	-1.03	8.12	17.02	0.71	-0.40	5.88	14.13	0.54
3	-0.30	7.79	13.84	0.71	-0.78	7.69	16.26	0.81	-0.58	8.17	15.28	0.70
4	-0.37	9.55	14.85	0.64	-0.89	9.79	17.31	0.71	-0.33	9.81	14.54	0.60
5	***-0.23***	7.51	14.32	0.69	-0.49	7.78	15.75	0.64	***-0.30***	5.83	14.70	0.81
6	-0.66	9.73	17.27	0.91	-1.38	12.90	20.46	1.02	-0.86	9.83	18.16	0.93
7	-0.61	13.99	17.97	0.49	-1.12	15.83	20.65	0.64	-0.66	15.42	18.23	0.52
8	-0.93	10.42	18.79	0.26	-1.77	15.67	22.60	0.44	-1.28	11.26	20.55	0.29
9	-0.32	10.22	14.57	0.17	***-0.30***	9.60	14.43	0.22	-0.35	9.23	14.73	0.18
10	-0.32	8.32	12.14	0.15	-0.68	8.90	13.54	***0.042***	***-0.30***	6.63	11.96	0.14
11	-0.39	8.60	11.81	0.15	-1.08	8.78	13.91	0.17	-0.42	8.96	12.03	0.15
12	-0.93	9.73	12.76	0.16	-1.71	10.97	14.69	0.18	-1.01	7.54	13.07	0.16
13	-0.86	6.29	12.22	0.15	-1.81	7.14	13.86	0.17	-0.94	7.69	12.12	0.15
14	-2.16	8.42	14.91	0.21	-3.49	10.55	17.04	0.29	-2.18	9.80	14.97	0.21
15	-1.76	9.60	14.40	0.18	-4.20	8.88	18.79	0.21	-2.61	9.78	16.44	0.19
16	-0.70	6.31	11.18	0.15	-1.22	7.28	12.42	0.16	-0.59	6.51	10.99	0.14
17	-0.60	16.04	19.16	0.19	-1.49	16.14	20.63	0.20	-0.84	17.03	18.98	0.19
18	-0.22	14.26	23.86	0.17	-0.88	13.61	29.02	0.19	-0.56	12.24	26.70	0.18
19	-0.81	10.19	18.04	0.15	-1.81	11.03	22.36	0.18	-1.22	9.04	19.95	0.16
20	-0.77	7.15	11.40	0.12	-1.52	8.31	13.39	0.14	-0.72	6.99	11.15	0.11
21	-1.18	5.99	7.73	***0.090***	-3.38	***4.71***	10.53	0.11	-1.88	5.50	8.67	***0.095***
22	-0.78	***4.04***	***6.90***	0.11	-1.65	5.48	***8.45***	0.15	-0.88	***4.45***	***7.08***	0.12
23	-0.81	9.11	11.98	0.18	-1.55	12.13	14.19	0.25	-0.82	9.05	12.03	0.19
Average	**-0.74**	8.98	**14.35**	**0.31**	-1.53	10.14	16.74	0.36	-0.90	**8.94**	14.90	0.32

Figure 7.2: Random Forest hourly persistence model on 16th March 2020

Figure 7.2 displays the actual and predicted prices on a randomly chosen date (16th March) for Random Forest hourly models to show how the model performs over one day. Even though the

Random Forest was the best performing algorithm, the hourly persistence models show poor accuracy performance as the predicted prices do not match the actual prices.

7.3 Technical Indicators

It is often appropriate to extract knowledge from raw data with feature engineering techniques rather than directly using raw data. In financial markets it is common to use a set of technical indicators to do this. Therefore, in this section a novel set of technical indicators is proposed for use with electricity market data to capture historical price behaviour. Trend price indicators specify whether the price increases or decreases by observing moving averages, oscillator price indicators represent periodic patterns, and momentum price indicators represent market power [3]. It is difficult to determine parameter optimisation in technical analysis [79] thus a dynamic window is used, and the window size, which corresponds to the amount of historical values used in the calculation of each indicator, should be carefully considered [123]. For example, for daily forecasting the window size may be 24 hours, for weekly forecasting the window size may be 168 hours, and for monthly forecasting the window size may be 744 hours. The typical requirement for ISEM is to trade in the Day-Ahead market, therefore the development of daily price technical indicators is of interest to increase prediction accuracy. In this section, a dynamic time window is applied which is not fixed and varies across a 24-hour period.

The eight innovative electricity price technical indicators developed in this section are underpinned by the standard financial indicators. In the financial market, technical indicators use historical price input combined with machine learning algorithms for stock price prediction [5]. The three core types of financial technical indicators are (i) trend, (ii) oscillator and (iii) momentum [3]. For this research, market behaviour was captured using only raw electricity price data to develop energy technical indicators based on the financial indicators. Designing novel indicators related to electricity price will support ISEM energy traders in forecasting, decision making and future planning. The technical indicator calculations are described below. For any indicator that involves a moving average, the calculation involves all the previous hours within the window size except for the current hour.

1. **Percentage Price Change Moving Average (PPCMA)**: A trend price indicator calculated over a rolling s-hour window size by averaging past prices where percentage price change is calculated to capture daily trend as the difference in current price ($Price_i$) and lagged price at the same time period from the previous day ($Price_{Lag\ 24}$), all divided by $Price_{Lag\ 24}$:

$$PPC_i = \frac{Price_i - Price_{Lag\ 24}}{Price_{Lag\ 24}} * 100 \quad (7.5)$$

$$PPCMA_s = \frac{1}{s}\sum_{i=1}^{s} PPC_i \quad (7.6)$$

2. **Moving Average Deviation (MAD)**: A trend price indicator calculated over a rolling s-hour window size for deviation rate of the current price within the window from **P**rice **C**hange **M**oving **A**verage (PCMA):

$$PCMA_s = \frac{1}{s}\sum_{i=1}^{s} \frac{Price_i - Price_{Lag\ 24}}{Price_{Lag\ 24}} \quad (7.7)$$

$$MAD_s = \frac{Price_s - PCMA_s}{PCMA_s} \quad (7.8)$$

3. **Average True Range (ATR)**: A trend price indicator that measures price volatility over three calculations of s-hour window size: (A) highest price within the window minus lowest price within the window, (B) highest price within the window minus a lagged (n) electricity price ($Price_n$), and (C) lowest price within the window minus a lagged (n) electricity price ($Price_n$). The maximum value from A, B, or C is selected for each trading hour (i) and averaged over a rolling s-hour window:

$$A_s = HighestPrice_s - LowestPrice_s \quad (7.9)$$

$$B_s = |\, HighestPrice_s - Price_n \,| \quad (7.10)$$

$$C_s = |\, LowestPrice_s - Price_n \,| \quad (7.11)$$

$$TR_i = MAX\{A_s, B_s, C_s\} \quad (7.12)$$

$$ATR_s = \frac{1}{s}\sum_{i=1}^{s} TR_i \quad (7.13)$$

4. **Average Directional Movement Index (ADX)**: A trend price indicator measuring strength of trend, with the two directional movement indexes grouped depending on if price change, calculated as current price ($Price_i$) minus a lagged (n) electricity price ($Price_n$), is a positive change ($Price\ Up$) or a negative change ($Price\ Down$). A piecewise function determined $Price\ Up$ and $Price\ Down$: if the difference from price change was greater than 0 then it was considered as $Price\ Up$, else $Price\ Down$. The two indexes are joined and smoothed over a rolling s-hour window:

$$DX\ Up_s = \frac{\frac{1}{s}\sum_{i=1}^{s} Price\ Up[Price_i - Price_n]}{ATR_s} \quad (7.14)$$

$$DX\ Down_s = \frac{\frac{1}{s}\sum_{i=1}^{s} Price\ Down[Price_i - Price_n]}{ATR_s} \quad (7.15)$$

$$ADX_s = \frac{|DX\ Up_s - DX\ Down_s|}{|DX\ Up_s + DX\ Down_s|} \quad (7.16)$$

5. **Relative Strength Index (RSI)**: An oscillator price indicator comparing recent price gains with recent price losses to measure price direction oscillating between 0 and 100; a value near to 100 signifies the majority of the electricity price units are $Price\ Up$ and a value near to 0 signifies the majority of the electricity price units are $Price\ Down$. $Price\ Up$ is calculated as the average of the previous s-hours with increased price difference and $Price\ Down$ is calculated as the average of the previous s-hours with decreased price difference:

$$D_s = \left(1 - \frac{\frac{1}{s}\sum_{i=1}^{s} Price\ Up[Price_i - Price_n\,]}{\frac{1}{s}\sum_{i=1}^{s} Price\ Down[Price_i - Price_n\,]}\right) \tag{7.17}$$

$$RSI_s = 100 - \left[\frac{100}{D_s}\right] \tag{7.18}$$

6. **Percentage Range (PR)**: An oscillator price indicator calculated over a rolling s-hour window to find a relationship between current price ($Price_i$) and the highest and lowest prices. PR oscillates between 0 and 100; a value close to 100 signifies the current price unit is nearer to the lowest price and a value close to 0 signifies the current price unit is nearer to the highest price:

$$PR_s = \left[\frac{HighestPrice_s - Price_i}{HighestPrice_s - LowestPrice_s}\right] * 100 \tag{7.19}$$

7. **Moving Average Convergence/Divergence (MACD)**: An oscillator price indicator which considers trend direction, trend duration, trend strength, and price momentum through short-term and long-term moving averages of previous prices; if a 24-hour model is used, the window sizes are *a*=12 and *b*=24 and if an hourly model is used, the window sizes are *a*=168 and *b*=336 :

$$MACD = \frac{1}{a}\sum_{i=1}^{a} Price_a \ - \frac{1}{b}\sum_{i=1}^{b} Price_b \tag{7.20}$$

8. **Price Momentum (PMOM)**: A momentum price indicator which evaluates market power through examining the current price with either the previous trading value ($n = 1$), if a 24-hour model is used, or if an hourly model is used with the previous trading value ($n =\ s\ minus\ the\ window\ size$):

$$PMOM_s = Price_s - Price_n \tag{7.21}$$

7.4 Technical Indicator Models

The proposed technical indicators were calculated with a window size of 24 hours to capture daily trends and were used as inputs into the three regression algorithms. This experiment used the same train/test set-up used in Section 7.2. The model inputs were the technical indicators at Hour T and the model target was the electricity price aligned at Hour T+1. The SISO persistence models in Section 7.2 were used for comparison with the technical indicator 24-hour models.

Table 7.3 presents the results for the technical indicator models for both the training and testing periods. EVS was 0.99 for all models during the training stage and ranged from 0.82 to 0.87 during the testing stage. Both these findings are a great improvement on the EVS values from the SISO persistence models. Gradient Boosting had the lowest RMSE for both training and testing with values of 1.76 and 5.02, respectively. The results using the SISO persistence models' indicated that Random Forest had the lowest RMSE for training and XGBoost had the lowest RMSE for testing. Here, Random Forest has the lowest MedAE (0.77) and RMSLE (0.088) during technical indicator model training, which is a similar finding to the persistence model. Gradient Boosting had the lowest MedAE (2.97) and RMSLE (0.17) during technical indicator model testing, which differs from the persistence model findings of XGBoost. Consistently Gradient Boosting has performed best when using the technical indicators as inputs, this differs from the results of the persistence models that found XGBoost to generally perform best. Comparing Table 7.1 and Table 7.3, when technical indicators are included as model inputs the overall model accuracy improves.

Table 7.3: Technical indicator 24-hour models summary results

Period	**Algorithm**	**EVS (↑)**	**MedAE (↓)**	**RMSE (↓)**	**RMSLE (↓)**
Training	Random Forest	0.99	**0.77**	2.18	**0.088**
	Gradient Boosting	0.99	1.12	**1.76**	0.091
	XGBoost	0.99	1.30	2.21	0.093
Testing	Random Forest	0.82	3.30	6.17	0.20
	Gradient Boosting	**0.87**	**2.97**	**5.02**	**0.17**
	XGBoost	0.84	3.24	5.59	0.19

This is also illustrated in Figures 7.3, 7.4, and 7.5 which use the first week of testing (168 hours) and display the actual electricity price, and the predicted electricity prices using both the SISO persistence models and the ML models with technical indicators as inputs for Random Forest, Gradient Boosting, and XGBoost, respectively. It can be seen in Figures 7.3, 7.4 and 7.5 that for each of the models the predicted prices using the technical indicators are more similar to the actual prices than the predicted prices from the persistence models.

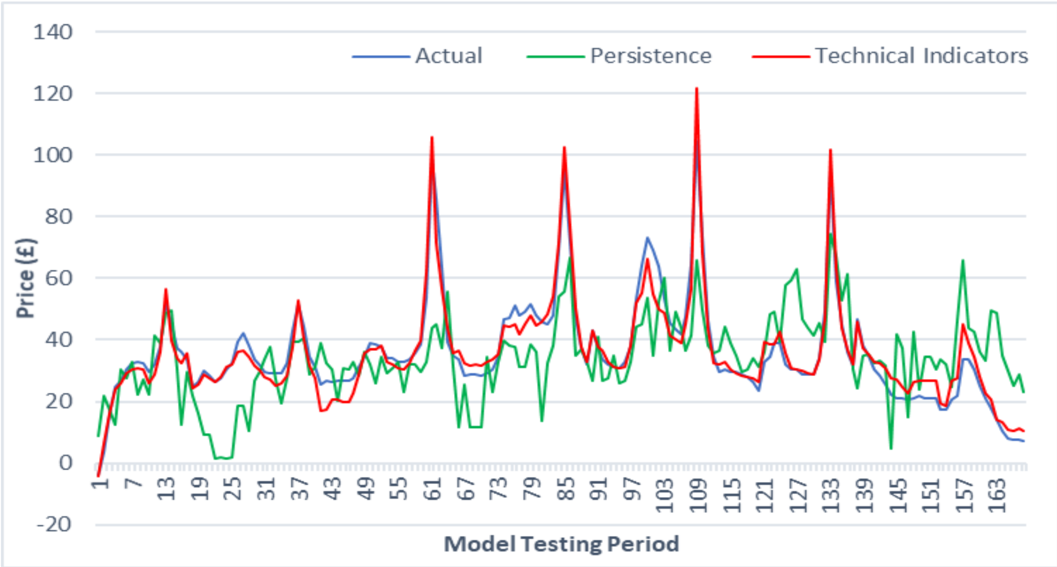

Figure 7.3: Random Forest 24-Hour model from 1st March 2020 to 7th March 2020

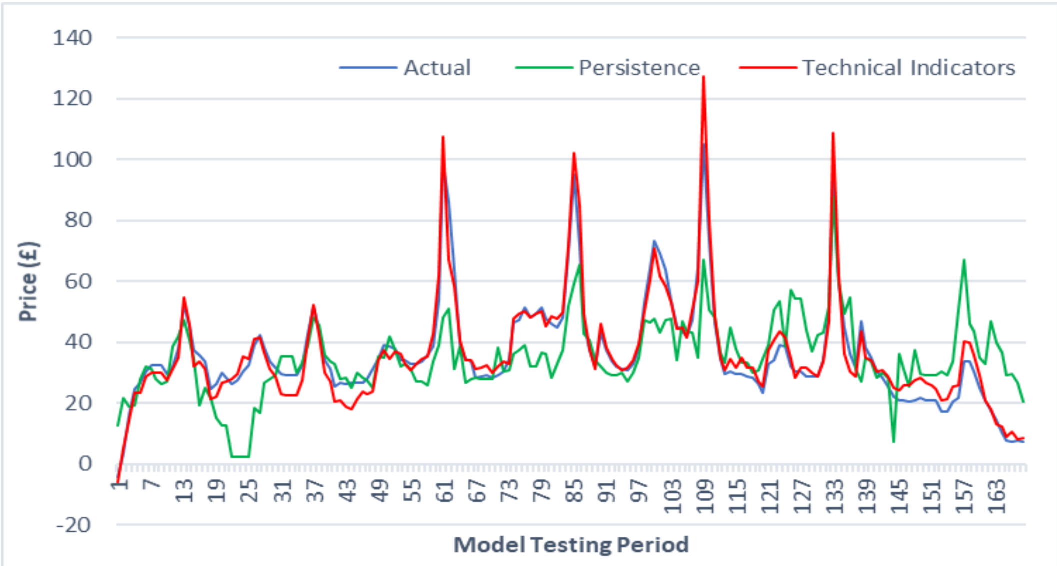

Figure 7.4: Gradient Boosting 24-Hour model from 1st March 2020 to 7th March 2020

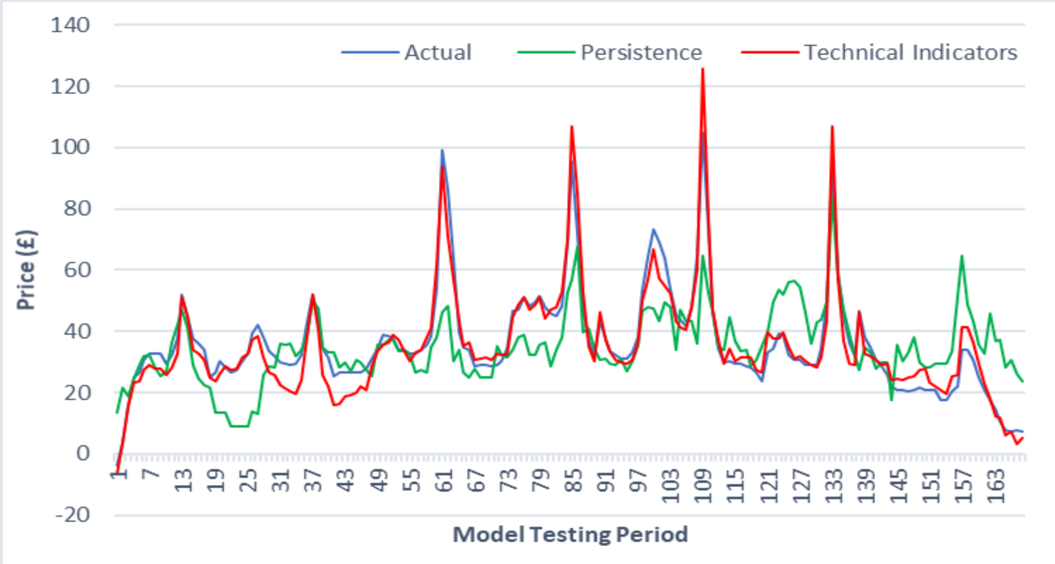

Figure 7.5: XGBoost 24-Hour model from 1st March 2020 to 7th March 2020

As the Gradient Boosting model displayed the most promising results using technical indicator inputs it was selected for further evaluation using additional unseen input data from the 24-hours on 1st April 2020 to predict the electricity price on that day. Figure 7.6 illustrates the performance where the actual price is plotted alongside the model predicted price. The RMSE value over this period was calculated as 2.82, demonstrating the robustness of the model for longer term predictions considering this period is over one month beyond the period on which the model has been trained.

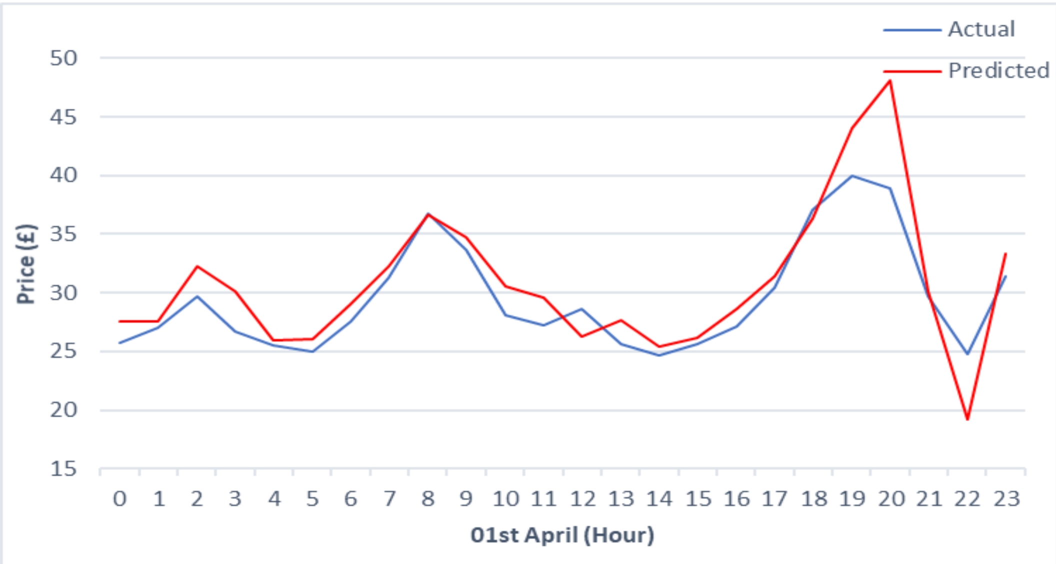

Figure 7.6: Results of Gradient Boosting model on 1st April 2020 prediction

Feature importance can be used to analyse variable significance where the output ranges from 0 to 1 with a value closer to 1 indicating variable significance is in predicting electricity price. Any score greater than 0.1, chosen empirically for this research, is considered to be significant in electricity price forecasting. Table 7.4 shows each technical indicator's feature importance for the three ML models. The three significant technical indicators for all ML models were MAD, PR, and PMOM suggesting that these indicators are key in electricity price forecasting. There is one of each type of price indicator (trend, oscillator, and momentum) considered important. PPCMA was also determined to be significant for Gradient Boosting.

Table 7.4: Technical indicators feature importance

Technical Indicator	Random Forest	Gradient Boosting	XGBoost
PPCMA	0.099	**0.113**	0.081
MAD	**0.144**	**0.181**	**0.111**
ATR	0.067	0.067	0.057
ADX	0.014	0.010	0.029
PR	**0.448**	**0.418**	**0.424**
RSI	0.048	0.063	0.088
MACD	0.034	0.021	0.039
PMOM	**0.144**	**0.128**	**0.170**

Individual hourly energy models have shown greater accuracy compared with a single 24-hour model [52]. Limited work to date has been conducted on developing technical indicators for the Day-Ahead energy market but recently Demir et al. [42] created individual hourly technical indicator forecasting models. Following on from [42], optimal versions of the technical indicator models from Section 7.3 were determined for each hour (Hours 0-23): first hyperparameters for lag and span were optimised for each individual hourly model. Once the optimal hyperparameters were selected, the 24 individually hourly models were created. The computation time was significantly longer, using Intel Pentium Quad Core Processor N4200, to create the technical indicators for each individual hour (725 seconds) compared to the single 24-hour (0.31 seconds). The reason for this is each hourly technical indicator is calculated 150 times to be used in the selection for the optimal hyperparameters.

Hourly data from 01st February 2019 until 31st March 2020 were used to select the optimal hyperparameters *n* (lag) and *s* (span) based on the approach in [42]. The lag is defined as the previous price values within the window and the span is defined as the length of the rolling window size. Technical indicator ML models outperformed the persistence models regardless of which ML algorithm was selected thus optimal *n* and *s* were determined for each individual hour through optimisation using the Random Forest algorithm, implemented using SKLearn. To find the optimal hourly technical indicators, a 2-step process was performed. Firstly, a grid-search was applied with all the possible combinations ranging from 1 to 150 during the training period to tune the hyperparameters *n* and *s* for the respective technical indicators. Secondly, for each hour, all possible combinations of the hyperparameters were generated and ranked by RMSE from lowest to highest, with the smallest RMSE during the testing period chosen as the optimal values for *n* and *s*. Applying this approach *n* was selected for ATR, ADX, PR, RSI, PMOM technical indicators and *s* was selected for PPCMA and MAD technical indicators. MACD already has parameters *a*=168 and *b*=336 in its calculation and therefore was not further optimised with *n* or *s*. Table 7.5 presents the optimal hyperparameters for each hour; for instance Hour 0 has *n*=24 and *s*=91 so the derived technical indicators for this hour are $PPCMA_{91}$, MAD_{91}, ATR_{24}, ADX_{24}, PR_{24}, RSI_{24}, MACD, and $PMOM_{24}$.

Table 7.5: Optimal hyperparameters

Hour	n	s
0	24	91
1	12	71
2	14	35
3	55	1
4	47	32
5	19	78
6	47	97
7	15	15
8	41	63
9	12	77
10	73	69
11	81	64
12	69	63
13	80	14
14	110	3
15	96	67
16	99	92
17	89	95
18	91	62
19	84	37
20	37	82
21	9	5
22	62	138
23	12	42

The hourly models were trained and tested with the same data period (09th September 2019 to 31st March 2020) and model split (85%/15%) as the 24-hour single models. The technical indicators at Hour T were the model inputs and the model output was the electricity price aligned with Hour T+24. Table 7.6 displays the hourly technical indicator summary results during the testing period for Random Forest, Gradient Boosting, and XGBoost. Observing the results in the table, the Random Forest models EVS ranged from 0.83 to 0.99. Hour 20 generated both the lowest MedAE (0.56) and RMSLE (0.010) while Hour 22 generated the lowest RMSE (0.90) for the Random Forest models. The Gradient Boosting models' EVS ranged from 0.49 to 0.99, the lowest MedAE (0.70) was generated at Hour 14, the lowest RMSE (1.04) at Hour 22, and the lowest RMSLE (0.013) at Hour 19. The XGBoost models' EVS ranged from 0.64 to 0.96, the lowest MedAE (0.88) was generated at Hour 15, the lowest RMSE (2.15) at Hour 20, and the lowest RMSLE (0.023) at Hour 19. From the overall averages, the Random Forest was the best performing out of the three: lowest EVS average (0.94), lowest MedAE average (1.63), lowest RMSE average (2.44), and lowest RMSLE average (0.089). It is shown in the results in Table 7.6 for all three regression algorithms that the novel technical indicators greatly improved model performance compared to Table 7.2 results using the raw electricity price data. This can be clearly seen by observing the EVS values between the two tables which go from negative for the best persistence models to close to 1 for the best technical indicator models. The four performance metrics all improved in the technical indicator models showing promising findings and concluding that hourly technical indicator models do enhance electricity price forecasting performance.

Table 7.6: Hourly technical indicator models summary results

Hour	**Random Forest**				**Gradient Boosting**				**XGBoost**			
	EVS (↑)	**MedAE (↓)**	**RMSE (↓)**	**RMSLE (↓)**	**EVS (↑)**	**MedAE (↓)**	**RMSE (↓)**	**RMSLE (↓)**	**EVS (↑)**	**MedAE (↓)**	**RMSE (↓)**	**RMSLE (↓)**
0	0.96	1.88	2.30	0.053	0.98	1.68	2.70	0.049	0.89	2.23	4.01	0.23
1	0.97	1.38	1.91	0.14	0.98	2.67	2.74	0.083	0.94	1.42	2.98	0.055
2	0.98	0.91	1.72	0.049	***0.99***	1.30	1.75	0.08	0.93	1.86	3.12	0.11
3	***0.99***	0.99	1.45	0.21	***0.99***	0.96	1.49	0.24	0.94	1.19	3.21	0.35
4	0.91	0.91	1.38	0.17	***0.99***	0.75	1.40	0.12	0.94	1.01	3.01	0.23
5	***0.99***	0.84	1.43	0.19	0.97	1.25	2.24	0.27	***0.96***	1.21	2.59	0.046
6	0.98	1.28	2.47	0.53	0.97	0.99	2.43	0.60	0.95	3.49	3.91	0.57
7	0.90	3.02	4.60	0.24	0.87	4.31	5.38	0.44	0.79	4.25	6.53	0.31
8	0.95	1.90	3.32	0.052	0.96	1.80	2.72	0.067	0.91	2.51	4.23	0.051
9	0.92	1.95	3.62	0.050	0.89	2.30	4.38	0.052	0.85	2.73	5.17	0.070
10	0.92	3.28	4.47	0.051	0.95	2.22	3.50	0.042	0.93	2.36	3.79	0.040
11	0.84	2.72	3.79	0.050	0.88	2.89	3.41	0.055	0.88	1.55	3.28	0.048
12	0.93	1.66	2.68	0.040	0.91	1.12	2.72	0.037	0.93	1.15	2.19	0.029
13	0.92	1.39	2.07	0.033	0.89	2.01	2.86	0.041	0.91	1.30	2.31	0.037
14	0.97	1.15	1.85	0.032	0.93	***0.70***	2.01	0.076	0.83	1.46	3.35	0.037
15	0.96	0.84	1.44	0.023	0.95	1.01	1.63	0.027	0.89	***0.88***	2.41	0.037
16	0.91	2.67	3.15	0.049	0.94	2.70	3.15	0.047	0.83	3.05	3.89	0.054
17	0.92	3.42	3.80	0.045	0.94	2.78	3.40	0.041	0.91	3.51	4.75	0.057
18	***0.99***	1.24	2.32	0.023	***0.99***	1.44	2.06	0.019	***0.96***	2.74	4.63	0.030
19	***0.99***	1.17	1.71	0.014	0.98	0.76	2.04	***0.013***	0.94	1.55	3.26	***0.023***
20	***0.99***	***0.56***	0.98	***0.010***	0.98	1.16	1.30	0.014	0.95	0.95	***2.15***	0.024
21	0.83	1.46	2.26	0.029	0.49	1.70	3.26	0.042	0.64	1.86	2.71	0.036
22	0.97	0.57	***0.90***	0.017	0.97	0.72	***1.04***	0.017	0.94	1.78	2.28	0.041
23	0.89	2.03	2.96	0.044	0.89	1.71	3.12	0.048	0.84	2.84	3.52	0.062
Average	**0.94**	**1.63**	**2.44**	**0.089**	0.93	1.71	2.61	0.11	0.90	2.04	3.47	0.11

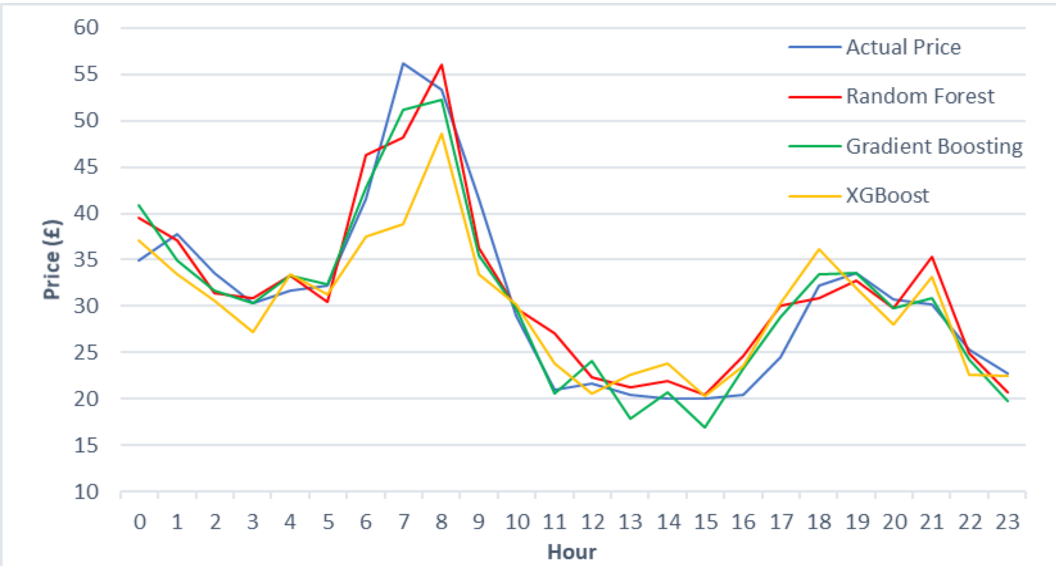

Figure 7.7: Hourly technical indicator models on 16th March 2020

Figure 7.7 displays the actual and predicted prices on a randomly chosen date (16th March 2020) for the three hourly technical indicator models: Random Forest, Gradient Boosting, and XGBoost. Each of the three regression algorithms show excellent model fits between actual and predicted electricity price demonstrating that when technical indicator models are split by hour and modelled separately, electricity price can be forecasted accurately. The Random Forest model with technical indicators is significantly better than the equivalent persistence model displayed in Figure 7.2. Therefore hourly machine learning models with technical indicator inputs would be worthwhile to consider for energy trading.

Sensitivity analysis determines which model inputs have the most influence by examining robustness and performance [124]. Sensitivity analysis was included for the Random Forest Hour 20 model to determine the significance of each of the technical indicators. One technical indicator was removed at a time and the model performance evaluated using the four summary metrics. This procedure is called parametric bootstrap as the model is re-evaluated after each replacement [125]. The summary results for the analysis are shown in Table 7.7.

Table 7.7: Sensitivity analysis results for Random Forest hour 20

Technical Indicator Removed	**EVS (↑)**	**MedAE (↓)**	**RMSE (↓)**	**RMSLE (↓)**
None	0.99	0.56	0.98	0.010
PPCMA	0.98	0.85	1.23	0.012
MAD	0.97	0.77	1.80	0.016
PR	0.72	2.76	4.40	0.047
ATR	0.98	0.69	1.02	0.010
RSI	0.99	0.61	0.99	0.010
ADX	0.99	0.68	1.03	0.010
MACD	0.99	0.72	1.03	0.010
PMOM	0.99	0.60	0.81	0.012

From the summary results, PR was the most significant as once the indicator was removed the model accuracy decreased significantly (EVS=0.72, MedAE=2.76, RMSE=4.40, RMSLE=0.047). RSI was the least significant as when the indicator was removed the model accuracy still resembled the original accuracy (EVS=0.99, MedAE=0.61, RMSE=0.99, RMSLE=0.010). These results indicate that Percentage Range is a strong technical indicator to use for electricity price forecasting.

7.5 Machine Learning Modelling with Click Energy Data

The electricity price data provided by Click Energy for 2020/2021 were used with the technical indicators and these indicators were applied as model inputs to the machine learning models. In the first experiment the model inputs were the technical indicators at Hour T and the model output was electricity price at Hour T+1 hours. 85% of the data were used for training (02nd December 2020 to 31st May 2021) and 15% of the data were used for testing (01st June 2021 to 30th June 2021).

Table 7.8 presents the results for the Gradient Boosting model (selected as it performed the best when using the technical indicators as inputs with 2019/2020 data) for both the training and testing periods. Compared to Table 7.3, the model performance is still highly accurate indicating strong robustness when predicting electricity price. This is also illustrated in Figure 7.8 which displays the actual electricity prices and the predicted electricity prices for June 2021. It can be seen in Figure 7.8 that the predicted prices are very similar to the actual prices.

Table 7.8: Gradient Boosting 24-Hour model summary results (2020/2021)

Period	EVS (↑)	MedAE (↓)	RMSE (↓)	RMSLE (↓)
Training	0.99	1.89	3.20	0.076
Testing	0.85	6.15	8.88	0.49

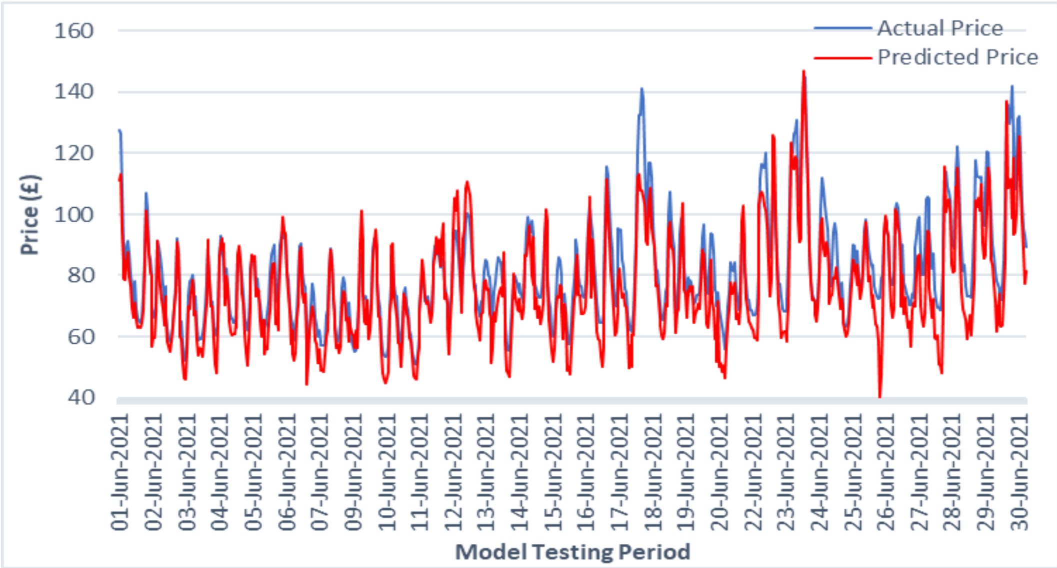

Figure 7.8: Gradient Boosting 24-Hour model from 1st June 2021 to 30th June 2021

For the second experiment, hourly data from 01st February 2020 until 30th June 2021 were used to determine the optimal hyperparameters *n* and *s* for Hour 20 using the Random Forest algorithm (the best performing hour from Table 7.6). The optimal hyperparameters from the grid-search for Hour 20 were *n*=1 and *s*=83 so the derived technical indicators were $PPCMA_{83}$, MAD_{83}, ATR_1, ADX_1, PR_1, RSI_1, MACD, and $PMOM_1$. The Hour 20 model was trained and tested using the same data period as the first experiment (02nd December 2020 to 30th June 2021) and model split (85%/15%). The technical indicators at Hour T were the model inputs and the model output was the electricity price aligned with Hour T+24.

Table 7.9 presents Hour 20 technical indicator summary results during the testing period for Random Forest. Observing the results, the Random Forest Hour 20 model performed well, generating much lower results than the metric values in Table 7.8 highlighting that a parsimonious model improves model performance. The findings are not as good as Table 7.6

with 2019/2020 data. This is due to electricity prices being low in Spring and Summer of 2020 and then a volatile market at the end of 2020 leading into 2021 with the beginning of Brexit. Nonetheless both models for the two data periods are consistent and this is shown in Figure 7.9 which displays a strong model fit between the actual and predicted prices.

Table 7.9: Random Forest hour 20 summary results (2020/2021)

EVS (↑)	MedAE (↓)	RMSE (↓)	RMSLE (↓)
0.89	2.00	4.63	0.022

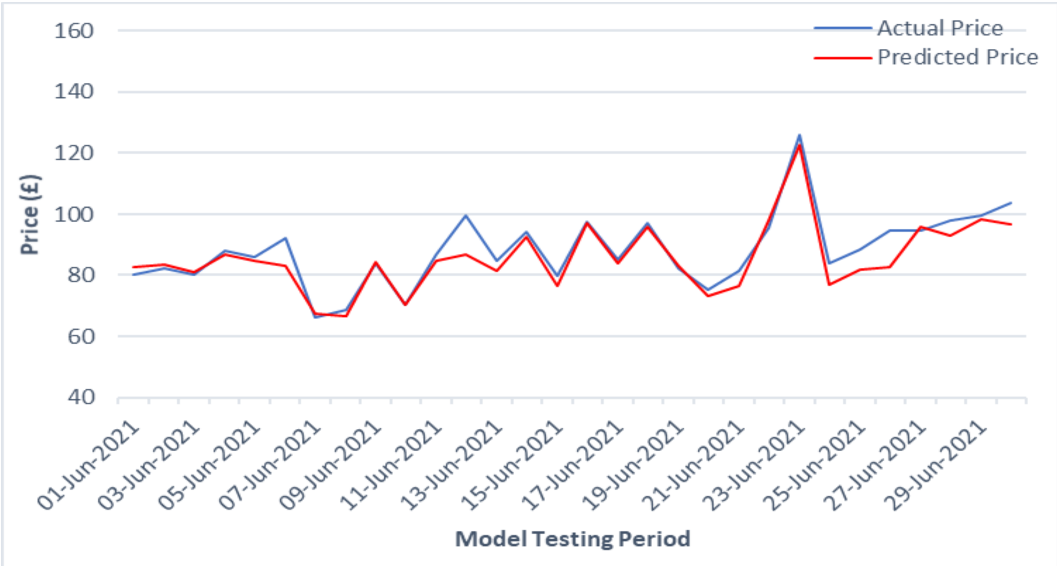

Figure 7.9: Random Forest hour 20 model from 1st June 2021 to 30th June 2021

7.6 Conclusion

This chapter explored the use of machine learning models algorithms (Random Forest, Gradient Boosting, and XGBoost) to determine if machine learning methods are more robust than traditional statistical techniques. In the first experiment, historical hourly ISEM price data were split 85% for training (09th September 2019 Hour 0 to 01st March 2020 Hour 5) and 15% for testing (01st March 2020 Hour 6 to 31st March 2020 Hour 23). The 24-hour model summary results found Random Forest to perform the best during the training period and XGBoost to perform the best during the testing period. Machine learning persistence models enhanced accuracy, when compared with the statistical model findings in previous chapters. In the second experiment, hourly persistence models were considered and, from the summary results, on average the Random Forest model was the best performing. Nonetheless, persistence model performance accuracy was poor when the predicted prices were compared against the actual electricity prices measured over one day, displayed in Figure 7.2, and the two lines did not match highlighting that the prices were not similar.

To capture market behaviour eight novel technical indicators (PPCMA, MAD, ATR, ADX, PR, RSI, MACD, and PMOM) were designed based on raw electricity price, originating from financial trading indicators, to aid in supporting ISEM energy traders with forecasting decisions. In this experiment the window size was 24 hours to detect daily forecasting and both the model input and output were aligned at Hour T. The EVS values greatly improved compared with the persistence model findings. Gradient Boosting performed the best for both the training and testing periods, nevertheless the model accuracy increased for Gradient Boosting, Random Forest and XG Boost as the predicted price for the technical indicators was more accurate than the predicted price for the persistence models. Feature importance analysis found MAD, PR, and PMOM to be the three significant indicators for all three regression algorithms considered for price forecasting.

The final experiment concentrated on hourly technical indicator models to develop an optimal forecasting model by selecting the hyperparameters *n* and *s* for each individual hour which generated the lowest RMSE value for testing when using the Random Forest algorithm. The technical indicators were calculated based on a window size for each hour and included as model input aligned at Hour T with the model output. Hourly persistence models with model input at Hour T and model output at Hour T+24 were generated for comparison. The summary findings showed that hourly technical indicators greatly enhanced model performance with Random Forest on average performing the best. Comparing these results with the persistence models results, it is clear that the models with technical indicators outputted better model accuracy as the performance metric values greatly improved and the predicted prices closely matched the actual electricity prices. These promising findings highlight that splitting by each individual hour is a more appropriate format and does improve price forecasting accuracy, therefore developing individual hourly technical indicator models would be helpful for energy trading instead of the 24-hour models considered as a benchmark in earlier chapters. Data from 2020/2021 were modelled for both the statistical and machine learning techniques with 6 months training and 1 month testing (June 2021). Comparing the two technique groups, the machine learning results are less volatile with significant model accuracy indicating that machine learning techniques are more robust for predicting electricity price.

To conclude, technical indicator price forecasting models, in particular optimal hourly models, would aid in following market trend patterns and over time reduce purchasing costs through accurate predictions.

Chapter 8

Conclusion and Future Work

8.1 Summary of Key Findings

Energy markets have a degree of unpredictability due to fluctuations between demand and supply making it difficult to forecast electricity prices. A robust system that can follow past trends, spot outliers, and make accurate predictions which over time will increase company profits and market share is ideal. This thesis explored the techniques of statistical regression, and machine learning models and considered how electricity price data can be used with these approaches for successful prediction. Stationarity checks, trends, and seasonality were important to consider for the statistical models, as well as exogenous input variables, to improve model accuracy. A non-linear regression model refined the statistical models by including only the key significant energy-related factors that influenced day-ahead electricity price. Machine learning models combined with price technical indicators capture market trends and significantly improved prediction performance of day-ahead electricity price.

Chapter 2 explained in detail the background, design, operating schedules, and challenges of the three energy markets that were examined in this thesis: **B**ritish **E**lectricity **T**rading and **T**ransmission **A**rrangements (BETTA) in Great Britain and both the **S**ingle **E**lectricity **M**arket (SEM) and **I**ntegrated **S**ingle **E**lectricity **M**arket (ISEM) in Ireland. The foundation of the BETTA and ISEM markets is the Day-Ahead trading period and thus literature regarding day-ahead electricity price forecasting models was examined for statistical regression and machine learning approaches. With similarities between stock markets and electricity price markets, forecasting techniques applied to financial stock markets were also reviewed.

Chapter 3 discussed the theory behind statistical regression and machine learning techniques in terms of data understanding and application in price forecasting. The strengths and weaknesses of each type of prediction model were discussed. It is clear from the literature that it is necessary to apply appropriate stationarity, integration, and seasonal checks on statistical models. It was highlighted that the transparency of non-linear models can identify key significant factors and improve overall model accuracy. Computational models, in particular machine learning algorithms, work well with complex data to spot patterns and should be considered to develop an optimal prediction model.

Single **I**nput **S**ingle **O**utput (SISO) models with historical BETTA and ISEM electricity price data were explored in **Chapter 4** for three statistical time-series models: **A**uto**R**egressive **M**oving **A**verage (ARMA), **A**uto**R**egressive **I**ntegrated **M**oving **A**verage (ARIMA), and **S**easonal

Auto**R**egressive **I**ntegrated **M**oving **A**verage (SARIMA). The key modelling stages were discussed for each statistical technique: identification, estimation, diagnostic testing, and forecasting. The best performance results generated from the SARIMA models where SARIMA(3, 1, 2)(2, 0, 2, 24) for the BETTA market and SARIMA(3, 1, 3)(2, 0, 2, 24) for the ISEM market, with the models having **R**oot **M**ean **S**quared **E**rror (RMSE) values of 9.67 and 14.12 respectively; nonetheless the model accuracy could still be improved further.

Multiple **I**nputs **S**ingle **O**utput (MISO) models were analysed in **Chapter 5** with multiple historical BETTA and ISEM energy related input data. The three statistical time-series models applied were **A**uto**R**egressive **M**oving **A**verage with e**X**ogenous inputs (ARMAX), **A**uto**R**egressive **I**ntegrated **M**oving **A**verage with e**X**ogenous inputs (ARIMAX), and **S**easonal **A**uto**R**egressive **I**ntegrated **M**oving **A**verage with e**X**ogenous inputs (SARIMAX). A **N**onlinear **A**uto**R**egressive **M**oving **A**verage model with e**X**ogenous inputs (NARMAX) was also considered to identify external factors influencing electricity price. Like the SISO model results, the seasonal MISO models generated the best performance results: the optimal BETTA market MISO model being SARIMAX(2, 1, 3) (2, 0, 1, 24) with a RMSE of 9.31 and the optimal ISEM MISO model being SARIMAX(2, 1, 2) (2, 0, 2, 24) with a RMSE of 14.32. These findings showed that for the BETTA market exogenous variables do improve model accuracy and should be considered as inputs in prediction models. However for the ISEM market the SARIMA SISO model had a slightly better model fit than the SARIMAX MISO model. This finding suggests that, due to multicollinearity in the data, including all exogenous variables in a MISO model might make it difficult to predict the ISEM market; for this reason it would be worthwhile to consider only the key factors identified from NARMAX and remove wind, East-West Interconnector and Moyle Interconnector as inputs in the ISEM prediction model. The NARMAX models identified the most significant energy related factors with the largest **E**rror **R**eduction **R**atio (ERR) values as historical electricity price, demand, and wind for BETTA market and as historical electricity price, demand, and system generation for ISEM market.

Correlated lags were examined for each individual exogenous variable with ISEM market data in **Chapter 6** and the model performance slightly improved; ARMAX(1, 9) generated a RMSE of 14.66, ARIMAX(7, 1, 6) generated a RMSE of 14.47, and SARIMAX(3, 1, 3)(2, 0, 1, 24) generated a RMSE of 14.36. The identified energy related factors from the NARMAX model were applied to refine the statistical original and correlated models. The refined models improved accuracy performance and the key significant energy-related factors were historical electricity price, system generation, demand, and temperature. Overall the optimal statistical models with ISEM data were a refined correlated ARIMAX(7, 1, 6) and a refined correlated SARIMAX(3, 1, 3)(2, 0, 1, 24) both with a RMSE of 13.99. Therefore the inclusion of both significant factors only and their respective correlated lags does improve model accuracy when forecasting day-ahead in the ISEM market.

Computational modelling was the focus in **Chapter 7** examining three regression machine learning algorithms: Random Forest, Gradient Boosting and E**x**treme **G**radient Boosting (XGBoost) with ISEM market data. First, persistence models were explored using raw electricity price data for a single 24-hour model and separate hourly models. For the 24-hour model, XGBoost performed the best during the testing period with an **E**xplained **V**ariance **S**core (EVS) of 0.10, **Med**ian **A**bsolute **E**rror (MedAE) of 7.53, RMSE of 12.89, and **R**oot **M**ean **S**quared **L**og **E**rror (RMSLE) of 0.37. For the hourly models, on average the Random Forest performed the best with an EVS of -0.74, MedAE of 8.98, RMSE of 14.35, and RMSLE of 0.31. Between the two approaches the 24-hour model performed slightly better than the average of the hourly models. Next, eight novel technical indicators were derived (Percentage Price Change Moving Average, Moving Average Deviation, Percentage Range, Average True Range, Relative Strength Index, Average Directional Movement Index, Moving Average Convergence/Divergence, and Price Momentum) and used as inputs to the price prediction models. For the 24-hour model, Gradient Boosting performed the best during the testing period with an EVS of 0.87, MedAE of 2.97, RMSE of 5.02, and RMSLE of 0.17. For the hourly models, on average Random Forest performed the best with EVS of 0.94, MedAE of 1.63, RMSE of 2.44, and RMSLE of 0.089. These promising results confirm that technical indicator inputs significantly enhance model accuracy and that hourly technical indicator models optimise prediction performance. Therefore these novel technical indicators, combined with machine learning are valuable to consider for energy trading forecasting.

Overall, this thesis demonstrates that many statistical regression and machine learning approaches are useful to consider when forecasting day-ahead electricity prices. The best statistical models were the refined correlated ARIMAX and SARIMAX and the best machine learning model was Random Forest with hourly technical indicators as model inputs. Over the last year the ISEM market has been quite volatile making it difficult to comment but results were validated with company feedback on electricity price data from 2020/2021. Referring back to the five research questions outlined in Chapter 1, the following conclusions can be stated:

1. Computational models are more appropriate than statistical methods for day-ahead electricity price forecasting. This finding was observed in Chapter 7 in the first experiment with the machine learning persistence models outputting a lower RMSE value compared to the statistical models.

2. Energy-related exogenous variables do improve model performance. This finding was noted in the results from Chapter 5 emphasizing that exogenous variables do help to improve model accuracy.
3. Transparent models do identify key factors that influence electricity price. This finding was noted in Chapter 6 as performance accuracy improved refining the statistical models to only include the significant factors identified by NARMAX.
4. Prediction accuracy is improved by developing representative energy-related technical indicators. This finding was highlighted in Chapter 7 with the inclusion of the eight novel technical indicators developed from raw electricity price data showing the improvement in model accuracy compared to the persistence models.
5. Model performance does improve by building on the strengths of statistical models and machine learning models. This finding was observed through the refined correlated models in Chapter 6 and through the hourly technical indicator models in Chapter 7.

The key findings from this research conclude that statistical models can spot market trends with energy-related inputs influencing model accuracy; non-linear regression techniques model input-output relationships and discover key significant exogenous factors; refining statistical models with the relevant energy-related factors can further improve model accuracy; and that computational models combined with novel price technical indicators offer the best prediction performance for the energy market.

8.2 Future Work

This thesis analysed various algorithmic approaches with energy market data to predict day-ahead electricity price. From addressing the research questions, computational techniques were noted to be more robust than statistical methods especially with the inclusion of price technical indicators. Nonetheless, there are still opportunities for future work to consider extending this research.

The first option to consider is including the other significant energy related data noted in Section 6.13 from the statistical model findings (system generation, demand, and temperature) as technical indicator hourly model inputs to determine if forecasting performance and accuracy can be improved further. Preliminary experiments have been carried out with the exogenous variables raw data included along with the eight technical indicators data as inputs for the 24-hour models. Table 8.1 displays these summary results, in which the EVS and RMSE values improved from Table 7.3, highlighting that energy related factors do influence machine learning forecasting models. Gradient Boosting still performed the best for the testing period. EVS ranged from 0.85 to 0.91 and the increase shows less overfitting of predicted against actual prices.

Table 8.1: Technical indicator 24-hour models with exogenous inputs summary results

Period	**Algorithm**	**EVS (↑)**	**MedAE (↓)**	**RMSE (↓)**	**RMSLE (↓)**
Testing	Random Forest	0.85	3.84	6.18	0.21
	Gradient Boosting	**0.91**	**3.09**	**4.55**	**0.19**
	XGBoost	0.90	3.20	4.76	0.23

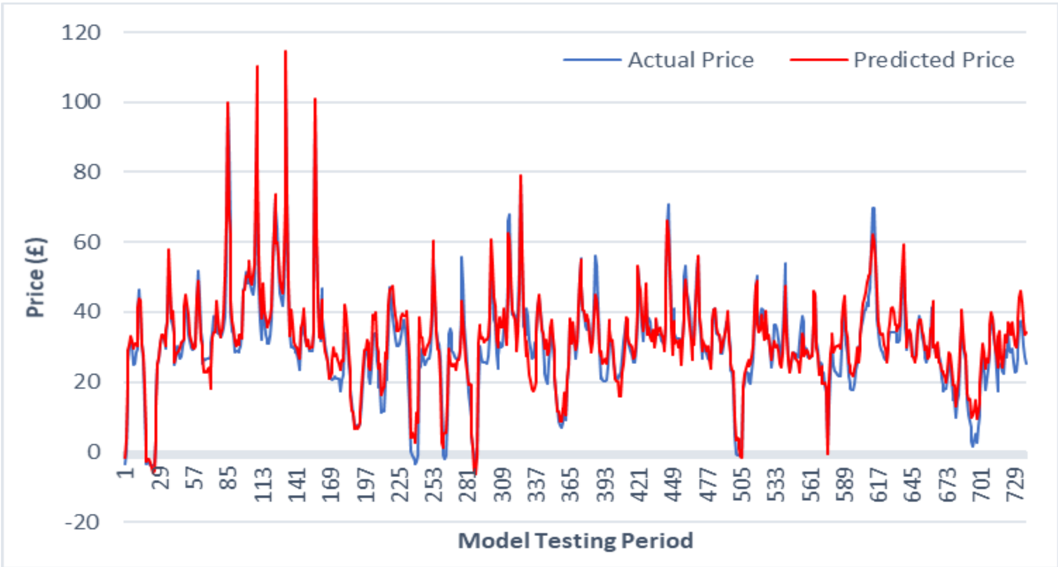

Figure 8.1: Gradient Boosting with exogenous inputs 24-hour model testing period

Figure 8.1 illustrates Gradient Boosting predictions within the testing period and the actual prices. It should be noted that model fit is excellent and predicted and actual values closely follow the same trend. It would also be interesting to explore these additional exogenous variables with the hourly models. As technical indicators have shown to enhance model performance, developing novel technical indicators for each of the exogenous variables might also be advantageous for electricity price forecasting. Creating technical indicators from a mixture of the energy-related factors could also be considered to optimise market behaviour.

The second option to consider is unique hyperparameters for each individual technical indicator. The work in this thesis split the technical indicators into two groups (lag, *n* and span, *s*) for optimisation. The current optimal hyperparameters provided excellent results and were determined for each hour by ranking the RMSE values of all combinations of the hyperparameters and selecting the combination which generated the smallest RMSE value.

However, having eight unique hyperparameters, one for each technical indicator, should make the prediction models more specific and further refine the tuning of each technical indicator.

The third option to consider is the correlated lags of each of the technical indicators. The findings in Chapter 6 demonstrated that the inclusion of correlated lags in statistical models greatly improved model accuracy. It would be interesting to only consider correlated lag prices for the technical indicators rather than all raw data in the rolling window to determine if this could further optimise the machine learning models.

The fourth option to consider is to extend the current approaches to on-line prediction models. The work in this thesis predicted electricity prices through applying historical data to observe market trends and adapt to current market conditions. The next step of the research would be to move the optimal machine learning prediction models to an online trading platform which is adaptable to current price changes in real-time.

In the future, a wider application for this research could be any field that uses time series data, e.g. sensor recorded data, robotics, environmental data. One area that could apply technical indicators for prediction is vehicle insurance through utilising speed, breaking, steering, weather conditions, etc. as model inputs and driver safety as the target output. The main contribution from this thesis for the energy market is the knowledge for energy traders to apply accurate forecasting models to optimise their costs. Machine learning combined with technical indicators offer the best accuracy for energy trading forecasting providing compact models that accurately represent the variability and dynamics within the energy market.

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Appendix

NARMAX Polynomial Models

Various NARMAX models were analysed with different low-order polynomials (linear, quadratic, and cubic) to predict day-ahead BETTA market electricity price. This research work was presented in the conference proceedings:

- C. McHugh, S. Coleman, D. Kerr, and D. McGlynn, “Daily Energy Price Forecasting Using a Polynomial NARMAX Model,” in Advances in Computational Intelligence Systems, UKCI 2018, pp. 71–82.

The research included daily data for a five week period in 2017 and observed from 1 hour to 12 hour input regression lags with historical electricity price and demand as model input factors. The BETTA market historical input data ranged from 01st May 2017 until 04th June 2017 and the target day-ahead BETTA price data ranged from 02nd May 2017 until 05th June 2017. As noted in Section 3.3.4, the NARMAX model has a model estimation and model validation stage. In this research, the 840 data records were split 50% with the initial 420 records used for model estimation and the remaining 420 records used for model validation.

Table A1 displays the percentage error values, which the closer the error value is to zero the better the accuracy, for both model estimation stage and model validation stage. For model estimation, the percentage error value decreased as the polynomial degree increased. For model validation it was the opposite, with the percentage error value lowest for linear polynomials. In particular, a linear Lag 2 model gives the lowest percentage error of 52.20 during model validation and the predicted values against the actual values are illustrated in Figure A1. The predicted values closely followed the actual values and reached the majority of the peaks.

This preliminary work found linear polynomial NARMAX models provided the best prediction when forecasting day-ahead electricity prices. The validation stage needs to display a suitable model fit and at times it was unable to fit the quadratic and cubic polynomial models due to the model overfitting. The findings highlight that a simpler polynomial model works better with unseen data and a complex polynomial model works better with training data. Therefore in this thesis, experiments involving NARMAX models used linear polynomials only.

Table A1: Percentage error for BETTA NARMAX model (historical electricity price and demand)

Lag	Polynomial	Model Estimation	Model Validation
1	Linear	22.68	54.43
	Quadratic	18.07	62.06
	Cubic	17.44	101.81
2	Linear	21.76	**52.20**
	Quadratic	17.91	67.57
	Cubic	16.69	74.54
3	Linear	21.38	53.21
	Quadratic	16.69	67.88
4	Linear	21.21	54.69
	Quadratic	16.35	74.53
5	Linear	21.29	54.29
	Cubic	**14.60**	93.66
6	Linear	21.30	53.62
	Quadratic	15.31	84.72
	Cubic	16.97	81.65
7	Linear	20.93	53.71
	Quadratic	15.70	75.52
	Cubic	15.81	113.86
8	Linear	20.79	53.05
9	Linear	20.68	53.20
	Quadratic	15.93	62.71
	Cubic	14.87	90.99
10	Linear	20.75	53.09
	Quadratic	15.15	72.01
11	Linear	20.43	53.68
	Quadratic	14.88	61.88
12	Linear	20.27	54.66
	Quadratic	15.14	75.31

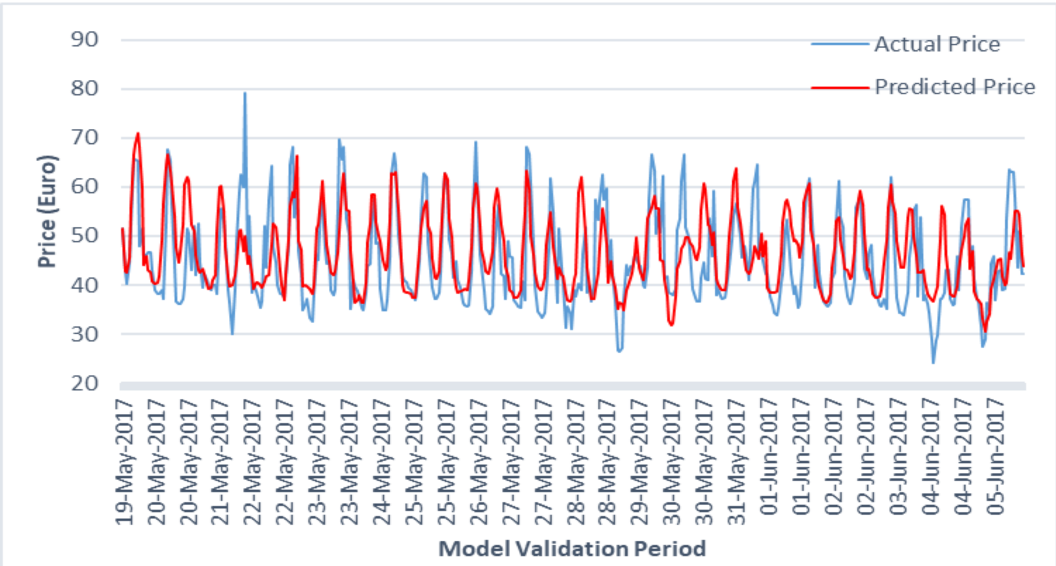

Figure A1: Best model validation for BETTA NARMAX model (historical electricity price and demand)