

# RAS Specialist Discussion Meeting report

Tom Elsdén, Matthew K. James, Jasmine K. Sandhu and Clare Watt report on the RAS Specialist Discussion Meeting ‘Planetary Ultra-Low Frequency Waves – Theory, Modelling and Observations’

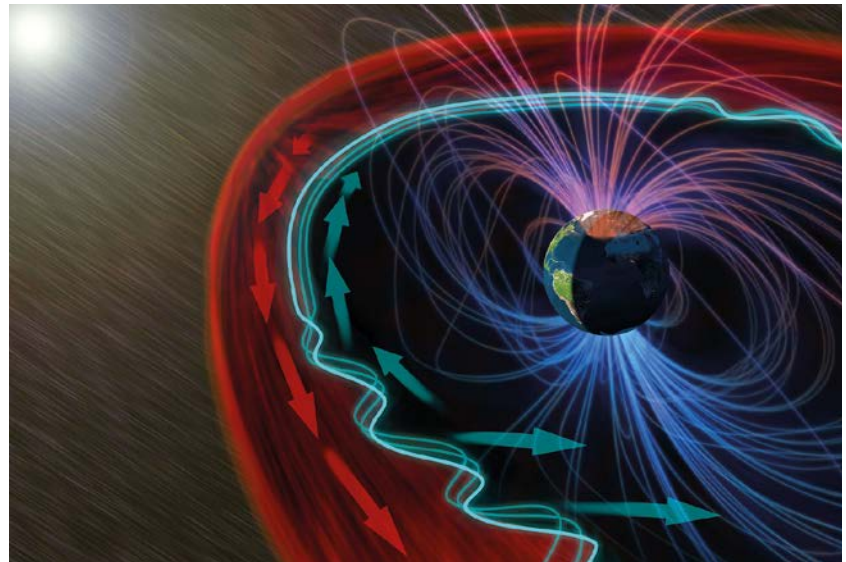
I was sitting in my bedroom (like many others working from home, I presume, my bedroom is also my home office) on the morning of 11 March 2022, eagerly awaiting the start of our specialist discussion meeting. My laptop was fully charged, I had all of the presentations backed up in case of technical issues, the co-chairs had been briefed, my Zoom background had been suitably manicured, and everything was ready.

Then the construction work started right outside my window. A pneumatic drill in full spate certainly is not what one wants to hear when chairing an online meeting. Thankfully, Zoom managed to cleverly apply an appropriate band-pass filter to mute the incessant buzzing from outside and we were able to successfully begin our meeting.

So what were we (around 50 planetary scientists from many different latitudes and longitudes) there to discuss? Well, ultra-low frequency (ULF) waves of course! These plasma waves pervade every planetary magnetosphere, representing the lowest frequency (~1 mHz – 1 Hz; Jacobs *et al.* 1964) and largest length-scale oscillations of the magnetic field. They are excited by a variety of mechanisms, the most blatant being the constant battering of the magnetosphere by the solar wind. Changes in the solar wind (e.g. through variations in density and pressure) disturb the magnetosphere from equilibrium. On large scales, such changes or disturbances are communicated as ULF waves. The Kelvin-Helmholtz unstable flanks of the magnetosphere, as well as wave-particle interactions provide further excitation mechanisms for these waves.

ULF waves tie in to several areas of importance in magnetospheric physics and are a key component in our attempt to understand space weather – the real-time plasma conditions in near-Earth space and their impact on humanity. For example, the field-aligned currents driven by these waves can generate auroral displays, with their characteristic periodic poleward moving arcs (e.g. Milan *et al.* 2001). They play a critical role in the dynamics of Earth’s radiation belts, modulating the behaviour of trapped energetic particles (e.g. Elkington *et al.* 2003). They have been employed extensively as a seismological tool, using frequencies and amplitudes of magnetic field fluctuations on the ground to infer characteristics of the plasma in near-Earth space (Waters & Menk 2013). They have also recently been shown to have an impact on geomagnetically induced currents (GICs) on the ground (Heyns *et al.* 2021; Shi *et al.* 2022), which in turn can adversely affect electrical power systems.

The purpose of the meeting was to provide a platform to discuss all aspects of ULF wave research, in particular facilitating discussion between those



1 An illustration of the magnetosphere, depicting how the solar wind and Kelvin-Helmholtz unstable flanks drive oscillations of the Earth's magnetic field, known as ULF waves.

(Martin Archer (Imperial) and Emmanuel Masongsong (UCLA))

working in theory, modelling and observations.

In this summary article, we will discuss the research presented and how it ties in with these higher level ULF wave research topics. Overall this will be somewhat of a whistle-stop tour of ULF wave research, but will hopefully provide enough background to be intelligible by those not directly in the field, as well as pointing to further resources for the interested reader.

## External drivers of ULF waves

There are several ways to excite ULF waves, which are often classified by whether the energy source originates from outside (external) or inside the magnetosphere. The five talks presented in this section all discuss the driving of ULF waves by sources external to the magnetosphere.

The day was started with a talk by **Martin Archer** (Imperial), who did a great job of balancing introducing ULF waves in a general sense with presenting his research, on global magnetohydrodynamic (MHD) simulations of ULF waves.

Illustrations like that shown in figure 1 are very useful at representing the different ULF waves of the magnetosphere. The figure displays the key boundaries of the magnetosphere, formed by the interaction with the solar wind. The light blue line depicts the magnetopause, the boundary of pressure balance between the incoming solar wind ram pressure and the Earth's outward magnetic pressure. This boundary or surface can support waves in the ULF frequency range, which Martin investigated.

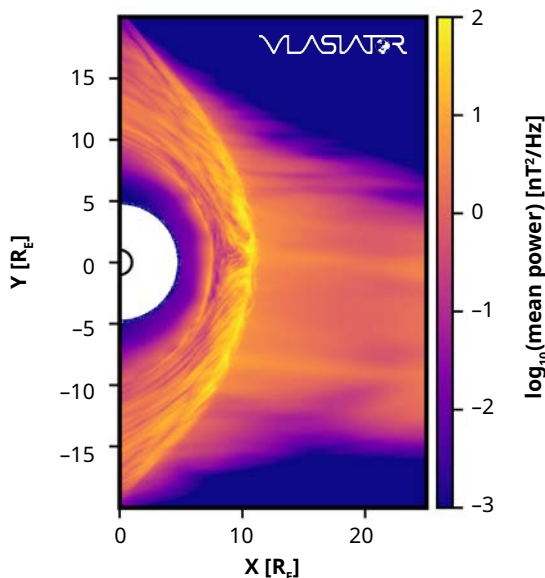
The simulation presented was driven by an isolated one-minute solar wind pressure pulse, which excited a hive of ULF wave activity in the magnetosphere. The key result of the simulation shows that surface waves that form on the magnetopause can in fact be stationary over a large portion of the dayside magnetopause – between 9 and 15 magnetic local time (MLT), as indicated in figure 1, by the portion of the magnetopause over which the blue and red

arrows oppose one another (Archer *et al.* 2021). This counters the previous paradigm that surface waves always propagate tailwards, driven by the advection of the magnetosheath (coloured red in figure 1) flow. This simulation and the resulting analysis advances our fundamental understanding of how the solar wind and magnetosphere interact.

A further point was addressed on the importance of using realistic magnetic geometries, like that provided by the global MHD simulation, instead of more simplified (e.g. straight or dipole) magnetic fields. It was shown that the nodal structure and polarization of the magnetic field perturbations are augmented by such a realistic geometry, particularly close to the cusps (Archer *et al.* 2022). Such geometrical effects should be taken into account when considering observations of ULF waves standing along geomagnetic field lines.

**Simone Di Matteo** (NASA-GSFC) then took centre stage to discuss the fascinating topic of ULF waves that are directly driven by periodic density structures in the solar wind.

To put this in context, there are two schools of thought (not by any means mutually exclusive) for generating periodic ULF waves in the dayside magnetosphere. The idea originally proposed by Southwood & Kivelson (1986), is that the magnetospheric cavity filters the broadband solar wind disturbances, responding with the preferred modes of oscillation depending on the size of the magnetosphere and the equilibrium (e.g. plasma density and magnetic field geometry/strength). This can be thought of in a similar fashion to blowing across the top of a half-empty bottle, which responds with a particular tone. These preferred modes are known as the ‘normal’ modes of the system, and have been employed to explain the observation of preferred or ‘magic’ frequencies (Samson *et al.* 1992). Alternatively, as Simone shows, periodic structures within the solar wind can directly drive periodic ULF wave activity in the magnetosphere at the same frequency (Di Matteo *et al.* 2022). The author establishes evidence for the periodic structures in the solar wind using data from the Wind spacecraft. These frequencies are then compared to those measured inside the magnetosphere at geostationary orbit as well as by ground magnetometers. Part of the problem is differentiating between waves generated by different processes, for which the authors showcase the use of a new, robust spectral analysis procedure. Work like this is critical in furthering our knowledge of how the magnetosphere responds

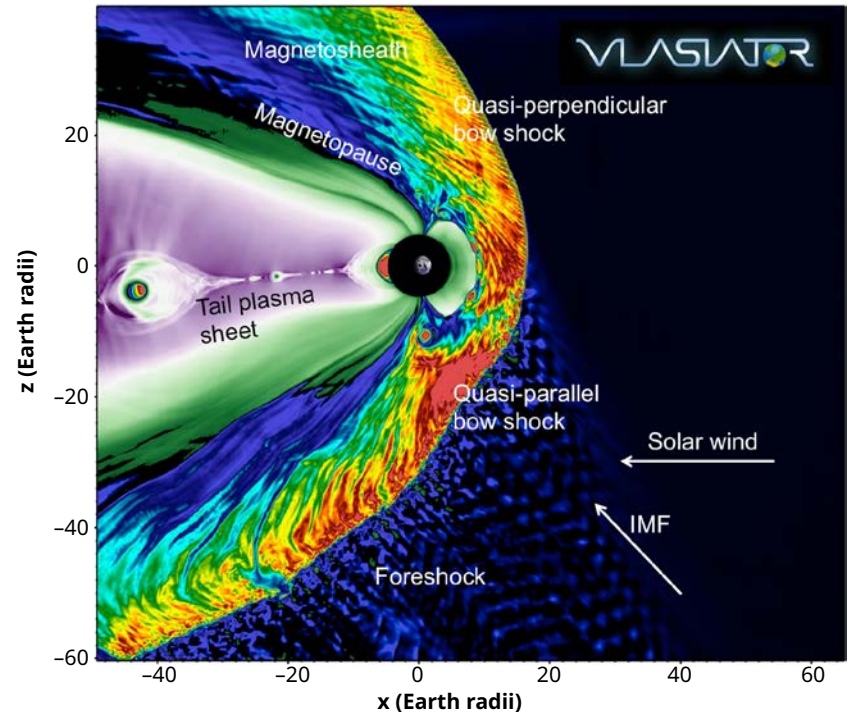


**2** Example of Vlasiator modelling of the Earth's magnetosphere, with key boundaries and features annotated. View in the noon–midnight meridian plane (i.e. the Earth–Sun meridian) from the morning sector. Reproduced from Palmroth *et al.* (2018).

**3** Mean Pc3 wave power from a Vlasiator simulation run, from Turc *et al.*, ‘A global view of Pc3 wave activity in near-Earth space: results from hybrid-Vlasov simulations’, in preparation.

to different inputs, as well as showing innovative ways to disentangle overlapping wave processes.

One of the most exciting new (and continuing) developments in magnetospheric physics is the Vlasiator code (Palmroth *et al.* 2018), which takes the first step towards a global magnetospheric simulation solving for particle distribution functions (i.e. solving the Vlasov equation for ions, treating the electrons as a MHD fluid, known as a ‘hybrid-



Vlasov’ approach) as opposed to making the MHD approximation unilaterally. This is important for areas where it is clear that MHD does not appropriately describe the plasma dynamics, where scales on the order of the ion Larmor radius become important. Figure 2 shows an example of the magnetospheric regions of interest for the Vlasiator simulation.

**Related to the previous talk on periodic density structures in the solar wind, Hongyang Zhou** (Helsinki) presented results from a Vlasiator simulation run where sinusoidal density variations were inputted from the upstream boundary.

Similarly to the previous observational study, it was demonstrated that such waves penetrate into the magnetosphere with the same frequency. The simulation also produced mirror modes and electromagnetic ion-cyclotron (EMIC) waves in the higher  $\beta$  (ratio of particle pressure to magnetic pressure) magnetosheath plasma.

**Lucile Turc** (Helsinki) also presented results from a Vlasiator run, this time considering the effect that varying solar wind conditions have on ULF waves driven by instabilities in the ion foreshock.

Indeed, foreshock waves are thought to be the main source of so-called Pc3 ULF waves (periods 10–45s) in the magnetosphere. Vlasiator critically provides the means for a self-consistent description of the ion kinetic processes that drive these waves. The simulation demonstrates that increasing the upstream Alfvén Mach number (ratio of solar wind speed to the local Alfvén speed) increases the wave power in the Pc3 band across all regions of near-Earth space. Figure 3 shows the mean Pc3 wave power from one of the simulation runs. Such an increase in power will then have further ramifications for the

energization of particles that these waves interact with. The simulation further shows the importance of the orientation of the interplanetary magnetic field (IMF) on the distribution of wave power, since the IMF orientation controls the position of the foreshock.

**Staying with kinetic modelling, Li-Jen Chen (NASA-GSFC) discussed the formation of solitary magnetic structures known as SLAMS (short large-amplitude magnetic structures) at Earth and Mars.**

These are thought to be driven by ULF waves which are gyroresonant with solar wind ions and propagate towards the magnetosphere (Chen *et al.* 2021). Presented were kinetic simulations and Magnetospheric Multiscale (MMS) observations which explained how this resonant interaction between the ULF waves and solar wind ions can result in a significant amplification of the magnetic field (Chen *et al.* 2022).

### Space weather and ULF waves

Although most ULF wave research relates to space weather, there were a few talks which had more direct relevance in this area.

**Xueling Shi (Virginia Tech) was invited to present her research on the direct driving of geoelectric and geomagnetic fields by magnetospheric ULF waves.**

These geoelectric fields in turn generate geomagnetically induced currents (GICs), which can then severely damage electrical power grids. The geoelectric fields which result are highly localized and depend critically on the local conductivity of the Earth. Xueling used one-second resolution geoelectric field data from the EarthScope magnetotelluric (MT) array across the United States, focusing on several geomagnetic storms. It was demonstrated that ULF waves during these storm events could drive significant geoelectric activity on the ground, as demonstrated in figure 4, reproduced from Shi *et al.* (2022). The electric fields (panels (c), (d)) and magnetic fields (panels (e), (f)) show clear long period oscillations, indicative of ULF waves. One of the features of ULF wave driven events is the possibility of a sustained peak in the geoelectric field over several wave cycles. Overall, this exciting new research is demonstrating the clear societal importance of understanding low-frequency wave activity in our magnetosphere.

Continuing with the more direct space weather applications, we move to the world of wave-particle interactions. ULF waves have been shown to play a critical role in the motion of energetic particles in Earth's radiation belts, primarily through the process of radial diffusion. This is a stochastic process, by which broadband ULF waves cause the diffusion of particles across magnetic field line shells, better known as L-shells (e.g. Elkington *et al.* 2003). Simplifying the process, moving particles into a region of different magnetic field strength changes their energy, which can result in the significant acceleration of radiation belt particles. Such particles are hazardous to spacecraft operations.

**Theodoros Sarris (University of Thrace, Greece) gave two presentations relating to ULF wave-particle interactions in the magnetosphere.**

First, he showed observations from the Van Allen Probes spacecraft of oscillations in electron flux at energies of order 100keV to a few MeV, in the ULF wave band (Sarris *et al.* 2020). Through comparison with particle-tracing simulations of the effect of broadband ULF waves on electrons, it was demonstrated that these flux oscillations are indeed driven by the resonant interaction of drifting electrons and ULF waves. The study found that parameters such as the local phase

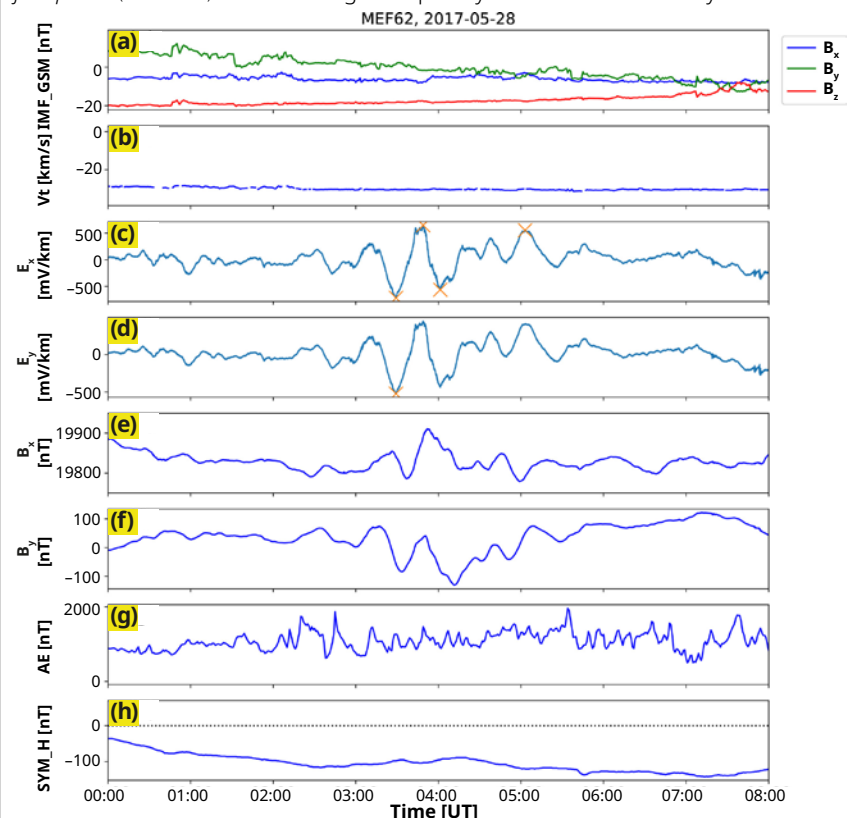
space density gradients as well as the width of the electron detector energy channels are important in setting the amplitudes of these flux oscillations.

Secondly, Theodoros presented a statistical analysis of how ULF wave power varies with magnetic latitude in the inner magnetosphere. The reason that quantifying the ULF wave power is important is that it is currently used as an input in analytical expressions determining the rate of radial diffusion. Often, the estimates of wave power come from spacecraft measurements close to the equator, therefore this study considers how the power changes with latitude. Based on THEMIS and Arase spacecraft measurements, it was found overall that ULF wave power increased away from the magnetic equator, which could have significant implications for the current estimation of radial diffusion rates.

**A novel approach taken by Shahbaz Chaudhry (Warwick) to assess the distribution of ULF waves during geomagnetic storms is to build a global dynamic network using hundreds of ground magnetometer measurements.**

Treating the magnetometers as nodes of a mathematical graph at their given geographic location, edges can be constructed between these nodes based on how well correlated the wave signatures are at each location. The idea of a direction associated with these connections (edges) can also be introduced, depending on the lag time of the signals appearing at each station, i.e. if there were no lag the signals would instantaneously appear at each node. Building up these connections between magnetometers forms the network, which allows for an understanding of the connection of different signals at different locations (Dods *et al.* 2015). In this case, the network was used to assess the Pc2 and Pc3 (frequencies of ~20mHz–0.2Hz) ULF wave response to a particular geomagnetic storm on 17 March 2015. The network clearly identifies the onset of the storm and is able to track the spatiotemporal distribution of wave power in the chosen frequency band. This test case clearly shows the potential for using this network technique on a catalogue of storms, to quantify the local and global spatially coherent storm time dynamics.

4 Ground-based measurements of parameters associated with intense geoelectric field events during the 24–25 October 2011 geomagnetic storm, driven by ULF wave activity. Orange crosses in (c) are identified geoelectric field peaks. (Shi *et al.* 2022)



## Field line resonance

Field line resonance (FLR) has been attracting researchers for almost 50 years. This process is attractive for its simplicity of understanding in terms of well-known resonant systems, like pushing someone on a swing. FLR involves the driving of a wave which has a standing structure along geomagnetic field lines (the MHD Alfvén wave, like a wave on a string) by a compressible wave which can propagate across the magnetic field (the MHD fast wave, which can be thought of as the plasma equivalent of a sound wave in a gas).

In a planetary magnetic field, the frequency of an Alfvén wave of a given field line is determined by the length of the field line, the magnetic field structure and strength, and the mass distribution (density) along the field line. This can be compared to the natural frequency of a guitar string, depending on the length of the string and mass distribution along it. When the global fast wave frequency matches the Alfvén frequency at a particular location, there is a resonance between the waves, with energy being transferred from the fast to the Alfvén wave. This process has several important ramifications. The Alfvén waves are resonantly driven, meaning that they can grow rapidly in amplitude. These waves carry a field-aligned current, which causes particle precipitation into the ionosphere and drives auroral displays. They further provide a mechanism for the transfer of energy from global to local scales, coupling different regions of the magnetosphere. Finally, these waves can of course interact with energetic particles, the importance of which has been discussed in previous sections.

**The seminal paper on FLR was written in 1974 by David Southwood (Imperial) (Southwood 1974), so it was only natural to have David give a presentation on his recent work on modelling and observing FLR at Saturn.**

Southwood *et al.* (2021) analyzed observations from the Cassini spacecraft on its final orbits before crash landing into Saturn. These now famous passes, known as the proximal orbits, showed standing Alfvén wave signatures jumping out of the background field on every pass inside Saturn's inner D-ring. An example of these signals, together with the spacecraft orbit are shown in figure 5. This established the systemic nature of the signals as opposed to being some transient phenomena. Using an MHD wave simulation in a background dipole magnetic field, the authors were able to reproduce several features of the observations. They surmised that the consistent frequency of the signals was produced by the normal MHD fast modes of a large cavity driving FLRs on the inner magnetic shells of Saturn. Additionally, the simulation results motivated the study of what a spacecraft observes when passing quickly through a FLR. The results showed a prominent Doppler shift in the observed signals, due to the wave phase motion inherent in a FLR. This can significantly augment the observed frequency.

A recent advancement in FLR research has been the extension of FLR theory to fully 3D inhomogeneous plasmas (Wright & Elsden 2016). Given the many asymmetries arising in the magnetospheric plasma, e.g. dusk side plasmaspheric plumes, it is clearly important to understand how this process operates in 3D. Numerical modelling has provided clear features regarding wave polarization and phase relations between electric/magnetic fields for observers to look for, in particular when there is significant azimuthal asymmetry.

**Matt James (Leicester) has bravely taken on the task of looking for evidence of 3D FLRs in ground magnetometer data.**

The most likely candidate identified in the modelling for 3D FLRs is at the boundaries of plasmaspheric plumes, where there are sharp changes in the plasma density with azimuth. Matt identified times where the Van Allen Probes were magnetically conjugate with a ground magnetometer array. It was then established whether a plume was present and if so, whether there was significant wave activity. This is currently work in progress.

**Related to the topic of 3D FLRs, Andy Wright (St Andrews) presented simulations describing how azimuthally small-scale Alfvén waves evolve in a dipole magnetic field.**

Waves with a small azimuthal scale, known as 'high-m', where m is the azimuthal wave number, tend to be generated in the magnetosphere by internal mechanisms, for instance through resonance with drifting particles. In a Cartesian, straight background magnetic field geometry, it was previously shown that an Alfvén wave with an initial poloidal polarization (i.e. dominant velocity/magnetic field perturbation in the radial direction) would rotate to a toroidal (azimuthal) perturbation over time (Mann & Wright 1995). Andy's work considers the extension of this theory to dipole magnetic fields. The curvature of the magnetic field present in the dipole was shown to critically affect the wave evolution (Elsden & Wright 2020). The initially poloidal waves will still evolve into more toroidally polarized waves in time, but will have an associated spatial motion across field lines as well. This temporal and spatial polarization variation will be important in determining the effectiveness of subsequent wave-particle interactions.

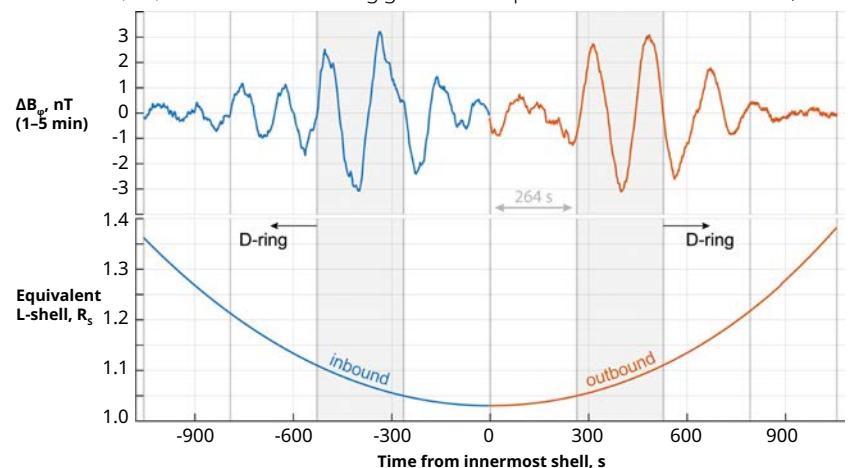
## Substorms, auroral beads and ULF waves

The final topic treated at our discussion meeting was the role of ULF waves in magnetospheric substorms and the auroral displays that they generate. Substorms are characterized by an explosive release of stored magnetic and thermal plasma energy in the Earth's nightside magnetosphere, converted into plasma kinetic energy, occurring over timescales of a few hours compared to the longer lasting (days) geomagnetic storms. Associated with substorms are a particular auroral form known as auroral 'beads', which describe the discrete azimuthal structuring of the aurora, causing an appearance in optical observations like bright pearls on a string necklace in the sky.

**Andy Smith (UCL, MSSL) discussed how these auroral forms are related to ULF waves.**

Using ground and space-based instrumentation,

**5 Top:** Filtered azimuthal magnetic field signals from the Cassini spacecraft at Saturn. **Bottom:** Cassini orbit in magnetic (*L*) shells. Reproduced from Southwood *et al.* (2021).



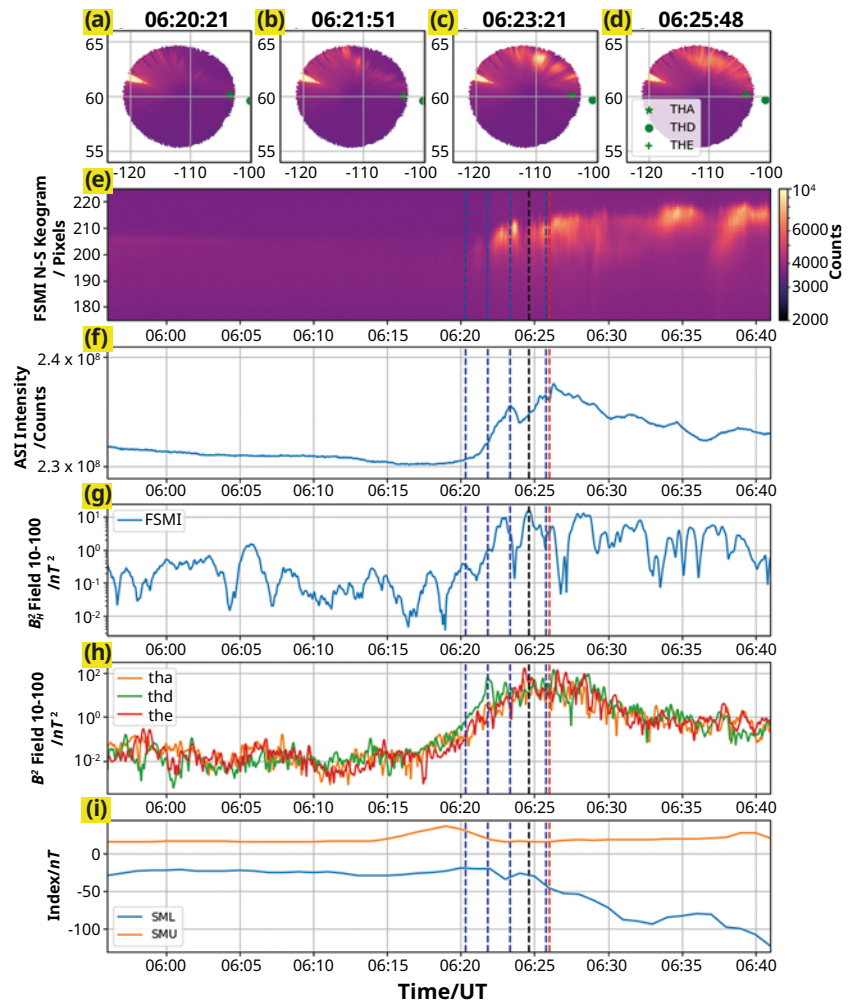
Andy showed how exponential increases in the auroral brightness occurred almost simultaneously with similar increases in ionospheric and conjugate magnetotail ULF wave power. This is demonstrated in figure 6, where the four images in the top panel (a-d) show the development of the discrete auroral beads. Quantities indicating the wave power on the ground (panel g) and in space (panel h) clearly indicate an increase in the wave power at a time corresponding with the bead formation. The important take away from this is the clear relation between the waves and the aurora. Indeed, recently the connection has been made between auroral beads and kinetic Alfvén waves originating in the magnetotail, which can efficiently accelerate electrons to form the visible aurora (Kalmoni *et al.* 2018).

**Jason Derr** (University of Texas at Austin) further describes the processes leading to the onset of a substorm and auroral bead formation, by considering an analytical treatment of the instability occurring in the magnetotail.

Derr proposes a new mechanism for initiating the onset of a substorm, namely the shear flow interchange instability (Derr *et al.* 2020). Beginning from the linearized MHD equations in a wedge model of the magnetotail, an equation is derived which governs the linear stage of the instability, providing a qualitative picture of the dynamics. This equation yields a description of an unstable propagating wave packet which grows as it travels along magnetic flux tubes towards the ionosphere. The growth rates and dispersion can be compared with observed auroral beads characteristics, though the authors note that a full nonlinear analysis would be required to gain a more quantitative understanding. It is fascinating to see that in the modern world of high-resolution observations and large-scale parallel computing, analytical (pen and paper) studies still provide great insight.

## Discussion

It feels only right to sign off with a discussion of the discussion! No formal discussion was organized, although I invited all attendees who wished to stay on the Zoom call, to grab a cuppa and chat all things ULF waves. The virtual environment really excelled at allowing informal questions to be asked of the speakers, something which a massive lecture hall does not promote. Furthermore, being virtual rather than in-person in London, allowed us to invite speakers from all over the world. A large proportion of researchers in ULF waves work in the USA, meaning that an in-person meeting on this topic in the UK would have attracted



**6 Top row (a-d):** images from the Fort Smith All Sky Imager close to substorm onset. Other panels further explained in the main text.

Reproduced from Smith *et al.* (2020).

far fewer participants. I certainly believe that these types of one-day focused workshops hosted virtually should remain a permanent fixture, even with the general return to more traditional conferences.

In terms of the overall outcomes of the meeting, the goal was to bring together researchers on all aspects of ULF wave research, as opposed to discussing one specific research question in detail. In this regard, certainly if the quality of the discussion is anything to go by, I believe the meeting was a great success in developing new connections between international researchers. The high attendance further highlights the widespread importance of this research field, and bodes well for the future of the discipline.

A full recording of the meeting is available on the RAS Youtube channel at [bit.ly/3RN8lpM](https://bit.ly/3RN8lpM). ●

## ACKNOWLEDGEMENTS

Many thanks to the RAS for hosting and recording this meeting, which can be viewed on the RAS Youtube channel: [bit.ly/3RN8lpM](https://bit.ly/3RN8lpM)

## AUTHORS

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spicy curries and midweek pub adventures.

**Matt James** (mkj13@leicester.ac.uk) is a postdoc at the University of Leicester investigating space plasmas using ULF wave observations and machine learning techniques. He likes beer and fast motorbikes (but not at the same time).

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or loss occurs.

**Clare Watt** (clare.watt@northumbria.ac.uk) is a Professor in Space Plasma Physics at Northumbria University. She specializes in kinetic plasma physics and the interaction between charged particles and electromagnetic waves in near-Earth space. Her favourite space plasma mission is the ESA Cluster mission that first probed the physics of reconnection in collisionless plasma, even though the new results required hasty revision of her PhD thesis!

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