Labour market extremes: A study of the high and low wage ends of the labour market

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Lay Summary

This thesis is comprised of three essays relating to the high and low wage ends of the labour market. As income inequality has increased over the past decades, the question of why some workers earn so much, whilst others earn so little, has attracted increasing attention. This attention is not confined to academic economists: both low-wage, insecure, jobs and extremely high wage jobs are the subject of political and media debate. In the first and third essay I develop theoretical models of the labour market that incorporate different types of 'non-standard' work. These models aim to shed light on why firms might offer workers non-standard jobs, and why workers might accept them. In the middle (co-authored) essay, we study very high, 'superstar' wages. Using data on the wages and performance of football players we study which of two main theoretical models is best suited to explain this phenomenon.

In the first essay I study low-paid, casual work, focusing on the UK and Australia. Casual jobs (also known as 'zero-hours', 'contingent', or 'on-demand' or 'on-call') provide flexibility to firms who do not need to guarantee casual workers fixed hours of work, and so can vary the size of their workforce cheaply and quickly. They also provide flexibility to workers, who are generally not required to accept any work they are offered, and in this paper I study this tradeoff between flexibility and certainty that exists for both firms and workers. There is evidence that the incidence of casual work has been increasing in many developed countries. However, this type of work remains controversial, with some commentators and politicians calling for an outright ban on casual jobs, and so I consider the effects that a ban might have on labour markets.

Although there is some empirical analysis of casual work, theoretical analysis is almost non-existent. This paper aims to fill this gap in the literature by developing a theoretical search and matching model of the labour market in which workers search for jobs and firms search for workers. Jobs in this model differ in their productivity, and productivity changes week to week. In the equilibrium of the model, low productivity jobs are more affected by these weekly changes and so low productivity firms offer casual jobs. On the other hand, high productivity firms want to ensure that their workers turn up every week and so they offer regular jobs. Some workers prefer casual jobs because they have other commitments (perhaps childcare, or studying), and want the flexibility to turn down work. Others would prefer regular work. When these workers meet a firm with a casual job, they accept the job but continue to search for a regular one. I compare this version of the model with a version in which firms cannot offer casual jobs. In this case, low productivity firms no longer find it profitable to create jobs at all, and unemployment increases. However, the remaining jobs are more productive. In addition, since all workers now have regular jobs, they no longer continue to search when they find a job. This decreases the number of workers searching for jobs and makes it easier for unemployed workers to find a job. These two effects offset, to some extent, the increase in unemployment caused by the ban.

In the second essay I turn to the other extreme, and consider why some individuals, such as finance executives, lawyers and media and sports stars, earn wages that are many orders of magnitude greater than the average. In this paper we study one particular labour market: Major League Soccer in the United States, and consider the question: why do some people earn such astronomically high wages? Major League Soccer (MLS) provides a good 'laboratory' to study this question for several reasons. Firstly, detailed salary data is available. Secondly, we can match this salary data with very granular data on productivity (in this context, footballing performance). Thirdly, the League has pursued a strategy of attempting to attract famous footballers from other leagues, particularly in Europe, by paying very high wages (one famous example is David Beckham's move to LA Galaxy). At the same time, most players in MLS earn comparatively little, so there is wide variation in wages.

There are two main theories of these 'superstar' wages. The first suggests that in industries such as sports, very small differences in talent can have an outsized effect on firm (in this context, team) revenue. For example, if a slightly better football player can score the crucial goal that helps the team win the League, the return to the team will be very high. It therefore makes sense for teams to pay slightly better players very high wages. The second theory suggests that consumers enjoy consuming the same product as their peers (e.g. supporting the same footballer). Thus, as a player becomes more popular, consumers prefer them even more. In this way superstars can emerge by chance, perhaps because they are initially slightly better known than others or through opportune timing, rather than because of small differences in talent. In this paper we attempt to distinguish between these two 'productivity' and 'popularity' based theories. We use a two step strategy to do so. Firstly, we use performance data to decompose player's salaries into the amount explained by on-the-pitch performance, and an unexplained amount, which may capture the 'superstardom' of some MLS players. We then aggregate this measure of ‘superstardom’ into a total amount for each team in each year. In the second step, we investigate the relationship between teams’ football performance and revenue and both the explained part of their players’ salaries, and the measure of ‘superstardom’. We find consistent evidence that the amount teams spend on the ‘superstardom’ salary component increases their revenue positively and significantly. However, it does not increase their performance in the League. For example, it does not appear to contribute to their average point score per game. This suggests that the top wages in MLS are a reflection of player popularity, rather than of current talent or performances on the football pitch. We conclude that our results support the ‘popularity’ based theory of high wages, at least in this particular labour market.

In the final essay I return to the low-wage end of the labour market, to study long-term trends in part-time work in the UK, which is concentrated in low-wage jobs. I show that there have been substantial changes to part-time work over the last 30 years. Firstly, there are more part-time workers. This increase is due to more men working part-time, whilst the percentage of women working part-time has decreased slightly. At the same time, part-time workers are now working longer hours. Secondly, I show that wages for part-time workers have been increasing relative to full-time workers. In this essay I consider whether these changes are the result of changes in workers’ preferences or changes in technology that mean that firms now prefer to employ more part-time workers. This is an important question from a policy perspective since the increase in part-time work could have implications for inequality.

In this essay I develop a flexible model of the labour market that incorporates both firm and worker preferences for part-time work. Since part-time work is associated very strongly with certain occupations, particularly those that require lower qualifications, I begin by assuming that firms require workers to do two types of tasks. The first type of task can be done in whatever time is available. The second type of task is more complex and has an associated hours cost. I show that workers with a greater preference for leisure will do the first type of task and work shorter hours, and those with a lower leisure preference will do the more complex tasks, and work longer hours. These tasks are more productive, and hence these workers will earn more per hour. I show how changes in the parameters of the model can affect the percentage of part-time workers, the hours that part-time workers choose to work, and the relative wages. Thus the model provides a framework that can be used to assess whether the trends observed in the data are the result of changes in worker or firm preferences.

Abstract

This thesis is comprised of three essays studying the high and low wage ends of the labour market. Historically there has been little theoretical analysis of low-paid, non-standard work and so in the first and third essays I develop theoretical models incorporating two types of non-standard work: casual and part-time. The second, co-authored, essay is empirical, using data on football players to study the wage determination of very highly paid workers.

Flexibility or certainty? The aggregate effect of casual work
In the first essay I study low wage, casual jobs. These jobs provide flexibility for firms to change the size of their workforce cheaply and quickly and for workers to choose whether to supply labour in every period. This flexibility comes at the expense of certainty for both firms and workers. I develop a search and matching model incorporating casual jobs, which I use to evaluate the effect of labour market policies on aggregate outcomes. The equilibrium of the model features the concentration of casual jobs at the bottom of the wage distribution. I find that a ban on casual jobs increases unemployment, but that the average wage of those employed actually increases. In addition, in the model with casual jobs, workers in low wage casual jobs continue to search for a higher quality match that will offer work more frequently. In a model with search frictions, this makes it harder for unemployed workers to match with a firm. This crowding out effect offsets some of the negative effects of a ban. I also consider the effect of a higher minimum wage for casual jobs. I find that the effects are limited. These results are due to an offsetting mechanism: although higher wages lead to higher unemployment, as firms offer more regular jobs, the number of workers called-up to work in any one period increases.

Extreme wages, performance and superstars in a market for footballers
In the second essay we turn to the other end of the wage distribution, to study the determinants of superstar wage effects, asking whether productivity or popularity-based explanations are more appropriate. We use longitudinal wage and performance data for workers (players) and firms (teams) from a particular market for sports talent: Major

League Soccer in the United States. We find evidence that the top earners, whose annual salaries are mostly not accounted for by their past MLS performances, when compared alongside other footballers, are paid more because they attract significantly higher stadium attendances and thus revenues. There is no evidence that higher residual salary spending by the teams affects their relative performance in football terms, or that the amounts the teams spend on actual talent affect attendances. Taken together, these results suggest that a popularity-based explanation of superstar wage effects is appropriate among the top earners in this labour market.

Long-term trends in part-time work in the UK

In the final essay I study long-term trends in part-time work. I show that there has been an increase in the part-time share over the past 30 years. There has also been an increase in the part-time work on the extensive margin; part-time workers on average work longer. Despite this, the difference in average hourly pay for part-time and full-time workers (the part-time pay penalty) has steadily decreased. I develop a flexible neoclassical model of the labour market which can explain firms' and workers' preferences for part- and full-time work. The equilibrium of the model matches key features of the labour market: full- and part-time workers undertake different tasks; there is bunching of workers at full-time hours; and full-time workers earn higher hourly wages than part-time. The model can be used to disentangle the effects of changes in workers' preferences and on firms' production technologies on the relative quantities and prices of part- and full-time labour. I provide an extension of the model which incorporates heterogeneity in workers' preferences, and will enable the study of gender differences in part-time work.

Declaration

I, Rachel Elizabeth Scarfe, confirm that the work presented in this thesis is my own. Where the research was carried out alongside others, or where information has been derived from other sources, I confirm that this has been indicated in the thesis. This work has not been submitted for any other degree or professional qualification.

Edinburgh, 31 March 2022

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Over the last five years I have been fortunate enough to enjoy the companionship and hospitality of a great many friends. I am thankful especially for Fiona and Susan, for my brother Daniel and for the whole Rennie Hill family. When I fell into a hole they never failed to reach out a hand and pull me back out. Finally, and most importantly, I thank my parents for 32 years of boundless practical, emotional, mathematical and nautical support. Without them I would never have started, let alone finished, this thesis and it is to them that it is dedicated, with all of my love.

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Chapter 1

Flexibility or certainty? The aggregate effects of casual work

An earlier version of this essay has appeared as Edinburgh School of Economics Discussion Paper Series, no. 294. It has also been presented at the 2021 Essex/RHUL/Bristol Junior Search and Matching Workshop; 2020 SOLE Conference; 2019 Scottish Economic Society Conference, and the 2019 Workshop on Zero Hours Contracts: Research, Measurement and Policy. This paper uses unit record data from the Household, Income and Labour Dynamics in Australia (HILDA) Survey. The HILDA Project was initiated and is funded by the Australian Government Department of Social Services (DSS) and is managed by the Melbourne Institute of Applied Economic and Social Research. The findings and views reported in this paper, however, are those of the author and should not be attributed to either DSS or the Melbourne Institute.

1.1 Introduction

From the agricultural day labourers of the nineteenth century to the agency and zero-hours workers of the modern day, casual work has long been a feature of labour markets around the world. Casual jobs provide flexibility to firms who do not need to guarantee casual workers certain, fixed, hours of work, and so can vary the size of their workforce cheaply and quickly. They also provide flexibility to workers, who are generally not required by law to accept any work they are offered. Thus both workers and firms face a trade-off. In a casual job, in each period the firm can choose whether to “call-up" a worker (i.e. whether to offer them work) and the worker can choose whether to supply labour. In contrast, in a standard job, firms must call-up their workers (and pay their wages), and workers must supply labour, to an agreed schedule.

These jobs are known by a variety of names, including “zero-hours", “contingent", “on-demand" or “on-call", and are used around the world. For example, approximately 4% of British and Finnish, and 5% of Irish employees have “zero-hours" contracts and 6% of Dutch employees are classified as “on-call" (Datta et al., 2020).[1] There is evidence that the incidence of casual work is increasing in some developed countries. However, these jobs are controversial and currently the subject of much policy debate, with some commentators calling for an outright ban.[2] Their proponents consider that casual jobs increase labour demand, by decreasing the risk firms face that future changes in productivity will make a job unprofitable. They may also increase labour supply, particularly amongst those with other responsibilities, such as studying or caring for relatives, who do not wish to commit to fixed hours of work. Critics argue that uncertainty about the availability of work is detrimental to employees, who would prefer a steadier income. There is also debate about whether casual jobs can provide “stepping stones" to regular employment.

There is currently very little theoretical analysis of casual jobs.[3] This paper aims to fill this gap in the literature by developing a search and matching model in which firms can offer regular and casual jobs, and in which the two types coexist in equilibrium. Such a model must incorporate the trade-offs between casual and regular work for both workers and firms. This is in contrast to the existing literature on two-tier labour markets, including models with temporary jobs, which assumes that all workers would prefer a standard job if available, and that workers consider non-standard jobs to be “inferior".[4] I can therefore consider the following key question: what are the effects of the availability of casual jobs on aggregate outcomes?

The model is based on the Mortensen and Pissarides (1994) model in discrete time. The equilibrium of the model reflects four key facts about casual work: (1) in equilibrium casual workers are concentrated in low-wage jobs and work less frequently than regular workers; (2) on average casual jobs are of a shorter duration; (3) workers with a preference for flexibility are concentrated in casual jobs; and (4) there is some mismatch between workers and jobs, as some casual workers would prefer to work more hours.

[1] Other countries which allow some form of casual work include Austria, Australia, Canada, the Czech Republic, New Zealand, Sweden and (to a limited extent) Germany and Italy.

[2] Countries where such jobs are already banned include France, Norway and Belgium.

[3] A key exception is concurrent, ongoing, work by Dolado et al. (2021), discussed in more detail below.

[4] Examples include Cahuc et al. (2016), Bentolila et al. (2012) Cahuc and Postel-Vinay (2002), and Blanchard and Landier (2002).

The model has three main features that reflect the trade-off between the two types of job. First, when a vacancy and worker meet, a permanent match-specific productivity, z, is drawn. A separate transitory productivity shock, x, is drawn at the beginning of each subsequent period. This productivity shock multiplies the permanent component of productivity, i.e. zx, so that productivity shocks are relatively more important for jobs with a lower permanent z and in equilibrium these jobs are casual. Second, I model different worker preferences for flexibility by assuming there are two types of workers. An exogenous fraction of workers have no disutility of labour, and wish to supply labour in every period. The other workers experience shocks to their disutility of labour, capturing any other time commitments that they may have. Finally I incorporate on-the-job search in a novel manner: workers can search only in those periods when they are not called-up to work and therefore have time to look for jobs. Thus, casual workers retain the flexibility to search for “better" jobs whilst regular workers do not. Differences in job length between casual and regular workers are endogenous, and in equilibrium casual jobs have a shorter average duration.

I show that, as a result of the structure above, there exists a unique reservation value of the permanent match-specific productivity component. Firms that draw a match quality greater than this reservation value will prefer to offer a regular job and firms that draw a permanent productivity lower than the reservation value will prefer a casual job. The proportion of casual jobs is thus an endogenous outcome of the model, and can change when labour market regulations, including minimum wages, change.

I solve the model numerically in steady state, and calibrate to moments of Australian data.[5] I find that a ban on casual jobs increases the unemployment rate and lowers the job finding rate dramatically. However, aggregate production and workers' utility fall only slightly. This is because, in the casual regime, firms only call-up casual workers in some periods. Additionally, workers sometimes decline offers of work when called-up. Thus there will be some workers employed in a casual job who are not actually supplying labour, producing, and earning wages. This offsets lower unemployment in the casual regime. At the same time, in the casual regime workers search on the job for a higher quality match that will offer work more frequently. In a model with search frictions, this makes it harder for unemployed workers to match with a firm. This crowding-out effect further offsets some of the benefit in terms of lower unemployment of the casual regime. I find that the effects of an increase in the minimum wage for casual jobs relative to regular jobs are limited. As the casual minimum wage rises there is an increase in unemployment

[5] As casual work is more prevalent in Australia, there is a clear definition that can be used to identify casual workers in Australian data. This is more difficult in data from other countries. Data is also available on workers' labour supply preferences.

and decrease in the job-finding rate. This is intuitive: as the minimum wage rises the expected value of a filled job decreases and so firms create fewer vacancies. However, there is little change in aggregate production and worker utility. This is due to the two offsetting mechanisms described above.

There is some empirical analysis of casual work in Australia (e.g. Buddelmeyer and Wooden (2010), Watson (2005) and Watson (2013)); of zero-hours jobs in the United Kingdom (Datta et al., 2020; Farina et al., 2019); and of on-call work in the Netherlands (Burri et al., 2018). However, to my knowledge, theoretical analysis of casual work is almost non-existent. An exception is Dolado et al. (2021), who also develop a search and matching model incorporating zero-hours jobs. Their focus is on the low-wage segment of the labour market in the United Kingdom, and they assume all workers are paid an exogenous minimum wage. Their model also features heterogenous workers but in a different way: workers are risk averse so most prefer the certainty of a regular job but there are a small number with very high disutility of labour that prefer zero-hours jobs, and will never take a regular job. However, workers in a zero-hours job must always accept offers of work. In my model, casual workers can turn down work. In addition, in my model the concentration of casual jobs in the low-wage segment is an equilibrium result. This allows me to consider the effect of changes in labour market regulations, such as minimum wages, on the proportion of casual jobs.

This paper is also related to models where firms adjust the size of their workforce along the intensive margin rather than along the extensive margin (such as Trapeznikova (2017) and Cooper and Willis (2009)), and to previous work on non-standard contracts, which has focused mainly on temporary jobs. In particular, Cahuc et al. (2016) and Cao et al. (2010) develop models where firms choose whether to offer permanent or temporary jobs based on match productivity. In these models workers would, all else equal, prefer standard jobs. They do not take into account workers who prefer to supply labour less frequently, and the consequences for aggregate outcomes.

The paper proceeds as follows. Section 1.2 discusses available data on casual work, and documents some broad findings about the characteristics of casual workers and jobs. Section 1.3 sets out the model, Section 1.4 discusses an indicative calibration and Section 1.5 presents the results. Section 1.6 concludes.

1.2 Descriptive statistics

In this section I compare casual and regular jobs, providing evidence for four key differences between them. First, casual jobs are concentrated in certain industries and occupations, particularly in low-skilled or service occupations and in industries with relatively large and frequent changes in demand. Second, there are differences in the characteristics of casual workers and jobs relative to regular workers and jobs. Casual workers are generally younger, with fewer years of education and shorter job tenure. They work fewer hours and for smaller firms. Third, casual workers are generally paid less than regular workers. Finally, they are more likely to transition to regular work than regular workers are to transition to casual work.

1.2.1 Data sources

I define a casual worker as one who usually works fewer than 35 hours a week, and whose usual hours of work and pay vary from week to week. There are several issues with currently available data on casual work. The percentage of casual workers in most countries is small, and there are concerns that survey data may not capture the full extent of casual work. Some countries with small numbers of casual workers do not collect data systematically (e.g. Finland), or may not have a strict legal definition that can be used to consistently identify casual workers (e.g. the Netherlands, Canada). Others have regulations that differ across states (e.g. the USA), or that have changed significantly in the last few years (e.g. Ireland, New Zealand). Finally, changes in the survey methods and low public awareness of different types of contracts may lead to under-counting (there is evidence this is the case in the UK, see Farina et al. (2019)). As a result, survey data on casual work in most countries may not give an accurate picture.

In Australia casual work is a more entrenched, and wider spread, phenomenon than in other countries. Therefore data from Australia is less likely to be subject to some of the issues described above. Data about casual work in Australia has been collected since 2001 in the annual Household, Income and Labour Dynamics in Australia Survey (HILDA). This is a panel survey, with information about transitions between different types of work and unemployment. It therefore provides data moments that can be used to calibrate the model in Section 1.3. A further advantage of the HILDA data compared to other sources is that it includes more questions about workers' labour supply preferences, including preferred hours of work and level of satisfaction with their working schedule.

Although the greater incidence of casual work in Australia may mean that data is more reliable, it could also mean that conclusions based on Australian data may not be applicable to other countries. This is especially likely if the high use of casual contracts in Australia is due to specific labour market regulations or conditions, such as firing costs. This section compares results for both Australia and the UK using data from the Labour Force Survey (LFS), as far as is possible given different survey designs. Australia and the UK are considered to have similar levels of labour market regulation (for example, they score similarly in the OECD's Employment Protection Index). The data presented below suggest that casual workers in the UK and Australian have similar characteristics. This section compares casual and 'regular' work, excluding other types of employment, such as self-employment or job sharing. Further details about the data sources and the construction of the sample are available in Appendix 2.

1.2.2 The distribution of casual jobs

Figure 1.1 shows the percentage of all workers in each industry grouping (Panel (A)) and occupation (Panel (B)) that are casual. Although there are differences in the level of casual work in Australia and UK, the distribution is similar. In both countries, workers in the Agriculture and Services industries and in Unskilled and Service occupations are much more likely to be casual. In the UK a higher percentage of workers in the Public Administration, Education and Health group are casual than in Australia. This is due to the higher number of care workers (most of whom have casual jobs) in the UK. These industries and occupations are all likely to be subject to relatively large and frequent changes in demand.

1.2.3 Characteristics of casual jobs and workers

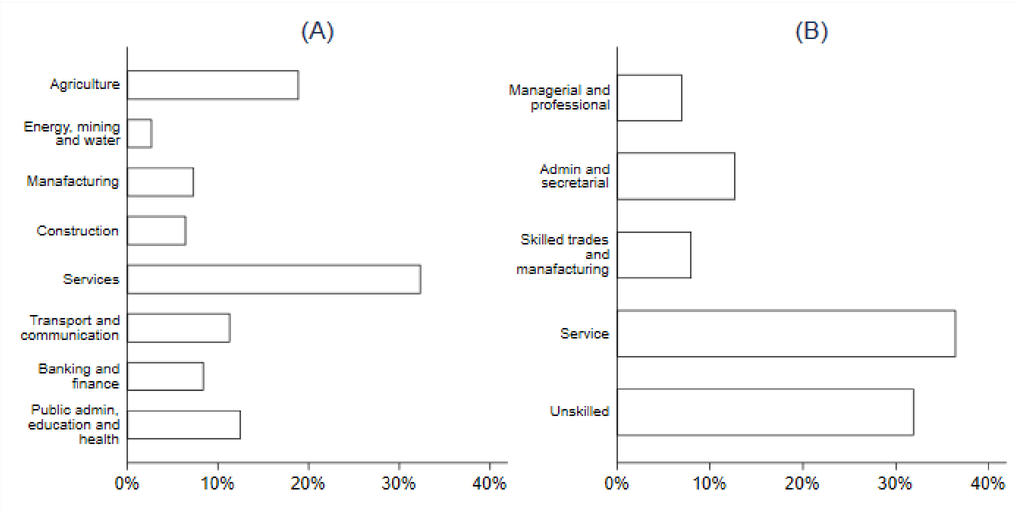
Figures 1.2 and 1.3 show some of the characteristics of casual and regular workers. Panels (A) and (B) show the actual usual weekly working hours of casual workers and the number of hours they would like to work. Casual workers generally work fewer hours than others, and are often offered no work at all. However, they would also prefer to work fewer hours in general, suggesting that they may have different labour supply preferences. This is supported by the proportion of casual workers with other commitments: in Australia 24% are currently studying and 16% have another job, compared to 9% and 7% of regular workers respectively.

Panels (C) and (D) show the percentage of casual workers in each age group and with each level of qualification, compared with the percentage of regular workers. Casual workers are generally younger, and have fewer years of education than regular workers. These differences are more pronounced in Australia, but are observable in the UK.

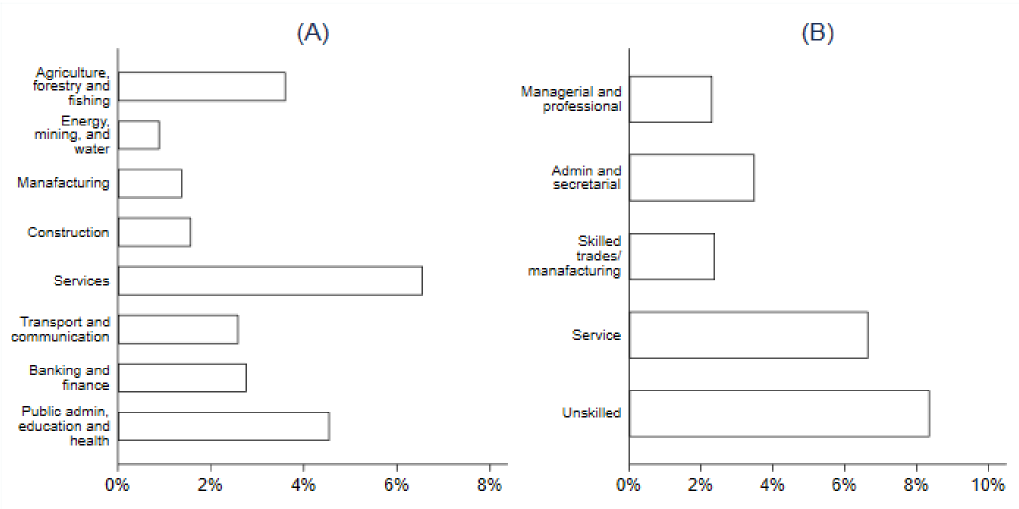
Panel (E) shows that casual workers generally have much lower tenure in their jobs, perhaps partly because they tend to be younger. Finally, Panel (F) shows the percentage of casual and regular workers split by the size of the firm they work for and shows that casual workers are concentrated in smaller firms.

FIGURE 1.1: Distribution of casual and regular jobs (% of all workers that are casual)

Australia



UK



Source: HILDA and LFS.

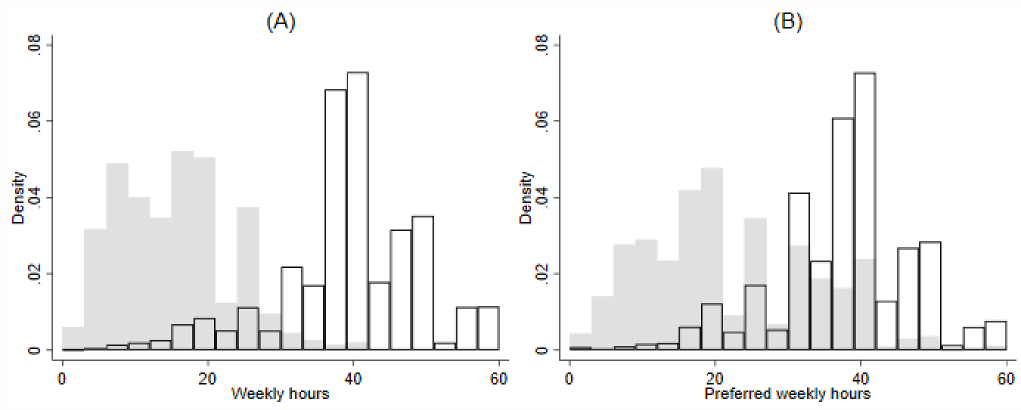
1.2.4 Wages of casual workers

The median hourly wage for a casual worker in Australia, at 2001 prices, was AUD 14.71, inclusive of a 25% pay premium for casual jobs. This pay premium is intended to compensate the worker for benefits such as holiday pay that are not available to casual workers. The median wage for regular workers was AUD 19.21, compared to a minimum wage for regular work between AUD 11.19 and AUD 12.49.[6]

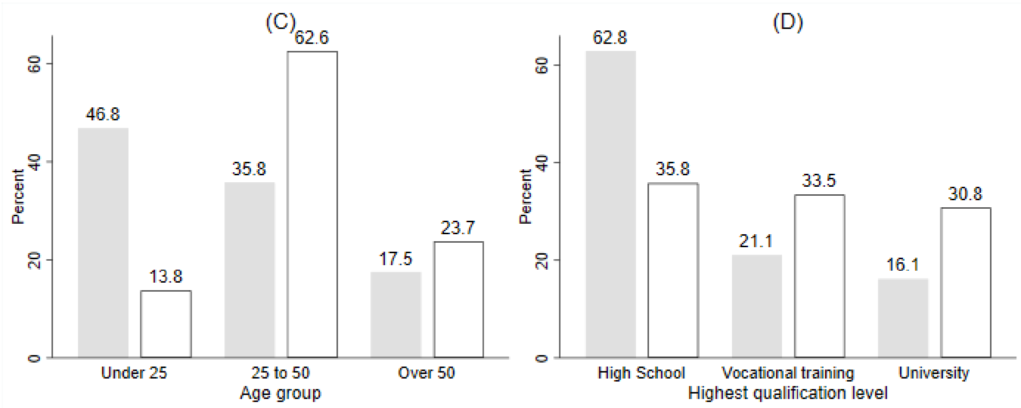
[6]These minimum wages are approximate as minimum wages in Australia vary across industries. This figure is the average federal minimum wage for workers aged over 21 at 2001 prices.

FIGURE 1.2: Characteristics of casual jobs and workers (Australia)

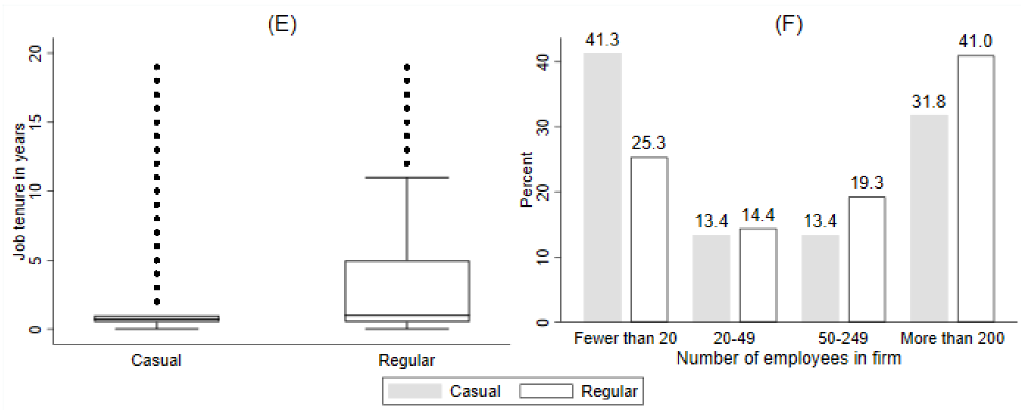
(a)



(b)



(c)



Source: HILDA Survey.

The hourly wage in the UK is also lower for casual workers; the median wage for casual workers was GBP 5.69, compared to GBP 7.60 for regular workers (in 2001 prices). Over the same period, the minimum wage ranged from GBP 3.80 to GBP 5.44.

1.2.5 Labour market transitions of casual workers

The table below shows workers' average annual transition probabilities in the HILDA survey.[7] 29% of casual workers move to regular jobs within a year, but only 3% move in the other direction. This provides some support for the argument that casual contracts can provide a "stepping stone" to regular jobs. Workers also transition between casual jobs: 22% of those who were in casual work for at least a year moved between (casual) jobs during the year. In contrast, only 15% of those who stayed in regular work for at least a year moved between (regular) jobs during the year.

TABLE 1.1: Average transition probabilities

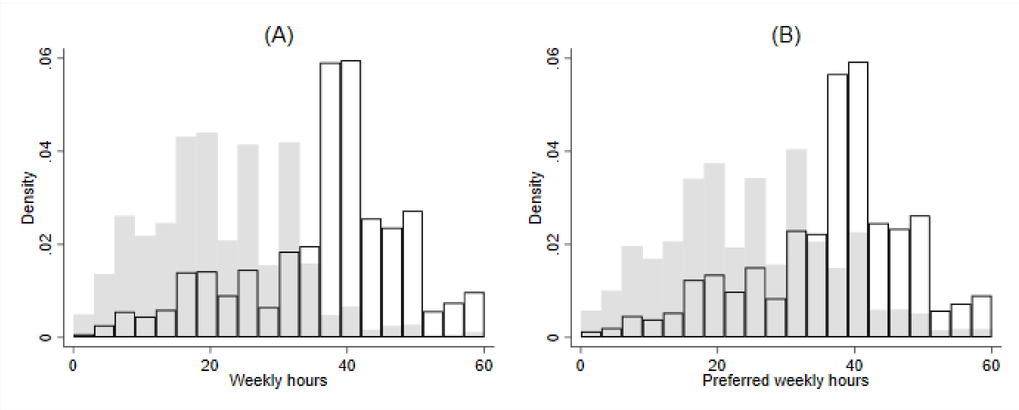
Year T+1	Inactive	Unemployed	Casual	Regular	Total
Year T					
Inactive	88%	3%	4%	5%	100%
Unemployed	26%	29%	16%	28%	100%
Casual	14%	4%	54%	29%	100%
Regular	4%	2%	3%	91%	100%
Total	36%	4%	9%	51%	100%

Source: HILDA survey.

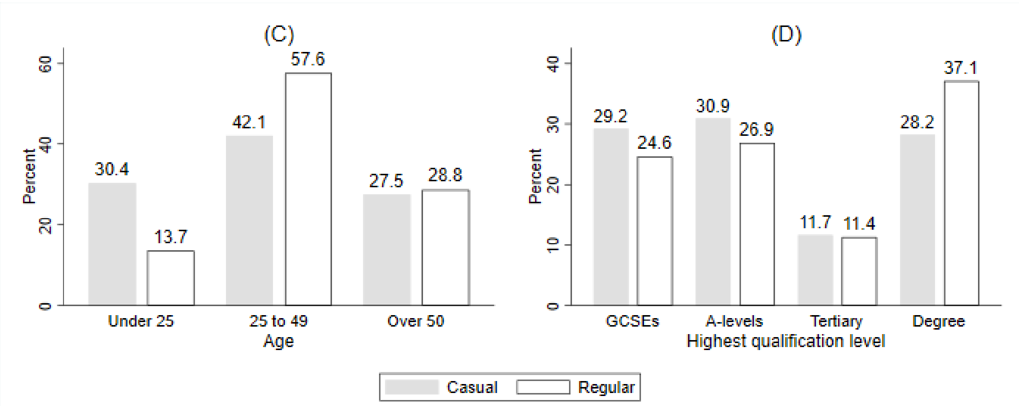
[7] Although there is a panel version of the UK LFS, the sample of casual workers is very small and so in this section I focus on Australian data.

FIGURE 1.3: Characteristics of casual jobs and workers (Australia)

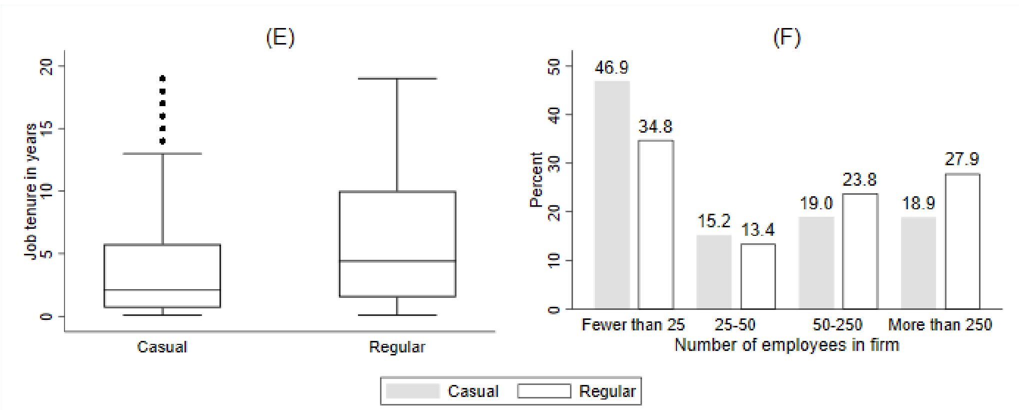
(a)



(b)



(c)



Source: LFS.

1.3 Model

In this section I introduce a model of the labour market in which firms decide whether to offer regular or casual jobs. I refer to this as the “casual" regime. I also set out a version of the model in which firms can only offer regular jobs, the “standard" regime. In order to compare aggregate unemployment, production and worker utility in the two regimes, the model should include four key features, reflecting the data discussed in Section 1.2. The aim of this section is to set out a model that incorporates these, whilst remaining as simple and tractable as possible. These key features are: (1) casual and regular jobs must coexist in equilibrium, with casual workers concentrated in low-wage jobs and working less frequently than regular workers; (2) the model must allow workers to transition from casual to regular jobs with casual jobs being shorter on average; (3) it must incorporate workers with different labour supply preferences with those who prefer flexibility concentrated in casual jobs; and (4) the data suggests there is some mismatch between workers and jobs, as some workers in casual jobs would prefer to work more hours and vice versa. The model equilibrium must incorporate this mismatch.

The model is based on the Mortensen and Pissarides (1994) model in discrete time. Firms and workers search randomly in a single labour market, and, on matching with a worker, a firm can offer either a casual or a regular job, denoted $j = \{c, r\}$. In a regular job, it must “call-up" the worker in every period (and pay them a wage), unless the job is destroyed. Similarly, the worker must supply labour in every period. In a casual job, the firm can choose whether to call-up the worker each period. If the worker is not called-up then the firm does not need to pay them a wage. If called-up, the worker can choose whether to supply labour or not.

There is a matching function $m(v, s)$ which governs the number of matches in the market, where s is the measure of workers searching for a job and v is the measure of vacancies. I make the standard assumptions that the matching function is increasing in s, v, concave, and has constant returns to scale. The probability that a vacancy matches with a worker is $q(\theta) = m(s, v)/v$ where $\theta = v/s$ is the market tightness. The probability that a worker matches with a vacancy is $f(\theta) = m(s, v)/s = \theta q(\theta)$.

For simplicity, one feature that is not captured in this model that could affect workers' decisions is their degree of risk aversion and their ability to save or borrow. Incorporating risk aversion would lower the relative value of a casual job to workers. However, if workers could save then they could, to some extent, insure themselves against the possibility of not being called-up to work. It is not clear therefore what the aggregate effect of these two factors would be.

Coexistence of regular and casual jobs

To ensure firms offer both types of job in equilibrium, I assume that the productivity of worker-job match has two components: a permanent productivity component (z) that is drawn once when the worker and firm match, and a temporary shock to productivity (x) that is drawn from the same distribution in each period, with no persistence, and which is independent of z. Total match productivity is xz. As I show later, this generates a channel of demand for casual contracts amongst firms that draw a low permanent productivity. The permanent component has distribution $F(z)$ and bounded support $[\underline{z}, \bar{z}]$, and the temporary component has distribution $G(x)$ and bounded support $[\underline{x}, \bar{x}]$. I assume that the mean of x is one. It acts as a multiplier of the permanent productivity z, so that productivity is “high" in periods when $x > 1$ and “low" in periods when $x < 1$.

Transitions

To allow for transitions between different types of job, workers search on the job. They can only search when they are not called-up to work, so that regular workers do not search and search intensity is endogenous. This specification has another benefit, which I discuss in more detail below, in that it ensures that workers will always take a casual job. For simplicity, I assume that firms are not able to renegotiate with workers who have been offered another job at another firm.

Workers’ preferences

Worker utility is $u(w) = w - \varepsilon$. The parameter ε is the disutility of labour, drawn at the beginning of each period, with distribution $H(\varepsilon)$. It captures any other time commitments that workers may have, such as caring or studying. In any one period, the disutility of labour can be high or low, so that $\varepsilon = \{0, \bar{\varepsilon}\}$ with $Pr(\varepsilon = 0) = \phi$. There are two worker types, $i = \{H, L\}$. “High Labour Supply" (type H) have no disutility of labour, so that $\phi_H = 1$. “Low Labour Supply" (type L) workers experience shocks to their disutility of labour, so that $\phi_L = \phi < 1$. A fraction γ of workers are type L, and workers cannot change types. In other respects, workers are ex-ante identical.

Mismatch

In the search and matching framework, search frictions generate unemployment. There is a single labour market without directed search, so that a worker may accept a casual job when they would prefer a regular one or vice versa. This ensures there is some mismatch between workers and jobs.

Finally, I assume that there is no endogenous job destruction.[8] I also assume that each worker can hold only one job at a time. Workers could otherwise hold multiple casual jobs, ensuring that at least one would call them up to work in every period. In practice, the percentage of casual workers who hold another job is quite small.[9]

Model timing

The model timing is as follows:

1. Firms decide whether to pay a flow cost in order to post a vacancy.

2. Workers and firms meet randomly, subject to a matching function.

3. Upon meeting a worker, firms and workers learn the permanent match productivity. Firms observe the type of the worker (H or L), and decide whether to offer a casual or regular job. I assume free disposal of matches, so that if the match productivity is extremely low, a firm can dispose of the match and create a new vacancy in the next period.

4. If the firm decides to offer the worker a job, at the beginning of each subsequent period, the firm and worker bargain over the wage for the period (see discussion below). After concluding the negotiations, the temporary productivity shock for the period is drawn. Type L workers learn their disutility of working for the period.

5. Production occurs. Regular jobs produce in every period. Casual jobs only produce if it is optimal for both worker and firm to do so. Otherwise the worker is (i) not called-up by the firm, or (ii) decides not to supply labour in this period and receives the per-period value of unemployment, b. Unemployed workers also receive b.

6. During the production phase employed workers who are not called up for work can search for a new job in the same labour market as unemployed workers.

7. At the end of each period, a job is destroyed with exogenous probability δ. Alternatively, a worker searching for another job may match with vacancy, and may quit for a new job.

[8] This is equivalent to assuming that there is a firing cost large enough that a firm will never wish to fire a worker.

[9] For example, it is 16% in Australia and 7% in the UK.

1.3.1 Workers

The measure of unemployed worker of each type is denoted u_i. The distribution of workers in casual jobs is denoted $n_{ic}(z)$ with cumulative distribution $N_{ic}(z)$ and the distribution of employed workers searching for a new job is $s_{ic}(z)$ with cumulative distribution $S_{ic}(z)$. The total measure of searching workers is $s = u_H + u_L + S_{Hc}(\bar{z}) + S_{Lc}(\bar{z})$.

The value of unemployment to a type i worker is

$$U_i = b + \beta\Big(f(\theta)\phi_i \int_{\underline{z}}^{\bar{z}} \int_{\underline{x}}^{\bar{x}} \int_0^{\bar{\varepsilon}} \big(\max\{\mathbb{1}_{\text{offer},ir}(z')W_{ir}(z',\varepsilon'), \mathbb{1}_{\text{offer},ic}(z')W_{ic}(z',x',\varepsilon'), U_i\}\big) dH_i(\varepsilon')dG(x')dF(z') + (1 - f(\theta)\phi_i p_i)U_i\Big) \quad (1.1)$$

where $\mathbb{1}_{\text{offer},ij}(z)$ is an indicator function that equals one if a worker of type i is offered a job of type j and zero otherwise. $p_i = \int_{\underline{z}}^{\bar{z}} (\mathbb{1}_{\text{offer},ir}(z') + \mathbb{1}_{\text{offer},ic}(z'))dF(z')$ is the probability that the firm offers the worker a job. Note that unemployed type L workers only search when $\varepsilon = 0$.

A worker in a regular job earns a wage w with certainty in each period. At the end of each period the job is destroyed with probability δ. Since regular workers are always working, they do not search on the job. The value of a regular job is therefore

$$W_{ir}(z,\varepsilon) = w - \varepsilon + \beta\Big((1-\delta)\int_0^{\bar{\varepsilon}} W_{ir}(z,\varepsilon')dH_i(\varepsilon') + \delta U_i\Big). \quad (1.2)$$

The value of a casual job depends on the realisation of x, which determines whether the firm calls-up the worker. Let $\mathbb{1}_{\text{prod}}(x,z,\varepsilon)$ be an indicator function that equals one if the firm calls-up the worker and the worker supplies labour, so that production occurs, and zero otherwise. If production occurs then the worker is unable to search for another job. Otherwise, they earn b and search. The value of a casual job is

$$W_{ic}(z,x,\varepsilon) = \mathbb{1}_{\text{prod}}(z,x,\varepsilon)w + (1 - \mathbb{1}_{\text{prod}}(z,x,\varepsilon))b + \beta\Big(\mathbb{1}_{\text{prod}}(z,x,\varepsilon)f(\theta)\Psi_i(z) + \quad (1.3)$$
$$\big(1 - \delta - f(\theta)\mathbb{1}_{\text{prod}}(z,x,\varepsilon)\big)\big) \int_{\underline{x}}^{\bar{x}} \int_0^{\bar{\varepsilon}} W_{ic}(z,x',\varepsilon')dH_i(\varepsilon')dG(x') + \delta U_i\Big).$$

where $\Psi(z)$ is the expected value of a match with a new firm, given by

$$\Psi_i(z) = \int_{\underline{z}}^{\bar{z}} \int_{\underline{x}}^{\bar{x}} \int_0^{\bar{\varepsilon}} \big(\max\{\mathbb{1}_{\text{offer},ir}(z,z')W_{ir}(z',\varepsilon'), \quad (1.4)$$
$$+ \mathbb{1}_{\text{offer},ic}(z,z')W_{ic}(z',x',\varepsilon'), W_{ic}(z,x',\varepsilon')\}\big) dH_i(\varepsilon')dG(x')dF(z').$$

1.3.2 Firms

A firm can post a vacancy with per-period cost k. With probability $q(\theta)$ the firm matches with a searching worker. The value of a vacancy to a firm is

$$V = -k + \beta\big(q(\theta)\int_{\underline{z}}^{\bar{z}}\int_{\underline{x}}^{\bar{x}}\int_{0}^{\bar{\varepsilon}}\big(\Omega_{Hu}(z') + \Omega_{Lu}(z') + \Omega_{Hc}(z') + \Omega_{Lc}(z')\big)dH_i(\varepsilon')dG(x')dF(z') + (1-q(\theta))V\big). \tag{1.5}$$

Ω captures the firm's decision about the type of job to offer an unemployed worker or one with a casual job with productivity z

$$\Omega_{iu}(z') = \frac{u_i}{s}\max\{\mathbb{1}_{\text{accept},ic}(z)J_{ic}(z',x',\varepsilon'), \mathbb{1}_{\text{accept},ir}(z')J_{ir}(z',x'), V\} \tag{1.6}$$

$$\Omega_{ic}(z,z') = \frac{1}{s}\int_{\underline{z}}^{\bar{z}} s_i(z)\max\{\mathbb{1}_{\text{accept},ic}(z,z')J_{ic}(z',x',\varepsilon'), \mathbb{1}_{\text{accept},ir}(z,z')J_{ir}(z',x',\varepsilon'), V\}dN_{ic}(z) \tag{1.7}$$

where $\mathbb{1}_{\text{accept},ij}(z,z')$ is an indicator function that equals one if a worker of type i will accept a job of type j with match productivity z' and zero otherwise. Since x is only drawn after the worker moves to a new firm, and because it has no persistence, the current x does not affect whether the worker accepts a new job, keeping the job search decision tractable.

The value of a regular job filled with a type i worker is

$$J_{ir}(z,x) = zx - w + \beta\Big(\delta V + (1-\delta)\int_{\underline{x}}^{\bar{x}} J_{ir}(z,x')dG(x')\Big) \tag{1.8}$$

The value of a casual job filled with a type i worker is

$$J_{ic}(z,x,\varepsilon) = \mathbb{1}_{\text{prod}}(x,z,\varepsilon)(zx - w - k_c) + \beta\Big((\delta + \mathbb{1}_{\text{prod}}(x,z,\varepsilon)f(\theta)p_i(z))V + (1-\delta-\mathbb{1}_{\text{prod}}(x,z,\varepsilon)f(\theta)p_i(z))\int_{\underline{x}}^{\bar{x}}\int_{0}^{\bar{\varepsilon}} J_{ic}(z,x',\varepsilon')dH_i(\varepsilon)dG(x')\Big) \tag{1.9}$$

k_c is the (very small) administrative cost of a casual job that applies even if the firm does not call-up the worker. This could be the cost to manage a more complicated schedule, for example.

1.3.3 Wage determination

I assume that wage bargaining happens after the vacancy and worker match and z is drawn. After bargaining, the transitory productivity shock x is drawn, and, if the job is a casual one, the firm decides whether to call-up the worker at the agreed wage. I use a bargaining mechanism proposed by Hall and Milgrom (2008). In this setting, firms and workers bargain over wages at the beginning of each period. Hall and Milgrom (2008) argue that the threat to quit negotiations entirely is not credible in the presence of search frictions, because the existence of a surplus means that both firm and worker will wish to resume negotiations in the next period. Instead, I assume that if they do not reach an agreement then no production occurs during that period and bargaining resumes in the next period. In this case the worker receives b and the firm receives nothing.

A benefit of this approach is that it captures the lack of commitment in the relationship between a firm and a worker with a casual job. On the firm side, the firm cannot commit to calling-up the worker before learning x. Regardless of what the firm and worker agree, when the temporary productivity shock x is drawn the firm will only choose to call-up the worker if $xz > w$, i.e. when the firm's per-period payoff is positive. The worker will only choose to supply labour if the payoff $w - \varepsilon$ is greater than b plus the value of searching on-the-job if not working. This bargaining mechanism therefore rules out upfront or lump-sum payments.[10] It also ensures that casual workers are not offered work in every period.

For a regular job, the firm's expected per-period payoff from agreeing a wage rather than delaying until the next period is the expected productivity minus the wage $z - w$.[11] The worker's payoff is $w - (1-\phi_i)\bar{\varepsilon} - b$. In equilibrium, firms and workers will never delay agreeing a wage and will bargain over the payoff. If the worker's bargaining power is η then maximising the Nash product gives the wage $w = \eta z + (1-\eta)(b + (1-\phi_i)\bar{\varepsilon})$. I assume there is a minimum wage $\underline{w}$ so that the wage is given by $w = \max\{\eta z + (1-\eta)(b + (1-\phi_i)\bar{\varepsilon}), \underline{w}\}$.

The correct wage bargaining mechanism for casual jobs is less obvious. To see why, consider the total expected per-period surplus for a casual job

$$Pr(xz > w)E[x|xz > w]z - (1 - Pr(xz > w))b. \tag{1.10}$$

[10] To maximise the surplus, the firm and worker could agree a wage of zero, and the firm could compensate the worker with an upfront payment at the beginning of a job. In this case the firm would call-up the worker in every period, maximising production. However, the firm and worker are not able to commit to such a contract. After receiving the lump sum, the worker could then turn down any offer of work, in favour of continuing to search for a better job.

[11] Recall that the expected value of x is one.

This is decreasing in w; a higher wage means that production will happen less frequently, decreasing the overall surplus. However, the worker's share of the surplus will be greater. The outcome of a bargain is thus unclear since workers would not necessarily prefer a higher wage. I therefore assume that firms cannot discriminate in how they bargain with workers with different contracts, and the wage for casual jobs is also $w = \max\{\eta z + (1-\eta)(b + (1-\phi_i)\bar{\varepsilon}), \underline{w}\}$.[12]

1.3.4 Equilibrium

An equilibrium consists of a market tightness θ, steady state stocks u_i, distributions $N_{ij}(z), S_i(z)$, reservation productivities $z_i^*, \hat{z}_{ij}$, value functions $W_{ir}(z,\varepsilon), W_{ic}(z,x,\varepsilon), U_i$, $J_{ir}(z,x), J_{ic}(z,x,\varepsilon), V$ and policy functions $\mathbb{1}_{\text{offer},ij}(z,z')$, $\mathbb{1}_{\text{accept},ij}(z,z')$, $\mathbb{1}_{\text{prod}}(z,x,\varepsilon)$, such that

1. Firms' and workers' value and policy functions satisfy Eqs. (1.1) to (1.9).
2. For each z, the flow of type i workers into jobs of type j with match productivity z is equal to the flow of type i workers out.
3. After meeting a worker of type i, a firm that draws $z = z_i^*$ is indifferent between posting a casual or regular job.
4. After meeting a worker of type i, a firm that draws $z = \hat{z}_{ij}$ is indifferent between offering a job of type j and destroying the match.
5. Free entry implies that $V = 0$.

Given θ^*, it is possible to find $z_i^*, \hat{z}_{ij}$ from Eqs. (1.8) to (1.9), and thus recover $u_i, S_i(z), N_{ij}(z)$. The algorithm for finding θ^* is set out in Appendix 3.

Reservation productivities

After meeting a workers and drawing z, a firm will be indifferent between offering a regular or casual job when the following condition holds

$$\int_{\underline{x}}^{\bar{x}} J_{ir}(z,x)dG(x) = \int_{\underline{x}}^{\bar{x}} \int_0^{\bar{\varepsilon}} J_{ic}(z,x,\varepsilon)dH_i(\varepsilon)dG(x) \tag{1.11}$$

[12] This assumption seems reasonable. In the UK, for example, the Part-time Workers (Prevention of Less Favourable Treatment) Regulations 2000 require employers to treat workers doing comparable work equally, regardless of the type of employment contract that they have. The European Union's Part-time Work Directive (97/81/EC) also aims to eliminate discrimination against part-time workers.

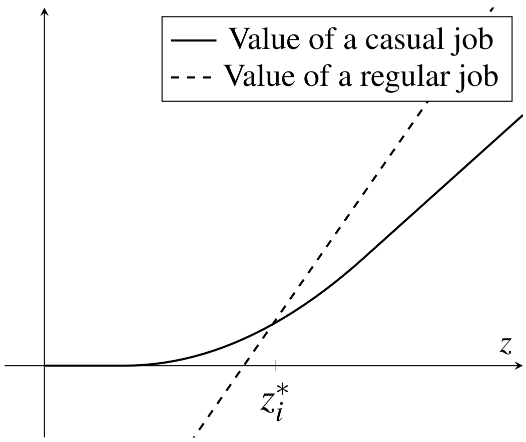
Similarly, the firm will be indifferent between offering a regular or casual job respectively and destroying the vacancy when

$$0 = \int_{\underline{x}}^{\bar{x}} J_{ir}(z,x)dG(x)$$
$$0 = \int_{\underline{x}}^{\bar{x}} \int_{0}^{\bar{\varepsilon}} J_{ic}(z,x,\varepsilon)dH_i(\varepsilon)dG(x) \tag{1.12}$$

Proposition 1 *For each worker type i, (a) there exists values $z = \hat{z}_{ij}$ that satisfy Eq.* (1.12) *and (b) if k_c is sufficiently small, there exists a value $z = z_i^*$ that satisfies Eq.* (1.11) .

Proving uniqueness of z_i^* is only possible after specifying properties of $F(z)$, or under certain circumstances, such as in the case with no on-the-job search. In Appendix 1 I provide a proof of Proposition 1 and a sufficient condition on $F(z)$ for the uniqueness of these reservation productivities. Note that z_i^* is decreasing in the market tightness, since when θ increases workers find jobs more easily and are more likely to quit a casual job. The firm therefore offers more regular jobs in order to retain workers. Fig. 1.4 below shows the value of the two types of job to the firm.

FIGURE 1.4: Reservation productivity z_i^*



Worker stocks in steady state

The steady state distributions of workers employed in a casual job, $N_{ic}(z)$, solve

$$\phi_i f(\theta)\min\{F(z) - F(\hat{z}_{ic}), F(z_H^*) - F(\hat{z}_{ic})\}u_i = \delta N_{ic}(z) + f(\theta)S_i(z) \tag{1.13}$$

where $S_{ic}(z)$ is the (cumulative) distribution of searching workers of type i, multiplied by the probability that they quit for a better job.[13] The distributions of workers employed in

[13]This happens if they find a casual or regular job with a higher worker value.

a regular job, $N_{ir}(z)$, solve

$$\phi_i f(\theta)\max\{F(z)-F(z_i^*),0\}u_i + S_{ir}(z) = \delta N_{ir}(z) \tag{1.14}$$

where $S_{ir}(z)$ is the (cumulative) distribution of searching workers of type i who will accept a regular job. The steady state measures of unemployed workers solve

$$\phi_i f(\theta)(1-F(\hat{z}_{ic}))u_i = \delta(N_{ic}(\bar{z}) + N_{ir}(\bar{z})) \tag{1.15}$$

It is not possible to solve this system of equations analytically without specifying a simple functional form for $F(z)$. However, it is possible to solve numerically by iterating over a grid of values for z, starting with the lowest value at which the firm chooses not to dispose of the vacancy, $\hat{z}_{ic}$, at which point $S_i(\hat{z}_{ic}) = 0$. I use Eq. (1.13) to find $N_{ic}(z)$ at this value, and find $S_{ij}(z)$ for the next value of z. This process can be repeated for all values of z on the grid.

1.3.5 Standard regime

In my policy experiment I compare the labour market outcomes in the “casual" regime described above with a “standard" regime, where casual contracts are banned, and firms can only offer regular contracts. The timing of the model remains the same: firms pay a cost k to create a vacancy and upon matching draw a permanent match productivity z. Workers search for vacancies subject to the matching function $m(s,v)$. Only unemployed workers search since all employed workers are called-up to work in every period. After drawing z, firms can choose to dispose of the match freely, or offer a regular job. Production happens in every period until the job is destroyed, which happens at the end of each period with probability δ. Wage bargaining follows the mechanism set out in Section 1.3.3.

The values of unemployment is

$$\begin{aligned} U_i =& b + \beta(f(\theta)\phi_i \int_{\underline{z}}^{\bar{z}} \int_0^{\bar{\varepsilon}} \max\{U_i, \mathbb{1}_{\text{offer},ir}(z')W_{ir}(z',\varepsilon')\} dH_i(\varepsilon')dF(z') \\ &+ (1-f(\theta)\phi_i p_i)U_i \end{aligned} \tag{1.16}$$

Once again, $p_i = \int_{\underline{z}}^{\bar{z}} \mathbb{1}_{\text{offer},ir}(z')dF(z')$ denotes the probability that the firm offers the worker a job. The value of a regular job is

$$W_{ir}(z,\varepsilon) = w - \varepsilon + \beta((1-\delta)\int_0^{\bar{\varepsilon}} W_{ir}(z,\varepsilon')dH_i(\varepsilon') + \delta U_i). \tag{1.17}$$

The value of a filled regular job to the firm is

$$J_r(z,x) = xz - w + \beta\big((1-\delta)\int_{\underline{x}}^{\bar{x}} J_r(z,x')dG(x') + \delta V\big). \tag{1.18}$$

The value of a vacancy is

$$\begin{aligned} V = & -k + \beta\Big(\frac{q(\theta)}{u_H + u_L}\int_{\underline{z}}^{\bar{z}}\int_{\underline{x}}^{\bar{x}}\int_{0}^{\bar{\varepsilon}} \big(u_H \max\{V, \mathbb{1}_{\text{accept},Hr}(z')J_r(z',x')\} \\ & + u_L \max\{V, \mathbb{1}_{\text{accept},Lr}(z')J_r(z',x')\}\big) dH_i(\varepsilon')dG(x')dF(z') + (1-q(\theta))V\Big). \end{aligned} \tag{1.19}$$

Equilibrium

An equilibrium consists of $\{\theta^*, u_i^*, N_{ir}(z)\}$, that satisfy:

1. Free entry implies $V = 0$.
2. The flow out of unemployment for each type of worker is equal to the flow into unemployment.
3. A firm that draws $z = \hat{z}_r$ is indifferent between destroying the match and offering the worker a job.

Since $q(\theta)$ is monotonically decreasing in θ, Eq. (1.19) has a unique solution for the equilibrium θ^*. The steady state measures of unemployed workers are

$$u_H = \frac{\delta(1-\gamma)}{\delta + f(\theta)(1-F(\hat{z}_r))} \qquad u_L = \frac{\delta\gamma}{\delta + \phi f(\theta)(1-F(\hat{z}_{Lr}))}. \tag{1.20}$$

The total measures of employed workers are

$$N_{Hr}(\bar{z}) = \frac{f(\theta)(1-F(\hat{z}_r))u_H}{\delta} \qquad N_{Hc}(\bar{z}) = \frac{\phi f(\theta)(1-F(\hat{z}_{Lr}))u_L}{\delta}. \tag{1.21}$$

1.4 Quanititative Analysis

I perform an indicative calibration of the model, in order to compare labour market outcomes in the casual and standard regimes. The benchmark economy is calibrated to match features of the Australian data. It is not possible to find closed form results for this model, so instead I solve the model numerically in steady state, using the parameters set out below and the solution algorithm described in Appendix 3. The length of a time period is one week.[14] I assume the matching function is Cobb-Douglas with efficiency parameter χ, so that aggregate matches are $m(s,v) = \chi s^{\alpha} v^{(1-\alpha)}$.

For tractability, I assume that x has a uniform distribution so that $x \sim U[\underline{x}, \bar{x}]$ with $m_x = \bar{x} - \underline{x}$, and a midpoint of one. I assume that the permanent productivity distribution $F(z)$ is lognormal, with $\ln(z) \sim N(\mu, \sigma^2)$.[15] This is a common assumption, motivated by evidence that firm productivity is distributed lognormally (Oulton, 1998; Cabral and Mata, 2003).

Table 1.2 summarises the pre-determined and calibrated parameters. It is not possible to associate each parameter in the model with a separate data moment. Instead, I choose the remaining parameters simultaneously to minimise the squared percentage distance between the model's predictions and the data moments listed in Table 1.3. Table 1.3 summarises the labour market outcomes in the benchmark economy. The model matches the data moments reasonably well, given the data limitations and the model's simplifications, in particular the division of workers into two discrete types, although the model somewhat overestimates the measure of unemployed type H workers.

[14] As labour demand and supply in my model are binary (a worker supplies one unit of labour in each period) it makes sense to choose a shorter period. The HILDA survey includes questions about weekly working hours, and so I use a period length of one week.

[15] The width of the support is set such that a firm that draws permanent productivity $\bar{z}$ will always want to call-up the worker, i.e. so that $\underline{x}$ solves $\bar{z}\underline{x} = w(\bar{z})$.

TABLE 1.2: Parameters

Parameter	Value	Source
Predetermined parameters		
Weekly interest rate, r	0.1%	Yearly interest rate of 5%
Weekly minimum wage $\underline{w}$	1	Normalised to one
Per period value of unemployment, b	0.2	Average replacement rate[a]
Proportion of type L workers, γ	0.11	HILDA survey[b]
Weekly job destruction probability, δ	0.5%	HILDA survey[c]
Elasticity of the matching function, α	0.7	Petrongolo and Pissarides (2001)[d]
Calibrated parameters		
Efficiency of the matching function, χ	0.18	
Mean of the distribution of $\ln(z), \mu$	-0.05	
Variance of the distribution of $\ln(z), \sigma^2$	0.14	
Per period vacancy cost, k_v	2.52	
Probability $\varepsilon = \bar{\varepsilon}$ (type L), ϕ	0.31	
Disutility of labour (type L), $\bar{\varepsilon}$	1.16	
Workers' bargaining power, η	0.39	

Notes: [a]Source: Nickell et al. (2005). [b]An important part of the quantitative analysis is the division of workers into two types based on their labour supply preferences. The HILDA survey includes questions about working hour preferences that I use to calculate an approximate proportion of type L workers (see Appendix 4). [c]Calculated from the average length of a regular job. [d]This is the upper end of the range of the estimates in Petrongolo and Pissarides (2001).

TABLE 1.3: Outcomes in the benchmark economy

	Model value	Data value	Source
Average job finding rate	6.7%	6.3%	Elsby et al. (2013)[a]
Measure of unemployed (type H), u_h	0.052	0.045	HILDA survey
Measure of unemployed (type L), u_l	0.018	0.020	HILDA survey
Measure of casual (type H), $N_{Hc}(\bar{z})$	0.092	0.086	HILDA survey
Measure of casual (type L), $N_{Lc}(\bar{z})$	0.052	0.072	HILDA survey
Average periods in casual job	63	63	HILDA survey
Percentage of casual jobs paid minimum wage	60%	58%	HILDA survey[b]

Notes: [a]This is higher than the implied job finding rate of 3.8% in the HILDA survey. However, the HILDA data includes some respondents who claim to have been searching for a job for a very long time (over ten years in some cases) . [b]Minimum wages in Australia vary across states, industries and occupations. In addition, the hourly wage in the data uses self-reported earnings and working hours, and may be subject to measurement error. I therefore use the percentage of workers paid a wage within 10 % of the federal minimum wage.

1.5 Results

This section presents a comparison of aggregate labour market outcomes in the casual and standard regimes described above, using the indicative calibration in Section 1.4. I also show the results of a policy experiment, varying the exogenous minimum wage for casual jobs relative to regular jobs.

1.5.1 Comparison of casual and standard regimes

Table 1.4 compares the steady states of both regimes. The standard regime is equivalent to a ban on casual work. Such a ban has already been implemented in some countries, including Belgium, France and Norway.

TABLE 1.4: Steady state comparison

	Casual regime	**Standard regime**	**% change**
Measure of unemployed workers (type H)	0.05	0.07	+38%
Measure of unemployed workers (type L)	0.02	0.06	+200%
Job-finding rate	6.3%	3.6%	-43%
Average measure of workers called-up and accepting work	0.85	0.86	+1%
Aggregate production	1.10	1.05	-5%
Aggregate production, less wages	0.20	0.14	-30%
Aggregate per-period utility (type H)[a]	0.91	0.87	-4%
Aggregate per-period utility (type L)	0.03	0.02	-33%
Average per-period earnings of employed worker[b]	0.98	1.05	+7%

Note: [a]Per-period utility is the expected wages and unemployment flow benefit paid to all workers, less the disutility of labour suffered by type L workers with regular jobs. [b]Earnings for employed workers consist of wages and unemployment benefits in periods where casual workers are not called-up. [c]All type H workers in casual jobs and type L in regular jobs are considered mismatched.

Result 1: Unemployment is higher, and the job-finding rate is lower in the standard regime

The intuition behind this result is simple: in the standard regime, if match productivity is very low the firm cannot offer a casual job, and instead may dispose of the job. Thus the unemployment rate is higher. This illustrates the effect that casual jobs have on labour demand as increased flexibility for firms increases their expected profits and they create more vacancies, leading to lower unemployment.

Clearly the increase in unemployment is very large. This is a result of two factors. First, the minimum wage means that, in the standard regime, firms choose to destroy low-productivity matches since they are unable to offer casual jobs. Without a minimum wage, the measure of unemployed workers in the standard regime is only 0.09, much closer to the casual regime.

Second, in the standard regime type L workers will only take high productivity, high wage, jobs that compensate them for their expected disutility of labour. The measure of unemployed type L workers is thus much higher than type H workers. In practice, if casual jobs were banned, some workers might choose not to enter the labour market at all, or switch to other types of work, such as work in the informal sector or self-employment (including work in the gig economy) that are not captured by this model. This would decrease the unemployment rate relative to the value in the table above, but also decrease the labour force participation rate. The response of unemployment above is therefore an “upper bound".[16] In addition, firms who choose to enter the labour market know that they may pay the vacancy cost, only to meet a type L worker who turns down a regular job and so they post fewer vacancies. If there were no type L workers, the measure of unemployed workers in the standard regime is 0.06, a more realistic figure. This is equivalent to assuming that all type L workers leave the labour force after the ban, and so can be thought of as a “lower bound" for the effect on unemployment.

Result 2: Production is (slightly) lower in the standard regime

Despite the large increase in unemployment in the standard regime, production decreases by a much smaller percentage. In the standard regime, employed workers are always called-up to work and must supply labour. In the casual regime, 16% of the workforce have casual jobs. These casual jobs are mostly those with low z, and hence casual workers are called-up infrequently (47% of the time, on average), and sometimes choose not to supply labour. The average measure of workers actually producing is therefore very slightly lower in the casual regime so the effect of banning casual jobs on production is small.

Result 3: Overall worker utility is slightly lower in the standard regime, but the average earnings of an employed worker are higher

This result is related to Result 2. Although the utility of type L workers is, relatively, much lower in the standard regime (since they would prefer the flexibility of a casual job), the utility of type H workers is only slightly lower (since they prefer regular jobs). The overall result is a small decrease in total utility in the standard regime, as the type H workers are

[16] In their paper, Dolado et al. (2021) consider the effect of zero-hours contracts on participation and conclude that it would fall if they were banned.

more numerous. Although fewer workers in the standard regime are employed, those who are employed are called-up in every period and thus earn more. In addition, firms create fewer low productivity jobs with low wages, so average wages are higher in the regular regime.

In summary, although the ban has a negative effect on unemployment and the job-finding rate, the effect is mitigated by the fact that, in the standard regime, everyone with a job is producing and earning a wage in every period. Thus, the negative effect on other aggregate outcomes is smaller. Although there are fewer employed workers in the standard regime, those who are employed earn higher wages, on average, and are more likely to be in their preferred type of job. The smaller effect on production illustrates another outcome of the model. In equilibrium, firms offer a casual job when match productivity is low. Hence the casual jobs are more 'marginal' jobs and the effect on aggregate productivity when they are banned is small.

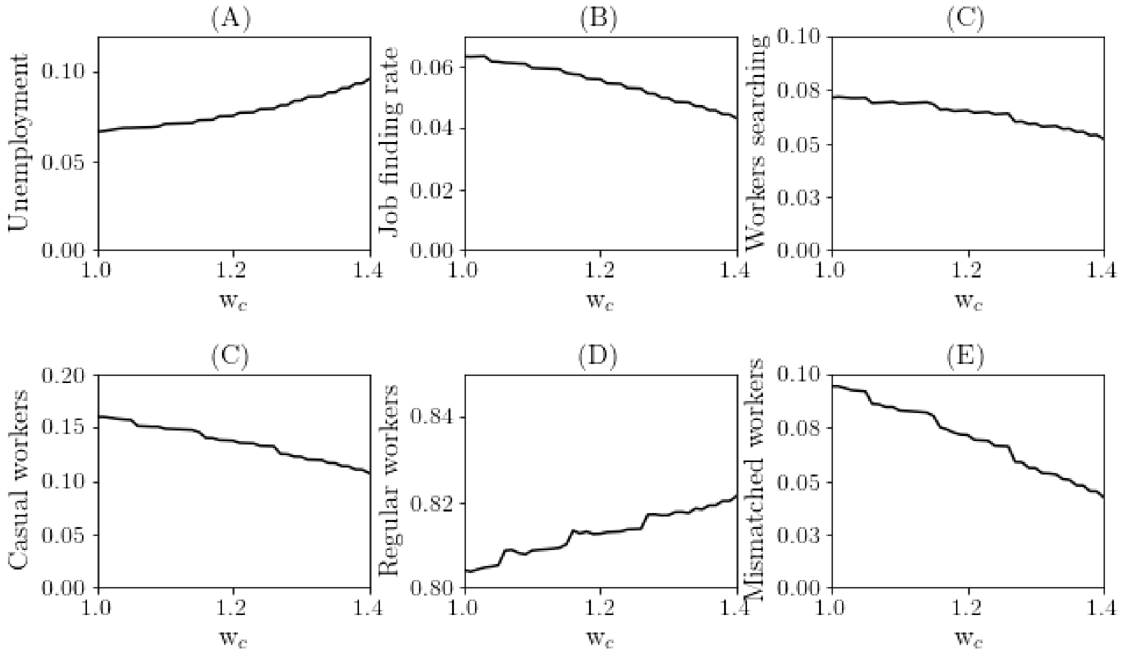
1.5.2 Varying wages

In this section I evaluate the effect of varying the minimum wage for casual jobs relative to regular jobs, w_c, so that for casual jobs the wage is $w_c = \max\{\eta z + (1-\eta)b, w_c\}$. The minimum wage for a regular job remains at $\underline{w}$. This policy has been proposed in the United Kingdom where an increase of 15% was recommended by the authors of a report commissioned by the government on non-standard work (Taylor, 2017, 2018). Figures 1.5 to 1.6 below show aggregate outcomes as w_c increases from $\underline{w}$ to $1.4\underline{w}$.

As w_c increases, unemployment (Panel (A)) increases and the job-finding rate (Panel (B)) decreases. However, these changes are fairly small, even with a substantial increase in the relative minimum wage for casual jobs. This sheds light on another mechanism at work: in the casual regime, on-the-job search means that both employed and unemployed workers are attempting to match with vacancies in a labour market with search frictions. As w_c increases, firms offer more regular contracts. Since workers in a regular job cannot search on-the-job, there are fewer employed workers searching. The overall measure of workers searching changes fairly little, as shown in Panel (C). In a market with search frictions, this makes it easier for unemployed workers to match with a firm.

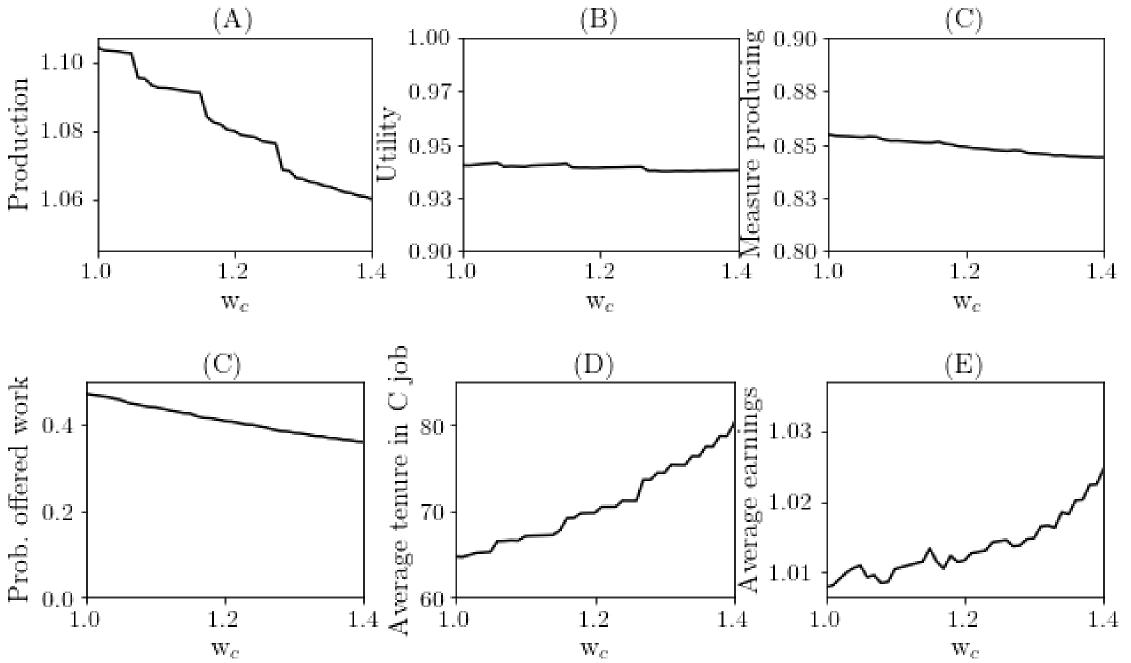
The measure of casual workers (Panel (D)) falls, and the measure of regular workers (Panel (E)) rises very slightly as the relative cost to the firm of a regular job decreases. Thus firms offer more casual jobs, and workers stay in those casual jobs for longer. Panel (F) shows mismatched workers: type H workers in casual jobs and type L workers in

FIGURE 1.5: Effects of varying w_c



regular jobs. These workers would prefer the other type of job, all else equal. As w_c increases, this measure of mismatch falls.

FIGURE 1.6: Effects of varying w_c



As Fig. 1.6 shows, there is a very small decrease in production (Panel (A)) as w_c increases. As the minimum wage increases, firms offer fewer jobs and workers in casual jobs are called-up less frequently. However, a larger proportion of jobs are regular, producing in every period. The net effect is very little change in production or the measure of workers producing. Since most jobs are regular jobs, and since casual jobs are in general lower productivity, the aggregate effects are very small. For the same reasons, overall worker utility (Panel (B)) changes very little.

The effect of an increase in w_c on employed workers is mixed. Workers in casual jobs are less likely to be offered work (Panel (C)), and, since the job-finding rate falls, they stay in casual jobs for longer (Panel (D)). However, the average earnings for an employed worker (Panel (E)) rises slightly. This reflects the fact that more workers are in regular jobs, where they are called-up in every period.

Once again, the effects of a substantial increase in w_c are very small, illustrating further that in equilibrium casual jobs are marginal jobs, with low productivity and low wages. Therefore policies specific to these jobs have small aggregate effects. In this situation, the effect of the increase in the minimum wage is to make casual jobs even more marginal, so that type H workers who cannot find regular jobs earn less and spend longer in casual jobs.

1.6 Conclusion

In this paper I first describe the characteristics of workers and firms with casual jobs. I show that casual workers are generally younger and less well-educated than regular workers. They are more likely to work for small firms, in industries where demand changes frequently. Low-skilled or service jobs are more likely to be casual, and on average wages for casual jobs are lower. There is evidence that a significant number of casual workers do not want full-time jobs and that their average preferred working hours are lower.

I develop a search and matching model of the labour market in which firms can offer casual or regular jobs, that reflects the facts described above. Firms face a trade-off between the certainty of a regular job and the flexibility of a casual job. The model shows that there exists a reservation productivity for the creation of a regular job. Thus firms drawing a low match-specific productivity prefer casual jobs and firms drawing a high productivity prefer regular jobs. In equilibrium both types of jobs coexist, and casual work is concentrated at the low end of the wage distribution, as observed in the data.

My model allows me to compare aggregate outcomes in this casual regime with a standard regime, in which casual jobs are banned. I find surprising effects of such a ban: despite a large increase in unemployment, in the standard regime workers' utility and production are only slightly lower. Some outcomes for workers, such as the average wage for employed workers, are higher. The same pattern is true when I impose a higher minimum wage for casual jobs: although there is a rise in unemployment, aggregate production and utility change very little. This illustrates two elements of the model. Firstly, in the standard regime, firms must call-up workers in every period, and workers must supply labour. This offsets some of the effects of the ban on casual jobs. Secondly, it is an equilibrium outcome of the model that casual jobs are marginal, so that casual workers earn less and work infrequently. Thus the effect on aggregate outcomes of policies aimed at casual jobs are small.

There are a number of avenues for future research and areas that need further consideration. If firms and workers were able to bargain more flexibly over wages and renegotiate contracts, then workers would be able to search on-the-job, and use job offers from another firm to capture some of the increased surplus that the firm receives with a casual contract, arising from the flexibility over whether to offer work. Equally, firms would be able to capture some of the surplus to a worker that arises from the flexibility to turn down work. It is not clear which effect would be stronger, and what the resulting effect on aggregate outcomes would be. Finally, it is important to note that both workers or firms in this model are risk neutral. As a result they make choices based only on the discounted income they expect from a match. If workers were risk averse then their decisions about the type of job to accept might change.

Chapter 2

Extreme wages, performance and superstars in a market for footballers

A version of this essay has appeared as an article in the January 2021 edition of Industrial Relations: A Journal of Economy and Society, DOI: https://doi.org/10.1111/irel.12270. An older version of this article appeared as Reading Department of Economics Discussion Paper No. 2020-04. This article was co-authored with Carl Singleton, Associate Professor at the University of Reading and Paul Telemo, PhD candidate at the University of Edinburgh. Carl and Paul have agreed that the essay can appear within this thesis, and that it represents a significant contribution on my part. Re-production of the essay here does not infringe the publisher's copyright policies. The version here has been rewritten and reformatted compared with the aforementioned article, and so they are not identical, though the main substance and results are. The data used in this study are publicly available from Major League Soccer (MLS) LLC and the MLS Player's Association. The use of these data does not imply the endorsement of the data owners in relation to the interpretation or analysis of the data.

2.1 Introduction

Some people have labour incomes so high that they are barely comprehensible to the average worker. These top earners, and the amounts they receive, attract widespread media, political and academic interest.[1] Over the past forty years and throughout the major developed economies, the differences between the wages of most workers and the

[1]Note the extensive coverage of the various Forbes rich lists, e.g., for sportstars: https://www.forbes.com/athletes/list/. Income taxes for the highest earners are often a subject of debate in national elections, e.g., in the December 2019 UK general election (see Adam et al., 2019).

few at the top have risen, especially in English speaking countries. For example, in the United States, the share of total income earned by the top 1% of earners more than doubled between the 1970s and the 2010s (Alvaredo et al., 2013). In addition, an increasing share of the wealthiest individuals in society have earned rather than inherited their wealth (Kaplan and Rauh, 2013). As a result, inequalities in labour income are an increasingly important driver of overall inequality. However, there is no clear consensus about why some individuals earn such enormous amounts or the causes of these ‘superstar wage effects’.

Top wage earners include business and finance executives, lawyers, medical professionals, and media and sports stars. We focus on the latter group, using longitudinal data on the salaries and productivity of both workers (players) and firms (teams) in US Major League Soccer (MLS), between 2007 and 2018. This is a specific group of workers, but it is nonetheless relevant to overall patterns in wage and income inequality. Media and sports stars are among the professions with the fastest growth in top incomes: the media and sports stars in the top 0.1% of US earners experienced a 5% compound annual growth in their incomes between 1979 and 2005 Bakija et al. (2012). They are also well represented among the wealthiest individuals around the world. For example, Franzini et al. (2016) report that two-thirds of the 120 top wage earners in Italy in 2003 were association footballers. In contrast to most other labour markets, there are detailed, accurate, linked, firm and worker productivity data in sports labour markets (Szymanski, 2007). We combine these with salary data, which are available for all players in the League due to MLS regulations. Because of this data availability and the clarity of the relevant rules and institutions, markets for sports talent provide a natural and convenient setting to answer a simple question: why do some individuals earn such astronomically high wages?

We use a two-step empirical strategy to approach this question. In the first step, we use past individual player performance data, such as the number of shots on target per 90 minutes on the football pitch, to decompose annual MLS salaries into two parts: (i) the amount explained or predicted by on-the-pitch performance or productivity; (ii) an unexplained or residual amount. We hypothesise that the latter part may capture the ‘superstardom’ of some MLS players. In the second step, we regress measures of teams’ annual football performance or revenue generation on the aggregate predicted and residual wages of their players, as estimated from the first step. Across various model specifications and robustness checks, we find consistent evidence that the amount teams spend on the residual wage component increases their revenue positively and significantly. A 1% increase in the residual salary spending of teams increases home stadium attendances by as much as 0.14%. This team-level spending on the residual salaries of players is not significantly associated with a team's relative performance in MLS, including whether they make the end-of-season playoffs. Conversely, the spending on the predicted part of player wages does significantly increase a team's points per game and chances of reaching the MLS playoffs, whereas the residual salary spending does not.

We relate these results to the two main theories of superstar wage effects, which are discussed in more detail in Section 2.2. One theory, proposed by Rosen (1981), suggests that in markets where there is a convex relationship between productivity and revenue, small differences in talent can result in large differences in wages. In contrast, Adler (1985) posited that superstar wages can result from differences in popularity rather than talent. Our results are more supportive of Adler's theory, as we find that teams do not perform better on the football pitch when they spend more on superstars, but they do attract more people to their stadiums. This suggests that the top wages in MLS are a reflection of player popularity, rather than of current talent or performances on the football pitch.

Our main contributions to the existing literature are twofold. First, we add to the debate on the causes of superstar wage effects. Our data allow us to both link a worker's wages with their individual performance (or productivity) and to accurately measure firm productivity over time, and thereby distinguish between productivity- and popularity-based theories of superstar wage effects. Another benefit of our data is that we observe approximately the universe of MLS players over the period we study, allowing further investigation by weighting each player by the time they spent on the football pitch in a season. Our results are robust to this weighting. This suggests that teams benefit from the popularity of their superstar players even when they are not playing that much, adding further support to Adler's theory of superstars. Second, we contribute new evidence about the determinants of pay in MLS and football more generally, as well as the impacts on teams of signing superstars. For example, by demonstrating the positive relationship between a team's spending on residual wages and its revenue, we are able to suggest a source for the large amount of unexplained variation in football player wages.

The remainder of the paper proceeds as follows: Section 2.2 discusses the related literature on superstar wage effects; Section 2.3 describes the estimation strategy and interpretation; Section 2.4 summarises the institutional setting of MLS and the data sources; Section 2.5 presents our results; and Section 2.6 concludes.

2.2 Related literature

2.2.1 Theories of superstar wage effects

Rosen (1981) defined the presence of superstars in a labour market as the “concentration of output among a few individuals, marked skewness in the associated distributions of income and very large rewards at the top." The two principal and competing theories to explain how some individuals become superstars and earn vastly greater income than most others were those proposed by Rosen (1981) and Adler (1985). The former’s theory states that superstars exist in markets where there is a convex relationship between productivity and revenue. This convexity arises because of the imperfect substitution between individuals of different productivity or talent, and because technology allows the joint consumption of output. In the music industry, for example, consumers prefer better musicians and are unwilling to substitute with others whom they consider to be inferior. At the same time, musicians can reach large audiences at a low cost, through technologies such as TV and the internet. Thus, small differences in talent can lead to large differences in revenue, and hence very large salaries for the most talented. In contrast, Adler (1985) argued that superstar wages can occur even without differences in individual talent. In his framework, there can be a number of equally talented individuals. However, consumers then derive utility from consuming the same product as others do (e.g., from listening to the same musician), and from knowing more about a particular individual. This knowledge is gained in discussion with other fans, so that as a musician becomes more popular, consumers prefer them even more. At the same time, it is costly for consumers to search for and identify the “very best" musician. Therefore, it is optimal for consumers to listen to the most popular musician, even when there are others, slightly more talented but less popular. In this way and unlike in Rosen’s theory, superstars can emerge by chance, simply because they are initially slightly better known than others or through opportune timing, rather than because of small differences in talent. Beyond these two main theories, Terviö (2009) suggests an alternative explanation of superstar wages, which may be relevant in markets where workers and firms are not able to commit to long-term wage contracts, and where talent is only revealed through actual performance on-the-job. In these markets, Terviö shows that firms will excessively bid for known talent rather than trying out new talent. This is because, although talent may not be scarce, the supply of workers whose talent has been revealed is scarce.

If superstar wages in football can be explained by Rosen’s theory, then we would expect to see small differences in individual productivity leading to differences in firm (team) revenue and large differences in player wages. If Adler’s theory provides a better description of superstar wages in football, then we would not expect such a clear relationship between individual productivity and revenue. Instead, we would expect that differences in an individual's popularity, measured as something unrelated to observed productivity, can explain differences in revenues and wages. In a particular football talent market featuring superstars, namely MLS, we can observe different measures of individual player and team productivity and revenues, and can therefore test whether the Adler or Rosen theories, or some combination of the two, are more relevant. We are generally silent on whether the Terviö (2009) theory is an appropriate description of superstars in football, other than to note that the players and teams can commit to long-term contracts, even when players are very young (e.g., Simmons, 1997; Frick, 2007) and that talent is normally revealed to teams through extensive scouting and coverage of player performances.

2.2.2 Evidence of superstar wage effects

There are a number of practical difficulties in identifying why some individuals attract superstar wages. First, in most settings it is difficult to objectively measure an individual's productivity and relate this to wages. Some authors have attempted to use other metrics as proxies for productivity. For example, Hamlen Jr (1991) used a singer's harmonic voice quality, and found that the elasticity of record sales with respect to voice quality is positive but small. He concluded that this suggested the superstar effects described by Rosen do not exist in the music industry. Similarly, Célérier and Vallée (2019) used the results of university entrance exams in France as a proxy for Chief Executive Officer productivity. They found that the returns to this measure were higher in finance than in other industries, and they interpreted this as evidence of superstar effects in the finance industry. However, these measures of talent are highly specific, and may miss other dimensions of productivity. As well as productivity, it can also be difficult to measure very high earnings accurately. Few people are required to publicly report their salary (sports labour markets, particularly in the US, are an exception), and data are often top-coded.[2]

A second problem is found in disentangling measures of productivity from measures of popularity. For example, Krueger (2005) used the amount of space devoted to musicians in an encyclopedia of music as a metric for "star quality", and found that this measure was associated with an increase in an artist's revenue. This measure was related to both talent and productivity, and it was not possible to identify which drove the higher revenue. Likewise, Hoffman and Opitz (2019) analysed superstar effects in the film industry. They showed that measures of popularity, such as internet search hits, positively affected

[2] Some studies, e.g., Hamlen Jr (1991); Krueger (2005), have used revenue (in these cases, from music sales) as a dependent variable, but do not observe exactly how much of this revenue is paid to the artists. Others, e.g., Célérier and Vallée (2019), have used self-reported data from questionnaires.

the earnings of highly paid film stars. They interpreted this as evidence of “publicity superstars”, as described by Adler. However, if those film stars had previously been nominated for an Oscar, then their earnings were not related to popularity. They suggested that this group are “talent superstars”, as described by Rosen.

Finally, it can be difficult to identify whether superstar effects are indeed a result of exogenous technologies that allow large numbers of people to consume the same output (a key component of both the Adler and Rosen’s theories). Instead, the adoption of new technologies may be endogenous, and driven by the presence or emergence of superstars within a market. Koenig (2019) tested for the mechanism creating superstar effects in the US labour market for entertainers. With a plausible source of exogenous variation in the adoption of a new technology, coming from when and where local TV stations launched, he showed that wages at the very top of the income distribution increased markedly due to the ensuing increase in audience sizes, and that the incomes and employment of less talented entertainers decreased.

Evidence of superstar wage effects in sport

Professional sport provides a setting to evaluate theories of superstar wages as both detailed productivity and wage data are available for both firms and employees. The literature exploiting this fact has focused on two main questions. First, a number of studies have asked what determines the wages of sports superstars. For example, Lucifora and Simmons (2003) found that the wages of Italian footballer players are highly convex in two particular measures of performance, namely the numbers of assists and goals scored per 90 minutes, suggesting that small differences in talent are indeed multiplied into large differences in wages, consistent with the Rosen theory. Franck and Nüesch (2012) considered the effects of both talent and popularity on player values in the German football transfer market. Their measure of player popularity was the residual from a regression of media citations on measures of a footballer’s talent. They interpreted their main findings, that both talent and popularity have positive effects on player values, as indicating that some footballers are superstars in both the Rosen and Adler senses. Also studying the wages in European football, Carrieri et al. (2018) showed that talent, popularity and bargaining power are all associated with higher wages. This association is stronger at the top of the wage distribution, and only popularity is significant at the 95th percentile of the wage distribution. The authors concluded, therefore, that a theory of superstars based only on talent differences is not adequate in this setting.

There is a second strand of research on the effect of superstar sportspeople on team (firm) revenue. In particular, there are several papers showing that the presence of superstars increases attendance and TV revenues in cricket (Paton and Cooke, 2005), baseball (Ormiston, 2014), basketball (Hausman and Leonard, 1997; Jane, 2016) and football (Brandes et al., 2008; Lawson et al., 2008). Particularly relevant to this paper are studies on the effects of so-called 'designated' players on the league results and attendances of MLS teams (Coates et al., 2016; Jewell, 2017). These designated players can be hired outside the salary cap that normally applies to teams. They tend to be the highest earners in MLS by some margin, with teams often signing well-known but ageing players, who were coming to the end of highly successful careers in the major Europe leagues and international competition (this is colloquially known as the 'Beckham' rule, since David Beckham was among the first beneficiaries). Jewell (2017) found that only a few of the designated players signed since the rule was introduced in 2007 had positive effects on stadium attendances (namely Beckham, Blanco and Marquez), that these effects diminished over time, and that they were larger when the superstars played away from their teams' home stadiums. In other words, these players generated a novelty factor in the League, which spilled over beyond the teams who signed the players. Coates et al. (2016) found that, whilst teams with a higher wage bill after the introduction of the designated player rule did perform better in the league, this was offset by a decrease in performance for teams with higher salary inequality among players. Although these studies suggested that superstar players do increase team performance and attendance (and hence revenues), they did not try to distinguish whether increased attendance was due to the players' superior talent or their greater popularity. Attempting to disentangle these mechanisms, Bryson et al. (2014) considered the effect of migrant players in the top tier of Italian football on both league points and attendance. They found that teams with more migrant players performed better in the league and had higher attendances. Using wage data, they also found evidence that these migrant players could be thought of as superstars in both the productivity-based (Rosen) and popularity-based (Adler) senses.

2.3 Empirical strategy

To measure how the allocation of playing talent in MLS affects a team's output, we use a two-step regression methodology. Our empirical strategy is similar to that used by Bryson et al. (2014), except here we introduce a manner of weighting the influence of each player in their respective team, potentially allowing us to dig deeper into whether players are superstars in the productivity- or popularity-based senses. In brief, in the first step we regress the contracted wages of football players on several measures of their performance during the past season and some other observable characteristics. We treat the predicted part of this regression as the amount of the wage that can be explained by factors relevant to a player's productivity on the pitch. The residual component of these regressions, which is by construction orthogonal to the productivity component, is the estimated amount of the wage which isn't accounted for by what players individually achieve on the pitch. In other words, it may reflect remuneration for some other contribution to the team's output, such as from the popularity or 'superstardom' of the player. We then aggregate these player-level measures to the team level for each season. In the second step, we regress the output of teams on the derived aggregates from the first step, which ought to reflect what their players are being paid for talent or for other unobserved contributions to their team.[3] In doing so, we look for evidence as to why some players are paid especially high wages in MLS, i.e., superstar wage effects.

We consider two principal measures of an MLS team's output, Y_{jt}: (i) a team's performance on the pitch (represented by its results in the League); (ii) its ability to generate revenue (represented by attendance at its home games). Consider the following general representation of the output of football team j in season t:

$$Y_{jt} = F_j\left(P_{jt}, Z_{jt}, \widetilde{Y}_{jt}, t\right) \ , \tag{2.1}$$

where F_j is the team-specific production technology, P_{jt} is the quantity of playing talent, and Z_{jt} is the amount of superstardom of the team's players. $\widetilde{Y}_{jt}$ represents other contemporaneous measures of output, which may affect Y_{jt}. For example, it is plausible that a team's success in the League, i.e., winning matches, would encourage higher stadium attendance and revenue.

To measure the quantity of playing talent and superstardom employed by a team, we assume that the log wages of player i in season t are given by:

$$w_{it} = G\left(p_{it}, z_{it}, t\right) \ , \quad p_{it} \perp z_{it} \ , \tag{2.2}$$

where p_{it} is the level of talent on the pitch and z_{it} is some uncorrelated other factor affecting wages. As with the team's output, player wages are affected by League-wide trends. We estimate Eq. (2.2) using least squares:

$$\ln\left(w_{it}\right) = \mathbf{x}'_{it-1}\beta_k + d_{kt} + z_{it} \ , \tag{2.3}$$

where $\mathbf{x}_{it-1}$ is a vector of lagged performance variables and personal characteristics, reflecting the importance to the team as a footballer, e.g., experience and whether they are the team captain, and β_k is the associated vector of coefficients. We allow the

[3]We make several other changes to the method used by Bryson et al. (2014). For example, in the first step, we estimate wage regressions separately by primary playing position, as the returns to different productivity measures may significantly differ in this way, i.e., between defensive or offensive players.

effects of each variable to vary by the player's primary position, where $k = K(i) = \{\text{Goalkeeper}, \text{Defender}, \text{Midfielder}, \text{Forward}\}$. To allow us to decompose player wages into a part paid in respect of playing talent, and a part paid in respect of other factors, it is important that the performance variables, $\mathbf{x}_{it-1}$, relate only to performance on the pitch. For this reason, we do not include characteristics such as a player's nationality, which has nonetheless been shown to relate to pay (e.g., Bryson et al., 2014; Thrane, 2019). We lag the performance variables since player contracts and wages are predetermined at the beginning of a season. This has two econometric benefits. First, it will provide more confidence in the second-step regression, since it is then not the case that the player performance measures (e.g., scoring goals) used in the first step are mechanically affecting the teams' current season outputs in the second step (e.g., winning football matches). Second, it helps to address the potential concern of reverse causality in the two-step model, which could occur if a team's current season output directly impacts the amount of talent they are able to hire. d_{kt} are season-position fixed effects and the remaining heterogeneity in player wages is in the residual term, z_{it}. This could include heterogeneity due to player popularity.

To generate estimates of P_{jt} and Z_{jt}, we aggregate within a team and season the estimates from Eq. (2.3) as follows:

$$\widehat{P}_{jt} = \sum_{i\in(j,t)} \widehat{p}_{it} = \sum_{i\in(j,t)} \omega_{it} \exp\left(\mathbf{x}'_{ikt-1}\widehat{\beta}_k\right) , \tag{2.4}$$

$$\widehat{Z}_{jt} = \sum_{i\in(j,t)} \omega_{it} \exp\left(\widehat{z}_{it}\right) , \tag{2.5}$$

where $i \in (j,t)$ denotes the players who are in team j in year t. ω_{it} is the weight of each player in the overall team-level aggregates. Thus, $\widehat{P}_{jt}$ is the part of the team-level aggregate wage bill that is explained by player performances, and $\widehat{Z}_{jt}$ is the part that is not explained by performances, and which may reflect 'superstardom'.

In our baseline estimation, we give each player on a team's roster in a given season the same weight, i.e., $\omega_{it} = 1$. We also consider whether players' contributions to their team's output depend on the amount of time they actually spend on the football pitch. To do so, we compare the baseline with results where player i's weight in year t is given by:

$$\omega_{it} = \frac{m_{it} I_{j(i)t}}{\sum_{i\in(j,t)} m_{it}} , \tag{2.6}$$

i.e., the share of the overall number of minutes played by all players on the team's roster that season, multiplied by the total number of players in the team and season, I_{jt}. Players who did not feature at all during the season would be given zero weight. This gives a low weight to young and inexperienced players, who play few minutes, and to players who are injured or suspended, even if they are highly paid. Any difference with the baseline results may tell us something about the mechanism by which superstardom affects a team's output, and may suggest whether the Rosen or Adler theories are more accurate in this setting. We also consider other values for ω_{it}, such as only giving weight to positive or very high values of $\widehat{z}_{it}$.

To relate these aggregate components of a team's salary spend to its output, as per Eq. (2.1), we estimate the following regression model:

$$\ln(Y_{jt}) = \alpha_j + \lambda \ln(\widehat{P}_{jt}) + \gamma \ln(\widehat{Z}_{jt}) + \mathbf{X}'_{jt}\delta + d_t + \upsilon_{jt} \ , \tag{2.7}$$

where α_j are team-specific fixed effects, such that we identify the model using variation over seasons in the output within teams. We are primarily interested in the coefficients λ and γ. These provide estimates of the elasticity of this season's output to the estimated overall quantities of playing talent and superstardom within teams, respectively. X_{jt} contains other relevant time-varying factors related to team output, with δ being the associated vector of coefficients. We also consider time fixed effects in the model, d_t, and υ_{jt} is the residual.[4]

The team output measure and model interpretation

In practice, we begin by using team performance on the pitch as the dependent variable inEq. (2.7). If we find that the total measure of residual wages across all players on a team, $\widehat{Z}_{jt}$, does not affect a team's relative on-pitch performance, i.e., $\widehat{\gamma}$ is small and insignificant, this suggests that we have indeed captured in $\widehat{Z}_{jt}$ a component of wages which is unrelated to how successful the team is in purely football terms. This would provide confidence in our decomposition of wages into a part explained by player productivity and a part that may reflect 'superstardom'. In effect, some players would be paid wages that are higher than we would expect based on their individual productivity, and these additional wages would not seem to be paid in respect of any unobserved characteristics that improve their teams' relative on-pitch performances.

We repeat the second step of the model, using the average home stadium attendance of teams as the dependent variable. If we find that $\widehat{Z}_{jt}$ positively affects a team's revenue

[4]We also estimate this equation including the lagged output variable on the right-hand-side, using two-step General Method of Moments (GMM), based on the approach suggested by Arellano and Bond (1991). This is to address concerns about reverse causality; teams with previous high attendance may be able to pay higher wages. We find that the lagged output variable is insignificant in all our model specifications, and so we omit it and estimate using OLS.

generation, i.e., $\widehat{\gamma}$ is large and significant, then it suggests that some players are paid especially high wages in MLS because of their popularity, and that teams (and the League) can financially benefit from this. This would be more consistent with Adler's theory of superstars rather than that of Rosen. In particular, if we also find that the aggregate level of playing talent on the pitch, $\widehat{P}_{jt}$, only increases revenue through the team's relative success in the League, and not in it's own right, i.e., $\widehat{\lambda}$ is small and insignificant, then this would provide further evidence supporting Adler's theory. Conversely, if we find that a team's aggregate playing talent positively affects revenue generation, this suggests that consumers (football fans) prefer to watch the most talented players, which would be more consistent with Rosen's theory.

2.4 Data and institutional setting

Major League Soccer is a talent market with several unique features. In this section, we discuss the relevant regulations in MLS, set out our data sources, and provide some descriptive statistics on player wages.

2.4.1 MLS institutions and regulations

MLS is different to the most popular association football leagues in Europe or elsewhere in a number of ways. MLS teams compete in two parallel closed leagues (i.e., no relegation or promotion): the Eastern and Western Conferences. In each season (calendar year), teams play each other team in their conference twice, and each team in the other conference once, known as the regular season. They earn points for winning or drawing a game, following which the top six teams in each conference advance to the MLS playoffs. These 12 teams play a direct elimination (knockout) series to determine the championship winner, known as the MLS Cup. Separately, the team with the highest regular season points across the two conferences is awarded the Supporters' Shield. Thus, both a team's points and whether or not it advanced to the playoffs are relevant measures of team performance

Unlike most other football leagues, MLS is a single-entity that owns a stake in all the teams, which are run as franchises. The teams receive some direct revenues, including local broadcast rights, all stadium revenue and 70% of match day revenues, such as ticket sales (Peeters, 2015). They also receive a share of the overall league's profits, including from national and international broadcast rights and league-wide sponsorship.

Salary regulations

Of particular importance to this study are the salary rules in MLS. Players sign a contract with the League, rather than with an individual team. This limits individual players' bargaining power, and has kept teams' salary costs low relative to their revenue, compared to other football leagues around the world (Twomey and Monks, 2011). There are lengthy rules governing salaries and which players a team can sign. These are subject to Collective Bargaining Agreements (CBAs) between the League and the MLS Players' Association (MLSPA). There were three CBAs during the period that we study, covering the periods 2004 to 2010, 2010 to 2015 and 2015 to 2020. Negotiations between the MLSPA and MLS have been fractious, as players have argued for higher salaries and more choice in which team they can play for.

The CBAs specify a minimum annual salary for every player and a total salary cap for every team.[5] These salary floors are binding for a small percentage of players. For example, in 2018, 26 players aged at least 24 were paid the minimum salary, out of a total of 537 in the League. Players earning the minimum salary are generally a homogeneous group: younger, new entrants to MLS, and often play no or few minutes during a season.

The salary cap is the total budget that teams have for all players on their roster, i.e., who can play for them during the season. Teams generally have the freedom to choose how to allocate wages within the cap, subject to minimum and maximum roster sizes, as well as some rules about the composition of player types on the roster, including international vs home-grown players. As MLS has become more successful, the MLSPA has been able to negotiate substantial increases in both the minimum salary, from $30,000 in 2007 to $70,250 in 2019, and the salary cap, from $2.1m in 2007 to $4.2m in 2019. There have been other changes during the period that we study, including an increase in job security. Both the 2010 and 2015 CBAs increased the percentage of players with 'guaranteed contracts'. These are contracts that cannot be terminated by MLS if the player performs badly or is injured during a season. Thus, every player will spend at least one full season in MLS. At the same time, the number of 'option years', where teams can extend a player's contract for an extra year, was reduced (Ferrari and Rueda, 2015).

There are two main ways that teams can spend money on salaries in excess of the cap. Teams can sign a number of players using the 'designated-player' rule. This was introduced in 2007 when David Beckham, who was at that time one of the highest paid footballers in the world, signed for LA Galaxy from Real Madrid of the Spanish La Liga. Until 2009, the rule allowed each team to sign one player whose salary did not count

[5]There is also a separate minimum salary for young players, below the age of 24, on a team's reserve roster.

toward the cap. In 2010, this was expanded to two designated players per team, with an option of paying a fee for a third.[6] The rule was designed to allow teams to sign high profile players from outside leagues, at salaries that would not be feasible under the salary cap (Coates et al., 2016). Teams also receive 'allocation money' from the League each season, which can be used to pay salaries above the cap but below the very large (essentially unregulated) salaries of designated players. This money is aimed at improving the quality of players in MLS, by increasing the overall amount spent on salaries, and at maintaining competitive balance in MLS, by allowing the younger or lower performing teams to purchase higher quality players (Major League Soccer (2017a,b)). MLS has a large amount of discretion in how much allocation money is given to each team. As a result, there is substantial dispersion in teams' overall salary costs. The percentage of players from outside the US and Canada has increased over time, from 32% in 2008 to 47% in 2018, as higher salaries have allowed MLS to attract more players from overseas. To account for the average effects of these changing regulations on player wages, we include season controls in our first- and second-step regressions.

Roster regulations

There are lengthy rules determining which players a team can sign and the make-up of its roster (see Major League Soccer, 2020, for details). For example, there is a maximum number of international players that a team can include on its roster. There is also a 'draft' system where teams can pick young players new to MLS in reverse order of the team's finishing position in the League in the previous year. In general, players have little control over which team they play for. The limited form of 'free agency' introduced in the 2015 CBA is an exception, whereby players aged over 28, who have played at least 8 seasons in MLS, can negotiate with any team when their contract expires.[7] By allowing players to negotiate wages directly with a team, and teams to compete to sign a player, free agency may increase player salaries. However, the number of players eligible for free agency during the period we analyse was small, ranging from 28 players in 2015 to 39 in 2020. There are a number of other ways that teams can acquire players and circumnavigate the rules governing team rosters and the salary cap. For example, they can trade players or their international roster allowance with other teams.

The League's structure and regulations are designed to ensure that it remains competitive. This appears to have been successful; the MLS Cup was won by 12 different teams in the 13 seasons between 2007 and 2018. This illustrates a further advantage of studying MLS: in this setting, without a small number of entrenched teams at the top of the League,

[6]This fee is shared among the teams that do not have three designated players.
[7]The 2020 CBA has expanded this to any player aged over 24 with at least 5 seasons in MLS.

managerial decisions, including on salary and the make-up of the team, can potentially have almost immediate effects on points and success, since the gaps between the teams are small and not persistent. A further advantage is that most players are still subject to the salary cap, although the teams with more resources can certainly afford to spend more on designated players. This implies that the richest teams in the League cannot always employ all the players they would wish to and cannot pay them as much as they would like to, easing concerns later about there being possible reverse causality between teams paying superstar salaries and performance.

2.4.2 Data sources

Our data primarily come from two main sources, the official MLS website and the MLSPA, and cover all players and teams in MLS from 2007 to 2018. Table B.1 contains further information on all the variables used in our analysis. As our analysis relates player wages to lagged performance variables, we use data on player performance from 2007 to 2017 and on wages and team performance from 2008 to 2018.

Wage data

Player wage data and team affiliations come from the MLSPA. The data refer to the players on a team's roster midway through the MLS season in August of each year, after the secondary transfer window when teams can sign players from abroad. The player performance data for each season were extracted from the MLS online database in September 2018. We merge these two sources of data together using player names and seasons, creating a dataset containing 6,194 player-season observations, representing 2,186 different players and approximately the universe of those contracted to MLS during 2007-18. After dropping the small number of observations where players had missing records for their age or season performance, or because records could not be matched between the two data sources, and focusing on 2008-18, our sample contains 5,458 player-season observations, representing 1,939 different players.[8] The wage measure we study is the guaranteed annualised compensation or salary, henceforth referred to as wages.

Our wage measure includes annualised payments for signing with a new team (loyalty bonuses) or related to marketing, but does not include performance-related pay. Although we do not have data on individual performance bonuses, we note the following: first,

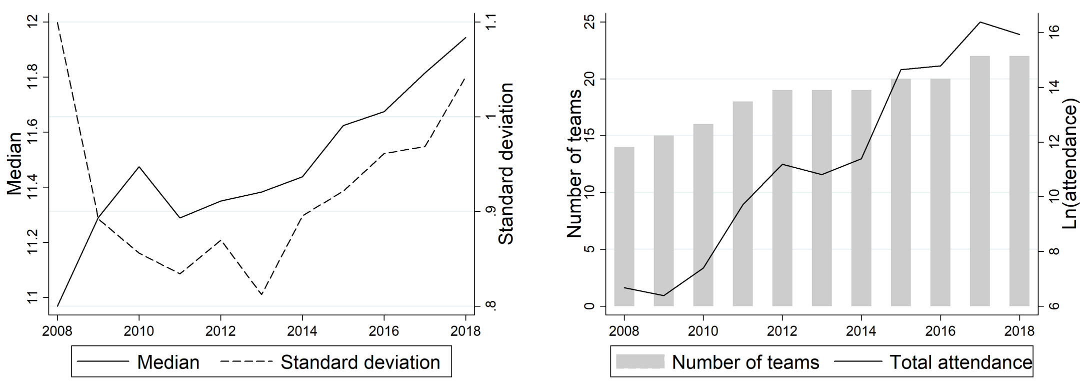
[8] There were some discrepancies when we merged the datasets, due to naming conventions and spelling, which we manually corrected as far as possible. There were a tiny number of obscure cases where we could not reconcile the two datasets, or where players appeared in one source but not the other. See Table B.2 for the number of players in the sample by year and position.

teams receive a bonus from the League for good performance, which is shared among all players in the team; second, bonuses are paid at the end of the season, while our data cover the wages agreed before the season; and third, according to MLS rules, any "readily achievable" individual bonuses are included in our wage measure for the purposes of calculating the contribution of each player's wage to the overall team-level salary cap. The total team wages that we observe (excluding any designated players) are generally higher than the salary cap (see Fig. 2.3 below). Although the rules for calculating exact contributions to the salary cap are complicated and require information which we are unable to access, this suggests that individual performance bonuses are not large compared to the wages that we observe.

The 2007-18 period was one of expansion in MLS and growth in wages, as shown in Fig. 2.1. Wage dispersion also increased during this time, partly due to the designated player rule (Fig. 2.1a). As we use wage data from 2008 onward, Fig. 2.2 summarises 'real' log wages in 2008-18 by primary playing position on the pitch, for the analysis sample of players, with wages adjusted to 2018 MLS-US$, to address average wage inflation over the period. Although the rules regarding wage determination are complex, there is nonetheless substantial variation. The variance is higher for forwards and midfielders, who are more likely to earn a very high wage than defenders or goalkeepers. Designated players make up 7% of the player-season observations, with some players switching to or from this status during their MLS careers.

FIGURE 2.1: MLS growth and expansion: player wages, teams and attendance, 2008-18

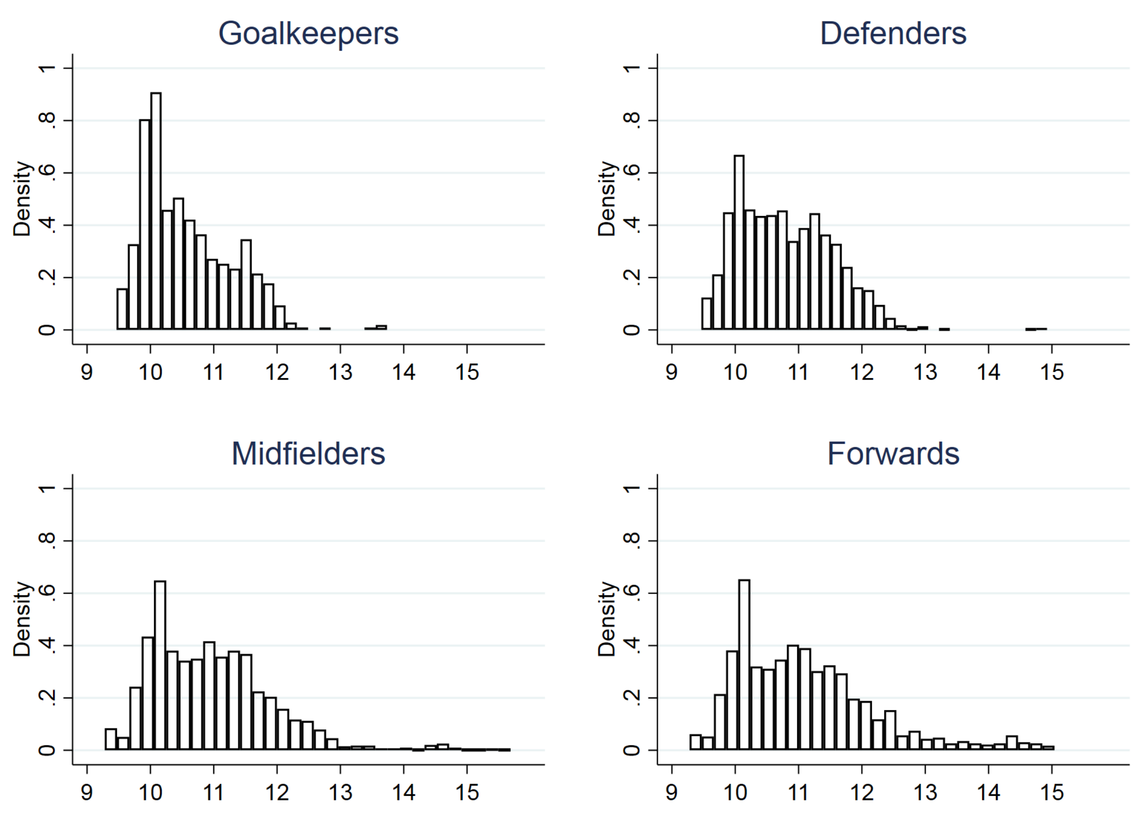
(a) Player log guaranteed wages (b) MLS expansion and attendance growth



Notes: Author calculations using MLS website and MLSPA data. Player wages are nominal US$.

Although there is a salary cap, in practice it appears that teams are able to spend very different amounts and make very different decisions regarding wages. Fig. 2.3a shows the total wage bill for each team in 2018. All teams spent more than the salary cap, with the extra coming from designated players and allocation money. The largest sum was spent by Toronto FC, nearly five times as much as Houston Dynamo. The wage dispersion

FIGURE 2.2: Distribution of MLS log guaranteed wages by player position, 2008-2018



Notes: Author calculations using MLS website and MLSPA data. To address general wage growth and inflation within the MLS when pooling over years, the wages of all players in each year were adjusted according to the average wage level in 2018, i.e., they are roughly adjusted to something akin to 2018 MLS prices.

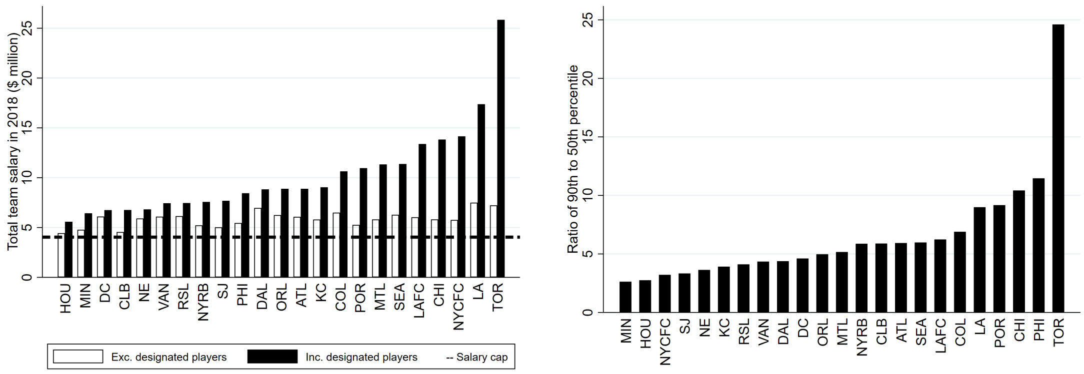
between players in the same team also varies substantially, as shown in Panel B, which plots for each team in 2018 the ratio of player wages at the 90th and 50th percentiles. To illustrate this heterogeneity over teams, the largest wage bill and highest variance in wages at Toronto FC in 2018 were due to three designated players (Sebastian Giovinco, Michael Bradley and Jozy Altidore), who together earned over US$18million, and were respectively the 1st, 2nd and 7th highest paid players in the League in 2018.

Player-season-level performance data

Table 2.1 summarises the player-season-level performance variables used in our analysis. For forwards, midfielders and defenders, we observe the number of goals, assists and shots on goal in the previous MLS regular season, i.e., excluding the playoffs. We also observe the numbers of red and yellow cards received, and the numbers of fouls committed by and against each player. To normalise and compare between players who spend different

FIGURE 2.3: MLS growth and expansion: player wages, teams and attendance, 2008-18

(a) Total team wages (b) Variance in wages among team players



Notes: Author calculations using MLS Player's Association data. Wages are nominal US$.

amounts of time on the pitch, we convert all these variables into the average per 90 minutes played in the previous regular season. For goalkeepers, we observe saves as a percentage of the shots on goal by the opposing teams.

One concern is that our performance variables are more relevant for forwards and midfielders than for defenders and goalkeepers. However, we can observe the minutes played in each regular season. Assuming that team managers choose players from their roster based on performance, this should provide a good proxy for individual productivity; better players will normally play more minutes, notwithstanding injuries.[9] There is substantial variation in minutes played during a season. On average, players are on the pitch for 44% of the total time that their team plays in a regular season, and the standard deviation is 29%. Another indicator of a player's productivity or value to the team, which we expect to affect wages, is whether they are the team's captain for the current season. We use data on historical team captains from Wikipedia, which we spot check against news articles and individual team websites.

We construct three dummy variables indicating new entrants to MLS, re-entrants, who were not signed by a team in MLS in the last season but who had previously played in the League, and transfers, who were traded to a new team in MLS between two consecutive seasons. Player turnover in MLS is high; 28% of our player-year observations are for players new to the League, and 14% are for players transferred to a new team. These variables may provide further useful information about player productivity. In particular, after a player's contract expires, the team for which he currently plays has the first right of refusal to re-sign him (Major League Soccer Players Union, 2015). If they choose not to

[9]This is potentially a strong assumption, as managers may wish to give more popular players more time on-the-pitch, regardless of their productivity, to keep fans happy.

do so, then this may indicate that he has performed below their expectations. Conversely, a highly productive player might be transferred to another team for a high price. We also construct a dummy variable indicating a player who tends to play in other positions besides his primary role, as this versatility could make him a more productive member of the team. This applies to 7% of the player-year observations.

TABLE 2.1: Summary of player-season-level variables

	Mean	St. Dev.	Min.	Median	Max.
All players: current season variables (2008-18)					
Wages (US$1,000s)	235.43	620.44	12.90	100.00	8,650.00
Actual age	25.91	4.45	15	25	42
Captain	0.03	-	-	-	-
Designated player	0.07	-	-	-	-
MLS new entrant	0.31	-	-	-	-
MLS re-entrant	0.03	-	-	-	-
MLS transfer	0.13	-	-	-	-
All players: lagged season variables (2007-2017)					
Mins played (%)	44	32	0	43	100
Multi-position	0.07	-	-	-	-
Defenders, Midfielders, Forwards: lagged season variables (2007-2017), per 90 minutes played					
Assists	0.11	0.14	0.00	0.07	0.65
Goals	0.12	0.18	0.00	0.04	0.79
Shots	0.43	0.49	0.00	0.23	2.12
Shots on goal	1.17	1.11	0.00	0.81	4.86
Red cards	0.01	0.03	0.00	0.00	0.18
Fouls committed	1.04	0.78	0.00	0.92	4.00
Fouls suffered	1.18	0.82	0.00	1.11	4.80
Goalkeepers: lagged season variables (2007-2017), per 90 mins played					
Saves	45%	33%	0%	64%	100%

Notes: For dummy variables (Captain; Designated player; MLS new entrant; MLS re-entrant; MLS transfer; Multi-position) the mean is the share of observations equal to one. See Table B.1 for variable definitions and sources. See Table B.2 for the number of players in the sample by year and position.

Team-season-level data

MLS expanded during the period we analyse, from 13 teams in 2007 to 22 teams in 2018 (see Table B.3). Table 2.2 summarises team-level variables. We use the average points per game played in the regular season of the League, taken from the MLS website, as our main team performance variable.[10] We also construct a dummy variable indicating

[10]Three points are awarded for a win, one point for a draw (tie) and zero for a defeat.

whether a team reached the playoffs in each season, as an alternative measure of success, and a dummy variable indicating a team new to MLS, i.e, an expansion franchise.

As explained above, the team operators are all joint investors in the League as a whole, and receive a share of the overall profits. Most team operators are individuals or private companies, and thus data on this particular component of their earnings are not available. However, operators do receive some direct revenues, in particular a majority share of any revenue derived from attendance at their team's games, such as ticket sales, parking, food and drink, and from local broadcast rights and local sponsorship revenue. The remainder of these revenues, as well as national broadcast rights, league-wide sponsorship etc., are distributed among all investors in the League (US Court of Appeals, 2002). We can, therefore, use attendance as another measure of team output. We check that attendance is not constrained by stadium capacity, which would lower the correlation between attendance and revenue, by comparing each team's average home attendance to the capacity of its home stadium (see Fig. B.1). In general, stadium capacities are substantially greater than average attendances, suggesting that team operators are rarely capacity constrained.

We use two further measures of team output in our robustness checks: team revenue and team value, estimated and published by the Forbes media company (Smith, 2013 to 2018). Forbes uses a range of information to construct these estimates, including attendance data, sponsorship deals, investments in stadiums, and broadcasting deals. Although they are only available for five seasons (2013 and 2015-18), they provide a cross-check for our results using average attendance as a measure of output. There is substantial variation in both these estimates. In these years, estimated team revenue ranged from \$13m to \$63m and estimated team value ranged from \$64m to \$330m. The Pearson correlation coefficient between annual average home attendance and estimated revenue is 0.69, and between attendance and value it is 0.57, providing reassurance that average attendance is a justifiable measure of team output.

2.5 Results

2.5.1 First-step regressions

We estimate the wage regression given by Equation (2.3) over the period 2008-18, separately for goalkeepers, defenders, midfielders and forwards. The results are shown in Table 2.3, with one column for each of these four playing positions. The fit of the estimated wage equations is low, with an R^2 between 0.47 and 0.62, which is in line with other studies of MLS (e.g., Kuethe and Motamed, 2010), suggesting that much of

TABLE 2.2: Summary of team-level variables

	Mean	St. Dev.	Min.	Median	Max.
Current season variables (2008-18)					
Points per game	1.37	0.30	0.47	1.41	2.09
Attendance (10,000s)	1.94	0.71	0.71	1.82	5.30
Expansion team	0.05	-	-	-	-
Revenue (US$millions)*	30	11	13	26	63
Value (US$millions)*	184	68	64	175	330

Notes: teams receive three points for a win, two points for a draw, and zero points for a loss. For dummy variables (Expansion team) the mean is the share of observations equal to one. See Table B.1 for variable definitions and sources. See Table B.3 for the number of teams and when they featured during the sample period.
*Nominal figures to nearest US$million, 2013 and 2015-18 only.

the variation in players' wages is not captured by some observable measures of their performance. Generally, there are statistically significant differences between the player positions in how the factors considered relate to wages.

Wages increase in age for outfield players (excluding goalkeepers). Unsurprisingly, the wage premium for a captain is large and significant for some positions, increasing the annual wage by 50 log points for defenders and 66 log points for forwards. For outfield players, the wage premium for a new entrant to MLS is also large and significant. However, players who return to MLS from playing elsewhere do not generally receive a significant premium. A transfer between MLS teams is associated with a wage cut of between 11 and 24 log points in the following season, which is probably a result of teams deciding not to re-sign players.[11]

For goalkeepers (column I, Table 2.3), wages are concave in the percentage of minutes played in the previous season, implying that wages significantly increase until a goalkeeper plays approximately 67% of the time, and then begin to decrease after. The estimated effect on wages of the lagged percentage of shots saved is small and insignificant. For defenders (column II, Table 2.3), none of the lagged performance variables, including the number of minutes played, are individually significant at the 5% level in the wage regression, after clustering standard errors at the level of the player-team pair.

[11]Players sign a contract with a team for a minimum of one year. A player's contract may give their team the option to extend after its initial term. If the team decides not to exercise that option, then the player enters a pool of players known as the 'Re-entry Draft', from which they can be signed by another team.

TABLE 2.3: First-step regression estimates: performance related determinants of log guaranteed salary, 2008-18

	Goalkeepers (I)	Defenders (II)	Midfielders (III)	Forwards (IV)
Age (years - 15)	0.025	0.134***	0.076**	0.102***
Age squared	0.001	-0.002**	0.001	0.000
Captain	0.592	0.497***	0.407*	0.660**
MLS new entrant	0.013	0.341***	0.500***	0.434***
MLS re-entrant	0.125	0.150	0.317**	0.355
MLS transfer	-0.181**	-0.107**	-0.150***	-0.242***
Multi position player		0.085	0.035	-0.161
Lagged performance:				
Mins played (% of team)	1.429***	0.360	0.185	-0.182
Mins played (% of team) sq.	-0.709**	0.357	0.602**	1.318**
Saves (% of shots)	0.001			
Assists		-0.212	0.023	0.138
Goals		0.738	0.713	-0.003
Red cards		0.457*	3.126*	2.735
Fouls committed		0.082	0.167***	0.105**
Fouls sustained		0.027	-0.078*	-0.107***
Shots		-0.041	-0.045	0.019
Shots on goal		0.088	0.112**	0.104**
Assists squared		-0.132	1.054	0.737
Goals squared		-0.480	-1.015	0.840
Red cards squared		6.456	-21.27*	-17.49
Year fixed effects	Yes	Yes	Yes	Yes
N	600	1,656	2,015	1,187
R^2	0.621	0.486	0.474	0.524

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, with standard errors robust to clusters at the player-team level.

Notes.- lagged performance explanatory variables are per 90 minutes played, e.g., red cards per 90 minutes played squared, besides minutes played and saves, which are percentages of total team minutes played in the season and the number of shots on target that the goalkeeper faced, respectively.

For midfielders and forwards (columns III and IV, Table 2.3), there is significant evidence that players who commit more fouls per game receive higher wages. This may relate to teams having a preference toward more aggressive players, or because fouls are associated with risk-taking or greater work effort. We also find that midfielders and forwards who suffer more fouls receive significantly lower wages on average. This may relate to the higher skill level of some players, if they are then better able to avoid tackles and challenges. The other performance variable which significantly affects wages is the number of shots on goal. An increase in one shot on goal per 90 minutes is associated with an 11 log point wage increase for midfielders and a 10 log point increase for forwards.

Conditional on shots on target and other variables, the number of goals scored per 90 minutes does not significantly relate to wages for MLS players.

Residual wages

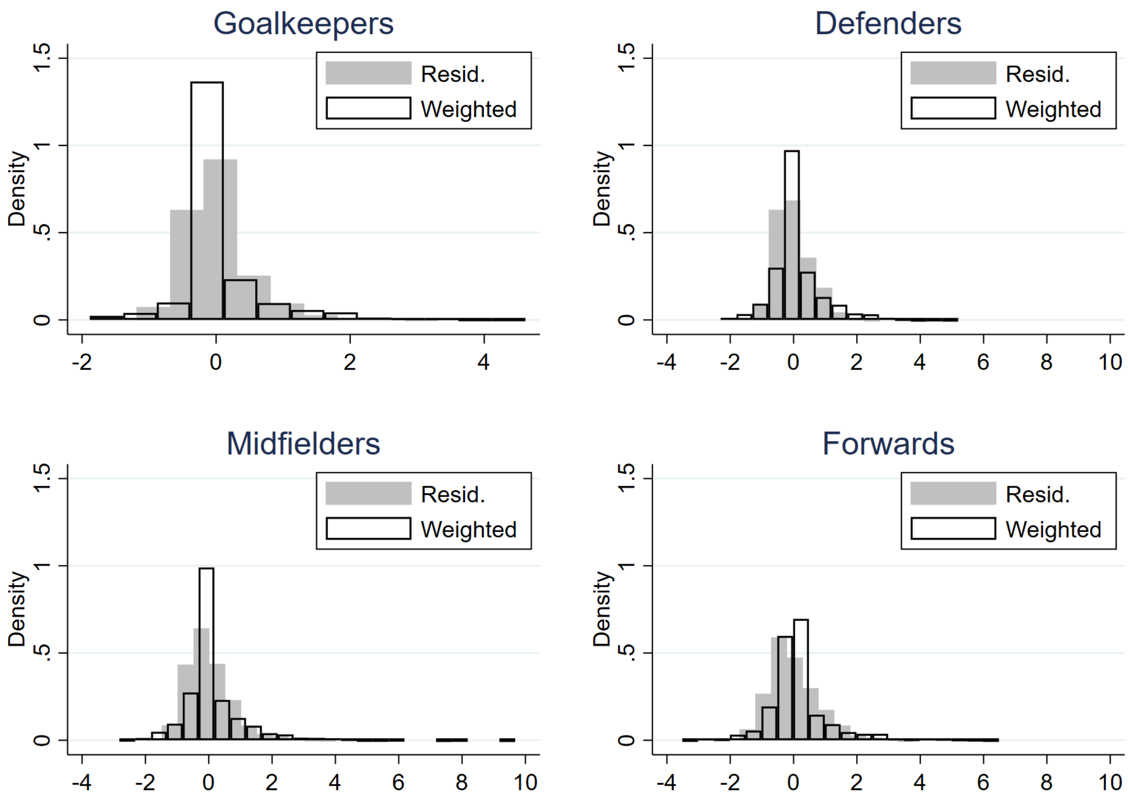
We proceed by collecting the predicted and residual wages from the first-step wage regressions. As Fig. 2.4 shows, there is substantial variation in the residual wages, i.e., the part of a player's annual guaranteed salary that is not explained by measures of individual player performance, and which may correspond to superstar quality. The variation in these residuals for midfielders and forwards is greater than for defenders and goalkeepers, especially with regard to the size of the right tail. This is consistent with the variation in actual wages by position (see Fig. 2.2). We also find that the variation is reduced when we weight each player's residual wage by the amount of minutes they played during the season, as per Eq. (2.6). This weighting decreases the mass of very low residual wages; players who are paid below the wage predicted by their performance also tend to play fewer minutes. This may reflect that these players have lower on-pitch productivity in ways that the wage regressions do not capture. The weighting also decreases the mass of players with very high residual wages, but who are injured or suspended during the season.

To determine whether a particular group of players could potentially drive the results of our second-step regressions, we plot the residuals for different types of player. Fig. B.2 displays the distributions of the estimated residuals, comparing new entrants with those who played in the previous season. The former are more affected by the weighting, given that new entrants generally play fewer minutes. Fig. B.2 also compares the estimated residuals for designated players with all other players. Designated players have considerably greater variation in their residual wages, especially after weighting by minutes played, with a generally greater mass of very high residuals in this case, i.e., designated players get time on the pitch even if their measured performances are relatively poor. Fig. B.3 shows the distribution of residuals by the region of a player's nationality. There is a greater mass of high residuals for players from Europe and Latin America. North American players are more likely to have residuals close to zero. This is consistent with an interpretation of residual wages as being related to player popularity.

Validity of the first-step results

To add confidence to our interpretation of residual wages as a potential measure of superstardom, we attempt to measure player popularity directly using Wikipedia page

FIGURE 2.4: Distributions of estimated residuals from first-step player wage regressions, by position and unweighted vs weighted.



Notes.- each sub-figure plots the residuals from the regression as per Equation (2.3) and the columns in Table 2.3. The weights are described in the text.

views.[12] The mean and median number of player page views per month in the sample is 2,763 and 1,027 respectively, and the most 'popular' player in any year was Ashley Cole, who had an average of 108,774 page views per month in 2016. From a regression of log residual wages, i.e., $\widehat{z}_{it}$, on log page views, a 1% increase in the latter is associated with a 0.09% increase in the former. Fig. B.4 displays the scatter plot and best linear fit corresponding to this regression, which is significant at the 1% level. To avoid omitted variable bias due to the Wikipedia page views of a player being correlated with the popularity of their team or with the league as a whole, which in turn may affect residual wages, we also report results including year and team fixed effects (Table B.4). Adding these controls does not alter the results in any significant way. This demonstrates that a player's popularity, as measured by Wikipedia page views, does increase his wages in a

[12]Data on page views for individual players are available at a monthly frequency going back to July 2015, and were found by collecting page view data for the URL https://en.wikipedia.org/wiki/[player first name]_[player last name]. To capture popularity at the time of contract negotiations, we use the average monthly page views in the off-season months of January and February.

way that cannot be explained by past performances, as captured by our first-step wage regressions.

The set of on-pitch productivity regressors included in Equation (2.3) is limited by data availability for the whole sample period. However, since the 2013 season, more information on MLS player performances is available from the website WhoScored.com. These include interceptions, attempted dribbles, pass completion rates, tackles and metres run during matches, obtained from Opta, the premier football analytics company. WhoScored.com amalgamates over 200 player performance statistics into an objective rating for every match.[13] We obtained a season average of this rating for as many MLS players as possible. We find that the previous season's value correlates strongly and significantly with the predicted component of the first-step wage regression, i.e., $\mathbf{x}'_{ikt-1}\widehat{\beta}_k$, though less so for defenders than other positions (Fig. B.5). We also find that a regression of $\mathbf{x}'_{ikt-1}\widehat{\beta}_k$ on a player's season-average WhoScored.com rating, including season and team fixed effects, yields a significant slope coefficient of 0.85 and an R^2 of 0.2 (Table B.5). This reassures us that the predicted component of the first-step wage regression is a meaningful measure of a player's on-pitch productivity.

2.5.2 Second-step regressions

Tables 2.4 and 2.5 show the main results of the second step in our analysis, in which we regress teams' output on their aggregate predicted and residual wage bills, i.e., Eq. (2.7), using the results presented in Table 2.3 and Fig. 2.4.

Team performance

First in Table 2.4, we use team performance in the League as the dependent variable in Equation (2.7). In columns I-III, we use the log of average points per game in a regular season as the dependent variable, and estimate using OLS. In columns IV-VI, we use whether a team reached the MLS playoffs as the dependent variable. As this is a binary variable, we estimate the probit model equivalent of Eq. (2.7). Columns I and IV show our results using the unweighted aggregate predicted and residual total wages for each team and season as regressors. In columns II and V, we use the predicted and residual wages from the first step for a team's designated players only, i.e., we give zero weight to non-designated players when aggregating. We do this to check whether these particular players are driving our results. Finally in columns III and VI, we use the predicted and residual wages for all players weighted by their contributions of time on the pitch, as per Eq. (2.5) and Eq. (2.6). Compared to the team production suggested

[13]This is explained here: whoscored.com/Explanations.

by Eq. (2.1) and Eq. (2.7), we also omit the log of team average attendance, season fixed effects and any other explanatory variables from the estimated models. We find no significant evidence, conditional on the included team fixed performance effects, that attendance affected performance on the pitch, that there were MLS trends in points per game (e.g., fewer drawn games), or that expansion teams had better or worse than average performances in their first season.

TABLE 2.4: Second-step regression estimates: log points per game and whether made the playoffs, 2008-18.

	Points per game			Playoffs		
	(I)	(II)	(III)	(IV)	(V)	(VI)
Log wages, all players ($m):						
Predicted ($\widehat{\lambda}$)	0.237*			1.938**		
	(0.122)			(0.786)		
Residual ($\widehat{\gamma}$)	0.065			0.531		
	(0.081)			(0.535)		
Log wages, designated players ($m):						
Predicted ($\widehat{\lambda}$)		-0.003			0.301	
		(0.028)			(0.189)	
Residual ($\widehat{\gamma}$)		0.025			0.027	
		(0.022)			(0.166)	
Log weighted wages, all players ($m):						
Predicted ($\widehat{\lambda}$)			0.221**			1.773***
			(0.092)			(0.484)
Residual ($\widehat{\gamma}$)			0.060			0.490
			(0.059)			(0.432)
Team fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
N	204	167	204	186	142	186
R^2	0.262	0.288	0.279			
Log pseudolikelihood.				-110.2	-85.3	-107.1

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors robust to team clusters [23 for Columns (I)-(III), 19 for (IV) and (VI), 18 for (V)] and displayed in parentheses.

Notes.- Columns (I)-(III) are estimated using OLS. Columns (IV)-(VI) give probit model estimates of slope coefficients.

When using all players' wages, including non-designated players, we find weak evidence that the estimated coefficients of predicted wages are positive and significant, whether using points per game or reaching the playoffs as the dependent variable. In Table 2.4 column I, for example, an increase of 1% in a team's aggregate predicted wages is associated with a 0.2% increase in points per game. In other words, teams that spend more on wages for talented players perform better in the League. This effect decreases slightly when using weighted wages but is more precisely estimated (column III). The coefficient on the residual wages, $\widehat{\gamma}$, is insignificant. This is robust to the choice of dependent variable and to whether we weight players by time on the pitch. This suggests that the unexplained part of player wages from the first-step regressions is not associated with better team performance. It also suggests that the residuals in our first step do indeed capture a component of wages that is unrelated, or orthogonal, to player talent, and gives us confidence that we are generally not missing important unobserved elements of individual productivity, e.g., team leadership qualities, in the predicted part of wages. If residual wages reflected these qualities, then we would have expected that higher residual wages should positively affect teams' on-pitch performances. Unsurprisingly, because reaching the playoffs is based on points achieved during the regular season, we find qualitatively similar results for the probit model estimates in columns IV and VI.

In Table 2.4 columns II and V, we check whether the results described above are driven by the highest-paid designated players. We find that the estimated effects of residual and predicted wages for these players on team performance are insignificant. This suggests that spending more on designated players is not associated with teams doing better on the pitch in MLS. This is unsurprising, given that these players can only plausibly have small impacts on a team's performance; there are typically only two such players per team, who will not play every game, and who usually appear for short spells in MLS, making it harder for them to integrate with their longer-serving teammates or into tactical setups.

Home attendance

In Table 2.5, we use the average attendance at home games for each team and season as our second measure of output. Column I shows our base specification of Eq. (2.7). This includes as a regressor the log points per game in the regular season, as we generally find this to be significant in explaining variation in home attendance within teams. This allows us to control for the effect of team performance on attendance; teams that win more games may attract more spectators. It also includes a dummy variable for expansion teams, as there is some significant evidence of higher attendances in a team's first year in MLS compared to later seasons.

Column I shows that aggregate predicted wages, without weighting, do not significantly predict home attendance within teams. However, aggregate residual wages do, with an elasticity estimate of 0.14. This result, when combined with the evidence we have described above, is consistent with the Adler theory of superstar wage effects. We find no evidence that aggregate residual wages predict a team's performance on the pitch. However, these residuals positively affect home attendance, and thus a team's revenues and profits. In other words, the results are consistent with the highest wages in MLS having little to do with the current talent of the players receiving them, but are instead a reflection of the fact that these players are popular and draw larger crowds into the stadiums. This is consistent with the previous literature that has studied superstar effects in MLS (e.g., Coates et al., 2016; Jewell, 2017), though these papers did not rule out on-pitch productivity-based explanations for very high wages at the same time.

In column II of Table 2.5, we estimate the same model as in column I except that we give non-designated players zero weight when aggregating predicted and residual wages from the first-step wage regressions. These results show that the residual wages of the designated players alone do not on average significantly affect home attendance. In other words, the superstar wage effects in MLS appear to be more general than just those implied by the 'Beckham Rule', which the previous literature has focused on (e.g., Jewell, 2017). In column III, we show estimates of the same model specification as column I, except that we use the predicted and residual wages for all players weighted by their contributions of time on the pitch, as per Equations (2.5) and (2.6). We find weaker positive effects of the team's residual wages when applying this weighting, with an elasticity of 0.06, which is only significant at the 10% level. This is further suggestive evidence in support of Adler's superstar wages theory over Rosen's, in this context, as teams benefit financially from spending on superstar wages largely regardless of the amount of time that the recipients are on the pitch displaying their footballing talent.

Finally in columns IV and V of Table 2.5, we investigate the response of attendance to different portions of the residual wage distribution. Instead of aggregating the residual wages of all players in a team, we separately aggregate the estimated residuals from the first step of players whose residual is below and above the 90th percentile (across all seasons and player observations). The latter set of players are those who are paid much more than their previous performances would suggest and who may be 'superstars'.[14] The 90% figure is largely arbitrary, so we vary it later, but it is also consistent with there being on average one or two superstars on a team's roster per season in MLS. We estimate the

[14] 24% of these players are designated players, higher than the 6% of designated players in the full sample. This is intuitive, as it is the designated players that are not subject to the salary cap, and can therefore be paid very high wages. However, there is significant salary overlap between designated and other players. Some designated players will be paid highly in recognition of high performance and will have low residual wages. Conversely, some other players will be overpaid relative to their performance, and will have high residual wages. This measure of 'superstardom' captures these players.

equivalent regressions as per columns I and III, except that we include the two separate measures of a team's 'high' and 'low' residual wages and omit the overall measure. When not weighting each player's influence in these measures by time on the pitch, we find that the coefficient estimate on the high residual wages of teams, $\widehat{\gamma}_H$, is positive and significant: a 1% increase in the amount of high residual wages increases home attendance by 0.06%. An increase in a team's overall wage bill that is spent on players below the 90th percentile of the estimated residuals does not significantly affect home attendance. As with the comparison between columns I and III, column V shows that the effect of the high residual wage measure is weaker and insignificant when we weight individual players by time on the pitch.

Fig. B.6 shows the coefficient estimates, $\widehat{\gamma}_H$, obtained by re-estimating the second-step regression presented in column IV of Table 2.5, now varying the cutoff level of what constitutes 'high' residual player wages. In general, the results show that residual wage spending above the 80th percentile by teams has a positive and significant effect on their home attendances. The coefficient estimate is only slightly decreasing as the cutoff increases from the 80th to the 95th percentile. Since this should be interpreted as an elasticity, and because we are accumulating the wage of fewer players as the cutoff increases, this would imply that the the absolute or relative responses of attendance to changes in residual wages are increasing with the cutoff value, i.e., players with higher residual wages have larger effects on home attendances at the margin.

In summary, we find that a team's residual wage bill has a significant positive effect on attendance and no effect on team performance. This suggests that footballers can be superstars of the type identified by Adler (1985). The unexplained part of a player's wages does not appear to be related to the unobserved elements of performance (or productivity) on the pitch as a footballer, but instead represents a popularity premium. This is slightly different to the findings of Bryson et al. (2014), who investigated whether the higher wages earned by migrants in the Italian football league could be explained by their superstar status. Using a similar empirical strategy, they found that migrants in Italian football earned a substantial wage premium (i.e., that the residual part of their wages was higher than for domestic players). They interpreted this wage premium as evidence of superstar wages, and found that teams with a higher residual wage bill for migrants enjoyed both better on-pitch results and higher attendance. In this paper we define a superstar as any player who earns much more than their on-pitch performance would suggest. Our findings are therefore more general: in MLS, on average, superstar wages are paid to players in respect of greater popularity, rather than higher productivity. There may exist specific groups of players in MLS, such as migrant players, who can command wage premia, or domestic players, who may suffer wage penalties, but we do not investigate this here.

2.5.3 Further robustness checks

To add to our confidence in these results, we perform three more sets of robustness exercises. First, we look for direct evidence that superstar effects can drive teams' revenue (and not only home attendance). Second, we use different methods of estimating the first-step regression to reduce the impact of outliers in the data. Third, we estimate the second-step regression giving zero weight to players who play less than 20% of the time, or who are paid a minimum wage, to ensure these observations are not influencing the main results.

Other measures of revenue

So far, we have only demonstrated that superstar wage effects in MLS are consistent with an interpretation whereby players receive those amounts due to a popularity premium, increasing home attendances and thus team revenues. However, for a small number of years we can check whether this is the only mechanism, using estimates of MLS teams' annual revenues and values. Table B.6 summarises the results from estimating variants of Equation (2.7) whereby the dependent variable is the team's overall revenue or value. We include in these regressions the log of home attendance as an explanatory variable. Conditional on this, we find no significant evidence that the measures of total team revenue or value are affected by the predicted or residual wages of teams from the first step regressions. Based on only a few years of data and broad financial estimates, from a source that may not be especially reliable, this suggests that the superstar wage effects in MLS significantly relate to stadium gate receipts but not the remainder of a team's revenue or value.

Outliers in the first-step regression

The first-step wage regression of Equation (2.3) may be affected by outliers among players for some of the past season performance measures, because some players spend little time on the pitch. For example, a player may come off the substitutes' bench in the final minutes of a game, score a goal with his only touch of the ball, and then never play again that season. Such a player would have an exceptionally high goal scoring rate in the wage regression, but in general may not be a high performance player. We address this issue in two ways, estimating the first-step using weighted least squares (WLS) and using robust regression as a non-parametric alternative. Using WLS, the weights for each player are proportional to the share of total minutes in the previous MLS regular season, before aggregating at the team level, thus generating alternative measures of team predicted and residual wages. We do this and summarise in Table B.7 the second-step estimates which are comparable with our main results. Compared with Table 2.4 column III, there is a significant response of points per game to a team's residual wages at the 5% level, with an elasticity of 0.12. However, the effect of predicted wages is still larger and significant, with an elasticity of 0.20. With regards to home attendance, there remains no impact from team spending on predicted wages, and the impact from residual wages is significantly positive at the 5% level, with a comparable magnitude to the main results. Overall, WLS in the first step improves the precision of the second-step estimates, suggesting that this approach generates less noisy estimates of a player's or team's residual wage amount.

We also use a robust regression to reduce the weight on outliers in the estimation of Equation (2.3) in a non-parametric way (see e.g. Berk, 1990, for details).[15] Reassuringly, we find that residual wages calculated in this manner still have a positive impact on attendance, with an estimated elasticity of 0.10, which is significant at the 10% level and quantitatively similar to our previous results (Table B.8, column 3). As before, the highest residuals drive the results (Table B.8, column 4). Using robust regression in the first step does, however, change the estimated effect of both the predicted and residual parts of wages on points per game and the probability of making the playoffs: the coefficient estimates are smaller and no longer significant compared with Table 2.4, suggesting that the robust regression performs worse at picking up the on-pitch talent of the players from their wages. A possible reason for this is that the highest performers, due to their very high wages, receive less weight in the robust regression and, therefore, their predicted wages fall in such a way that their importance to the team is no longer captured. Indeed, we do find that the robust regression weights are decreasing with predicted wages, and that the predicted wages of the high performers are shifted down in the robust regression.

Players at the 'bottom' of the roster

An MLS team can comprise up to 30 players. At the bottom of a team's roster are a number of players who may not play at all during a season, and who often earn the minimum wage specified in the CBA. To ensure that our results are not influenced by these players, we perform two final robustness checks.

First, to check whether our results are determined by 'benchwarmers' who play few minutes, we give zero weight in the team wage aggregates to players who played for less

[15] Specifically, we use Stata's *rreg* command with the default setting of a tuning constant equal to 7, meaning that residuals which exceed 7 times the median deviation receive zero weight, while other residuals receive a weight which decreases with their distance from the regression line.

than 20% of the total time that their team spent on the pitch. As in our main specification in Table 2.4, the coefficient of a team's aggregate predicted wages on performance is positive and significant (Table B.9, columns I and II). However, the effect of residual wages is also positive and significant. Using home attendance as the dependent variable, the coefficient of residual wages remains significant, providing further confidence that higher spending on 'superstar' players translates into higher home attendances (Table B.9, columns III and IV).

Second, there is a group of players earning the minimum wage solely determined by MLS salary regulations. The relationship between their wages and lagged performances is less clear. We perform a robustness check whereby we exclude these players from the first-step regression. As nearly 20% of our sample are paid at, or below, the senior minimum wage, excluding these players from the aggregate team wage bills in the second step is problematic.[16] We therefore impute these players' predicted wages, based on their performance in the previous season, and calculate their implied residual wages. We use these results to calculate new measures of each team's aggregate predicted and residual wage bills. Table B.10 shows the second-step estimates using this method. Compared with the main results (Table 2.4), the evidence that predicted wages affected team performance is weaker. However, the coefficient of a team's aggregate predicted wages is significant at the 10% level when the dependent variable is whether a team reached the playoffs. As with the main results (Table 2.5), the estimated elasticity of home attendance to residual wages is statistically significant and driven by the 10% of players with the highest residual wages.

2.6 Conclusion

It has generally proven difficult for economists to answer why some individuals attract astronomically greater wages than their peers. We have used the market for football players in the US to investigate this phenomenon. As we were able to link player wages with measures of performance, or productivity, we could construct a measure of each player's 'predicted' wage, the part of his salary that was explained by his past performance, and his 'residual' salary. We aggregated these measures for each team, to generate their overall spending on predicted and residual wages in each season.

We found that a high spend by a team on predicted wages led to a better performance in the League, while a high spend on residual wages did not. We also found that a high spend on residual wages increased attendance at home games (a proxy for revenue),

[16]This includes players aged under 24 on a team's reserve roster, who have a lower minimum wage.

while a high spend on predicted wages did not. This latter result was driven by the players who earned the highest residual wages, above the 80th percentile. These results suggest that some players were paid large amounts because of their popularity, rather than because they were supremely talented. This is consistent with Adler's theory of superstar wages. Our results are also broadly consistent with those of Bryson et al. (2014), who used a similar two-step empirical strategy to investigate whether a particular sub-group of players, migrant players in Italian football, could be thought of as superstars. In addition, our results provide a possible explanation for the low model fit typically found in wage regressions within sports labour markets. We performed a number of robustness checks, exploiting the fact that our data cover almost all the players in MLS, and our main conclusions are robust to all these different model specifications.

Despite being robust, we must apply some caveats on why our conclusions may not be widely applicable. First, MLS has complicated rules regarding player salaries that are probably unique to this market. Second, a footballer's career is short, and they may only spend a few years in MLS before or after playing in other leagues. Third, MLS is still relatively new, and is different to other long-established sports leagues, particularly those in European football. Nonetheless, some features of MLS make it a useful setting to investigate the questions surrounding superstar wages. For example, it is especially competitive when compared with most other major sports leagues, such that a team's decisions about wages potentially affect its results more immediately. Likewise, MLS teams do not yet have a significant unmet demand for matchday tickets, unlike the top teams in Europe, where the majority of the global football superstars play.

Our results suggest that MLS consumers prefer to watch popular 'superstar' players. MLS competes for these players with other football leagues. It competes for consumers both with other football leagues (such as the UK's Premier League) and other sports leagues in the US (such as the National Basketball Association League). Despite this, the League's salary regulation limits the amounts that teams can spend on superstar players. It is likely, therefore, that MLS could attract more viewers by loosening this regulation and thus attracting more superstars. MLS recognised this in 2007, when it introduced the designated player rule. However, the aim of the salary regulation is to ensure that the League remains competitive, i.e., that a small number of well-funded teams are not able to spend vastly more on actual talent than other teams and thus dominate. If MLS consumers also value a competitive league, then the salary regulation may benefit the League as a whole. Quantifying which of these effects dominates is beyond the scope of this paper, but it is an interesting area for future research.

TABLE 2.5: Second-step regression estimates: log home attendance (10,000s), 2008-18.

	(I)	(II)	(III)	(IV)	(V)
Log points per game ($\widehat{\phi}$)	0.139**	0.145***	0.131*	0.109	0.114*
	(0.065)	(0.049)	(0.071)	(0.066)	(0.063)
Expansion team ($\widehat{\delta}$)	0.106*	0.085	0.126**	0.073	0.153**
	(0.060)	(0.063)	(0.056)	(0.053)	(0.057)
Log wages, all players ($m):					
Predicted ($\widehat{\lambda}$)	0.166			0.076	
	(0.127)			(0.121)	
Residual ($\widehat{\gamma}$)	0.136**				
	(0.053)				
Log wages, designated players ($m):					
Predicted ($\widehat{\lambda}$)		0.026			
		(0.027)			
Residual ($\widehat{\gamma}$)		0.020			
		(0.014)			
Log weighted wages, all players ($m):					
Predicted ($\widehat{\lambda}$)			0.133		0.156*
			(0.093)		(0.088)
Residual ($\widehat{\gamma}$)			0.062*		
			(0.033)		
Log wages, split by the residual percentile ($m):					
Residual below 90th percentile ($\widehat{\gamma}_L$)				-0.044	
				(0.146)	
Residual above 90th percentile ($\widehat{\gamma}_H$)				0.057**	
				(0.053)	
Log weighted wages, split by the residual percentile ($m):					
Residual below 90th percentile ($\widehat{\gamma}_L$)					0.011
					(0.079)
Residual above 90th percentile ($\widehat{\gamma}_H$)					0.028
					(0.019)
Team fixed effects	Yes	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes	Yes
N	204	167	204	195	191
R^2	0.862	0.895	0.859	0.891	0.846

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors robust to team clusters (21 for Column (V), 23 otherwise) and displayed in parentheses.

Chapter 3

Long-term trends in part-time work in the UK

3.1 Introduction

The study of labour supply and the length of the working week is as old as the study of economics itself. Before the Industrial Revolution, weekly hours of work for the majority of the population varied with the seasons. Early economists such as William Petty lamented the fact that the labour supply curve at low incomes was backwards bending, and argued that wages should be kept low to encourage workers to supply as many hours of labour as possible (Hatcher, 1998). With the Industrial Revolution, average weekly working hours in the UK increased, reaching a peak of 66 in 1830. By this time, Adam Smith was arguing that working too many hours was detrimental to productivity, writing that 'the man who works so moderately as to be able to work constantly not only preserves his health the longest, but, in the course of the year, executes the greatest quantity of work' (Smith, 1776).

It was when women began to enter the labour force during the two World Wars that the formal idea of part-time work entered public discourse, as a way of enabling women with children to contribute to the war effort. After World War II, most women returned to home production, but by the 1960s labour shortages led the government to encourage part-time working again (Dale and Holdsworth, 1998). The percentage of women aged 20 to 64 with a job who were working part-time (the 'part-time share') increased from 16% in 1951 to 52% in 1991 (Bryson et al., 2021). This increase in women working part-time has attracted much academic attention (see, for example, Connolly and Gregory (2008); Hirsch (2005); Manning and Petrongolo (2008); Matteazzi et al. (2014). However, there has been comparatively little research into part-time work amongst men. Historically, very few men have worked part-time. In 1983, the male part-time share in the UK was only 3%, comparable with other OECD countries. However, it nearly doubled from 7% in 1994 to 13% in 2010. As (Belfield et al., 2017; Blundell et al., 2018) note, this increase has mostly occurred at the bottom of the wage distribution. It has been somewhat offset by a decrease in the female part-time share over the same period, so that from the early 1990s to 2010 the overall part-time share increased very slightly (see Fig. 3.1.)). Similar changes have occurred in other countries, as shown in Fig. C.5. An exception is the US, which has not seen any long-term increase in the male part-time share.[1]

Part-time work is concentrated in service industries, and in low-wage, non-managerial, occupations. There is a strong relationship between hourly wage and weekly hours: on average, part-time workers earn approximately 20% to 30% less per hour than those with full-time jobs. This is often referred to as the 'unadjusted' part-time pay penalty.[2] As Manning and Petrongolo (2008), Matteazzi et al. (2014) and Hirsch (2005) show, the majority of the part-time penalty is accounted for by the occupational segregation between part- and full-time work. This is consistent with Connolly and Gregory (2008), who find that 14% of women moving from full- to part-time work in the UK moved to a job that required a lower level of qualifications than their original full-time job. Different characteristics of part- and full-time workers, such as education and experience, also account for some of the pay penalty, although estimates of how important this factor is differ (Manning and Robinson, 2004; Devicienti et al., 2020). After adjusting for all these factors, the remaining 'adjusted' penalty in the UK is approximately 2-3%.[3]

A number of reasons why part-time work is more prevalent in some industries and occupations have been suggested. Fixed costs, such as hiring or administration costs, could lead firms to hire fewer workers who each work longer hours. There could also be differences in daily or weekly start-up costs, so that in some jobs those who work longer hours benefit from increased productivity later on in the day. Another potential explanation is that workers with lower commitment to the labour force may select into part-time work so that both workers and firms are less likely to invest in training for part-time workers, and in equilibrium those jobs which require less human capital are

[1]In this paper I consider long-term trends. However, it has long been known that weekly hours are procyclical. Borowczyk-Martins and Lalé (2019) documents that this is driven by cyclicality in the part-time share. Recently there has been increasing interest in explaining this pattern (see, for example Kang et al. (2020); Mukoyama et al. (2021); Warren (2017)).

[2]Since part-time work is more prevalent in low-wage jobs, and less in high-wage managerial jobs, the unadjusted part-time pay penalty for men is higher than for women (Nightingale, 2019).

[3]This varies substantially across countries, and some authors have found that, conditional on worker and job characteristics, part-time workers earn more than their full-time counterparts in some countries, particularly in Italy (Matteazzi et al., 2014; Devicienti et al., 2020) and Australia (Booth and Wood, 2008).

more likely to be part-time (Garnero et al., 2014). Part-time work is heavily concentrated in some industries and occupations and there is evidence that this is linked to the skills that they require. Hirsch (2005) finds that part-time jobs are more prevalent in occupations that require lower verbal, mathematical, problem solving and technical skills, but that there is little difference in the requirement for spatial skills. Elsayed et al. (2017) finds that part-time workers are less likely to use computers compared to full-time workers in the same occupation.

In the first part of this paper I analyse the changes that have occurred in part-time work in the UK since the early 1990s. I focus on changes in the quantity and relative wages of part-time work. The first contribution of this paper is to document some novel facts about aggregate part-time work. Firstly, conditional on working part-time, weekly working hours have increased. In 1994, mean weekly hours for part-time workers were 16.2, by 2020 this had increased to 18.3. Fewer people now work very low hours (fewer than 10 hours per week). Instead, more people work between 21 and 30 hours per week. As a consequence, the proportion of total hours done by part-time workers has increased (I refer to this as the 'part-time share of all hours'). Secondly, I show that the part-time pay penalty has steadily decreased since the 1990s, reversing the increase observed by Manning and Petrongolo (2008) and Harkness (1996). This result holds after adjusting for the different characteristics of full- and part-time jobs and workers. Finally, I show that, conditional on worker characteristics, the part-time pay penalty is approximately constant across the hours distribution. In other words, part-time workers doing very few hours (under 10 per week) do not earn significantly less per hour than those doing more hours (20 to 30 per week). However, there is a sharp discontinuity in hourly wages between 30 and 35 hours a week.

Previous research has mostly focused on part-time work amongst women.[4] However, as discussed above, the long-term trend in the part-time share is opposite for men and women: the male part-time share has increased markedly, whilst the female part-time share has slightly decreased. In the first part of this paper I therefore also consider the differences in part-time work by gender. Male part-time workers are more heavily concentrated at the bottom of the wage distribution, and the unadjusted male part-time pay penalty is much larger than for women. However, it has also decreased much more quickly than for women. After adjusting for worker characteristics such as education, the penalty is the same for both genders, suggesting that men are more likely to sort into part-time work based on their individual productivity. An analysis of part-time work in aggregate, or focusing on one gender, would miss these differing trends.

[4] An important exception is Belfield et al. (2017) although this paper does not focus on part-time work.

Taken together, these motivating facts suggest that there has been long-term, structural change in part-time work, in addition to changes over the business cycle that have been more widely studied. There have been changes in both the quantity and the relative price (or wages) of part-time work. Observing these equilibrium outcomes does not identify whether changes are caused by changes in workers' preferences or in firms' production technology. The question of what has caused the increase in quantity and relative wages of part-time work has both theoretical and practical importance. From a theoretical perspective, many models incorporating workers' labour supply choices along the intensive margin assume that earnings are linear in hours (so that hourly wages do not depend on hours worked). These models may not be adequate to consider the effect of policies such as taxes and transfers or restrictions on hours worked (Prescott et al., 2009). From a policy perspective, the increase in part-time work could have implications for inequality. For example, Checchi et al. (2016) calculate that up to a third of income inequality in the US, UK, France and Germany is due to dispersion in working hours. As Belfield et al. (2017) note, 'changes along the intensive margin of male labour supply...have played a more important role in explaining recent changes in inequality than one might have expected, given the traditional view that male labour supply varies little along the intensive margin'. In addition, if part-time workers accumulate less human capital, either through learning on-the-job or because they receive less training, then episodes of part-time work could cause income inequality even amongst workers with different work histories (Biewen et al., 2018).

Despite the importance of this question, there have been few theoretical attempts to explain why full- and part-time jobs differ so much, and why hourly wages are not constant. There is previous work focused on labour supply, assuming that the mapping between hours worked and production is non-linear, and hence that earnings are non-linear in working hours, and analysing the effect of this non-linearity on workers' labour supply decisions (Bick et al., 2022; Prescott et al., 2009). However, the authors of these papers do not attempt to explain the source of the non-linear relationship between working hours and production. An alternative approach is to assume that there is an exogenous minimum hours constraint below which workers are less productive, which can generate a non-linearity in hourly wages (Card, 1990). There have also been attempts to incorporate part-time work into the firm's production function by assuming that full- and part-time workers are not perfectly substitutable for some reason (see, for example, (Kang et al., 2020; Lariau, 2018)). The source of this complementarity between full- and part-time work is not discussed in these papers. There is another strand of literature analysing possible complementarities in production between workers (Battisti et al., 2020; Yurdagul, 2017; Rogerson, 2011). These models aim to explain why there is substantial bunching at 'full-time" hours, and part-time workers earn much less. They highlight the problems this causes for estimating the parameters that govern workers' labour supply choices: whatever causes the bunching will dampen the labour supply response to changes in these parameters.

This paper aims to fill this gap in the theoretical literature by developing a flexible neoclassical model in which firms' requirements for workers doing different types of task results, in equilibrium, in the coexistence of full- and part-time work. Importantly, the selection of workers doing different tasks into full- and part-time work is an endogenous outcome of the model, rather than being exogenously imposed. The equilibrium of the model matches key features of the labour market: full- and part-time workers undertake different tasks; there is bunching of workers at full-time hours; and full-time workers earn higher hourly wages than part-time. I show that the model can match the aggregate increase in the quantity of part-time work and the decrease in the part-time pay penalty described above, and can therefore be used to analyse the effect of changes in both firm technology and worker preferences on the part-time share and part-time pay penalty. I next extend the model to incorporate heterogeneity in workers' disutility of labour. This will provide a way to incorporate gender into future research, since men and women are likely to have different preferences, that may be changing in different ways.

The key feature of the model is the introduction of two types of task, which I label 'divisible' and 'complex'. Workers are ex-ante equally productive in either type of task, and must choose which to work in (for simplicity they cannot combine tasks). Divisible tasks can be done in whatever time available, and workers in these tasks are perfectly substitutable. I introduce a new parameter, a stochastic "hours requirement". Each worker doing complex tasks draws an individual hours requirement. Possible interpretations of this parameter include a start up time to do complex tasks (e.g. time to set up a machine); customers' preferences for a specific employee (e.g. a client wants to be able to call their lawyer); or workers' specific human capital that may be required (e.g. someone who knows how to use a particular piece of equipment or technology). Whilst previous research has used a task-based framework to explain trends in automation and substitution between capital and labour (see, for example, Acemoglu and Restrepo (2019)), to the best of my knowledge there is no theoretical research considering the effect of changes in tasks on the intensive margin.

The key requirements are firstly that production in complex tasks depends on whether the worker is working for longer than the hours requirement. For example, someone working 10 hours per week will produce less if the hours requirement in that week is 15 hours (since the hours requirement has not been met), than if the hours requirement in that week is 5 hours (in which case the hours requirement has been met). Secondly, I assume that divisible and complex tasks are not perfect substitutes in aggregate production. These two conditions create a convexity in overall earnings in complex tasks: hourly wages are increasing in hours worked. The intuition behind this result is that, as hours increase, workers are more likely to satisfy the hours requirement. They are therefore more likely to produce at the higher level. This convexity ensures that, in equilibrium, earnings for divisible tasks are greater at low working hours than earnings for complex tasks, whilst earnings for complex tasks are greater at high working hours than earnings for divisible tasks. I show that, in equilibrium, workers doing fewer hours will choose divisible tasks (and can be considered “part-time"), whilst workers doing more hours will choose complex tasks (and can be considered “full-time"). Thus the equilibrium of the model features coexistence of part- and full-time work, higher hourly wages for full-time workers and differences in tasks for the two different types of worker. The model can be thought of as providing a microfoundation for the assumption in Bick et al. (2022) and Prescott et al. (2009) that earnings are non-linear in hours.

I begin the exposition of the model by setting out a simple example, in which workers are homogenous in productivity and disutility of labour, and the hours requirement is uniformly distributed. I show that this model has a unique solution for the part-time share. I provide a simple calibration of the model, and show how it can be used to assess how the ‘demand’ parameters (relative productivity and substitutability of complex and divisible tasks) are changing, compared to the ‘supply’ parameter (the disutility of labour) and total factor productivity. I then present an extension of the model, with heterogeneity in disutility of labour. I show that there is still a unique equilibrium, in which workers doing divisible tasks earn less per hour. In equilibrium workers with a higher disutility of labour choose these tasks. Workers with a low disutility of labour choose complex tasks, and earn more per hour. This model is much more general, with few restrictions on the distribution of the hours requirement or the production function for complex tasks, and this flexibility enables it to reflect a range of features of the labour market, such as the bunching of workers at full-time hours.

This paper is comprised of two parts. In Part 3.A I present the stylised facts about part-time work described above. In Part 3.B I present the simple model, with a quantitative illustration of the effect of changing supply and demand parameters, and discuss the extended version of the model.

3.A Motivating facts

In this section I describe the differences in part- and full-time workers and jobs, and how they have changed since the 1990s. I focus on two areas: the quantity of part-time work (both in the percentage of workers in part-time work and in their hours) and the relative price of part-time work compared to full-time work. Both the aggregate quantity and the average price of part-time work have changed substantially over this period. I also consider gender differences in part-time work, and show that the trends in part-time work have been very different for men and women. Previous research has focused on female part-time work, but, as this section shows, it is important to consider changes in male part-time work as well.

3.A.1 Data description and definitions

There is no legal definition of part-time work in the UK. The UK government states that someone who works less than 35 hours per week is usually considered part-time (UK Government, 2022). However, some benefits, such as the Working Tax Credit discussed below, define full-time work to be above 30 hours. In my analysis I define part-time workers as those working 30 hours a week or fewer. There is a sharp increase in the percentage of respondents who define themselves as working part-time from 47% who usually work 30 hours to 85% who usually work 29 hours.

I use data from the UK's Labour Force Survey (LFS), published by the Office for National Statistics (ONS). The LFS is a quarterly household survey, covering approximately 37,000 households, and covers the employment and personal characteristics of all individuals in a household. In my sample I include all adults aged between 17 and 64. I use the quarterly weights provided by the ONS. The LFS records actual and usual hours worked. I use usual hours worked in an individual's main job as my measure of working hours (the variable TTUSHR). This measure of hours is self assessed, and may be subject to reporting error. Respondents are asked (1) how many hours per week they usually work, excluding meal breaks and overtime; (2) how many hours of paid overtime they usually work; and (3) how many hours of unpaid overtime they usually work. I include paid and unpaid overtime in order to reflect all the hours worked. For example, some workers may be paid high wages per contracted hour but also expected to work unpaid overtime, so that excluding overtime would overstate their hourly wage. I exclude anyone working more than 84 or less than 4 hours per week from the sample.

The measure of wages I use is the variable GRSSWK which is the gross weekly pay calculated using the most recent pay period, and including overtime pay, divided by working hours. Respondents are also asked whether their most recent pay was the same as they usually receive. Where the answer is no, I replace actual gross pay with usual gross pay (the variable USUGPAY). I calculate real hourly wages using the CPI index published by the ONS. Respondents remain in the sample for five quarters, and are asked about their income in the first and fifth quarter that they are surveyed.

One advantage of the LFS is that it samples all households. An alternative data source would be the Annual Survey of Hours and Earnings, which uses data reported by employers. However, this is likely to capture contracted, rather than actual, hours worked. In addition, firms with a turnover below the VAT threshold, or without employees earning more than the National Insurance Threshold do not have to register with the tax authorities and are not surveyed. Since part-time workers are more likely to be earning less, this suggests that the ASHE may not capture part-time workers as well as the LFS. Until 2004 the predecessor to the AHSE, the National Earnings Survey, did not make any adjustments or provide weights to correct for these issues (Bird, 2004).

3.A.2 Government policy regarding part-time work in the UK

It is clearly important to consider the role that working-time regulations, taxes and transfers may have had on the demand, supply and relative price of part-time labour in the UK. In general, these policies have been aimed at encouraging workers to supply more labour, whilst maintaining the birth rate by making work more flexible (United Nations Department of Economic and Social Affairs, Population Division, 2015).

Since 2000, the Part-time Workers (Prevention of Less Favourable Treatment) Regulations have required firms to treat part-time workers equally to comparable full-time workers so that, in theory, part-time workers should receive the same hourly pay as full-time workers with the same characteristics doing the same or similar job. Manning and Petrongolo (2008) note that since the part-time pay penalty (PTPP) after adjusting for selection in part-time work was already low, these regulations are unlikely to have had much effect on relative pay. However, they may have had an effect on other work amenities that are not captured by the LFS, and thus made part-time work more attractive to workers. On the other hand, these regulations may have made part-time work less attractive to firms, who must now pay part-time workers the same hourly wages and provide the same benefits.[5]

[5]The legal database LexisNexis lists 237 cases involving the legislation between 2000 and February 2002. Many of these involved other benefits such as pensions or paid breaks.

The other changes that may have affected the extent of part-time work in the UK are the successive increases in the categories of employees with the right to request flexible working (including part-time work).[6] The firm is able to deny such a request on a number of grounds, including if they believe it will negatively affect the employee's productivity. There is evidence that firms are doing so: surveys suggest that between 10% and 20% of requests were denied (Hegewisch, 2009), and in a 2015 survey, only 42% of fathers thought that they would be able to request flexible working, compared to 78% of mothers (Cook et al., 2021). However, these regulations may have made it easier for some employees to work part-time.

Unlike many other countries, in-work benefits in the UK depend on the number of hours worked. In 1999 the Working Families Tax Credit was introduced, which provided transfers to low paid workers with children. In order to be eligible, individuals had to work at least 16 hours a week, and there was an additional transfer for those working more than 30 hours. In 2003 this was extended to all adults aged over 25. As a result, the budget constraint of low-paid workers has kinks at 16 and 30 hours a week, which results in bunching (see Fig. C.6 in Appendix 5). Blundell et al. (2008) analyse the effect of these changes on womens' labour supply. They find that they led to higher working hours, but that most hours adjustments occurred when individuals changed jobs. They conclude that there is evidence that production technology leads to hours constraints in some jobs.

3.A.3 Changes in the quantity of part-time work

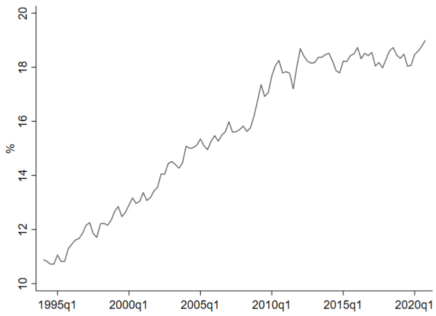
Historically, part-time work was a female phenomenon. However, there is evidence that this is changing. As Fig. 3.1 shows, whilst the female part-time share slowly decreased between 1994 and 2009, the male part-time share increased from 11% to over 18%. Since the majority of part-workers are still women, the net result has been a slight increase in the overall part-time share, from 30% in 1994 to a peak of 34% in 2010.

As Fig. 3.2 below shows, while there was no obvious trend in the percentage of the total male working-age population working full-time, until it decreased sharply during the financial crisis, the percentage working part-time has been steadily increasing since 1994 so that, by 2020, the percentage of all working age men working part-time had nearly doubled. This suggests that some of the long term increase in male part-time work may have come from men who previously would not have worked at all, rather than from those who would have previously worked full-time. The trend for women is very different; the percentage of women working full-time increased slowly until the financial

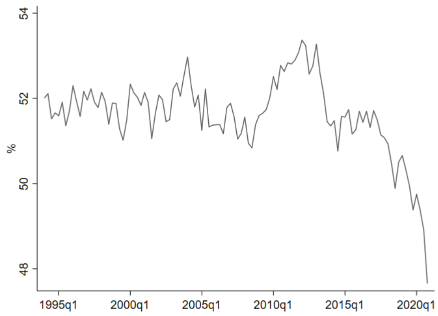
[6] In 2000 this right was introduced for parents of young and disabled children. In 2003 and 2006 it was extended to carers and parents of older children and in 2014 to all employees (Pyper, 2018).

FIGURE 3.1: Part-time workers (as % of working population)

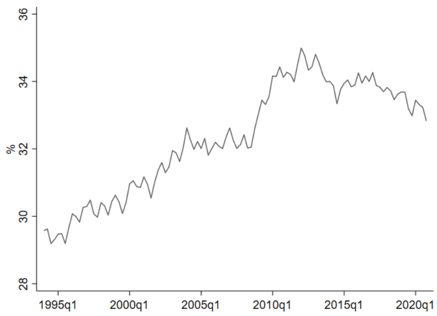
(a) Male



(b) Female



(c) All



Source: LFS

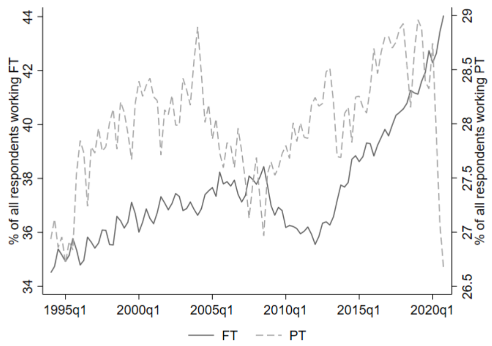
crisis, before falling from 2010 to 2012, and has increased sharply since then. In contrast, the percentage of all women working part-time has not changed very much over the time period.

As well as changes in the number of part-time workers, there have also been changes in their working hours, conditional on working part-time. The decrease in average working hours across all workers has attracted much attention. Across Europe, the percentage working very long hours (generally defined as more than 48 hours per week, in line with the European Working Time Directive) has fallen since the mid 1990s (Messenger, 2010). However, conditional on working part-time, hours have actually *increased*. Fig. 3.3 below shows the proportion of part-time workers doing 1-10, 11-20 and 21-30 hours. There has been a clear decline in the proportion working a very low number of hours per week, and a clear increase in the proportion working 21-30 hours per week. This pattern holds for both men and women, despite the different trends in male and female part-time work discussed above. In order to confirm that this increase was not driven by workers affected by the policy changes described above, in particular the introduction of working tax credits, I repeat this analysis using only the respondents in the upper quartile of the hourly wage distribution, who are unlikely to be eligible for such transfers.(see Fig. C.7 in Appendix 5).

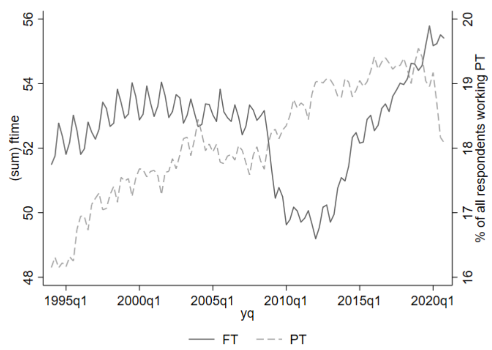
FIGURE 3.2: Part-time and full-time workers (as % of whole population)

(a) Male

(b) Female



(c) All



Source: LFS

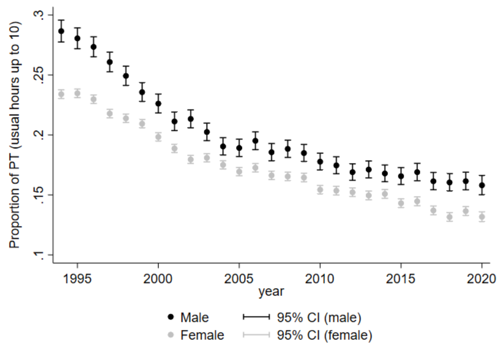
The pattern of changes is the same, suggesting that the observed increase is not solely due to the tax schedule. In addition, the increase in the percentage of part-time workers doing 21 to 30 hours shown in Fig. 3.3c has happened steadily, rather than in the year or two following policy changes.

As a result in the decline in very long hours, and the increase in hours conditional on working part-time, the proportion of total hours done by part-time workers,the 'part-time share of all hours' has increased. In 1994 Q1, 16.7% of all working hours were done by part-time workers. As Fig. 3.4c shows, by 2013 Q1 this figure had increased to a high of 21.9%, and has remained at approximately at this level since. Here the gender differences have become clear since the financial crisis; whilst the male part-time share of all hours for men has continued to grow, the opposite has happened for women.

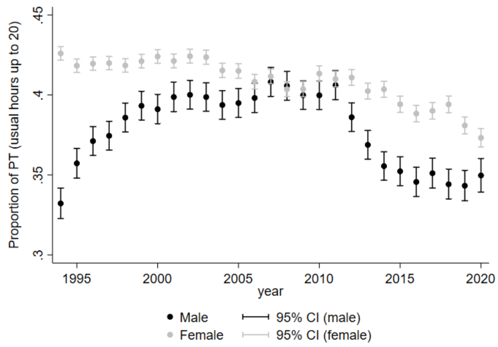
One possibility is that these changes are due to structural changes in the distribution of occupations. Part-time work is heavily concentrated in certain occupations, particularly service occupations. It is also concentrated in occupations with lower skill requirements. In Appendix 3 I provide more information on the link between occupations and their skill

FIGURE 3.3: Trends in the distribution of part-time workers by usual weekly working hours

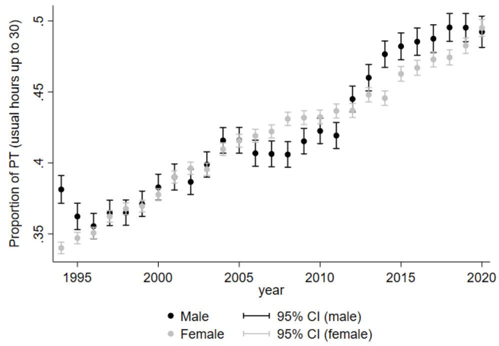
(a) Usual weekly hours 1-10



(b) Usual weekly hours 11-20



(c) Usual weekly hours 21-30



Source: LFS.

requirements and part-time work. As I show in **??**, a higher requirement for non-routine, cognitive skills is particularly strongly associated with a lower part-time share, even accounting for differences in worker characteristics across occupations. This is consistent with the findings of Elsayed et al. (2017); Hirsch (2005).

If it is the case that working hours within different occupations are constrained, then a change in the distribution of occupations may be the reason for the observed trends in part-time work. As Goos and Manning (2007), amongst others, explain, since the 1980s there has been an increase in low-wage service and unskilled jobs and in high-wage service and finance jobs. At the same time, there has been a decline in manafacturing jobs, which tend to be in the middle of the wage distribution. This phenomenon is known as job polarisation. If, for some reason, some of these growing jobs are more likely to be part-time, as suggested by the evidence above, then this may explain some of the increase in part-time work.

FIGURE 3.4: Percentage of all hours done by pt workers

(a) Male (b) Female

(c) All

Source: LFS.

To test this hypothesis, I carry out a shift share analysis. This shows that approximately a third of the increase in the male part-time share has been due to an increase in employment in industries and occupations which have historically had a higher part-time share. The remaining two-thirds is due to more men working part-time within each industry and occupation. This contradicts to some extent the findings of Borowczyk-Martins and Lalé (2019) who find that changes in the composition of jobs by industry and occupation did not contribute to the increase in part-time work during the financial crisis. However, they compare the period before and after the financial crisis, rather than analysing the cumulative increase in part-time work over a period of time. For women, the pattern is less clear. Since 2000, there has been a decrease in female employment in jobs that have historically had a high part-time share. In other words, women moved away from service sector jobs and towards jobs with a lower part-time share. However, between 2000 and 2010, for women there was actually an increase in the part-time share within occupations. After the financial crisis the female part-time share within occupations fell. In addition, I find that the growth in the part-time share within an occupation is not strongly associated with an occupation's skills requirements. To summarise, while part-time work is strongly associated with certain occupations and their skill requirements, the changes in part-time work described above are not solely the result of changes in the distribution of workers across occupations.

3.A.4 Changes in the relative wages for part-time work

Part-time work has long been concentrated in low wage jobs. However, there is some evidence that this concentration is increasing for men. Table 3.1 below shows the part-time share across the wage distribution in 1994, before the cyclical increase in 2009, and in the most recent year, 2020. The table shows that the decline in the female part-time share has occurred in the upper two quartiles of hourly wages, whilst the increase in the male part-time share has been greatest in the bottom wage quartile. Very few men in high-wage jobs work part-time.

TABLE 3.1: Part-time share (%) by wage quartile

	Female			Male			All		
Wage quartile	1994	2009	2020	1994	2009	2020	1994	2009	2020
1	59.1	61.37	54.03	17.19	29.60	24.69	45.12	49.06	42.02
2	40.62	41.40	33.84	5.67	8.77	9.20	24.32	25.90	21.55
3	29.16	26.46	25.31	2.61	4.08	4.78	13.67	14.36	14.69
4	24.10	24.83	24.01	2.50	3.78	4.05	9.49	11.68	11.85
All	41.32	41.16	36.87	5.5	10.8	10.27	22.7	25.45	23.51

Source: LFS.

Alongside the increase in the part-time share of all hours, there has also been a steady decrease in the part-time pay penalty (PTPP). To estimate the average PTPP in year t I use the following model, estimated using OLS

$$\log w_{it} = X_{it}\gamma + \beta_t PT_{it} + \varepsilon_{it} \tag{3.1}$$

where w_{it} is the hourly wage, and X_{it} is a vector of controls, including a constant. PT_{it} is a dummy variable equal to one if an individual works part-time in year t. The coefficient of interest is β_t, which I allow to vary by year. β_t measures the difference in percentage points between average part- and full-time hourly wages, after any adjustments, so that $\hat{\beta}_t < 0$ indicates that part-time workers earn less. Fig. 3.5 below shows the estimate of the PTPP over time. The top left panel (Fig. 3.5a) shows the 'unadjusted' PTPP, including only controls for year and quarter (to capture the general and seasonal trend in wages).

Fig. 3.5b adds controls for 2-digit occupation and industry and other job characteristics.[7] Fig. 3.5c includes worker-specific controls instead, and Fig. 3.5d includes the full set of job- and worker-specific controls.[8]

The figures show that a large part of the PTPP is accounted for by occupational and industry segregation of part- and full-time work.[9] Controlling instead for worker specific characteristics also decreases the PTPP, although not as much. Across all model specifications, there is a steady decline in the PTPP over the time period. In other words, the hourly wage for part-time workers has increased, relative to full-time workers. To confirm that this result is not due mechanically to the introduction and subsequent increases in the minimum wage since 1999, I repeat the estimation excluding anyone earning less than 5% above the minimum wage from the sample. The trend in the PTPP is the same, although the level is lower.

The unadjusted PTPP (Fig. 3.5a) is much larger for men, reflecting the fact that part-time work is concentrated at the bottom of the wage distribution whilst jobs with a higher hourly wage are less likely to be part-time. There are more men doing these highly paid, full-time, jobs, so that the PTPP for men is greater. Despite evidence that the increase in male part-time work is greater at the bottom of the wage distribution (see Table 3.1), the PTPP for men has decreased. The male PTPP remains larger after adding controls for industry and occupation. However, after controlling instead for worker characteristics the PTPP for both genders is approximately the same. This suggests that men may be more likely to sort into part-time work based on their individual characteristics than women.

Clearly, there are concerns that the PTPP may be a result of sorting into part- and full-time work on the basis of some unobserved variable (perhaps ability, or motivation) that also affects wages. In this case, the estimate $\hat{\beta}_t$ would be biased. To check whether this is the case I estimate the PTPP after adjusting wages for selection into part- and full-time work using a Heckman sample selection model. I provide further details in Appendix 2. Both the level and the trend in the PTPP are very similar are adjusting for potential selection bias.

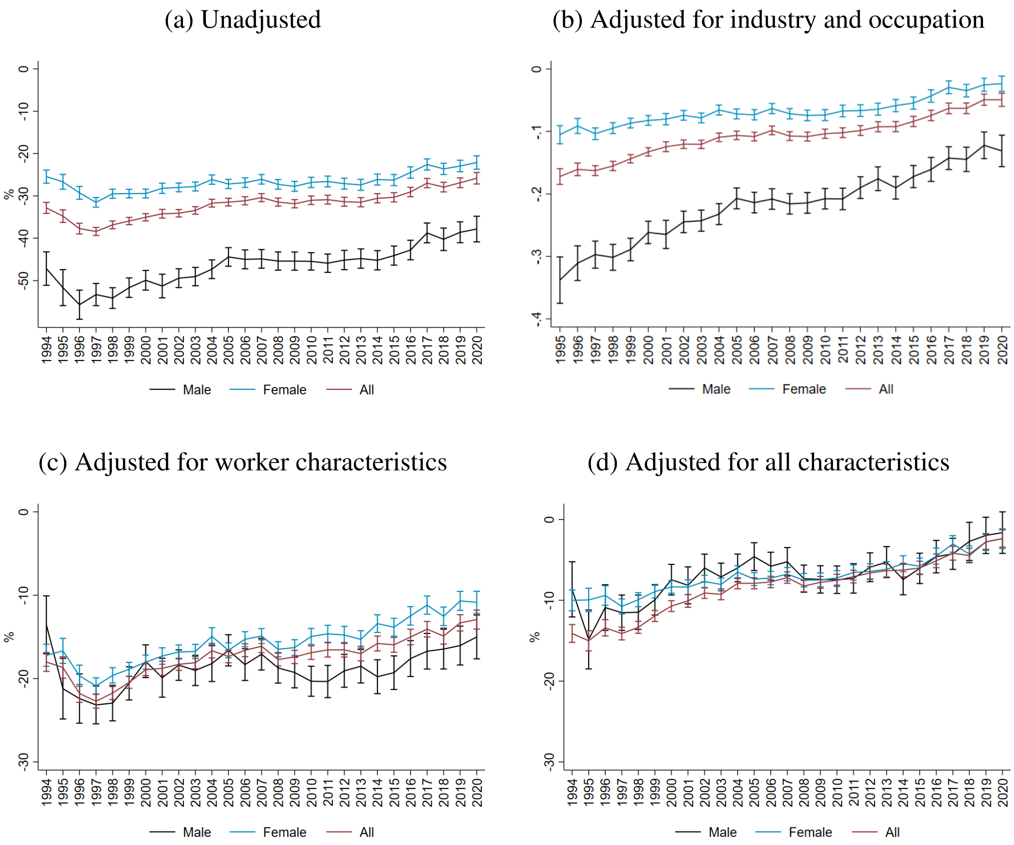
As there has been a clear change in working hours conditional on working part-time, it is instructive to consider the full hours-wage profile, as well as the average pay penalty discussed above. Bick et al. (2022) show that, in the US, the relationship between

[7]Controls for job characteristics are for region; firm size; whether private or public sector; whether the worker is employed through an employment agency; and whether the job is temporary.

[8]Worker-specific controls are age; age squared; tenure; highest level of qualification; family type (single, single with children, living with a partner without children, living with a partner with children); whether currently studying; and whether the job is the worker's main job.

[9]This is consistent with previous research by Manning and Petrongolo (2008), Matteazzi et al. (2014).

FIGURE 3.5: Trend in the part-time pay penalty



Source: LFS.

hours and wages is non-monotonic; mean hourly wages increase up to 50 hours, and then decrease. Their analysis focuses on long working hours, and they do not consider anyone working less than 20 hours. I repeat their analysis, but as the focus of this paper is on part-time work, I include all workers with at least 5 hours. I also add controls for occupation and industry, job and worker characteristics, as described above. To analyse the hours-wage profile I estimate the following (cross-sectional) model using OLS

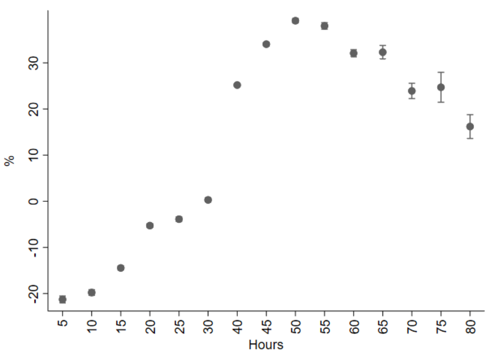
$$\log w_{it} = X_{it}\gamma + \sum_{h\in H} \beta_h d_{ih} + \varepsilon_{it} \tag{3.2}$$

I partition weekly hours into groups of five indexed by $h \in H = \{5, 10, 15, ..., 80\}$ where h is the minimum hours for that group. For example, the group of workers with $h = 10$ includes all workers doing between 10 and 14 hours. d_{ih} equals one if worker i is in group h and zero otherwise. X_{it} is a vector of controls. I allow β_h to vary across the hours distribution to capture any non-linearity in the relationship. Figure 3.6 below shows the estimated coefficients $\hat{\beta}_h$. The 'base' group are those working 35-39 hours, so that $\hat{\beta}_h$ is an estimate of the pay penalty in percentage points relative to this group. The figure shows $\hat{\beta}_h$ estimated pooling all editions of the LFS, and for both genders. However, the results splitting the sample by year or gender are very similar.

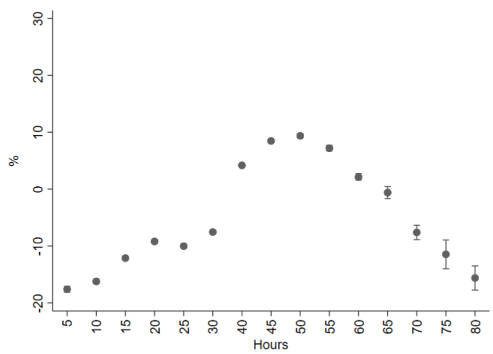
Across all specifications, hourly wages are higher for workers in higher hours groups. There is also a sharp increase in wages between 30 and 35 hours. As I explain above, this is approximately the threshold commonly considered to reflect ‘full-time’ work.[10] Including only controls for year and quarter, and for occupation and industry, hourly wages for part-time workers increase as hours increase from 5 to 30. However, when controls for worker and job characteristics are included (Figs. 3.6c and 3.6d), the increase is much smaller, and in fact, in Fig. 3.6c, $\hat{\beta}_h$ is approximately constant from 5 to 30 hours. This would correspond to hourly wages that, conditional on worker characteristics, are constant between 5 and 30 hours.

FIGURE 3.6: Cross-sectional hours-wage profile

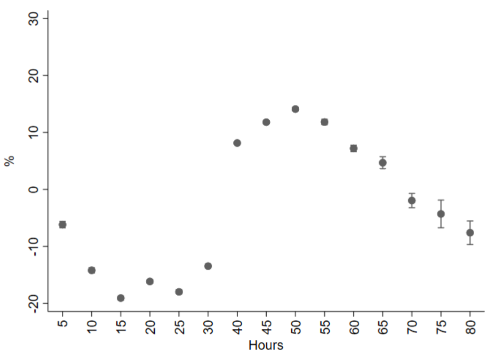
(a) Unadjusted



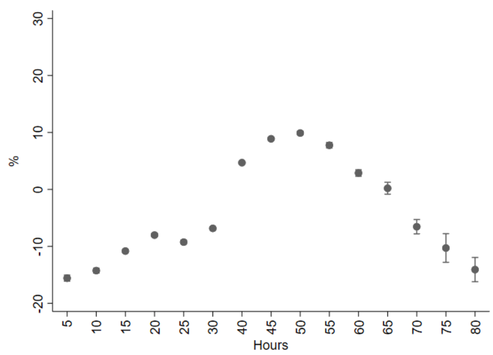
(b) Adjusted for industry and occupation



(c) Adjusted for worker characteristics



(d) Adjusted for job characteristics



To summarise, the stylised facts presented in this section suggest that there have been long-term, structural, changes in part-time work in the UK, in addition to the changes over the business cycle that are more widely studied. In aggregate, there has been an increase in both the quantity of part-time work, both through an increase in the part-time share

[10] Since Bick et al. (2022) only consider hours groups greater than equal to 20, it is not possible to see clearly from their analysis whether this jump occurs in the US as well as in the UK.

and through an increase in hours conditional on working part-time. At the same time, there has been an increase in the price of part-time work relative to full-time work, which manifests in a fall in the part-time pay penalty. In the next section, I set out an example model that can be used to rationalise these aggregate changes in quantity and price. I then consider an extended version of the model, incorporating worker heterogeneity. This will be required in order to analyse the gender differences in part-time work.

3.B Theoretical framework

In the first part of this paper I show that there have been long-term, structural, changes in part-time work in the UK, and that there are significant gender differences in part-time work. It is not a priori clear whether these changes are the result of changes in technology that have affected firms' demand for part-time work or a result of worker's preferences changing. In this section I set out a flexible neoclassical model of the labour market which can explain firms' and workers' preferences for part- and full-time work. The equilibrium of the model features the coexistence of part- and full-time work, the concentration of part-time work at the lower end of the (hourly) wage distribution, and the bunching of workers at "full-time" hours. Importantly, these outcomes are endogenous features of the equilibrium, rather than being exogenously imposed on the model. Given that part-time work is concentrated in some occupations, and particularly those with lower skill requirements, it is natural to begin with a production function that features different types of tasks, which have different characteristics. Again, I do not impose that part-time workers do certain types of task. Instead, in equilibrium, part- and full-time workers *choose* to do different types of tasks, which generates the concentration of part-time work in occupations that require a lower skill level.

I first consider a setting in which workers and firms are homogenous. This can be used to explain the aggregate increase in the quantity of part-time work and the decrease in the part-time pay penalty. I provide a quantitative example, and show that the model can approximately reflect the trends described in Section 3.A above. I also provide some suggestions for disentangling the effect of changes in the various parameters of the model. However, this simple model cannot explain gender differences in part-time work, and hence the differences in trends between men and women. This requires a more general model, incorporating worker heterogeneity. In this chapter I set out one possible extension, incorporating heterogeneity in the disutility of labour, which provides a natural starting point to explore the observed gender differences in trends in part-time work.

The model is a one-period model in a simple, closed, economy with perfect competition. There is a continuum of workers of measure one, indexed by $i \in [0,1]$. They are able to choose weekly hours of work $h \in [0,1]$ hours, so that the maximum available working time is normalised to one. Workers gain utility from consumption and suffer disutility from working. A representative firm produces a final good using labour supplied by the workers. In the remainder of this chapter, I refer to the total sum that the worker receives for all hours worked as their *earnings*, which I denote $e(h)$, and earnings divided by their weekly hours as their hourly *wage*, denoted $w(h) = e(h)/h$. Note that I allow the hourly wage to depend on weekly hours.

3.B.1 Production technology

Production of the final good is a CES aggregate of output from two types of task: output from divisible tasks, Y_d, and output from complex tasks, Y_c, so that production is given by

$$E[Y] = \Gamma E[(\alpha Y_c^\rho + (1-\alpha)Y_d^\rho)^{\frac{1}{\rho}}]. \tag{3.3}$$

α governs the relative productivity of complex tasks, relative to divisible tasks. The elasticity of substitution between the two types of tasks is given by $\sigma = \frac{1}{1-\rho}$. As ρ approaches one and σ approaches infinity, the two tasks become perfect substitutes. As ρ approaches negative infinity and σ approaches zero, the two tasks become perfect complements. As ρ tends to zero, production is Cobb-Douglas, with $\sigma = 1$. I assume that there is some substitutability between complex and divisible tasks but that they are not perfectly substitutable, so that $\rho \in (0,1)$. Γ is the total factor productivity. $m_{h,j}$ denotes the measure of workers doing hours $h \in [0,1]$ in task type $j \in \{d,c\}$.

Divisible tasks can be done in whatever time available, and workers are perfectly substitutable in these tasks. These tasks are more ‘defined’ and the same task can be repeated many times. This would include tasks in many service sector occupations, for example hairdressing, or serving coffees in a cafe. Production in divisible tasks is equal to the hours of work, giving[11]

$$Y_d = \int_0^1 h m_{h,d} dh \tag{3.4}$$

[11] Provided that an individual’s production in divisible tasks is concave, the qualitative properties of the equilibrium discussed in the introduction to this section will hold. However, linear production is intuitive for a worker that is doing lots of short repetitive tasks.

I introduce a new parameter, a stochastic “hours requirement" for workers doing complex tasks, denoted by the letter x. There are several situations that might lead to such a cost. These include:

1. a start up time to do complex tasks. An example would be machinery that requires setting up every day. This is similar to the minimum hours thresholds in papers including Card (1990);

2. specific human capital that may, or may not be required. This could result from demand for the firm's services. For example, a client might wish to be able to call their lawyer during the working week. If the lawyer is not available when the client calls, this will have a negative effect on their output. A further example would be a worker with specific human capital that might be required by other employees at the firm. For example, a worker might have specific technical knowledge that other workers can draw on.[12]

Workers who do complex tasks draw an individual hours requirement, x_i. The worker's output in complex tasks depends on whether the worker is working for longer than the hours requirement. In other words, someone working 10 hours per week will produce less if the hours requirement in that week is 15 hours (since the hours requirement has not been met), than if the hours requirement in that week is 5 hours (in which case the hours requirement has been met). As I explain below, this cost is required to ensure that an equilibrium exists. Mathematically,

$$y(h,x) = \begin{cases} y_1(x,h) & \text{if } x < h \\ y_2(x,h) & \text{if } x \geq h \end{cases}$$

The firm employs a continuum of workers, so does not face any uncertainty, giving total production in complex tasks

$$Y_c = \int_0^1 E_x[y(h,x)] m_{h,c} dh. \tag{3.5}$$

3.B.2 A simple example

I begin with a simple example to fix ideas. In this example, $x_i \sim U[0,1]$, i.e the hours requirement is uniform between 0 and 1 (the maximum available hours). Workers in

[12]Note that this could affect the productivity of other workers. For tractability, I do not consider this possibility in this model.

complex tasks who draw an hours requirement that is below the number of hours they work, so that $x_i < h_{ic}$ have production equal to the number of hours they work. Workers who draw an hours requirement greater than the number of hours they work cannot produce at all.[13] In other words,

$$y(h,x) = \begin{cases} h & \text{if } x < h \\ 0 & \text{if } x \geq h \end{cases}$$

Thus an individual i, working for h_{ic} hours in complex tasks has expected production equal to the probability that they meet the hours requirement, $P(x_i < h_{ic})$ multiplied by their production conditional on meeting the hours requirement, h, so that

$$\begin{aligned} E_x[y(h_{ic},x_i)] &= \int_0^{h_{ic}} h_{ic} dx \\ &= h_{ic}^2. \end{aligned} \tag{3.6}$$

In this example, therefore, expected production in complex tasks is convex for all $h \in [0,1]$. The intuition for this result is that, in this framework, an increase in h has two effects. Firstly, it increases the probability that $h > x$ and the worker will be able to produce. Secondly, it increases the workers productivity if the hours requirement is satisfied. This generates the convexity in expected production seen above.

Firms

The representative firm decides the measure of workers to employ in complex and divisible tasks, $\{m_{h,d}, m_{h,c}\}$ for each $h \in [0,1]$. The firm's problem is

$$\begin{aligned} \max_{\{m_{h,d},m_{h,c}\}_{h\in[0,1]}} \quad & E[Y] - \int_0^1 e_d(h) m_{h,d} dh - \int_0^1 e_c(h) m_{h,c} dh \\ \text{s.t.} Y &= \Gamma(\alpha Y_c^\rho + (1-\alpha) Y_d^\rho)^{\frac{1}{\rho}} \end{aligned} \tag{3.7}$$

The amount the firm pays the worker does not depend on the realisation of x. This assumption can be interpreted as the firm providing full insurance to workers against their stochastic productivity. Assuming perfect competition between firms and normalising the price of the final good to one, the firm's FOCs give earnings for divisible tasks which are linear in hours and depend on production in divisible tasks and on total output. This reflects the data presented in Fig. 3.6, which shows that the wage-hours profile is flat for

[13]This is clearly an extreme example, but will serve to illustrate the mechanisms of the model. In Section 3.B.3 below I show that a much more less restrictive production function results in an equilibrium with the same qualitative properties.

part-time workers with constant hourly wages w_d.

$$e_d(h) = \Gamma(1-\alpha)hY_d^{\rho-1}(\alpha Y_c^\rho + (1-\alpha)Y_d^\rho)^{\frac{1-\rho}{\rho}} \tag{3.8}$$

However, earnings for complex tasks are non-linear in hours. In this example, they are convex for all hours. Thus the introduction of the parameter x has endogenously generated a convexity in overall earnings, so that hourly wages, $w_c(h)$ are no longer constant. This convexity has been generated without requiring any assumptions on the worker side of the model.

$$e_c(h) = \Gamma\alpha h^2 Y_c^{\rho-1}(\alpha Y_c^\rho + (1-\alpha)Y_d^\rho)^{\frac{1-\rho}{\rho}}. \tag{3.9}$$

The partial equilibrium effect of an increase in α will be to increase earnings in complex tasks and decrease earnings in divisible tasks as they become relatively less productive. An increase in Γ will increase earnings in both types of task.

Workers

Assume initially that workers are homogenous. They must choose whether to work on complex or divisible tasks and cannot combine the two. For now, I assume that all workers are equally productive in both. Workers have utility that is linear in their earnings and separable in consumption and hours. Their utility maximisation problem is

$$\begin{aligned} \max_{j=\{c,d\}} \{U_c(h_c), U_d(h_d)\} \quad \text{s.t.} \quad U_c(h_c) &= \max_{h_c} e_c(h_c) - \frac{\phi h_c^{1+\theta}}{1+\theta} \\ U_d(h_d) &= \max_{h_d} e_d(h_d) - \frac{\phi h_d^{1+\theta}}{1+\theta} \end{aligned} \tag{3.10}$$

ϕ indicates the disutility from labour. Since workers are homogenous, ϕ is the same for all workers. However, allowing ϕ to vary across workers would summarise individual's different preferences for leisure. $\frac{1}{\theta}$ is the Frisch elasticity of labour supply.

Workers equate the marginal disutility of more hours to marginal earnings in task j and then pick whichever type of task offers the highest utility, so that (assuming an interior solution)

$$h_d = \left(\frac{1}{\phi}w_d\right)^{\frac{1}{\theta}} \tag{3.11}$$

$$h_c = \left(\frac{1}{\phi}w_c\right)^{\frac{1}{\theta}}. \tag{3.12}$$

Clearly the partial equilibrium effect of a higher disutility of labour (higher ϕ_i) will be fewer hours in both types of task. Working hours will also decrease in θ. Note that the choice of functional form for the utility function is not immaterial, especially when earnings are non-linear in hours.[14]

Equilibrium

An equilibrium consists of earnings functions $e_c(h), e_d(h)$, working hours h_c, h_d and an allocation of workers $m_{h,c}, m_{h,d}$ that satisfies the following conditions

1. firms solve the profit maximisation problem Eq. (3.7)
2. households solve the utility maximisation problem Eq. (3.10)
3. the market for labour clears

Note that there are four types of possible solution to the workers' problem:

1. An interior solution as shown in Fig. 3.7 above, with $h_d < h_c < 1$
2. A corner solution for complex workers, with $h_d < h_c = 1$
3. A corner solution for complex and divisible workers, with $h_d = h_c = 1$

In Appendix 1 I derive a sufficient parameter restriction that ensures the equilibrium is of type (1) or (2). Type (3) is uninteresting for the purposes of this paper, as it does not feature any difference in hours between workers doing the two types of task.

Proposition 2
(i) In equilibrium, there exists $\hat{h}$ where $e_c(\hat{h}) = e_d(\hat{h})$
(ii) For $h < \hat{h}, e_c(\hat{h}) < e_d(\hat{h})$ and for $h > \hat{h}, e_c(\hat{h}) > e_d(\hat{h})$
(iii) Given the earnings schedules $e_c(\hat{h}), e_d(\hat{h})$ there exists a unique equilibrium with $h_c > h_d$.

A full proof is given in Appendix 1 but the intuition is as follows: first, assume that the earnings for workers doing divisible (complex) tasks are always greater than for complex (divisible) tasks. Then everyone would do divisible (complex) tasks. The

[14] In Appendix 4 I discuss the choice of utility function in more detail. I show that, in a static model such as this one, there is a unique solution for optimal hours in both types of task.

marginal product of labour doing complex (divisible) tasks would be infinite so everyone would switch to complex (divisible) tasks. Hence, this cannot be an equilibrium. Thus, there must be some $\hat{h}$ where earnings in both types of tasks are equal. Since $e_c(h)$ is convex, and $e_d(h)$ is linear, it must be the case that, for $h < \hat{h}, e_c(\hat{h}) < e_d(\hat{h})$, and for $h > \hat{h}, e_c(\hat{h}) > e_d(\hat{h})$.

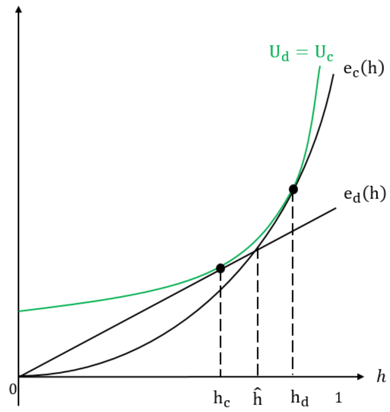
As a result, the worker effectively faces an hours-earnings schedule that is the upper envelope of the two earnings functions. Since workers are homogenous, in equilibrium all complex workers will do $h_c > \hat{h}$ hours (and can be thought of as full-time workers) and all divisible workers will do $h_d < \hat{h}$ hours (and can be thought of as part-time workers). For the remainder of this section, I will therefore use the term 'part-time' to refer to workers in divisible tasks, and 'full-time' to refer to workers in complex tasks.

Production in divisible tasks and expected production in complex tasks is given by $h_d i^*, h_c^2(1-i^*)$ respectively, where i^* is the share of workers doing divisible tasks (the part-time share). I show in Appendix 1 that there is a unique solution for $i^* \in (0,1)$. To summarise the argument, first note that substituting Eq. (3.11) into the equilibrium condition $U_d(h_d) = U_c(h_c)$, workers must be indifferent between full- and part-time work. If this were not the case, all workers would wish to do the same type of task. As the two types of task are not perfect substitutes in production, the wages for the other task would increase until workers became indifferent. Substituting optimal hours into Eq. (3.10) and equating utility in the two tasks allows me to write the ratio of output in the two tasks Y_c/Y_d as a function of the model's parameters. Given Y_c/Y_d, optimal hours in both tasks can be written in terms of the parameters, and the part-time share recovered from the relationship $\frac{Y_c}{Y_d} = \frac{h_c^2(1-i^*)}{h_d i^*}$. Unfortunately, it is not possible to find Y_c/Y_d analytically, and so the model must be solved numerically.

Fig. 3.7 below shows the equilibrium earnings and the workers utility indifference curve in equilibrium. It is clear from the diagram that full-time workers will earn higher hourly wages than part-time workers. Thus, even this very simple version of the model can reflect two key aspects of the data: the difference in tasks undertaken by part- and full-time workers, and the part-time pay penalty. The equilibrium also features the bunching of workers at 'full-time' hours (approximately 38 in the UK data.)[15] The earnings functions imply a wage-hours profile that approximately matches the empirical wage-hours profile discussed in Section 3.A.4 above; hourly wages are constant (earnings are linear in hours) for part-time workers, and there is a discontinuity, caused by the convexity of $e_c(h)$. This figure highlights the importance of the hours requirement x. Without x, any concave

[15] The bunching of workers at h_c is a result of homogeneity of workers. However, I show below that, in the more general model with heterogenous workers, this bunching can still occur.

FIGURE 3.7: Equilibrium in simple example



production function $y(h)$ in complex tasks would result in an equilibrium where either (i) the marginal productivity of a worker doing complex tasks would be higher at low h so that $h_c < h_d$ and part-time workers would earn more per hour; or (ii) if $y(h)$ were linear, then the ratio Y_c/Y_d would adjust so that in equilibrium, earnings would be the same in both tasks of task, resulting in $h_c = h_d$ and no difference in hourly wages for the two types of task.[16] The figure also shows the benefit of assuming that x is stochastic. Whilst this assumption appears to make the model more complicated, the alternative would be assume production in complex tasks of $y(h,x) = h - x$, with constant x. This would require the calibration of the parameter x. Instead, in this simple example, I do not need to make any further assumptions about x other than that it is uniform.

Effect of the model parameters

The model features five key parameters: α and ρ are 'demand' parameters, governing the firm's preferences for complex and divisible tasks, whilst θ and ϕ are 'supply' parameters, governing how many hours of labour the worker wishes to supply. Finally, there is the 'aggregate' parameter Γ.

In Table 3.2 I show the general equilibrium effect of an increase in each parameter on the equilibrium ratio Y_c/Y_d. As I explain above, it is this ratio that I use to solve for the other quantities: h_c, h_d, i^* and the part-time pay penalty. Equilibrium hours in divisible and complex tasks are

$$h_d = \left(\frac{\Gamma(1-\alpha)}{\phi} Y_d^{\rho-1} \left(\alpha Y_c^\rho + (1-\alpha) Y_d^\rho\right)^{\frac{1-\rho}{\rho}}\right)^{\frac{1}{\theta}} \tag{3.13}$$

$$h_c = \left(\frac{2\Gamma\alpha}{\phi} Y_c^{\rho-1} \left(\alpha Y_c^\rho + (1-\alpha) Y_d^\rho\right)^{\frac{1-\rho}{\rho}}\right)^{\frac{1}{\theta-1}}. \tag{3.14}$$

[16]Fig. C.8 in Appendix 5 shows the possible equilibria in both these cases.

The ratio between the wages of divisible and complex workers is equivalent to the part-time pay penalty

$$\frac{w_d}{w_c(h_c)} = \frac{1-\alpha}{2\alpha h_c}\left(\frac{Y_d}{Y_c}\right)^{\rho-1}. \tag{3.15}$$

This exercise highlights how interrelated the parameters are, it is only possible to isolate the effect of one parameter in general equilibrium in a few cases. For example, in partial equilibrium, an increase in α unambiguously increases h_c and decreases h_d. However, it also means that firms wish to employ more labour in complex tasks, which are relatively more productive. This decreases the relative earnings for complex tasks (which would instead decrease h_c), and increases the relative earnings in divisible tasks (which would increase h_d.) The net effect of these two offsetting effects is ambiguous. It also highlights that it is impossible to determine which parameters are causing changes in aggregate outcomes.

In Table 3.3 I also show the effect of an increase in each parameter when the equilibrium of the model has a corner solution, i.e. when $h_c = 1$. This analysis presupposes that any change in the parameters is sufficiently small that there is still a corner solution. Effectively fixing hours in complex tasks means that there is less ambiguity in the comparative statics exercise.

TABLE 3.2: General equilibrium effects of an increase in parameters, interior solution

	$\frac{Y_c}{Y_d}$	h_c	h_d	**PTPP**	**Part-time share**
ρ	Increase	Decrease	Effect ambiguous	Decrease	Effect ambiguous
α	Effect ambiguous	**Decrease in** $\frac{Y_c}{Y_d}$**:** h_c increases **Increase in** $\frac{Y_c}{Y_d}$**:** Effect ambiguous	**Decrease in** $\frac{Y_c}{Y_d}$**:** h_d increases **Increase in** $\frac{Y_c}{Y_d}$**:** Effect ambiguous	**Decrease in** $\frac{Y_c}{Y_d}$**:** PTPP increases **Increase in** $\frac{Y_c}{Y_d}$**:** Effect ambiguous	Effect ambiguous
ϕ	Effect ambiguous	**Decrease in** $\frac{Y_c}{Y_d}$**:** Effect ambiguous **Increase in** $\frac{Y_c}{Y_d}$**:** h_c decreases	**Decrease in** $\frac{Y_c}{Y_d}$**:** h_d decreases **Increase in** $\frac{Y_c}{Y_d}$**:** Effect ambiguous	**Decrease in** $\frac{Y_c}{Y_d}$**:** PTPP increases **Increase in** $\frac{Y_c}{Y_d}$**:** PTPP decreases	Effect ambiguous
θ	Effect ambiguous	**Decrease in** $\frac{Y_c}{Y_d}$**:** h_c decreases **Increase in** $\frac{Y_c}{Y_d}$**:** Effect ambiguous	**Decrease in** $\frac{Y_c}{Y_d}$**:** Effect ambiguous **Increase in** $\frac{Y_c}{Y_d}$**:** h_d decreases	**Decrease in** $\frac{Y_c}{Y_d}$**:** PTPP increases **Increase in** $\frac{Y_c}{Y_d}$**:** PTPP decreases	Effect ambiguous
Γ	Decrease	**Decrease in** $\frac{Y_c}{Y_d}$**:** h_c increase **Increase in** $\frac{Y_c}{Y_d}$**:** Effect ambiguous	**Decrease in** $\frac{Y_c}{Y_d}$**:** Effect ambiguous **Increase in** $\frac{Y_c}{Y_d}$**:** h_d increase	**Decrease in** $\frac{Y_c}{Y_d}$**:** PTPP increases **Increase in** $\frac{Y_c}{Y_d}$**:** PTPP decreases	Effect ambiguous

TABLE 3.3: General equilibrium effects of an increase in parameters, corner solution

	$\frac{Y_c}{Y_d}$	h_c	h_d	**PTPP**	**Part-time share**
ρ	Increase	Decrease	Effect ambiguous	Decrease	Effect ambiguous
α	Increase	Effect ambiguous	Effect ambiguous	Effect ambiguous	Effect ambiguous
ϕ	Effect ambiguous	Effect ambiguous	Effect ambiguous	Effect ambiguous	Effect ambiguous
θ	Increase	Decrease	Effect ambiguous	Decrease	Effect ambiguous
Γ	Increase	Effect ambiguous	Increase	Decrease	Effect ambiguous

Quantitative illustration

To illustrate the example above, I now compare the predictions of the model with the LFS data. This requires assigning values to the parameters $\rho, \alpha, \phi, \theta, \Gamma$. In this simple example, I calibrate the model anew each year.

As discussed above, there are three possible equilibria. The fact that there has been bunching of workers at hours between 38 and 40 per week throughout the period I analyse suggests that the second is most likely: an interior solution for h_d, and a corner solution for $h_c = 1$, and so I focus on parameter combinations that lead to this type of equilibrium. If this were not the case, then h_c would depend on parameters and equilibrium quantities and we would expect it to change over time.

I begin by setting a value for θ. There is a large literature concerned with estimating the Frisch elasticity of labour supply, $1/\theta$. Whalen and Reichling (2017) document estimates from the late 1980s to 2012, focusing on the intensive margin, and find that they range between 0 and 0.8. I therefore begin by choosing $\theta = 1.5$, corresponding to a Frisch elasticity of 0.67, which is near the middle of this range.[17]

The demand parameters, α and ρ, and total factor productivity, Γ are more complicated. Unfortunately the three are not separately identified from the two available data moments: part-time and full-time wages. As far as I am aware, there are no recent attempts to estimate the elasticity of substitution between part- and full-time labour in the UK, $\sigma = 1/(1-\rho)$. However, there are older estimates for other countries, including Montgomery (1988) who used a survey of American employers to find an elasticity of 1.5 (corresponding to a value of ρ of 0.33) and Hitoshi and Toshiyuki (2001) who find an elasticity of 5 (corresponding to a ρ of 0.8) using data from Japan. Finally, Kang et al. (2020) calibrate an elasticity of 5.3 for the US. Instead, I proceed by assuming that the elasticity of substitution between complex and divisible tasks is constant, so that ρ is constant. The firm's first order condition for workers in complex tasks, given in Eq. (3.9) can be written

$$\log w_c(h_c) = \log\alpha + \log\Gamma + (\rho - 1)\log\left(\frac{Y_c}{Y}\right) - \log h_c \tag{3.16}$$

[17] I experiment with smaller and larger values of θ and find that a smaller θ fits the data better, although $\theta > 1$ is required to induce some curvature in the worker's utility function.

where w_c is the hourly wage. I begin by estimating a basic equation for the wage of full-time worker i in year t and quarter q using OLS.[18]

$$\log w_{c,iqt} = \delta_t + (\rho - 1)\log\left(\frac{Y_{c,qt}}{Y_t}\right) + \beta X_{iqt} + \varepsilon_{iqt}. \tag{3.17}$$

δ_t is a year fixed effect that incorporates changes in the relative productivity of complex and divisible tasks, α and aggregate productivity Γ, i.e. $\delta_t = \log\alpha + \log\Gamma$. The underlying assumption is that these changes occur more slowly than changes in Y_c/Y. In other words, I assume that firms make decisions based on the current (quarterly) amount of labour, but that productivity changes more slowly. As discussed above, I assume a corner solution, where h_c is not changing over time. I therefore divide hourly wages in the data by 38, which is approximately a full-time week. If the length of a full-time week had changed substantially over the time period, I would need to include h_c in the model.

This very simple example does not take any heterogeneity between workers into account. The observed hourly wage in the data will reflect differences in worker productivity. In the model above, this will be captured in the error term ε_{iqt}. If a worker's individual productivity is correlated with aggregate output Y_c, Y then the estimate of ρ will be biased. I therefore include a set of worker characteristics in order to minimise this bias.[19] Using OLS, I find an estimate of $\hat{\rho} = 0.72$, corresponding to an elasticity of 3.57. I show the results of this estimation in Column I of Table C.4. To further minimise any bias that may result from differences in productivity between full- and part-time workers I repeat the estimation using the wage adjusted for selection into full-time work, as described in Section 3.A.4. This results in an estimate for $\hat{\rho}$ of 0.59 (Column II in Table C.4).

I also consider an alternative specification, where I regress the mean wage for full-time workers on Y_{ct}/Y_t, and the mean characteristics of full-time workers, giving

$$\log \overline{w_{c,iqt}} = (\rho - 1)\log\left(\frac{Y_{c,qt}}{Y_{qt}}\right) + \delta_t + \beta\overline{X_{iqt}} + \varepsilon_t. \tag{3.18}$$

Once again, I include a year fixed effect, δ_t. $\overline{X_{iqt}}$ are the mean characteristics for all workers in the sample. This results in an estimate of $\hat{\rho} = 0.83$, although it is extremely imprecise, with a 95% confidence interval that includes zero (Column III in Table C.4). This is unsurprising, given the small sample size of 107 periods, but it is roughly the same as the estimate of ρ discussed above. Finally, I repeat the estimation using the mean wage adjusted for selection into full-time work, as described above (Column IV in

[18] I use full-time workers because there is less heterogeneity in their hours in the data.

[19] As in Section 3.A these include: age, age-squared, tenure; highest level of qualification; family type (single, single with children, living with a partner without children, living with a partner with children); whether currently studying; and whether the job is the worker's main job.

Table C.4). This gives an estimate of $\hat{\rho} = 0.86$. Taking these four specifications together, I set ρ equal to 0.75. This is between the estimates of Montgomery (1988); Hitoshi and Toshiyuki (2001) and Kang et al. (2020).

Given the estimate of ρ, I choose the remaining two parameters, α and Γ to match the part-time pay penalty in each year t[20] Since the PTPP was very volatile in the first two years of the LFS, I begin in 1996.

$$PTPP_t = \frac{1-\alpha_t}{\alpha_t}\left(\frac{Y_{dt}}{Y_{ct}}\right)^{\rho-1} \tag{3.19}$$

and average part-time wages

$$w_{dt} = \Gamma_t(1-\alpha_t)Y_{dt}^{\rho-1}(\alpha_t Y_{ct}^{\rho} + (1-\alpha_t)Y_{dt}^{\rho})^{\frac{1-\rho}{\rho}}. \tag{3.20}$$

These relationships result in an estimate for α that is slowly decreasing, from 0.66 in 1996 to 0.62 in 2020. In the context of this model, this would imply that the relative productivity of complex tasks is decreasing relative to divisible tasks. It would also imply that complex tasks are approximately twice as productive as divisible. This appears large, and suggests that there may be unobserved heterogeneity in productivity between workers that the model fails to reflect adequately. The estimate of Γ is increasing from 1994 to 2009, before falling and beginning to increase again in 2015. Although this does not match exactly the dates of the financial crisis, it is encouraging that the model can reflect, to some degree, the business cycle. Figure 3.8 below shows the estimates of ϕ, Γ, α over time.

Finally, I set ϕ to match average hours and wages in divisible tasks, using the worker's first order condition,

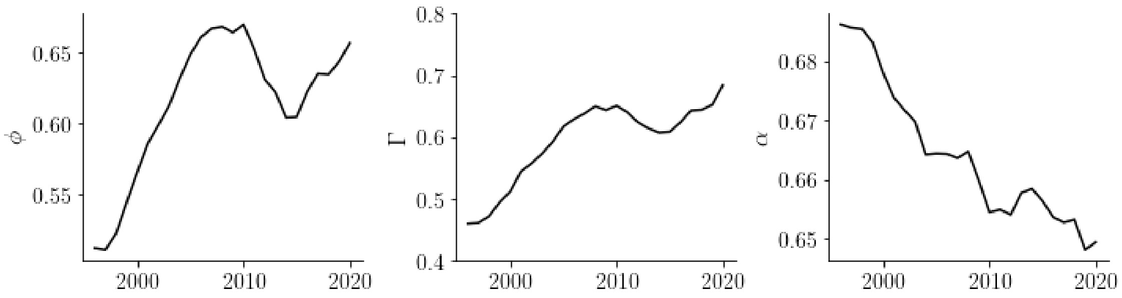
$$w_d = \phi h_d^{\theta}. \tag{3.21}$$

This suggests a disutility of labour that increased from 0.51 to 0.67 from the early 1990s until 2010, before dipping during the financial crisis. The large decrease after 2010 may reflect the fact that the model does not account for other income or workers' assets. Thus, the decrease in (real) wages observed after the financial crisis feeds through mechanically into a decrease in ϕ.

The fit of the model is reasonably good, given the simplicity of the example, reflecting the increase in part-time hours and decrease in the PTPP. The model derived part-time

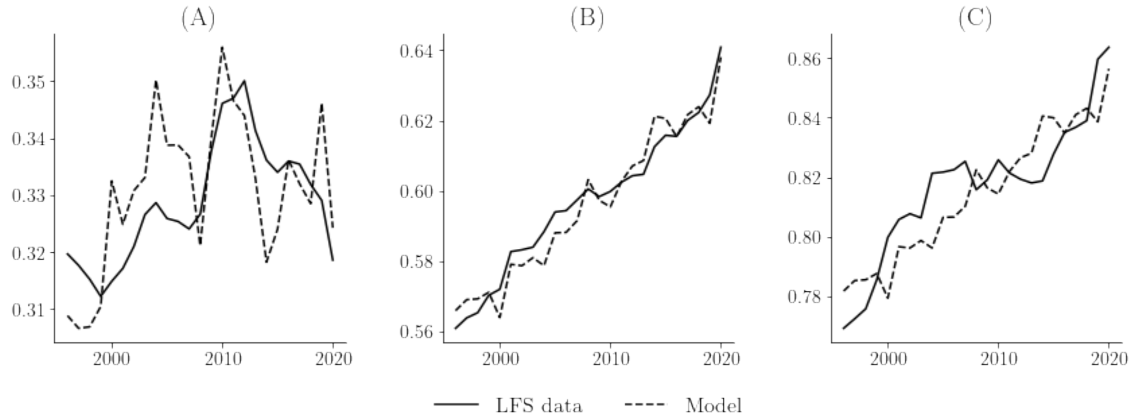
[20] I use the PTPP after adjusting for worker characteristics, as described in Section 3.A.4. This should result in a PTPP that is not so affected by differences in worker productivity, which is not captured in this model.

FIGURE 3.8: Yearly parameter calibration



share is of the right magnitude, but is more volatile than in the LFS data. Fig. 3.9 below shows the part-time share (Panel A), the average hours for part-time workers (Panel B), and part-time pay penalty (Panel C) in the data, and the model outcome.

FIGURE 3.9: Comparison of data and model outcomes



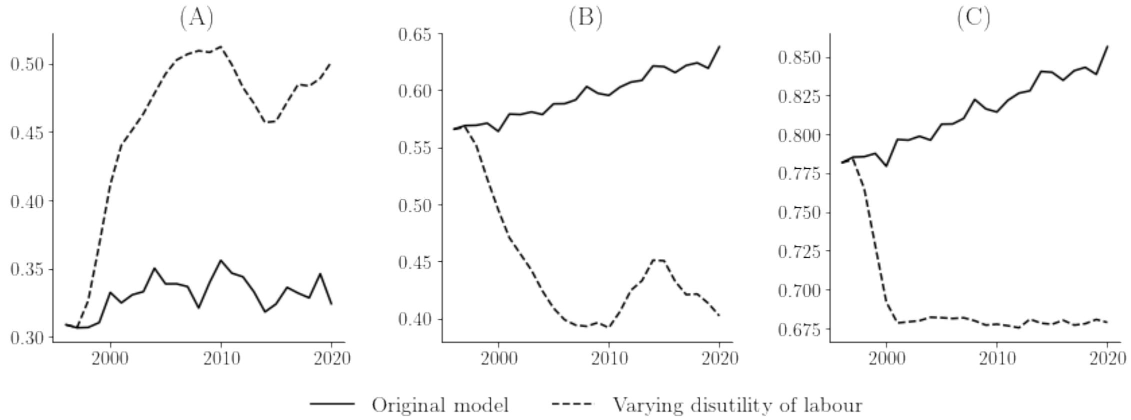
I now turn to a counterfactual exercise, in which I sequentially fix two of the three non-constant parameters at their 1994 value, varying only the third parameter. The aim of this exercise is to assess the effect that each parameter has separately. I solve the model in each period allowing only one parameter to vary and compare the part-time share (Panel A), hours for part-time (Panel B), and part-time pay penalty (Panel C) with the values using the original model.

Fig. 3.10 shows the results of this exercise. Firstly, Fig. 3.10a shows the effect of the estimated increase of disutililty of labour, which with this parameterisation, increases the part-time share, and decreases part-time hours. The net result of this is an increase total production by part-time workers Y_d, leading to a decrease in the relative wages of part-time workers. Note that, as the disutility of labour increases holding the other parameters fixed, there comes a point where there is no longer a corner solution for hours in complex tasks. As a result, h_c begins to fall as ϕ increases as well, and the result is that the PTPP stops falling so steeply. In this parameterisation this occurs in the year 2000. Fig. 3.10b shows that an increase in the relative productivity of complex tasks would, in this model, lead to an increase in the part-time share. It would also, holding the disutility of labour fixed, lead to a decrease in part-time hours. Again, the net result is to increase total production by part-time workers Y_d, leading to a decrease in the relative wages of part-time workers. Finally, the increase in aggregate productivity suggests a large decrease in the part-time share, as workers wish to work full-time in order to benefit from the higher productivity. The same effect causes part-time hours to increase. However, since the net effect is a decrease in total production by part-time workers, the result is an increase in their relative wages. The inverse relationship between the part-time share and the part-time pay penalty is simply a result of the fact that divisible and complex tasks are not perfect substitutes in production.

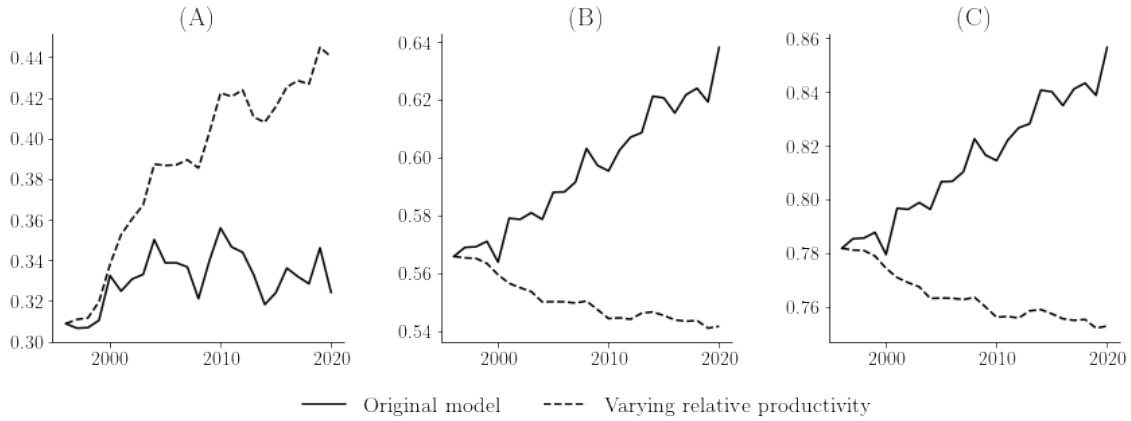
In summary, this simple example shows that a task-based framework can generate the coexistence of part- and full-time work and the lower hourly pay of part-time workers. However, it cannot help to analyse gender differences in part-time work. Calibrating the model also requires me to make several very strong assumptions (such as a constant elasticity of substitution between complex and divisible tasks). However, the exercise above shows that an appropriately calibrated model would be able to shed light on the driving forces behind the trends in part-time work. In the next section I set out a more general version of the model that will help to address these issues in future research.

FIGURE 3.10: Part-time pay penalty

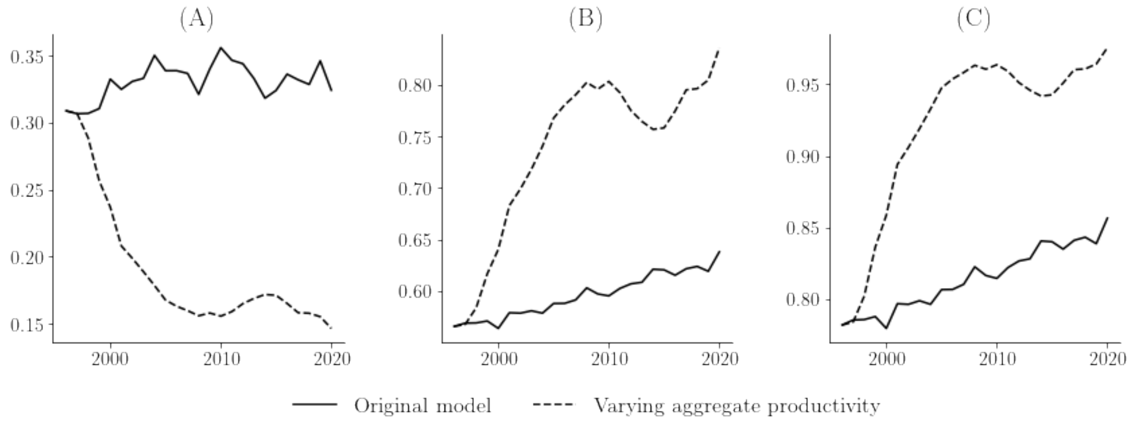
(a) Allowing ϕ to vary



(b) Allowing α to vary



(c) Allowing Γ to vary



3.B.3 General model

In the example above, workers are homogenous and therefore all work the same part- and full-time hours and earn the same hourly wages. There are two possible dimensions of heterogeneity between workers, in disutility of labour and in productivity (either absolute, or relative productivity in complex and divisible tasks.) There is also, of course, the possibility that the two are correlated. I begin by considering heterogeneity in the disutility of work. Specifically, I assume that worker i has disutility of labour ϕ_i, distributed $G(\phi)$ with support $[\phi_{min}, \phi_{max}]$. This will provide a natural way for future research to incorporate gender differences, by allowing men and women to have different preferences.

In addition, I consider a general case, where the hours requirement has distribution $F(x)$ with support $[\underline{x}, \bar{x}]$ and allow a more general form of production in complex tasks. The aim is to develop a general model which requires as few assumptions as possible, whilst retaining the attractive features of the example above. Specifically, the equilbrium of the model should feature the coexistence of part- and full-time work, the concentration of part-time work at the lower end of the (hourly) wage distribution and in occupations that require a lower skill level, and the bunching of workers at “full-time" hours as endogenous outcomes. The core ‘building blocks’ of the model remain the same, although the environment is now much richer.

Firms

The representative firm once again decides the measure of workers to employ in complex and divisible tasks, $\{m_{h,d}, m_{h,c}\}$ for each $h \in [0,1]$. The firm’s problem is

$$\max_{\{m_{h,d}, m_{h,c}\}_{h\in[0,1]}} E[Y] - \int_0^1 e_d(h) m_{h,d} dh - \int_0^1 e_c(h) m_{h,c} dh \tag{3.22}$$
$$\text{s.t.} Y = \Gamma(\alpha Y_c^\rho + (1-\alpha) Y_d^\rho)^{\frac{1}{\rho}}$$

where

$$Y_d = \int_0^1 h m_{h,d} dh \tag{3.23}$$
$$Y_c = \int_0^1 E_x[y(h,x)] m_{h,c} dh. \tag{3.24}$$

Earnings for divisible tasks are still linear in hours

$$e_d(h) = \Gamma(1-\alpha) h Y_d^{\rho-1} (\alpha Y_c^\rho + (1-\alpha) Y_d^\rho)^{\frac{1-\rho}{\rho}} \tag{3.25}$$

However, it is no longer clear that earnings for complex tasks are convex for all $h \in [0,1]$.

$$e_c(h) = \Gamma \alpha E_x[y(h,x)] Y_c^{\rho-1} (\alpha Y_c^\rho + (1-\alpha) Y_d^\rho)^{\frac{1-\rho}{\rho}}. \tag{3.26}$$

The shape of the hours-earnings profile is determined by the form of the expectation $E_x[y(h,x)]$. To see this, write the expectation as

$$E_x[y(h,x)] = Pr(x < h)E_x[y_1(h,x|x < h)] + Pr(x \geq h)E_x[y_2(h,x|x \geq h)] \tag{3.27}$$

Thus expected production depends on the probability that the hours requirement is met, as well as the two parts of the production function $y_1(x,h), y_2(x,h)$.

Production in the general model

Recall that output for a worker doing complex tasks for h hours is given by

$$y(h,x) = \begin{cases} y_1(x,h) & \text{if } x \leq h \\ y_2(x,h) & \text{if } x > h. \end{cases}$$

Conditions for production function

I make the following (non-restrictive) assumptions about output, which I explain in more detail in Appendix 1:

1. $y_1(x,h) = y_2(x,h) = 0$. At least some working hours are required for production to take place.

2. Production if the hours requirement is met is greater than if it is not, $y_1(x,h) > y_2(x,h)$. This is a very natural assumption. It requires, for example, that someone working 10 hours per week will produce less if the hours requirement in that week is 15 hours (since the hours requirement has not been met), than if the hours requirement in that week is 5 hours (in which case the hours requirement has been met).

3. The marginal product of h if the hours requirement is met is at least as great as when it is not, so that

$$\frac{\partial y_1(x,h)}{\partial h} \geq \frac{\partial y_2(x,h)}{\partial h} \quad \forall h \in [0,1]. \tag{3.28}$$

   This is also a natural assumption. It requires, for instance that the difference between working 10 and working 11 hours is greater if the hours requirement is 5 hours than if the hours requirement is 15 hours.

4. Either:

(a) Production in both cases, $y_1(x,h), y_2(x,h)$, is linear in hours. This requires that marginal productivity in complex tasks does not decline as hours increase;[21] or

(b) $y_2(x,h)$ has the form $y_2(h) - c(x)$, so that there is the possibility that $y_2(x,h)$ is negative. I assume that, the cost is sufficiently high that, for low values of h, $E_x[y(h,x)]$ is negative. This condition implies an hours requirement sufficiently onerous that working very short hours becomes unproductive.

One example of a production function that would meet these assumptions is the following, where production in both cases is linear, but the constant $a < 1$ ensures that the marginal product is lower if the hours requirement is not met.

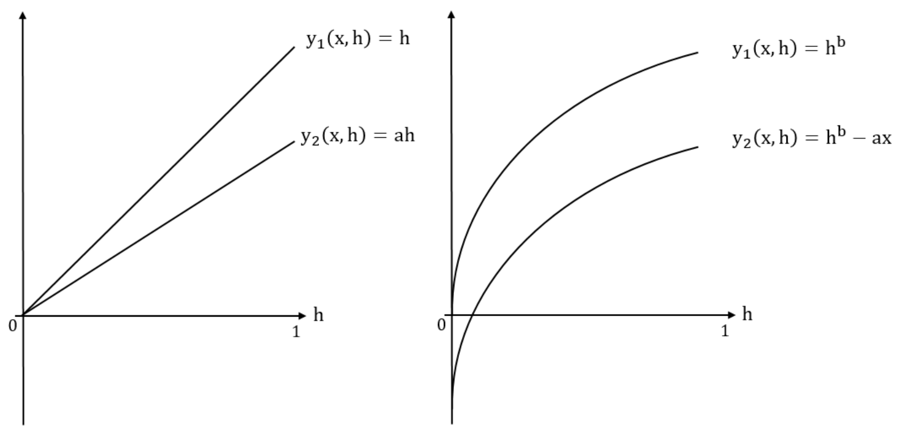
$$y(h,x) = \begin{cases} h & \text{if } x \leq h \\ ah & \text{if } x > h. \end{cases} \tag{3.29}$$

An alternative example production function could involve a linear cost if the hours requirement is not met, where $a < 1$ and b are constants.

$$y(h,x) = \begin{cases} h^b & \text{if } x \leq h \\ h^b - ax & \text{if } x > h. \end{cases} \tag{3.30}$$

These two example production functions are shown in Fig. 3.11 below.

FIGURE 3.11: Example production functions



[21] Although there is evidence that productivity declines at very long working hours, in this paper I focus on part-time work, and I therefore consider this a reasonable assumption.

Conditions for hours requirement

I also make three assumptions about the distribution $F(x)$. These conditions are not required for the existence of an equilibrium with part- and full-time work coexisting, but they are sufficient to show that the equilibrium is unique, and ensure that it has the properties specified in the introduction to this section.

1. $F(x)$ is continuous and twice-differentiable. This rules out distributions with mass points.[22]
2. $F(0) = 0$. In other words, complex tasks always have a strictly positive hours requirement. This is a natural assumption. If the interpretation of x is as a start-up cost, then this assumption requires that the start-up cost is positive. If the interpretation is that complex tasks are a result of specific human capital, then this assumption requires that this capital is required at least some of the time.
3. $F(x)$ is unimodal. This assumption prevents the hours-earnings profile from oscillating “too much".

Proposition 3

Assume that the conditions for $y(h,x)$ and $F(x)$ set out above are met. Then
(i) The marginal product $e_c'(h)$ is strictly positive for all $h \in [0,1]$
(ii) For small $h < \tilde{h} \in (0,1)$, earnings are convex, so that $e_c''(h) \geq 0$

See Appendix 1 for a proof. Part (i) of the proposition is intuitive since working more hours implies greater production. The intuition for part (ii) is as in the simple example above; an increase in hours worked increases productivity but also increases the probability that $x \leq h$ and hence that the worker can produce at the higher level $y_1(x,h)$.

Workers

Workers continue to choose optimal hours in each type of task, and pick the task that offers them the highest utility, so that their maximisation problem is

$$\max_{j=\{c,d\}} \{U_c(h_c), U_d(h_d)\} \quad \text{s.t.} \quad U_c(h_c) = \max_{h_c} e_c(h_c) - \frac{\phi_i h_c^{1+\theta}}{1+\theta} \tag{3.31}$$

[22] It does allow for the non-stochastic case, where $x = \bar{x}$ where $\bar{x} < 1$ is a constant, provided that $y_1(x,h)$ is also linear in hours. This would correspond to a fixed start-up time for complex tasks, similar to that in Card (1990). Such an assumption has the benefit of simplicity, but would not be able to generate bunching in hours at a “full-time" level.

$$U_d(h_d) = \max_{h_d} e_d(h_d) - \frac{\phi_i h_d^{1+\theta}}{1+\theta}$$

However, the worker's choice of optimal hours will now depend on their *individual* disutility of labour

$$h_d(\phi_i) = \left(\frac{w_d}{\phi_i}\right)^{\frac{1}{\theta}} \tag{3.32}$$

$$h_c(\phi_i) = \left(\frac{w_c(h_c(\phi_i))}{\phi_i}\right)^{\frac{1}{\theta}}. \tag{3.33}$$

For an individual worker, hours in divisible tasks are decreasing in ϕ_i. Thus workers with a higher disutility of labour will, as is standard, work shorter hours.[23] There is not necessarily a one-to-one mapping between ϕ and $h_c(\phi)$, but given a worker's hours h, ϕ_i is unique. This relationship could, in theory, be used to recover the distribution of ϕ.[24]

Equilibrium

An equilibrium consists of earnings functions $e_c(h), e_d(h)$, a distribution of working hours $h_c(\phi), h_d(\phi)$ for $\phi \in [\phi_{min}, \phi_{max}]$ and an allocation of workers $m_{h,c}, m_{h,d}$ that satisfies the following conditions

1. firms solve the profit maximisation problem in Eq. (3.22)
2. households solve the utility maximisation problem in Eq. (3.31)
3. the market for labour clears for all $h \in [0,1]$

Proposition 4

Assume that the conditions in Section 3.B.3 are met. Then
(i) In equilibrium, there exists at least one $\hat{h} \in (0,1]$ where $e_c(\hat{h}) = e_d(\hat{h})$.
(ii) For the smallest $h < \hat{h}, e_c(\hat{h}) < e_d(\hat{h})$.
(iii) If $y_1(x,h), y_2(x,h)$ are linear, then $\hat{h}$ is unique.

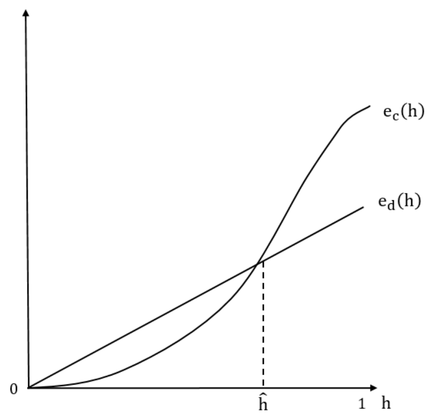
See Appendix 1 for a proof. In other words, overall earnings for divisible tasks are greater when hours are low, and earnings for complex tasks are greater when hours are high. For

[23] As written, the model does not incorporate the extensive margin. It would be fairly simple to include a third choice for workers not to enter the labour force at all, which would give them some utility U_n, which would depend on ϕ.

[24] An exception would be if x were bounded with $\bar{x} < 1$. In this case, there would be bunching of workers doing complex tasks for $\bar{x}$ hours (I discuss this case in more detail below). However, the distribution of hours for divisible workers could still be used to provide information about $G(\phi)$.

a given h, workers will always choose the task with higher earnings. As a result, workers with hours below $\hat{h}$ will do divisible tasks. Thus, there will be some workers working low hours in divisible tasks, and earning low hourly wages. There will also be workers doing higher hours, above $\hat{h}$ in complex tasks, and earning more per hour. Thus the equilibrium retains the properties discussed in the introduction to this section: people working low hours do different tasks, and earn less per hour, than those working longer hours. Fig. 3.12 below shows the hours-earnings profile for the first example production functions in Eq. (3.29) above, with an example distribution for x: $x \sim N(\mu, \sigma^2)$.

FIGURE 3.12: Example hours-earnings profile



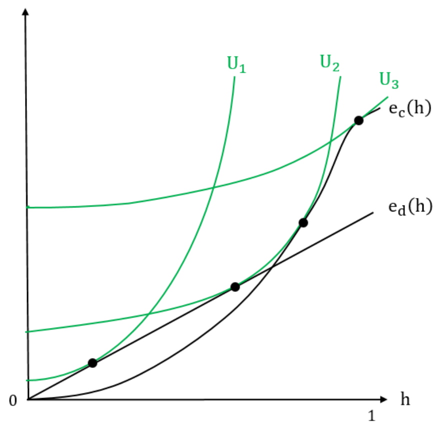
Corollary 1

If $\hat{h}$ is unique, then
(i) there is a unique $\hat{\phi} \in [\phi_{min}, \phi_{max}]$ that satisfies $U_c(h_c(\hat{\phi})) = U_d(h_d(p\hat{h}i))$;
(ii) for $\phi_i > \hat{\phi}, U_c(h_c(\hat{\phi})) > U_d(h_d(\hat{\phi}))$ and for $\phi_i < \hat{\phi}, U_c(h_c(p\hat{h}i)) < U_d(h_d(\hat{\phi}))$

See Appendix 1 for a proof. As a consequence, in equilibrium workers with high disutility of labour will choose to do divisible tasks for fewer hours, and those with a lower disutility of labour will choose to do complex tasks for longer hours. Thus the model accounts for the fact that workers who switch from full- to part-time work on average move from jobs that require more formal education to those that require less (Connolly and Gregory, 2008). This has implications for inequality, since those workers with caring responsibilities cannot access high paying jobs, even if they have the required education.

Fig. 3.13 below shows the equilibrium hours-earnings profiles as in Fig. 3.12 above, and the utility indifferent curves for three different workers. The first has a high disutility of labour and hence chooses divisble tasks, receiving utility U_1. The second is the marginal worker who is indifferent between the two tasks and receives U_2. The third has a low ϕ and receives U_3.

FIGURE 3.13: Example equilibrium



The model can also account for the fact that there are comparatively few workers working between 31 and 35 hours per week, and that this corresponds to the discontinuity in the hours-wage profile shown in Fig. 3.6. The marginal worker with $\phi = \hat{\phi}$ will be indifferent between $h_d(\hat{\phi}) < \hat{h}$ hours in complex tasks, and $h_c(\hat{\phi}) > \hat{h}$ and it is the gap between h_c and h_d that generates the discontinuity in hourly wages observed in the data.[25]

Bunching of workers at full-time hours

It is well known that there is substantial bunching of workers at 'full-time' hours, and that this will affect estimates of labour supply parameters (see Battisti et al. (2020); Bick et al. (2022); Rogerson (2011), for example). The framework set out here can generate this bunching by choosing an appropriate distribution $F(x)$. If the hours requirement is constant and not stochastic, then bunching can only occur where there are kinks in the production function or mass points in the distribution $G(\phi)$, which must be specified exogenously. Allowing x to be stochastic makes the model more flexible. For example, if x is bounded $x < \bar{x} \in (0,1)$ then this will generate a kink in the earnings function at $h = \bar{x}$ that will generate bunching (see Fig. C.1 for an example of what this would look like in equilibrium).

3.B.4 Conclusion

In this paper I examine long term trends in part-time work. I begin by documenting important aggregate changes in part-time work over the last 30 years. Firstly, the quantity of part-time work has increased. In aggregate, this increase has mostly occurred at the 'extensive' margin, in the sense that weekly hours, conditional on working part-time have

[25]I note that some of the spike in the hours distribution at approximately 30 hours seen in Fig. C.6 is due to the tax credits described in Section 3.A.2. However, this spike occurs before the tax credits were introduced as well so is not solely due to them.

increased. However, for men there has also been an increase at the 'intensive' margin, in the sense that the share of men employed who are working part-time has grown. In aggregate, some of this increase has been offset by a slight decline in female part-time work. Secondly, wages for part-time workers relative to full-time workers have increased for both genders, even after adjusting for differences in the characteristics of part- and full-time workers and jobs.

Taken together, these trends suggest changes in both firm technology and worker preferences, and the aim of this paper is to improve the understanding of the forces driving these trends. Changes in working hours could have effects on inequality, since part-time work is most often found at the bottom of the wage distribution, and it is therefore important to understand what is causing them. I develop a model which can explain both firm's and worker's preferences for part- versus full-time work. To do so, I use a task based approach, in which production is a CES aggregate of two types of task. More complex tasks are associated with a cost, that generates a convexity in the hours-earnings profile for these tasks. As a result, in equilibrium, workers choose to work longer hours in complex tasks and shorter hours in simpler, divisible tasks and hence earn less per hour. I show that even a very simple version of the model can match the observed increase in the part-time share, increase in part-time hours, and simultaneous decrease in the part-time pay penalty. A quantitative exercise attempting to disentangle changes in the parameters that affect demand and supply for part-time work highlights the identification problem that the quantity and price (here, relative wages) of part-time work are simultaneously determined. In the example in this paper, I assume that the substitutability of divisible and complex tasks is fixed, and use this assumption to consider the effect of the other parameters.

I extend the model to incorporate workers with heterogenous preferences. Those with a higher disutility of labour choose to do simpler divisble tasks, and those with a lower disutility of labour choose complex tasks, and earn more per hour. This model will provide a setting to study changes in part-time work by gender in future research since it allows for differences between different types of workers. A further advantage of the more complex model is that it provides more data moments for the identification of changes in the supply and demand parameters.

Appendix A

Flexiblility or certainty? The aggregate effects of casual work

Appendix 1 Proofs

Proof of Proposition 1 (a)

Firstly, write the expected value, before the realisation of x and ε, of a regular and casual job to the firm as

$$E[J_r(z)] = \frac{\int_{\underline{x}}^{\bar{x}}(xz-w)dG(x)}{1-\beta(1-\delta)} \tag{A.1}$$

$$E[J_{ic}(z)] = \frac{\phi_i \int_{\frac{w}{z}}^{\bar{x}}(xz-w)dG(x) - k_c}{1-\beta\big(1-\delta-\phi_i f(\theta)G(\frac{w}{z})p_i(z)\big)} \tag{A.2}$$

To show existence of $\hat{z}_{ij}$ it is sufficient to show that there exists at least one region in the domain of z where the expected value of a type j job to the firm is zero, and at least one region where it is less than zero. Firstly, if $z < \underline{w}/\bar{x}$ then both Eq. (A.1) and Eq. (A.2) are less than zero. Conversely, for $z > \underline{w}/\underline{x}$ both equations are positive and hence there exists $\hat{z}_{ij}$ where Eq. (1.12) is satisfied.

Proof of Proposition 1 (b)

To show existence of z_H^*, it is sufficient to show that there exists at least one region in the domain of z where the expected value of a casual job is greater than or equal to the value of a regular job, and vice versa. If $z < \underline{w}/\bar{x}$ then for any realisation of x, $xz < w$ and a firm with a casual job will never call-up the worker. The expected value of such a casual job is bounded below by

$$E[J_{ic}(z)] = \frac{-k_c}{1-\beta\left(1-\delta\right)}. \tag{A.3}$$

The value of a regular job is

$$E[J_r(z)] = \frac{\int_{\underline{x}}^{\bar{x}}(xz-\underline{w})dG(x)}{1-\beta\left(1-\delta\right)}. \tag{A.4}$$

Provided that $\int_{\underline{x}}^{\bar{x}}(xz-w)dG(x) < -k_c$ at this point, the expected value of a casual job is greater than of a regular job. Secondly, for $z > w/\underline{x}$ a firm with a casual job will always want to offer work. Assuming that the minimum wage is not binding at this point, the expected value of a casual job in this case is

$$E[J_{Hc}(z)] = \frac{\int_{\underline{x}}^{\bar{x}}(zx-w(z))dG(x)-k_c}{1-\beta(1-\delta)} \tag{A.5}$$

The inclusion of the (small) administrative cost k_c ensures that this is always less than the expected value of a regular job, and hence there exists at least one reservation productivity z_H^*. Similar arguments hold for z_L^*.

Conditions for the uniqueness of z_i^*

To prove uniqueness of z_i^*, I assume that $F(z), G(x)$ are both continuous and twice differentiable distributions, with probability density functions $f(z), g(z)$. I show that $E[J_r(z)]$ and $E[J_{ic}(z)]$ are linear and convex functions of x respectively. For small z, $E[J_r(z)] < E[J_{ic}(z)]$ and for large z, $E[J_r(z)] > E[J_{ic}(z)]$. Hence these two functions can only cross at a single point and z_i^* is unique.

$E[J_{ic}(z)]$ depends on the probability that the worker will accept another job

$$\psi_i(z) = \begin{cases} 1-F(\hat{z_{ir}}) \text{ if } W_{ic}(z) < W_{ir}(\hat{z_{ir}}) \\ 1-\min\{F(z), F(\tilde{z}_{ir}(z))\} \quad \text{else} \end{cases} \tag{A.6}$$

where $\tilde{z}_{ir}$ is such that $W_{ic}(z) = W_{ir}(\tilde{z}_{ir}(z))$. $\psi(z)$ is constant when $W_{ic}(z) < W_{ir}(\hat{z_{ir}})$. Otherwise, $\psi_i(z)$ is decreasing and concave when $\min\{F(z), F(\tilde{z}_{ir}(z))\}$ is convex and decreasing and concave otherwise.

Case 1: minimum wage binds. From Eq. (A.1) it is clear that $E[J_r(z)]$ is linear in z. $E[J_{ic}(z)]$ depends on the probability that a casual worker will accept a new job, denoted $\psi(z)$. To see that $E[J_{ic}(z)]$ is convex, first write it as the ratio of two functions i.e.

$E[J_{ic}(z)] = A_i(z)/B_i(z)$. The first and second differentials of $A_i(z)$ are both strictly positive and hence $A_i(z)$ is convex and increasing. Similarly,

$$B_i'(z) = \beta f(\theta)\Big(G\big(\frac{w}{z}\big)\psi_i'(z) - \frac{w}{z^2}g\big(\frac{w}{z}\big)\psi_i(z)\Big) \tag{A.7}$$

$$\begin{aligned} B_i''(z) =& \beta f(\theta)\Big(G\big(\frac{w}{z}\big)\psi_i''(z) - \frac{w}{z^2}\Big(g\big(\frac{w}{z}\big)\psi_i(z) + \frac{2g\big(\frac{w}{z}\big)\psi_i(z)}{z} \\ &+ \frac{wg'\big(\frac{w}{z}\big)\psi_i(z)}{z^2} - g\big(\frac{w}{z}\big)\psi_i'(z)\Big)\Big) \end{aligned} \tag{A.8}$$

$B_i'(z)$ is strictly negative. If $B_i''(z)$ is also positive, then the reciprocal $1/B_i(z)$ is convex and increasing. When this condition holds, $E[J_{ic}(z)]$ is the product of two functions which are weakly greater than zero, increasing and convex, and hence $E[J_{ic}(z)]$ is convex. A sufficient (although not necessary) condition for $B_i''(z) > 0$ is that $\psi_i''(z) > 0$. If the cumulative distribution $F(z)$ is concave over the region where the firm chooses not to destroy a vacancy, i.e. for $z > \hat{z_{ic}}$ then this condition holds.[1]

Case 2: minimum wage binds. Similar arguments show that $E[J_r(z)]$ is linear and $E[J_{ic}(z)]$ is convex in this case. Hence there only cross at one point, z_i^*.

Appendix 2 Further data description

The Australian HILDA survey

The HILDA survey is an annual household survey, covering all individuals over the age of 15 in each household. Children are included when they reach the age of 15. In my sample I include all adults aged 16 or over. The survey covers individual and employment characteristics.

It is important to define casual work clearly in order to identify casual workers in the data consistently. In Australia, casual work is a legally recognised state, where the worker

> (i) has no guaranteed hours of work (ii) usually works irregular hours (but can work regular hours) (iii) doesn't get paid sick or annual leave (iv) can end employment without notice, unless notice is required by a registered agreement, award or employment contract (Australian Government, 2015).

To compensate workers for the lack of sick and holiday pay, firms must pay casual workers a "casual-loading", a premium of 15-25% above the wage of regular workers doing

[1]In Section 1.4 I assume that z is lognormally distributed. The lognormal distribution is concave for most values of z and so this condition is likely to hold.

the same job. It is therefore possible for firms to designate workers who usually work full-time as “casual" in order to avoid paying for these benefits (see Campbell (2018) for a more detailed discussion). Thus someone who works regular hours may appear as “casual" in the data. In my analysis I distinguish between “regular" workers who are guaranteed certain hours of work (although they may work overtime), and “casual" workers, who are not. I therefore relabel any casual workers who usually work over 35 hours a week, and whose hours do not vary, or who usually work over 35 hours a week, as regular workers. The percentage of casual workers using the definition above (15% of employed workers) is thus slightly lower than the percentage reported in official statistics (between 20-25% over the period of the survey). I exclude workers with other types of non-standard jobs, such as flexitime or job sharing, and those who are self-employed.

The UK LFS

The LFS is a quarterly survey, and covers the employment and personal characteristics of all the individuals in a household. In my sample I include all adults aged 16 or over. Since 2000 the Spring and Autumn editions of the LFS have included a question asking whether or not each individual has a zero-hours contract. They are also asked whether their weekly hours vary.

There was a large increase in respondents with a zero-hours contract between the Spring and Autumn 2013 editions. The ONS attributed this to measurement issues in previous editions, and to the increase in awareness of zero-hours contracts following media coverage (Chandler, 2014). I therefore use data from Autumn 2013 to Autumn 2017.[2] Unlike the HILDA survey, the LFS relies on workers self-identifying as zero-hours workers. I once again relabel any casual workers who usually work over 35 hours a week, and whose hours do not vary, as regular workers.

Data on minimum wages and inflation is taken from the OECD website.

Appendix 3 Algorithm for equilibrium solution

I use the following algorithm to find the equilibrium values of $\theta, z_i^*, \hat{z}_{ij}$, the steady state stocks u_i and distributions $s_i(z), N_{ij}(z)$:

1. Set an initial value for $\theta = \theta_0$

[2]It is possible to link some observations into a five-quarter panel. However, as not all questions are asked in each quarter, it is only possible to observe zero-hours workers at two points over the five quarters, and the number in this dataset is very small (fewer than 100 workers).

2. Using θ_0, find the workers' and firms' values using Eqs. (1.1) to (1.9). From these values, find the reservation productivities $z^*_{i0}, \hat{z_{ij0}}$ and the policy functions $\mathbb{1}_{\text{offer},ij}(z), \mathbb{1}_{\text{accept},ij}(z)$

3. Using the worker and firm values, find the steady state $u_{i0}, s_{i0}(z), N_{ij0}(z)$ using Eqs. (1.13) to (1.15)

4. Given the firm's values of a filled job, and $u_{i0}, s_{i0}(z), N_{ij0}(z)$, find the updated value θ_1 that satisfies the free entry condition

5. Update the initial guess to the new guess $\theta_0 = \theta_1$ and continue from Step 2

6. After each iteration calculate the difference between the current and previous guess for θ^* (i.e. after n iterations, calculate $|\theta_{n+1} - \theta_n|$). Continue until the difference is lower than some ε. Stop when $|\theta_{n+1} - \theta_n| < \varepsilon$

7. Confirm uniqueness of reservation productivities $z^*_i, \hat{z}_{ij}$

Appendix 4 Worker types

The HILDA survey contains the following questions about preferred working patterns for employed and unemployed workers:

> **(A) Please pick a number between 0 and 10 to indicate how satisfied or dissatisfied you are with [the hours you work/ the flexibility to balance work and non-work commitments].**
> **(B) You have said that (currently) you usually work fewer than 35 hours per week. What is the main reason for your working part-time hours?**
>
> (i) Own illness or disability
> (i) Caring for children/ disabled or elderly relatives
> (iii) Other personal or family responsibilities
> (iv) Could not find full-time work
> (v) Prefer part-time work
> (vi) Involved in voluntary work
> (vii) Attracted to pay premium attached to part-time or casual work
> (viii) Getting business established
> (ix) Prefer job and part-time hours are a requirement of the job
> (x) Other
>
> **(C) At any time during the last 4 weeks have you looked for paid work?**

(i) No, have not looked for work in last 4 weeks

(ii) Yes, looked for full-time work only

(iii) Yes, looked for part-time work only

(iv) Yes, looked for any work, FT or PT

(D) What is the main difficulty you have had in getting a job?

(i) Hours were unsuitable

(ii) Difficulties in finding child care

(iii) Other family responsibilities (not child care difficulties)

(iv) [Variety of other reasons]

[8pt]

Source: HILDA survey (edited for clarity).

As Fig. A.1 shows, there are noticeable differences in reported satisfaction with hours and flexibility between regular and casual workers. This suggests it is possible to use these questions to identify workers who have a strong preference for flexibility. This is done as follows:

Casual workers

Type L: satisfaction with hours and flexibility of at least 5; would prefer to work fewer than 30 hours (4 days) per week; and any answer to question B, other than (iv), (vii), (ix). Type H: satisfaction with hours and flexibility of below 5; answered (iv), (vii), or (ix) to question B; or would prefer to work more than 28 hours a week

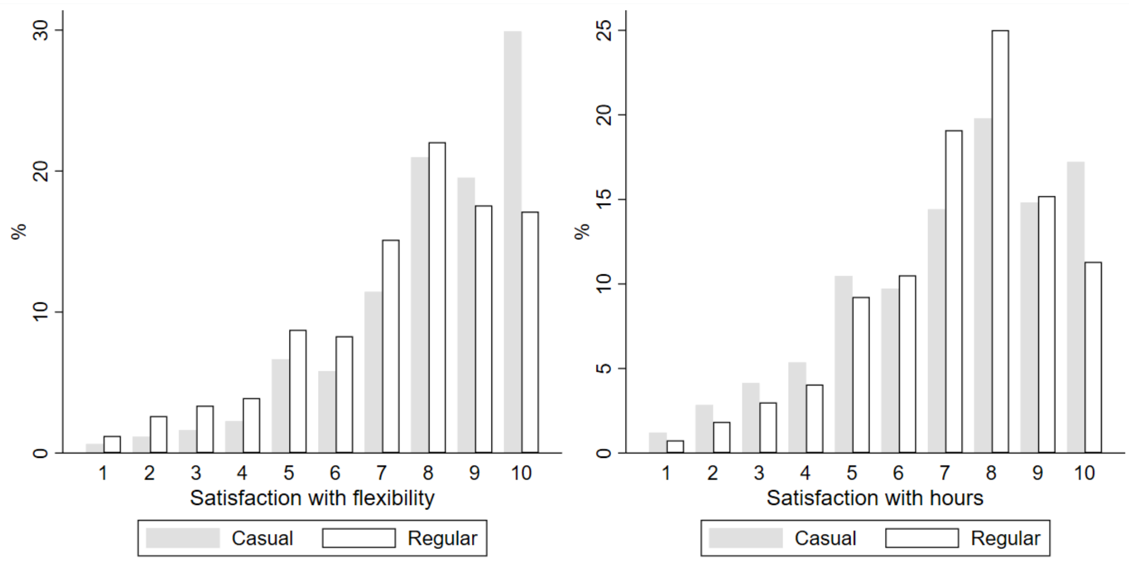
Regular workers

Type L: satisfaction with hours and flexibility of below 5; and would prefer to work fewer than 28 hours a week. Type H: all other regular workers

Unemployed workers

Type L: answered (i), (ii) or (iii) to question D; or answered (iii) to question C. Type H: all other unemployed workers

FIGURE A.1: Level of satisfaction (as % of workers)



Source: HILDA survey.

Appendix B

Extreme wages, performance and superstars in a market for footballers

Appendix 1 Further details on data and variables

TABLE B.1: Notes on variables

Variable name	Notes/Description	Source
Player-level variables:		
Position	Derived primary position as of MLS records in 2018. The positions are forward, midfielder, defender (outfield players) or goalkeeper; some players may have played in multiple positions throughout their MLS careers.	MLS
Multi-position	Dummy variable for a player who can play in more than one primary position	MLS
Player-season-level characteristic variables:		
Wages/salary (w)	Log guaranteed annual salary (US$)	MLSPA
Age	Player age in years (minus 15) at the beginning of each season	MLS
Captain	Dummy variable for the team captain	Various, MLS website
Designated	Dummy variable for a designated player	MLS
MLS new entrant	Dummy variable for an entrant to MLS, who had not played in the MLS before (during the sample period)	Derived
MLS re-entrant	Dummy variable for an entrant to MLS who had previously played in MLS during the sample period but who was not signed to a team in the previous season	Derived
MLS transfer	Dummy variable indicating a player's first season at a new team, after playing for another MLS team the previous season.	Derived
Mins played (%)	Percentage of the team's regular season minutes played (i.e. time on the football pitch)	MLS
Rating	A combination score of a player's average performance in the season	WhoScored.com via Opta
Page views	Average of views of a player's Wikipedia profile page in January and February	Wikipedia API

(Table continued)

Variable name	Notes/Description	Source
Player-season-level performance variables (given as the average per 90 minutes played):		
Goals	Goals scored (outfield players only)	MLS
Assists	Number of passes to another player that result in a goal being scored (outfield players only) - a higher value would indicate a more productive player, especially for forwards and midfielders	MLS
Shots	Number of shots toward goal, both on and off target - a higher value would indicate a more productive player, especially for forwards and midfielders	MLS
Shots on goal	Number of shots on target that could have resulted in a goal, including those that were saved by a goalkeeper or blocked by another player - a higher value would indicate a more productive player, especially for forwards and midfielders	MLS
Fouls committed	Number of fouls committed by the player, including minor fouls. A higher value may suggest a player prepared to take more risks - a priori it is not obvious whether this is indicative of a higher productivity player	MLS
Fouls suffered	Number of times the player suffered a foul committed by another player - higher value suggests a player may be adept at attracting fouls and may indicate higher productivity, though conversely it may indicate a lack of assertiveness by players	MLS
Red cards	Number of red cards for serious fouls, that result in the player being sent off the pitch. A higher value may suggest a player prepared to take more risks - a priori it is not obvious whether this is indicative of a higher productivity player	MLS
Saves	Saves as a percentage of shots on goal by opposing players (goalkeepers only) - a higher value would indicate a more productive player	MLS

(Table continued)

Variable name	Notes/Description	Source
Team-season-level variables:		
Log points per game	Natural logarithm of the total points achieved over the regular season divided by the number of games played	MLS
Log attendance	Log of average attendance (10,000 persons) at home games during the season	MLS
Playoffs	Dummy variable indicating whether a team qualified for the MLS playoffs due to their performance during the regular season	MLS
Expansion team	Dummy variable for a team's first season in MLS	Derived
Log revenue	Log of estimated team revenue (US$, millions), available for 2013, 2015-18 seasons	Forbes
Log value	Log of estimated team value (US$, millions), available for 2013, 2015-18 seasons	Forbes

TABLE B.2: Number of players by year and position in analysis sample

Year	**Defender**	**Forward**	**Goalkeeper**	**Midfielder**	**Total**
2008	126	80	43	146	395
2009	123	75	40	127	365
2010	118	82	39	135	374
2011	142	113	53	194	502
2012	142	123	58	202	525
2013	148	130	59	197	534
2014	161	127	55	187	530
2015	149	119	58	206	532
2016	157	99	60	197	513
2017	184	108	61	213	566
2018	206	131	74	211	622
Total	1656	1187	600	2015	5458

Notes: Player positions are defined according to the primary record as observed on the MLS website in 2018.

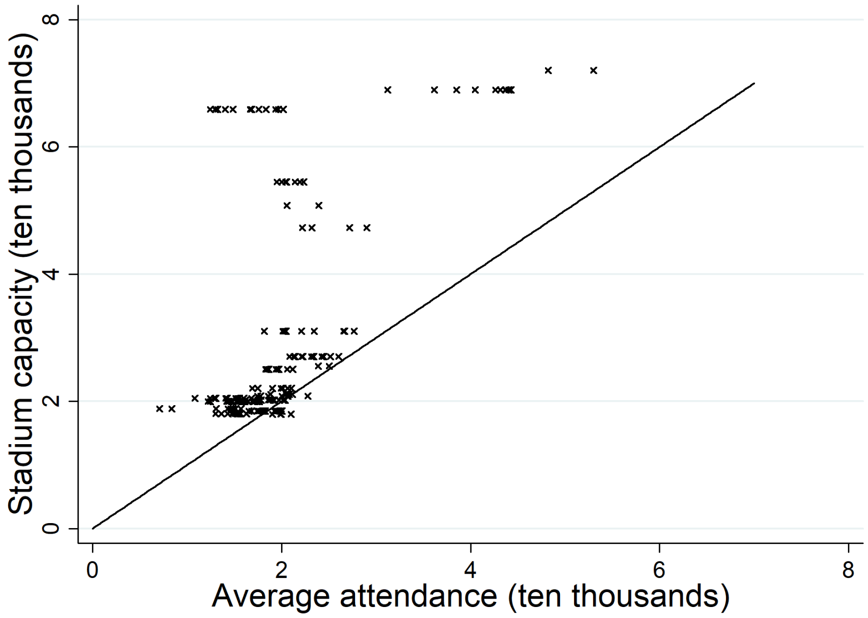
TABLE B.3: MLS teams/franchises & abbreviations, 2007-18

Abbreviation	**Latest team name**	**Years active in sample period**
ATL	Altanta United FC	2017-18
CHI	Chicago Fire	2007-18
CHV	Chivas USA	2007-14
CLB	Columbus Crew	2007-18
COL	Colorado Rapids	2007-18
DAL	FC Dallas	2007-18
DC	D.C. United	2007-18
HOU	Houston Dynamo	2007-18
KC	Sporting Kansas City	2007-18
LA	LA Galaxy	2007-18
LAFC*	Los Angeles FC	2018
MIN	Minnesota United FC	2017-18
MTL	Montreal Impact	2012-18
NE	New England Revolution	2007-18
NYCFC	New York City FC	2015-18
NYRB	New York Red Bulls	2007-18
ORL	Orlando City SC	2015-18
PHI	Philadelphia Union	2010-18
POR	Portland Timbers	2011-18
RSL	Real Salt Lake	2007-18
SEA	Seattle Sounders FC	2009-18
SJ	San Jose Earthquakes	2007-18
TOR	Toronto FC	2007-18
VAN	Vancouver Whitecaps FC	2011-18

Notes: *LAFC joined MLS in 2018, the last year of our sample period. At the time of writing, team-level data was not available for LAFC in 2018, so we do not include it in the second step of our analysis. We do include its players in the first step of our analysis.

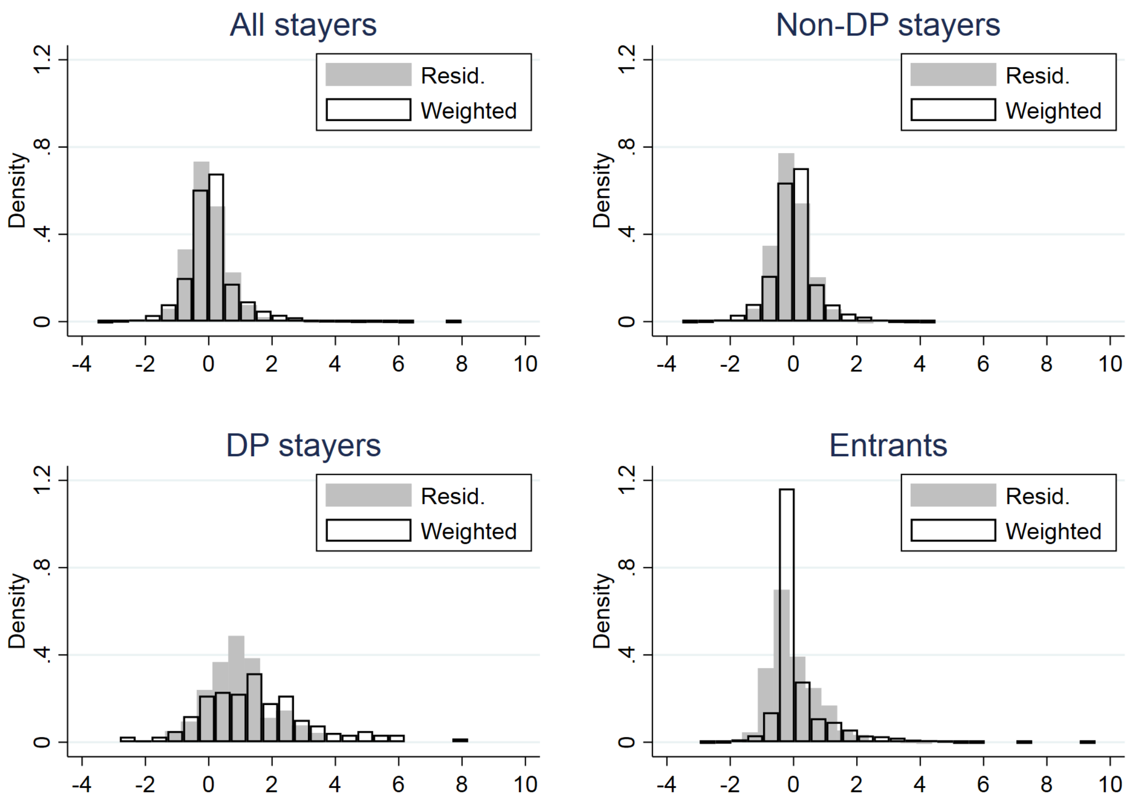
Appendix 2 Additional figures

FIGURE B.1: Average home attendance vs stadium capacity, by team-season, 2007-18



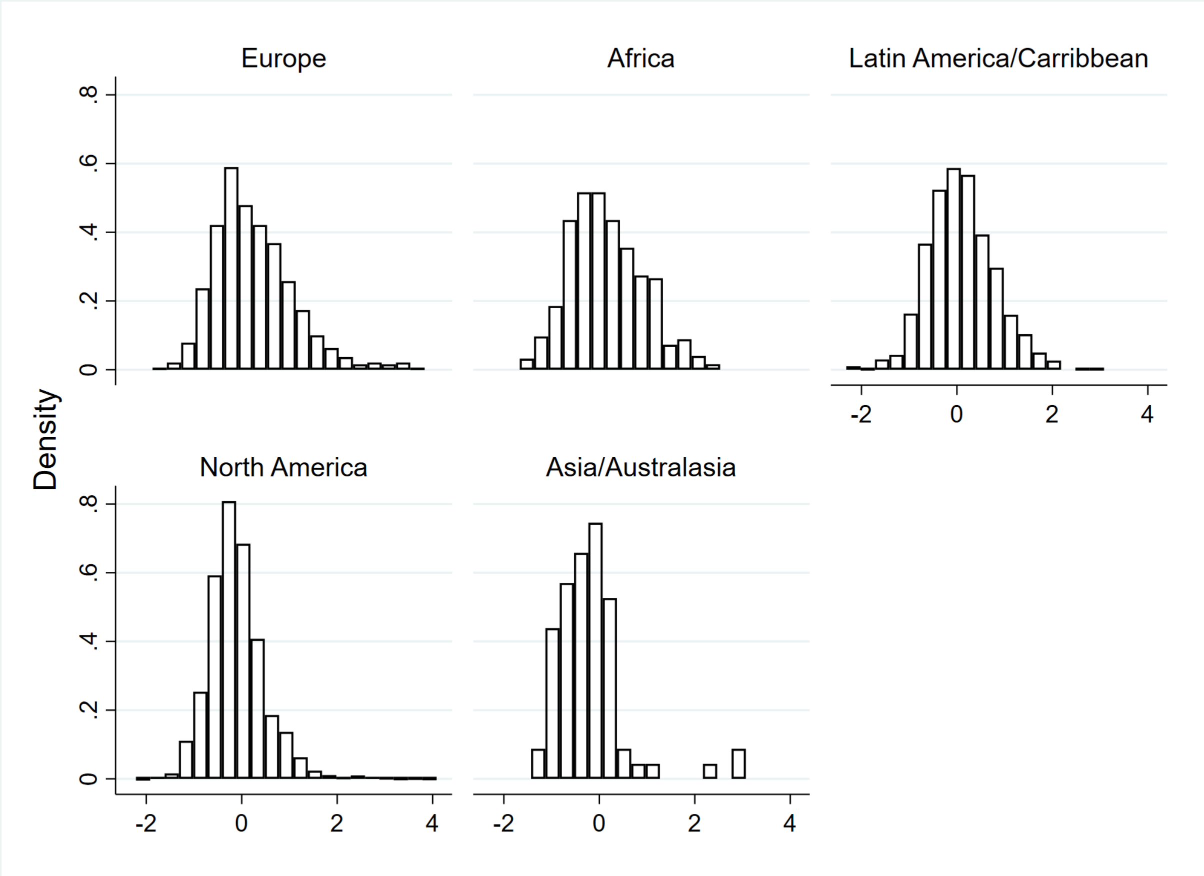
Notes: author calculations using MLS and other sources. It is possible to be below the 45-degree line; many teams have the potential to increase their stadium capacity within a season, by agreeing with stadium owners for one-off matches to sell tickets in parts of the stadia that are normally not used.

FIGURE B.2: Distributions of estimated residuals from first-step player salary regressions, stayers vs entrants to MLS and unweighted vs weighted models, 2008-18



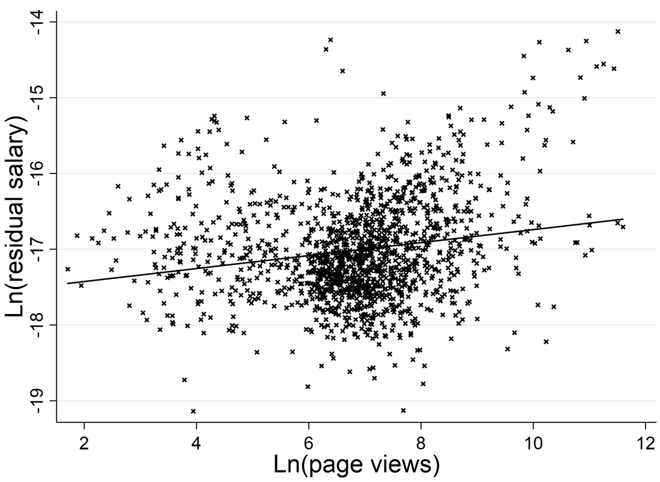
Notes: See Fig. 2.4. A 'stayer' is a player-year observation who was in MLS the preceding season. An 'entrant' is a player-year observation who was not in MLS the preceding season, either because they have entered for the first time or have returned to the league. 'DP' refers to players with designated player status.

FIGURE B.3: Distributions of estimated residuals from first-step player salary regressions, by player region, 2008-18.



Notes: See Fig. 2.4.

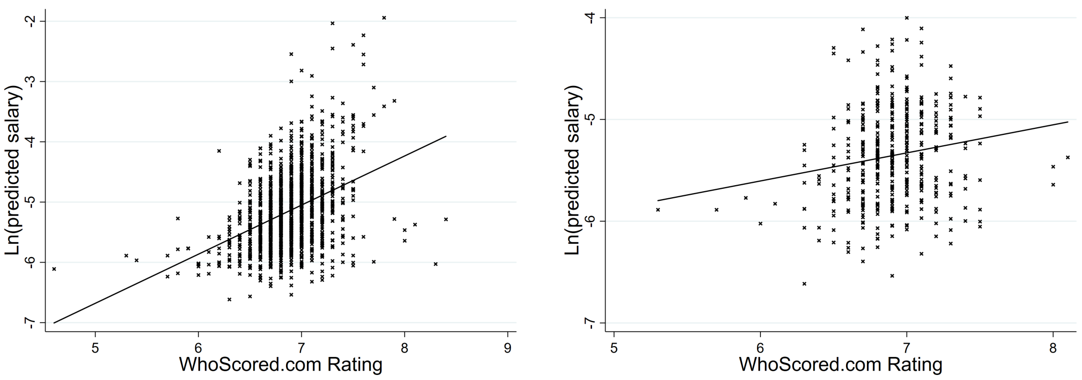
FIGURE B.4: Correlation between the residual salaries and Wikipedia page views of players in MLS, 2016-18



Notes: solid line plots the line of best fit. Residual salary is measured for each player-year observation from estimates of Equation (2.3). Wikipedia page views are only for the off-season months of January and February.

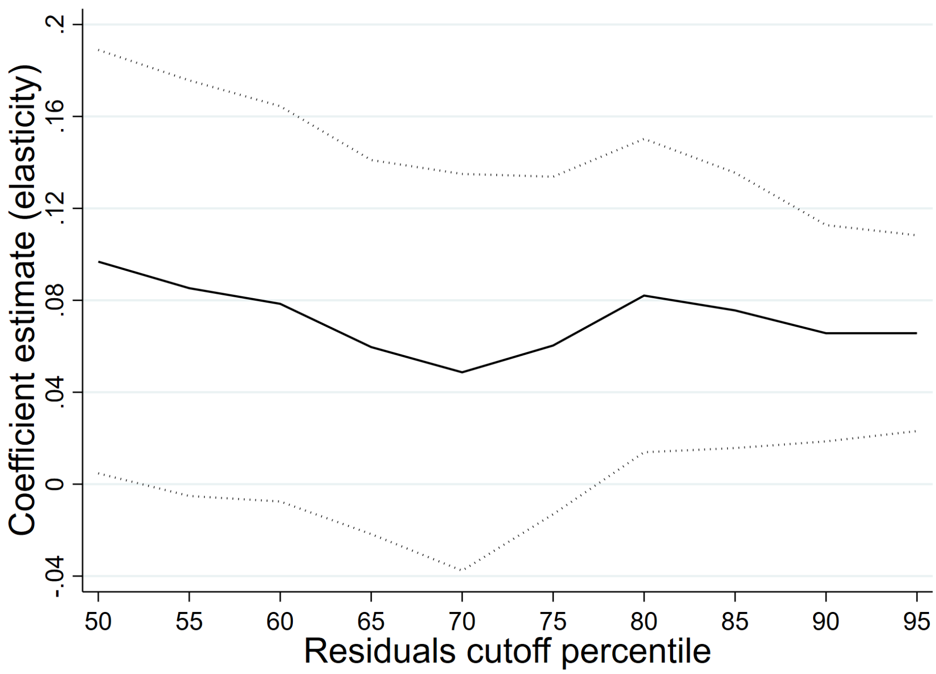
FIGURE B.5: Correlation between the predicted salaries and previous season whoscored.com rating of players in MLS, 2014-18

(a) All positions (b) Defenders only



Notes: solid lines plot the lines of best fit. Predicted salary is measured for each player-year observation from estimates of Eq. (2.3). Shows the subsample of 1,299 player-year observations (428 defenders) in MLS seasons 2014-18 who had a Whoscored.com rating in the previous season.

FIGURE B.6: Sensitivity analysis of $\widehat{\gamma}_H$: varying the definition of 'high' residual salary



Notes: the solid line plots the estimated effects of aggregate residual salary above the Xth percentile on team average home attendance, i.e., varying the cutoff, equivalent to the results presented in column IV of Table 2.5, which used the 90th percentile (also shown here). Dotted lines show 95% confidence intervals robust to team-level clusters.

Appendix 3 Additional tables

TABLE B.4: Robustness check: regression of log residual salary on players' Wikipedia page views

	(I)	(II)	(III)
Ln(page views)	0.086***	0.085***	0.084***
	(0.020)	(0.020)	(0.020)
Constant	-17.60***		
	(0.135)		
Year fixed effects	No	Yes	Yes
Team fixed effects	No	No	Yes
R^2	0.036	0.036	0.048
N	1,474	1,474	1,474

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors robust to player clusters and displayed in parentheses. See also Appendix Figure B.4.

TABLE B.5: Robustness check: regression of log predicted salary on previous season's WhoScored.com player rating, 2014-2018

	(I)	(II)	(III)	(IV)
Rating	0.815***	0.847***	0.849***	0.348***
	(0.094)	(0.094)	(0.090)	(0.092)
Constant	-10.80***			
	(0.631)			
Year fixed effects	No	Yes	Yes	Yes
Team fixed effects	No	No	Yes	Yes
R^2	0.159	0.170	0.197	0.105
N	1,299	1,299	1,299	429

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors robust to player clusters and displayed in parentheses. Column (IV) is only for players whose main position is defender. See also Appendix Figure B.5.

TABLE B.6: Robustness check, second-step regression estimates: log annual revenue and value (Forbes, \$millions), 2013, 2015, 2017 and 2018

	Revenue		Value	
	(I)	(II)	(III)	(IV)
Log points per game	0.061		0.112	
	(0.135)		(0.100)	
Log attendance (10,000s) ($\widehat{\phi}_2$)	0.677***	0.694***	0.666***	0.633***
	(0.181)	(0.192)	(0.198)	(0.166)
Log wages, all players (\$m):				
Predicted ($\widehat{\lambda}$)	0.564*	0.558*	0.225	0.216
	(0.280)	(0.276)	(0.204)	(0.201)
Residual ($\widehat{\gamma}$)	0.055		-0.007	
	(0.082)		(0.062)	
Residual above 90th percentile ($\widehat{\gamma}_H$)		0.009		0.008
		(0.029)		(0.021)
Year fixed effects	Yes	Yes	Yes	Yes
Team fixed effects	Yes	Yes	Yes	Yes
N	95	94	95	94
R^2	0.845	0.841	0.949	0.948

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors robust to team clusters (20) and displayed in parentheses.

TABLE B.7: Robustness check, second-step regression estimates: weighted least squares in first step, 2008-18

	Points per game	Playoffs	Attendance	
	(I)	(II)	(III)	(IV)
Log points per game			0.147**	0.132**
			(0.069)	(0.065)
Expansion			0.082	0.079
			(0.052)	(0.051)
Log salary ($m):				
Predicted			0.015	0.004
			(0.057)	(0.060)
Residual			0.122***	
			(0.043)	
Bottom 90% residual				0.102
				(0.091)
Top 10% residual				0.051***
				(0.018)
Log weighted salary ($m):				
Predicted	0.199***	1.853***		
	(0.069)	(0.498)		
Residual	0.117**	0.854*		
	(0.057)	(0.475)		
Year fixed effects	No	No	Yes	Yes
Team fixed effects	Yes	Yes	Yes	Yes
N	204	186	204	199
R^2	0.284		0.858	0.865

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors robust to team clusters (19 for Column (II), 23 otherwise) and displayed in parentheses.
Columns here are the equivalents of Table 2.4:(III), Table 2.4:(VI), Table 2.5:(I) and Table 2.5:(IV), respectively. First step regression models estimated using the team-season percent of minutes played for each observation as weights.

TABLE B.8: Robustness check, second-step regression estimates: robust regression in the first step, 2008-18.

	Points per game	Playoffs	Attendance	
	(I)	(II)	(III)	(IV)
Log points per game			0.137**	0.107
			(0.068)	(0.065)
Expansion			0.105**	0.095*
			(0.049)	(0.048)
Log salary ($m):				
Predicted			0.125	0.071
			(0.103)	(0.094)
Residual			0.101**	
			(0.049)	
Bottom 90% residual				0.075
				(0.123)
Top 10% residual				0.051**
				(0.021)
Log weighted salary ($m):				
Predicted	0.055	0.447		
	(0.054)	(0.307)		
Residual	0.052	0.439		
	(0.056)	(0.366)		
Year fixed effects	No	No	Yes	Yes
Team fixed effects	Yes	Yes	Yes	Yes
N	204	186	204	196
R^2	0.254		0.860	0.889

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors robust to team clusters (19 for Column (II), 23 otherwise) and displayed in parentheses.
Columns here are the equivalents of Table 2.4:(III), Table 2.4:(VI), Table 2.5:(I) and Table 2.5:(IV), respectively. First step regression models estimated using robust regression.

TABLE B.9: Robustness check, second-step regression estimates: assigning players who played less than 20% of total time a weight of zero, 2008-18.

	Points per game	Playoffs	Attendance	
	(I)	(II)	(III)	(IV)
Log points per game			0.146**	0.135**
			(0.068)	(0.061)
Expansion			0.082	0.079
			(0.052)	(0.051)
Log salary ($m):				
Predicted			0.009	-0.003
			(0.059)	(0.063)
Residual			0.127***	
			(0.045)	
Bottom 90% residual				0.112
				(0.090)
Top 10% residual				0.061***
				(0.020)
Log weighted salary ($m):				
Predicted	0.212***	1.840***		
	(0.068)	(0.511)		
Residual	0.116**	0.867*		
	(0.054)	(0.464)		
Year fixed effects	No	No	Yes	Yes
Team fixed effects	Yes	Yes	Yes	Yes
N	204	186	204	197
R^2	0.288		0.858	0.868

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors robust to team clusters (19 for Column (II), 23 otherwise) and displayed in parentheses.

Columns here are the equivalents of Table 2.4:(III), Table 2.4:(VI), Table 2.5:(I) and Table 2.5:(IV), respectively. First step regression models estimated using the team-season percent of minutes played for each observation as weights.

TABLE B.10: Robustness check, second-step regression estimates: excluding minimum wage players from first-step estimation, 2008-18.

	Points per game	Playoffs	Attendance	
	(I)	(II)	(III)	(IV)
Log points per game			0.143**	0.144**
			(0.063)	(0.056)
Expansion			0.085	0.086
			(0.052)	(0.054)
Log salary ($m):				
Predicted			0.035	0.012
			(0.178)	(0.138)
Residual			0.176***	
			(0.060)	
Bottom 90% residual				0.188
				(0.142)
Top 10% residual				0.071***
				(0.022)
Log weighted salary ($m):				
Predicted	0.193	1.518		
	(0.166)	(1.019)		
Residual	0.124	1.000*		
	(0.091)	(0.548)		
Year fixed effects	No	No	Yes	Yes
Team fixed effects	Yes	Yes	Yes	Yes
N	204	186	204	196
R^2	0.265		0.862	0.872

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors robust to team clusters (19 for Column (II), 23 otherwise) and displayed in parentheses.
Columns here are the equivalents of Table 2.4:(III), Table 2.4:(VI), Table 2.5:(I) and Table 2.5:(IV), respectively.

Appendix C

Long-term trends in part-time work in the UK

Appendix 1 Proofs

Proof of Proposition 2

Firstly, note that, since all workers are homogenous, in equilibrium they must all choose the same hours in complex and divisible tasks, and receive the same utility from each type of task, so that $U_d(h_d) < U_c(h_c)$ for all workers. From Eqs. (3.8) and (3.9) we can see that earnings in divisible tasks are linear in hours, whilst earnings in complex tasks are convex between 0 and 1. At $h = 0$ both are zero. Thus there can be at most one other solution to $e_d(h) = e_c(h)$ for $h > 0$.

1. Assume $e_d(h) = e_c(h)$ has one solution, at $h = 0$ and $e_d(h) < e_c(h)$ for all $h > 0$. As h tends to zero, $e'_c(h)$ also tends to zero. Thus, this can only occur when $e'_d(h) = 0$. This requires $1 - \alpha = 0$, or $\alpha = 1$. If $\alpha < 1$, then $e_d(h) < e_c(h)$ results in $U_d(h) < U_c(h)$ for all $h > 0$ and $Y_d = 0$. This requires that

$$(1-\alpha)\left(\frac{Y_d}{Y}\right)^{\rho-1} > \alpha h\left(\frac{Y_c}{Y}\right)^{\rho-1}. \tag{C.1}$$

   With $\rho < 1$ and $Y_d = 0$, the left hand side of this inequality is undefined. This contradiction ensures that this cannot be an equilibrium solution, unless $\alpha = 1$.

2. Assume there is a solution $e_c(\hat{h}) = e_d(\hat{h})$ with $\hat{h} > 1$. This is greater than the maximum bound of available hours. In this case, $U_d(h,\phi) < U_c(h,\phi)$ for all $h \leq 1$ and $Y_d = 0$. Similar arguments ensure this cannot be an equilibrium solution.

Thus there must be exactly one solution $e_c(\hat{h}) = e_d(\hat{h})$ with $\hat{h} \in (0,1]$. It only remains to find the parameter restrictions necessary such that the solution is not at $\hat{h} = 1$. Assume a corner solution with $e_c(\hat{h}) = e_d(\hat{h})$ and $\hat{h} = 1$. This requires

$$(1-\alpha)\Big(\frac{Y_d}{Y}\Big)^{\rho-1} = \alpha\Big(\frac{Y_c}{Y}\Big)^{\rho-1} \tag{C.2}$$

and hence

$$\frac{Y_c}{Y_d} = \Big(\frac{1-\alpha}{\alpha}\Big)^{\frac{1}{\rho-1}}. \tag{C.3}$$

It must be the case in this scenario, that optimal hours for workers in both types of task are greater than 1 so that workers choose a corner solution. Otherwise, all workers would do whichever type of task has optimal hours less than one. By the same arguments made above, this cannot be an equilibrium. A corner solution requires $e_c'(h) \geq \phi$ and $e_d'(h) \geq \phi$. Since $e_c'(h) > e_d'(h)$ at this point, it is only necessary to check that $e_c'(h) \leq \phi$. This requires

$$\alpha\Gamma(\alpha + (1-\alpha)(\frac{Y_c}{Y_d})^{\rho})^{\frac{1-\rho}{\rho}} \leq \phi \tag{C.4}$$

subsituting Eq. (C.3) into the condition above gives the parameter restriction

$$\alpha\Gamma(\alpha + (1-\alpha)(\frac{1-\alpha}{\alpha})^{\frac{\rho}{\rho-1}})^{\frac{1-\rho}{\rho}} \leq \phi. \tag{C.5}$$

IF this condition is met, then we cannot have a corner solution with $\hat{h} = 1$. Hence, there must be a single solution to $e_c(\hat{h}) = e_d(\hat{h})$ with $\hat{h} \in (0,1)$.

To show that optimal hours $h_c > h_d$ so that workers in complex tasks do longer hours, recall that, in equilibrium, there must be some workers in both types of task. Otherwise, the marginal product (and hence the earnings) of the other type of task would be infinite. For $h < \hat{h}$, earnings in divisible tasks are higher than those for complex tasks. Thus, from the first order conditions to the utility maximisation problem for divisible tasks, there must be a solution to

$$e_d'(h_d) = \phi h_d^{\theta} \tag{C.6}$$

with $h_d < \hat{h}$. Similarly, there must be a solution to

$$e_c'(h_c) = \phi h_c^{\theta} \tag{C.7}$$

with $h_c > \hat{h}$. This results in an equilibrium in which workers either do few hours in divisible tasks, or more hours in complex tasks.

Proof of Proposition 3

Using Leibniz's rule, marginal earnings are given by

$$e_c'(h) = \Psi_c\Big((y_1(h,h) - y_2(h,h))f(h) + E[\frac{\partial y_1(x,h)}{\partial h}|x \leq h] + E[\frac{\partial y_2(x,h)}{\partial h}|x > h]\Big)\Big) \tag{C.8}$$

where Ψ_c is the constant (from the worker's point of view)

$$\Psi_c = \alpha Y_c^{\rho-1}(\alpha Y_c^\rho + (1-\alpha)Y_d^\rho)^{\frac{1-\rho}{\rho}}. \tag{C.9}$$

If the conditions in Section 3.B.3 are met then $y_1(h,h) > y_2(h,h)$ and hence $e_c'(h)$ is strictly positive. Differentiating Eq. (C.8) with respect to h gives

$$\begin{aligned} e_c''(h) = &\Psi_c\Bigg((y_1(h,h) - y_2(h,h))f'(h) + 2\Big(\frac{\partial y_1(h,h)}{\partial h} - \frac{\partial y_2(h,h)}{\partial h}\Big)f(h) \\ &+ E\Big[\frac{\partial^2 y_1(x,h)}{\partial h^2}|x \leq h\Big] + E\Big[\frac{\partial^2 y_2(x,h)}{\partial h^2}|x > h\Big]\Bigg) \end{aligned} \tag{C.10}$$

There are two situations in which this will be convex for small h, as set out in Section 3.B.3:

1. If $y_1(x,h), y_2(x,h)$ are linear, then

   $$E_x\Big[\frac{\partial^2 y_1(x,h)}{\partial h^2}|x \leq h\Big] = E[\frac{\partial^2 y_1(x,h)}{\partial h^2}|x \leq h] = 0 \tag{C.11}$$

   In this case, the condition $F(0) = 0$ ensures that $g'(h)$ is weakly positive as h tends to zero. Thus the first term of Eq. (C.10) is weakly positive, and $e_c''(h)$ is convex as h tends to zero.

2. in the alternative case, for small h, the expected output of a worker doing complex tasks is negative. Let $\tilde{h}$ be the point where $E_x[y(h,x)] = 0$. Assuming that earnings cannot be negative, then $e_c(h) = 0$ for $h \leq \tilde{h}$ and thus earnings in this interval are convex.

Proof of Proposition 4

First, note that there must be at least some workers doing each type of task. If all workers do one type of task, then the marginal productivity of the other type of task would be infinite. This cannot be an equilibrium. Thus, there must be some interval in which $e_c(h) > e_d(h)$, and vice versa.

Earnings in divisible tasks as a function of hours are linear, with derivative

$$e_d'(h) = \Gamma(1-\alpha)\left(\frac{Y_d}{Y}\right)^{\rho-1} \tag{C.12}$$

From Proposition 3, $e_c(h)$ is convex for all h smaller than some $\tilde{h}$. Assume $e_d(h) < e_c(h)$ for some $h < \tilde{h}$. Since $e_d(h)$ is linear and $e_c(h)$ is convex, this would require that $e_d(h) < e_c(h)$ for *all* $h < \tilde{h}$. From Proposition 3, $e_c(h)$ is strictly increasing, and thus, in this case, $e_d(h) < e_c(h)$ for all feasible $h \in [0,1]$. In other words, earnings for divisble tasks would always be lower than earnings for divisible tasks. If this were the case, the same argument as above hold: all workers would choose complex tasks, and earnings in divisible tasks would tend to infinity. Thus this cannot be an equilibrium. Hence, there must be some value $\hat{h}$ below which $e_c(h) < e_d(h)$. This proves part (ii) of Proposition 4.

However, if $e_c(h)$ is not uniformly convex, then it is possible that $\hat{h}$ is not unique. The assumption that the distribution $F(x)$ is unimodal ensures that $e_c(h)$ has one inflection point for $h \in (0,1)$, and so there can be at most two points with $e_c(h) = e_d(h)$ in this interval (i.e. two crossing points). However, if parameter restrictions are such that $e_c(1) > e_d(1)$ then $\hat{h}$ is unique. Substituting in Eqs. (3.25) and (3.26) and simplifying gives

$$\left(\frac{Y_c}{Y_d}\right)^{\rho-1} > \frac{1-\alpha}{\alpha E_x[y(1,x)]} \tag{C.13}$$

If the equilibrium ratio Y_c/Y_d satisfies this condition, then $\hat{h}$ is unique.

Alternatively, assume that $y_1(x,h), y_2(x,h)$ are linear and $y_1(x,h) = ah$. Then, from Eq. (3.27), $e_c(h)$ is bounded above by a straight line running through the origin and the point $(\bar{x}, e_c(\bar{x})$ (the dotted line in Fig. C.1). This would correspond to the situation where there was no hours requirement and $y(h,x) = y_1(x,h)$. This line has equation

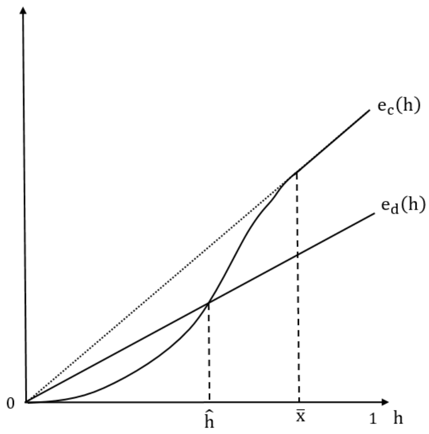
$$\Gamma\alpha a\left(\frac{Y_c}{Y}\right)^{\rho-1} \tag{C.14}$$

and $e_c(h)$ approaches this line as h tends to $\bar{x}$. It must be the case that $e_c(\bar{x}) > e_d(\bar{x})$ in equilibrium. Otherwise, $e_d(h) > e_c(h)$ for all h and no worker would choose complex tasks. As discussed above, $e_c(h)$ has one inflection point, and so $e_c(h), e_d(h)$ cross at one point, $\hat{h}$. Note that this does not require that $\bar{x} < 1$, as shown in Fig. C.1.

Proof of Corollary 1

The parameter ϕ governs the steepness of the utility indifference curve. Workers with higher ϕ have a greater marginal cost of work, and thus work shorter hours. The

FIGURE C.1: Example equilibrium with linear production in complex tasks



arguments discussed above ensure that, in equilibrium, there must be at least one worker who is indifferent between the two types of task. In addition, the two earnings functions cross at only one point. $e_c(h) > e_d(h)$ for $h > \hat{h}$ and so workers with a higher ϕ will do complex tasks, and vice versa.

Appendix 2 Selection into part-time work

The assumption underlying the model is that an individual's part-time status is determined by a latent variable

$$PT_i^* = \gamma Zi + \eta_i \tag{C.15}$$

where Z_i is a vector of characteristics that affect the propensity of individual i to work part-time. PT_i^* is not observed. Instead, the dummy variable PT_i is observed, with

$$PT_i = \begin{cases} 1 & \text{if} \quad PT_i^* > 0 \\ 0 & \text{if} \quad PT_i^* \leq 0 \end{cases} \tag{C.16}$$

Using a standard Heckman sample correction, I calculate a selection term for each individual, so that Eq. (3.1) becomes

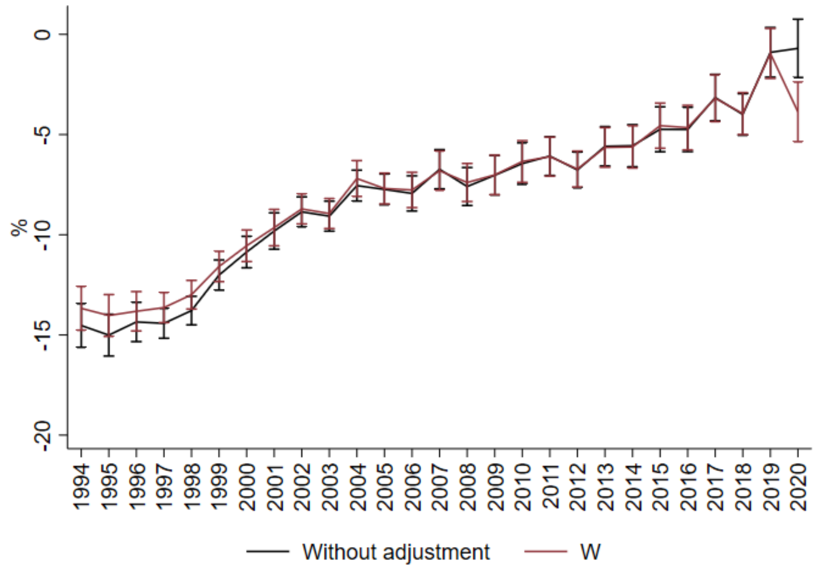
$$\log w_{it} = X_{it}\gamma + \beta_t PT_{it} + \delta\lambda(\gamma Zi) + \varepsilon_{it} \tag{C.17}$$

where

$$\lambda(\gamma Zi) = \frac{\phi(\gamma Zi)}{\Phi(\gamma Zi)} \tag{C.18}$$

is the inverse Mill's ratio. The parameter γ is estimated by probit maximum likelihood. A disadvantage of this method is that, to avoid collinearity, there should be at least one characteristic in Z that affect the propensity to work part-time, but don't directly affect the wage. Following the literature, I include an individual's family status in Z_i and the number of children they have, but exclude both from X_i. Fig. C.2 below shows the PTPP with the full set of firm and worker characteristics, with and without the Heckman sample selection adjustment.

FIGURE C.2: Trend in the part-time pay penalty, Heckman selection adjustment



Appendix 3 The role of occupations in part-time work

In Table C.1 I decompose the increase into growth 'between' and 'within' industry and occupation sectors. The 'between' component (ΔP_t^B) is the increase in the part-time share that is due to changes in the composition of sectors, i.e. a shift in workers towards sectors that have traditionally had more PT workers. The 'within' component (ΔP_t^W) is the increase in the part-time share that is due to growth in the part-time share within sectors. The overall change in the part-time share from year 0 to year t can therefore be expressed as follows

$$\begin{aligned} \Delta P_t &= \Delta P_t^B + \Delta P_t^W \\ &= \Sigma_j \bar{\lambda}_j \Delta E_{jt} + \Sigma_j \lambda_{jt} \bar{E}_j \end{aligned} \tag{C.19}$$

Here $\lambda_{jt} = P_{jt}/L_{jt}$ denotes the PT share of employment in industry/occupation sector j at time t, whilst $\bar{\lambda}_j$ is the average over time. $E_{jt} = L_{jt}/L_t$ is the share of total employment working in industry/occupation sector j at time t, whilst $\bar{E}_j$ is the average over time. Thus ΔP_t^B is the between component of the increase in the PT share, and ΔP_t^W is the within component.

The occupation classifications used by the ONS, the Standard Occupation Classification (SOC) codes were updated twice in this period, so that the LFS data uses three different classifications: SOC1990, SOC2000 and SOC2010. The ONS does not publish a mapping between the old and new codes, although some of the LFS files include respondents' occupations using the SOC2000 and SOC2010 codes. To deal with these issues I adopt two approaches. First, I follow the approach in Schaefer and Singleton (2019), by converting the SOC2010 codes to the 2008 International Standard Classification of Occupations (ISCO) developed by the International Labour Organization, using a mapping available from the ONS website, and updated by the authors. I then convert the SOC1990 and SOC2000 codes to ISCO1988, using conversion tables from the Cambridge Social Interaction and Stratification Scale (CAMSIS) project. I use a ISCO2008 to ISCO1988 cross-walk, available from the International Labour Organization to map the ISCO2008 codes to the ISCO1988 codes (International Labour Organization, 2012). I also repeat the analysis using the original SOC codes, but split into three periods, corresponding to when the codes were changed.

Growth in part-time work within occupations contributed more to the growth in the male part-time share (except before 2000, when the contribution was slightly lower). For women the pattern is less clear. Until 2010, the changes in the part-time share due growth within and between occupations is roughly the same. The fall in the part-time share that has occurred since the financial crisis is due both to a decrease in the part-time share within occupations, and to a shift towards occupations that have traditionally had a lower part-time share.

TABLE C.1: Shift share of occupations and industries

	1994-2000	2001-2009	2011-2020	All years
Percentage point growth in male part-time share				
Total	1.48	3.58	0.17	5.38
Between ind/occ growth	0.84	1.22	-0.59	1.23
Within ind/occ growth	0.64	2.36	0.77	4.10
Percentage point growth in female part-time share				
Total	-0.31	-0.02	-5.98	-5.51
Between ind/occ growth	0.12	-0.87	-2.45	-2.59
Within ind/occ growth	-0.43	0.85	-3.53	-2.92

Note: The column 'All years' uses the SOC to ISCO mapping described above. The other columns use the original SOC codes.

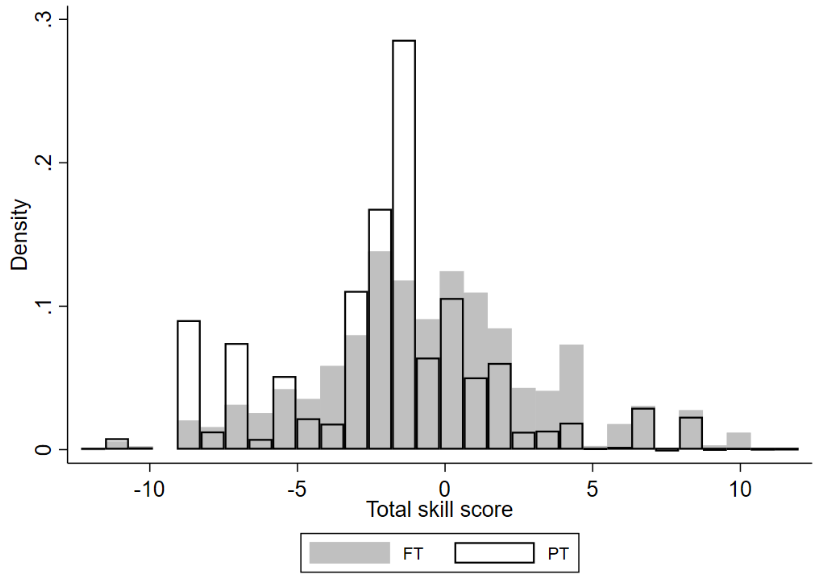
The occupations with the highest contribution to the 'within' increase in each of the three periods were:

- **1994 - 2000**: Sales assistants; Drivers of road goods vehicles; Waiters, waitresses; Postal workers, mail sorters; Storekeepers, warehousemen/women.
- **2001 - 2010**: Taxi, cab drivers and chauffeurs; Postal workers, mail sorters, messengers, couriers; Customer care occupations; Cleaners, domestics; Labourers in building and woodworking trades.
- **2011 - 2020**: Shelf fillers; Managers and proprietors in other services; Elementary storage occupations; Customer service occupations; Sales and retail assistants.

These are mostly service occupations, and many are associated with changes in technology, or the gig economy. I have therefore investigated whether there is any relationship between part-time work within an occupation and the task content of that occupation, as it may be the case that occupations that require certain types of task lend themselves more easily to part-time work. To do so, I use the O*NET task database. For every occupation in the US classification system, the database provides a set of variables measuring the extent to which the occupation involves over 200 tasks. In their work on job polarisation, Acemoglu and Autor (2011) combine these variables to create a standardised measure of each occupation's requirements for six categories of tasks: Routine cognitive, Routine manual, Non-routine cognitive (analytical), Non-routine cognitive (interpersonal), Non-routine manual (physical) and Non-routine manual (interpersonal). Since the O*NET database uses the US occupation classification system, I map the US occupation codes to the international ISCO occupation classifications, modifying code written by Hardy et al. (2018). The scores for each score are standardised, so that they have mean zero and standard deviation of one.

I first consider the distribution of skill requirements across part- and full-time workers. Using a composite measure (summing all six categories of skills), Fig. C.3 shows that part-time workers are more likely to work in occupations with a lower total skills requirement. Considering the different categories of skill separately, Fig. C.4 shows that part-time workers are concentrated in occupations with a lower requirement for non-routine cognitive and non-routine manual skills.

FIGURE C.3: Distribution of skills requirements across occupations, all skill categories



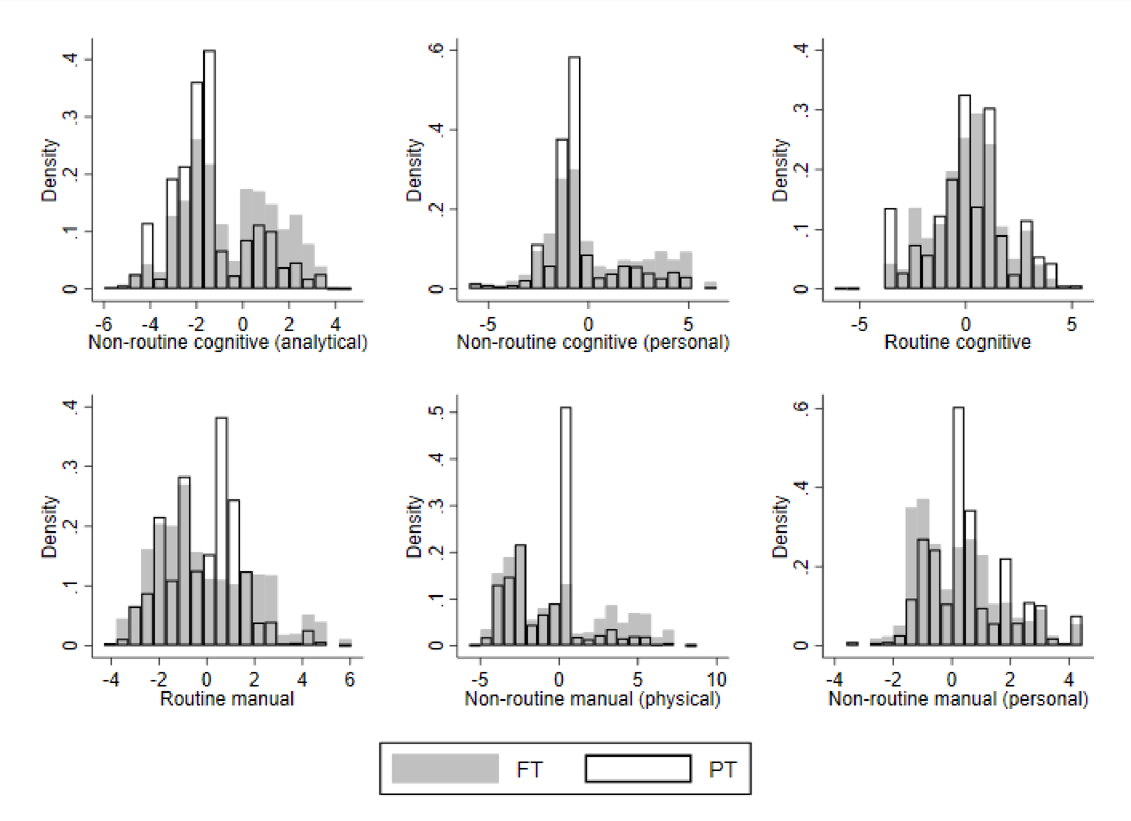
Source: LFS and O*NET database.

I first estimate, using pooled OLS, a simple linear probability model in which the dependent variable is a dummy variable indicating an individual i's part-time status in occupation j, at time t

$$PT_{ijt} = \beta X_{it} + \gamma \alpha_j + \delta_t + \varepsilon_{ijt} \tag{C.20}$$

X_{it} is a vector of worker and firm characteristics as described in Section 3.A.4. α_j is a vector of O*NET skills measures for occupation j, and thus δ is the coefficient of interest. Column I in Table C.2 shows the estimated $\hat{\gamma}$, excluding controls, and shows that workers in occupations with a higher requirement for cognitive skills (routine and non-routine) and non-routine manual (physical) skills are less likely to work part-time. Those whose occupations require greater non-routine manual (personal) are more likely to work part-time. These results are robust to the inclusion of worker and firm characteristics (Model II).

FIGURE C.4: Distribution of skills requirements across occupations, by skill category



Source: LFS and O*NET database.

TABLE C.2: Regression of part-time status on occupational skill requirements

	I	II
Non-routine cognitive (analytical)	-0.07***	-0.05***
	(0.000)	(0.000)
Non-routine cognitive (personal)	-0.02***	-0.02***
	(0.000)	(0.000)
Routine cognitive	-0.02***	-0.02***
	(0.000)	(0.000)
Routine manual	0.01***	0.02***
	(0.000)	(0.000)
Non-routine manual (physical)	-0.05***	-0.04***
	(0.000)	(0.000)
Non-routine manual (personal)	0.05***	0.03***
	(0.000)	(0.000)
Include controls?	No	Yes
Observations	5549781	5549781
R^2	0.110	0.183

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors in parentheses.

However, it is likely that there are unobserved worker characteristics (e.g. unobserved productivity) that are correlated with both the occupational choice and the decision to work part-time. I therefore regress the part-time share in occupation j (the mean of the dummy variable PT_{ijt}) on a vector of the mean characteristics of workers in occupation j and on α_j

$$PT_{jt} = \beta\overline{X_{it}} + \gamma\alpha_j + \delta_t + \nu_{jt}. \tag{C.21}$$

Since α_i is constant, a fixed effects or first differences model is not appropriate, and so I first estimate the model using random effects. However, it is still likely to be omitted variables that are correlated with X_{jt} or α_j and with the part-time share. I therefore perform a cross-check using the Hausman-Taylor method. This two-step estimator requires at least one time-invariant exogenous variable for each potentially endogenous time-invariant variable.[1] Unfortunately, it is very difficult to find variables that are plausibly exogenous. To avoid having to specify six such variables, I use the composite measure of all skills as α_j. I follow the literature, and use the share of workers in occupation j who are married in the first time period as the necessary time-invariant exogenous variable.

Columns I and II of Table C.3 show the results of the random effects model described above, with and without controls. As we would expect, the coefficients are smaller, but are mostly of the same sign. The exception is routine manual skills: occupations that require greater routine manual skills are associated with a *lower* part-time share. Columns III and IV show the results of the Hausman Taylor model. Here a greater overall demand for skills implies a lower overall part-time share, although column IV makes the further strong assumption that the controls are all exogenous.

In summary, the analysis above provides evidence that, even after accounting for differences in worker and firm characteristics, there is a link between skills requirements and part-time work and that, in general, occupations that require higher skill levels, particularly non-routine and cognitive skills, are less amenable to part-time work. This complements the work of Elsayed et al. (2017), who looked specifically at computer use,

[1] The algorithm first estimates the model using only the within-occupation variation (and hence excluding the time invariant α_j). It then regresses the residuals from the first step on all the time-invariant variables, using the time-invariant and time-variant exogenous variables as instruments.

and found that jobs where computers are used more intensively are more likely to be part-time.

TABLE C.3: Panel regression of part-time status on occupational skill requirements

	(I)	(II)	(III)	(IV)
Non-routine cognitive (analytical)	-0.04***	-0.01***		
	(0.001)	(0.001)		
Non-routine cognitive (personal)	-0.01***	-0.00		
	(0.001)	(0.001)		
Routine cognitive	-0.00**	-0.00***		
	(0.001)	(0.000)		
Routine manual	-0.01***	-0.01***		
	(0.001)	(0.001)		
Non-routine manual (physical)	-0.03***	0.01***		
	(0.001)	(0.001)		
Non-routine manual (personal)	0.05***	0.00*		
	(0.001)	(0.001)		
Composite skill measure			-0.02	-0.02**
			(0.017)	(0.006)
Include controls?	No	Yes	No	Yes
Observations	23704	23704	23704	23704
R^2	0.334	0.706		

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors in parentheses.

Appendix 4 Nonlinear earnings

Divisible tasks

A more general form of the utility function in Eq. (3.10) is

$$\frac{e_d(h)^{1-\xi}}{1-\xi} - \frac{\phi h^{1+\theta}}{1+\theta} \tag{C.22}$$

with $\xi > 0$. The worker's first order condition implies that the marginal benefit of working an extra hour equals the marginal cost

$$w_d^{1-\xi} h^{-\xi} = \phi h^{\theta} \tag{C.23}$$

The parameter ξ measures the strength of the income response to an increase in the wage. When ξ tends to one, and utility is log in consumption, workers exhibit KPR preferences and the income and substitution effects of an increase in hourly wages will cancel out. This is unattractive for the purposes of this model, since it would not allow us to analyse the effect of changes in technology on hours worked in divisible tasks. I make the simplification that $\xi = 0$, so that utility is linear in hours. This implies that the marginal benefit of working an extra hour, the marginal utility of consumption, is constant and equal to the wage.

$$e_d(h) - \frac{\phi h^{1+\theta}}{1+\theta} \tag{C.24}$$

with first order condition

$$w_d = \phi h^{\theta} \tag{C.25}$$

This has a unique solution since the marginal benefit of an extra hour of work is constant, whilst the marginal cost is increasing in h. In equilibrium, the lack of perfect substitutability between complex and divisible tasks ensures that optimal hours h_d lie in the interval $[0,1]$.

Complex tasks

Utility in complex tasks is given by

$$\frac{e_c(h)^{1-\xi}}{1-\xi} - \frac{\phi h^{1+\theta}}{1+\theta} \tag{C.26}$$

with $\xi > 0$. When earnings are potentially non-linear in hours, the first order condition becomes

$$C \frac{\partial E_x[y(h,x)]}{\partial h} (E_x[y(h,x)])^{-\xi} = \phi h^{\theta} \tag{C.27}$$

where C is the constant (from the individual worker's point of view), $C = Y_c^{\rho-1}(\alpha Y_c^{\rho} + (1-\alpha)Y_d^{\rho})^{\frac{1-\rho}{\rho}}$. In this case, the restriction $\xi = 0$ is attractive because it allows a simplification of the condition to

$$C\frac{\partial E_x[y(h,x)]}{\partial h} = \phi h^\theta \tag{C.28}$$

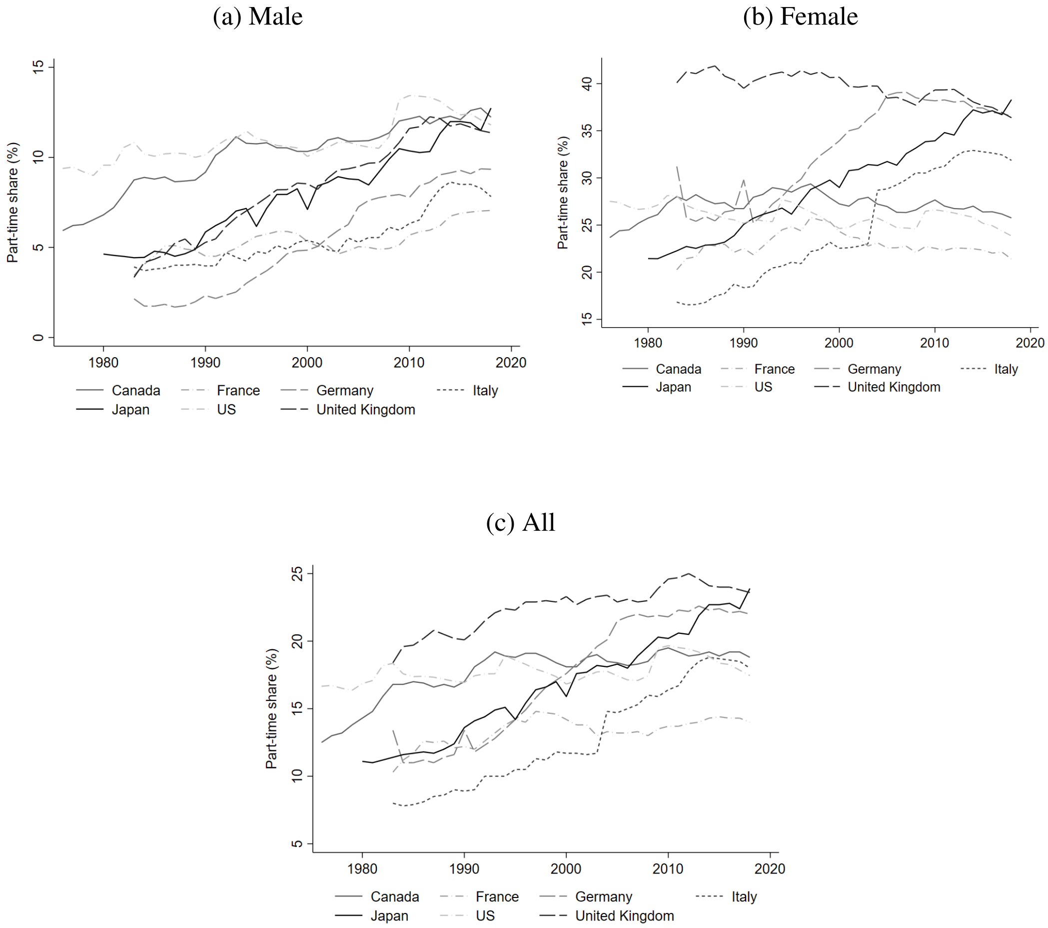
Note that Eq. (C.28) is always satisfied by setting $h = 0$, but I do not consider this solution. In the simple example, the left hand side of the equation is linear in h and hence there is a unique choice for h_c. This is no longer guaranteed in the general model in Section 3.B.3, since $\partial E_x[y(h,x)]/\partial h$ is not necessarily constant. In fact, for values of h for which $e(h)$ is convex, the marginal benefit of working an extra hour is increasing in h. Once again, lack of perfect substitutability between complex and divisible tasks ensures that at least one solution exists, since C will adjust accordingly in equilibrium. A functional form for $y(h,x)$ is therefore needed to ensure that the solution for hours is unique. The assumptions in Section 3.B.3 ensure that the left-hand side of Eq. (C.28) has at most one turning point. Then, as long as

$$C\left.\frac{\partial E_x[y(h,x)]}{\partial h}\right|_{h=1} < \phi \tag{C.29}$$

there will be a unique solution to Eq. (C.28) for $h \in (0,1)$. Alternatively, if $y_1(x,h), y_2(x,h)$ are linear, then there will also be a unique solution.

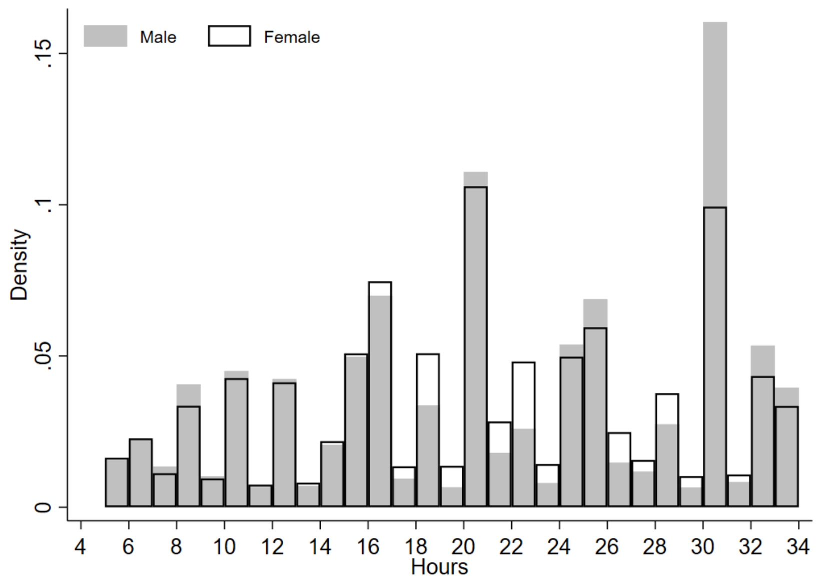
Appendix 5 Additional figures

FIGURE C.5: Part-time share across countries



Source: OECD (2019).

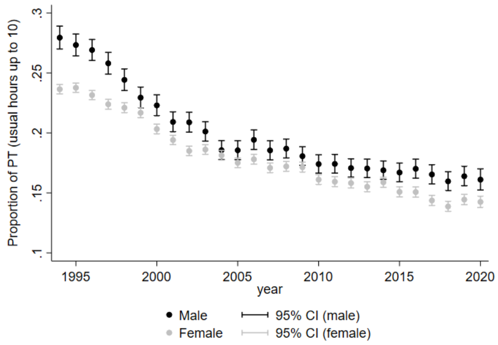
FIGURE C.6: Distribution of working hours for part-time workers, 1994-2020



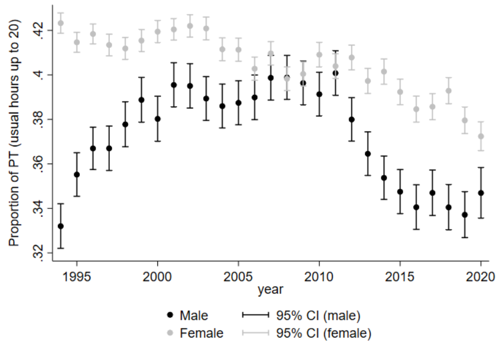
Source: LFS.

FIGURE C.7: Trends in the distribution of part-time workers by usual weekly working hours

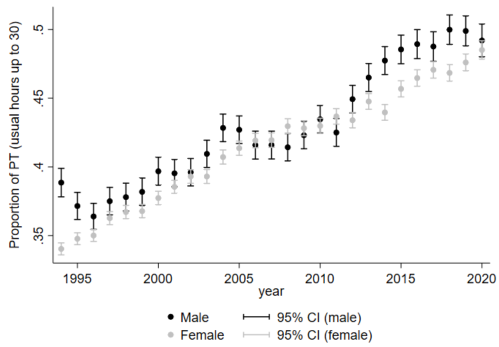
(a) Usual weekly hours 1-10



(b) Usual weekly hours 11-20

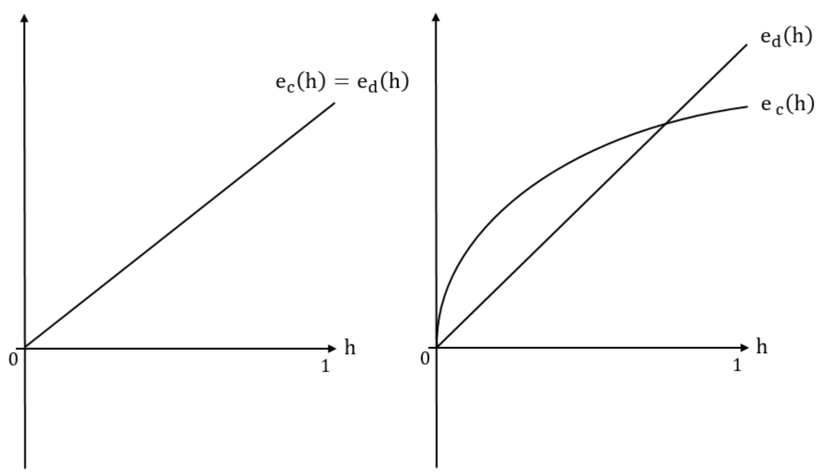


(c) Usual weekly hours 21-30



Source: LFS. Includes only workers whose hourly wages are in the top 25th percentile.

FIGURE C.8: Possible equililbria without an hours requirement



Appendix 6 Additional tables

TABLE C.4: Regression estimates of $\hat{\rho}$

	(I)	(II)	(III)	(IV)
Y_c/Y_d	-0.279**	-0.4067**	-0.166	-0.144
	(0.101)	(0.119)	(1.645)	(1.613)
N	761,028	761,028	106	105
R^2	0.982	0.883	1.000	0.997

***,**,* indicate significance from zero at 1%, 5% and 10% levels, respectively, two-sided tests, standard errors displayed in parentheses.

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