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Optimal Controller Design for Non-Affine Nonlinear Power Systems with Static Var Compensators for Hybrid UAVs

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Abstract: A generalized non-affine nonlinear power system model is presented for a single machine bus power system with a Static Var Compensator (SVC) or State Var System (SVS) for hybrid Unmanned Aerial Vehicles (UAVs). The model is constructed by differential algebraic equations on the MATLAB-Simulink platform with the programming technique of its S-Function. Combining the inverse system method and the Linear Quadratic Regulation (LQR), an optimized SVC controller is designed. The simulations under three fault conditions show that the proposed controller can effectively improve the power system transient performance.

Key words: Static Var Compensator (SVC); Unmanned Aerial Vehicles (UAV); power system; non-affine nonlinear control; inverse system method; Linear Quadratic Regulation (LQR)

1 Introduction

The use of new technologies and the consideration of cost-effectiveness have resulted in the composition of electric power systems has become increasingly complex[1]. At the same time, the stable operation of power systems at largely varying loads has become important but difficult to maintain. The Static Var Compensator (SVC) or State Var System (SVS), also known as the static var generator, is a reactive power electric equipment used in transmission networks[2, 3]. As a part of flexible AC transmission systems[4, 5], the SVC is generally used to adjust the voltage, power parameters, harmonics content, and stability of systems.

In long-distance power transmission systems, SVCs are installed in a line to maintain a constant voltage at the installation point by absorbing from the grid or introducing into it a continuously adjustable reactive power. This operation not only helps ensure the balance of grid reactive power, but also improves the stability and voltage quality of long-distance power transmission subjected to various interferences. SVCs offer broad application prospects because of their good characteristics, including fast control, simple maintenance, and remarkably positive effects. In small-sized power systems, such as those of airplanes, the effective control of energy transport is especially important.

A Hybrid Electric Unmanned Aircraft (HEUA) uses an airborne thermal engine and Energy Storage System (ESS) to drive an electric motor that meets the load demand. Thus, the HEUA electric system can be viewed as a "mobile multi-energy microgrid", as suggested in Refs. [6–8]. The low-voltage energy from renewable sources is applied to low-voltage microgrids and suffers from a high degree of uncertainty; an alternative is to use SVC components, as described in Refs. [9–14].

Introducing SVCs into any power system is a cost effective and efficient means to improve voltage stability.

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The same effect is expected in the microgrids of Unmanned Aerial Vehicles (UAVs). SVCs maintain voltage stability at any point of the installation by continuously absorbing from or injecting into the grid a portion of adjustable reactive power. SVCs benefit the correct balance of grid reactive power. Different SVCs with different characteristics and control methods have various functions in power systems. In low-voltage power distribution systems, SVCs can reduce voltage flicker caused by heavy impact loads, balance asymmetric loads, etc. In power transmission systems, SVCs can reduce instantaneous overvoltage peaks, so as to greatly improve the active power transmission capacity and transient stability of such systems. Figure 1 shows a typical configuration of an HEUA[15], in which three types of energy are interchanged during operation[8].

In other implementations, some hybrid UAVs abandon thermal engines and directly use Permanent Magnet (PM) motors and generators for reducing weight (Fig. 2). The energy performance in such cases is satisfactory, but the electronic control of entire energy systems, their energy storage and energy-saving properties, and the control of energy consumption become exceptionally difficult.

Figure 3 shows the insertion of an SVC into the transmission line of a UAV's microgrid power system. Through the SVC, the line voltage can be quickly and reliably regulated. The SVC usually maintains the voltage at the required setpoint under normal steady-state and transient conditions. The SVC responds to system emergencies, e.g., short circuit and open circuit, and provides dynamic and fast-responding portions of reactive power. In addition, the SVC can increase power transmission capacity, reduce losses, active power oscillations, and prevent overvoltage during loss of load.

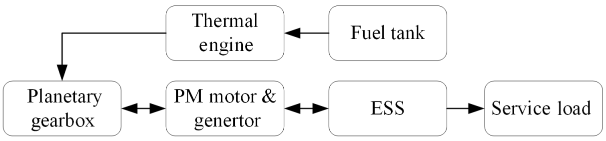

Fig. 1 A typical configuration of an HEUA energy system.

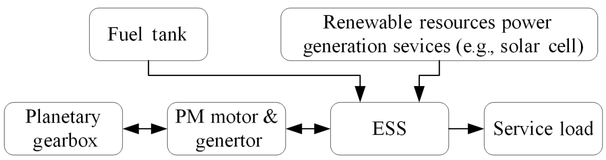

Fig. 2 Structure of hybrid electric control UAVs.

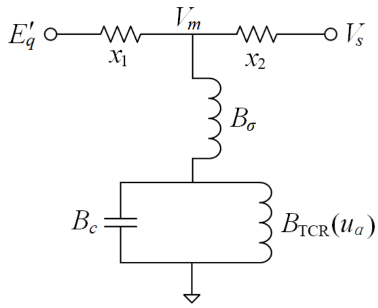

Fig. 3 Equivalent circuit of a single machine microgrid power system bus with the SVC.

Table 1 lists definitions of some parameters.

At present, the research on SVC control mainly covers single variable feedback control[16], auxiliary control[17], linear optimal control[18], adaptive control[19], and nonlinear control[20]. The model of controlled objects must be established as accurately as possible, as it is a prerequisite for implementing advanced control strategies, and thus obtaining high-quality control effects. The identification of the control model used in practice is associated with the concept of control design and the level of development of related theories. Traditional controllers are designed on the basis of equilibrium point neighborhood linearization. This method is based on a concise theory, but its working range is small. Moreover, it cannot easily guarantee the stability of systems when they are subjected to large interferences or failures. The development of nonlinear control theory, especially the systematic and in-depth development of design theory represented by nonlinear differential geometry methods, has facilitated the establishment of control theory based on affine nonlinear models. Hence, affine models have been fully and effectively applied to power systems[20].

By applying the inverse system method to the nonlinear reactive power compensation controller, the nonlinear model of the object can be compensated

Table 1 Parameter definition.

Parameter	Definition
E_q' (V)	Generator transient potential
V_s (V)	Bus voltage of the power system
V_m (V)	Node voltage of the power system
x_1, x_2 (Ω)	Equivalent impedance
B_σ (S), B_c (S)	Admittance of capacitance
$B_{\mathrm{TCR}}(u_\alpha)$ (S)	Admittance of Thyristor Controlled Reactor (TCR)
B_{cn} (S)	Total admittance of n TSCs in parallel
B_L (S)	Admittance of TCR
P_e (W)	Generator output power
T_θ	Time constant of firing angle controller

into a linear model. The relevant literature presents an indirect design for a thyristor-controlled reactor-fixed capacitor-static var compensator[21]. In this design, the fixed capacitor bank is connected in parallel to the TCR, and the passage through the total admittance is controlled as an input value. The research object in the current work is the SVC composed of a TCR-type device and multiple Thyristor-Switched Capacitor (TSC)-type devices. The total admittance of this SVC has a larger range of adjustment than those of other SVCs.

2 Application of Nonlinear Control in SVC System

2.1 Application of recently introduced control technology in SVC system

The power system is one of the largest man-made nonlinear dynamic systems. It is a massive system with strong nonlinearity, time-varying properties, parameter uncertainty, and unmodeled dynamics. The power system is widely distributed, and most of the components have complex nonlinear characteristics, such as delay, hysteresis, and saturation. With the emergence of high-power power electronic devices and the development of computers, nonlinear control theories and methods have also been applied to control reactive power compensation systems.

Optimal control theory is an important part of modern control theory. Since the 1970s, optimal control theory has been widely used in power systems. This theory includes generator excitation control, fast valve control for generator sets, and optimal time control; it also involves a combination of devices that are applied to control operations. The work in Ref. [22] comprehensively discussed the application of linear optimal control theory in power systems to many fields, including excitation control, valve control, single-machine infinity models, multimachine models, optimal control, suboptimal control, and optimal time control.

Differential geometric control theory has been widely used in actual control problems, and it exposes unique nonlinear phenomena. In control research, this theory is also called precision linearization or feedback linearization. Researchers have applied differential geometric control theory to the stability control of multimachine power systems[23]. Moreover, monographs[20] have provided a detailed introduction to various applications of differential geometric control, such as excitation control, valve opening control, DC system control in AC/DC transmission systems, and SVCs. In Ref. [23], the researchers proved that the optimal control law for a transformed linear system is the optimal control law in a certain sense of the original system.

In addition to the aforementioned control theories that have been widely used in the control of power systems, adaptive control theory, artificial neural networks, fuzzy set theory, expert systems, and many others, have been applied to power systems. For the sake of convenience, this work compares the application research status of several linearization methods in power system control. The differential geometry method effectively solves a series of nonlinear control problems in power systems, but the process of solving the control law is cumbersome, and the method is only suitable for affine nonlinear systems.

The physical meaning of the inverse system method is intuitive and is not limited to affine nonlinear systems. It only needs to perform derivative operations and algebraic operations when used, and the process of solving the control law is relatively simple and convenient for engineering applications[24]. In recent years, the inverse system method has been widely used in the nonlinear control of power systems because of its intuitive concept, simple mathematical tools, and easy understanding.

In Refs. [25, 26], the inverse system method was used to simplify the process of solving problems, such as excitation control, valve control, converter station control, and reactive power compensator control. Sun et al.[27] used the inverse system method in power systems, combined it with the ITAE optimal control law, and derived the nonlinear excitation control law of the generator. The results showed that the generated nonlinear excitation controller designed on the basis of the inverse system method can greatly improve the dynamic performance of systems. At the same time, the linearization process of this method is much simpler than that of the differential geometry method, and it is more suitable for engineering applications. Reference [28] used the inverse system method to transform the nonlinear control problem of an SVC device into the corresponding linearization control problem. A nonlinear SVC controller was designed, and it was found to significantly improve the stability of the power system. In Ref. [29], the neural network-order inverse system control strategy was used in the control of a controllable series capacitor compensation device. Aiming at a single-machine infinite double-circuit power system simplified from an actual equivalent system, they designed a neural network-order inverse system with strong adaptive identification and additional linear controllers to greatly improve the transient stability of power systems. Table 2 presents the details of several linearization methods.

2.2 Application of inverse system method in SVC system

The differential geometry method effectively solves a series of nonlinear control problems in power systems, but the process of solving the control law is very cumbersome, and it is only suitable for affine nonlinear systems.

The inverse system method[37] is a feedback linearization method that differs from the differential geometry method. The physical meaning of the inverse system method is intuitive and is not limited to affine nonlinear systems. This method only needs to perform derivative operations and algebraic operations during use, and the process of solving the control law is relatively simple and convenient for engineering applications[24]. The basic idea of the inverse system method is as follows. The inverse system of an object is used to form an order integral inverse system that can be realized by the feedback method. The object is then compensated into a system with a linear transfer relationship, that is, the actual system equivalent, which has the same nonlinear properties as the original system. A neural network-order inverse system with strong adaptive identification and additional linear controllers is designed to greatly improve the transient stability of power systems. Ge and Li[21] established a non-affine nonlinear model for a single-machine infinite bus system with an SVC, and then designed a controller on the basis of nonlinear feedback linearization and linear quadratic optimal control law according to the inverse system method.

Generally, for a nonlinear system Σ with a general form, there are the following forms:

$$\Sigma: \begin{cases} \dot{\boldsymbol{X}} = F[\boldsymbol{X}, \boldsymbol{U}], \quad \boldsymbol{X}(t_0) = \boldsymbol{X}_0 & (1) \\ \boldsymbol{Y} = G[\boldsymbol{X}, \boldsymbol{U}] & (2) \end{cases}$$

where $F[\,]$ is the model function of the system, and $G[\,]$ is the output function of the system. $\boldsymbol{X} \in \mathbf{R}^n$ is the state vector, $\boldsymbol{U} \in \mathbf{R}^m$ is the input vector, and $\boldsymbol{Y} \in \mathbf{R}^r$ is the output vector.

Obviously, the output of Eq. (2) can also be written as follows:

$$\begin{cases} y_1 = g_1(\boldsymbol{X}, \boldsymbol{U}) \\ \quad \vdots \\ y_r = g_r(\boldsymbol{X}, \boldsymbol{U}) \end{cases} \tag{3}$$

Derivative transformation is performed on Eq. (3), $g_1, g_2, \ldots, g_r$ are the components of the system output function. For $y_1, y_2, \ldots, y_r$ in turn, $\alpha_1, \alpha_2, \ldots, \alpha_r$-order derivative with respect to time t is calculated as follows:

Table 2 Application of several linearization methods in power system control.

Method	Control problem	Reference	Research content and result
Local linearization	Excitation control	[30]	Early application of optimal control to power systems
Differential geometry method for precise linearization	Excitation control	[23]	Multi-machine system distributed control
Differential geometry method for precise linearization	Various control issues	[24]	Give an easy-to-implement excitation control law
Differential geometry method for precise linearization	Various control issues	[20]	The nonlinear control theory and application of power system are systematically expounded.
Differential geometry method for precise linearization	Integrated control	[31]	Control quantity is the excitation voltage, and the output is the voltage and power angle.
Direct feedback linearization	Voltage control	[32]	Decentralized control of multi-machine power system with robust control
Direct feedback linearization	Valve control	[33]	Easier to use than nonlinear differential geometry theory conditionally
Direct feedback linearization	Reactive power compensator	[34]	Enhance system damping and voltage accuracy
Inverse system approach	Excitation control	[27]	The inverse system method of nonlinear system is applied to power system, and the nonlinear excitation control law of generator is deduced.
Inverse system approach	Various control issues	[25–28, 35, 36]	Static reactive power compensation, excitation, and valve control

$$\begin{cases} y_1^{(\alpha_1)} = h_1(\boldsymbol{X}, \boldsymbol{U}) \\ \quad\vdots \\ y_r^{(\alpha_r)} = h_r(\boldsymbol{X}, \boldsymbol{U}) \end{cases} \tag{4}$$

where $h_1, h_2, \ldots, h_r$ are the derivatives of α_1-order to α_r-order of each output component of the system.

The value of $\alpha_i\,(1 \leqslant i \leqslant r)$ is defined as follows:

$$\begin{cases} \dfrac{\partial\left[F^k g_i\right]}{\partial \boldsymbol{U}} \equiv 0,\ \ k = 0, 1, \ldots, \alpha_i - 1; \\ \dfrac{\partial\left[F^k g_i\right]}{\partial \boldsymbol{U}} \neq 0,\ \ k = \alpha_i \end{cases} \tag{5}$$

where $Fg_i = \left[\dfrac{\partial g_i}{\partial \boldsymbol{X}^{\mathrm{T}}}\right]F$, and $F^k g_i = F\left(F^{k-1} g_i\right)$. Moreover, when $k = 0$, $F^0 g_i = g_i$. Suppose that in the above derivation process, for $1 \leqslant i \leqslant r$, exists $\alpha_i < \infty$.

In Eq. (4), if $\boldsymbol{U}$ is a function of $\boldsymbol{X}$ and $(y_1^{(\alpha_1)}, y_2^{(\alpha_2)}, \ldots, y_r^{(\alpha_r)})^{\mathrm{T}}$, a new system equation can be formed, that is the inverse system of the original system Σ^{-1},

$$\boldsymbol{U} = H^{-1}\left(\boldsymbol{X}, \boldsymbol{Y}^{\alpha}\right) \tag{6}$$

$$\Sigma^{-1}: \begin{cases} \dot{\boldsymbol{X}} = F\left[\boldsymbol{X}, H^{-1}\left(\boldsymbol{X}, \boldsymbol{Y}^{\alpha}\right)\right],\ \boldsymbol{X}(t_0) = \boldsymbol{X}_0 & (7) \\ \boldsymbol{U} = H^{-1}\left(\boldsymbol{X}, \boldsymbol{Y}^{\alpha}\right) & (8) \end{cases}$$

where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_r)$ is an r-dimensional vector, and $H = (h_1, h_2, \ldots, h_r)^{\mathrm{T}}$.

Replace $\boldsymbol{Y}^{\alpha}$ in Eqs. (7) and (8) with $\boldsymbol{\varphi} = (\varphi_1, \varphi_2, \ldots, \varphi_r)^{\mathrm{T}}$ to further form the α-order integral inverse system Σ_{α}^{-1} of the original system,

$$\Sigma_{\alpha}^{-1}: \begin{cases} \dot{\boldsymbol{X}} = F[\boldsymbol{X}, H^{-1}(\boldsymbol{X}, \boldsymbol{Y}^{\alpha})], \\ \dot{\boldsymbol{X}} = F[\boldsymbol{X}, H^{-1}(\boldsymbol{X}, \boldsymbol{\varphi})] & (9) \\ \boldsymbol{U} = H^{-1}(\boldsymbol{X}, \boldsymbol{\varphi}) & (10) \end{cases}$$

Then, by connecting Σ_{α}^{-1} in series before the original system Σ, the corresponding pseudolinear system can be obtained. The transfer relationship can be expressed by the following equation:

$$\operatorname{diag}\left(D^{\alpha_1}, D^{\alpha_2}, \ldots, D^{\alpha_r}\right)\boldsymbol{Y} = \boldsymbol{\varphi} \tag{11}$$

$\boldsymbol{X}$ in Eqs. (9) and (10) is replaced with the feedback of the corresponding quantity in the original system. The structure diagram of the pseudolinear system corresponding to the original system Σ after equivalent feedback realization is shown in Fig. 4.

For the resulting pseudo-linear system, linear control theory can be used to complete the system integration.

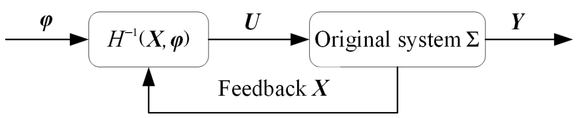

Fig. 4 Structure diagram of pseudo-linear system.

3 SVC Structure and Its Mathematical Model

The simple form of SVC is a continuously adjustable reactor TCR connected to the system (Fig. 3). The admittance of TCR is[20]

$$B_{\mathrm{TCR}} = \frac{2\pi - 2\theta + \sin 2\theta}{\pi} B_L = \frac{B_L}{\pi}[2\pi - \Psi(\theta)] \tag{12}$$

where $\Psi(\theta) = 2\theta - \sin 2\theta$, $B_L = -1/(\omega L)$, L is the inductance of the reactor, and θ is the trigger angle of the thyristor. When θ continuously changes between 90° and 180°, B_{TCR} continuously changes between B_L and 0.

In actual systems, one TCR device and multiple TSC devices are also commonly used to form another type of static reactive power compensation system TCR-TSC SVC.

The design of the SVC controller is usually concentrated on an approximate linearization model or an affine nonlinear model[16–18]. This paper aims to establish a more general non-affine nonlinear control model and design a combined controller, based on the inverse system method and the quadratic optimization for this model.

The single machine microgrid power system considered herein is shown in Eq. (13), which is described by a set of variables and parameters (Table 3).

$$P_e = \frac{E_q' V_s \sin\delta}{x_1 + x_2 + x_1 x_2 \dfrac{B_{\sigma}\left[B_{cn} + \dfrac{B_L}{\pi}[2\pi - \Psi(\theta)]\right]}{B_{\sigma} + B_{cn} + \dfrac{B_L}{\pi}[2\pi - \Psi(\theta)]}} \tag{13}$$

As shown in Fig. 3, x_1 is the sum of generator transient reactance, transformer reactance, and transmission line reactance between the node of the power system and the generator, and the x_2 is the reactance of the transmission

Table 3 System parameter list and definition.

Parameter	Definition
P_m (W)	Generator mechanical power
δ (°)	Generator rotor operating angle
ω (rad/s)	Generator speed
H (kg·m^2)	Unit moment of inertia
D	Damping coefficient
B_{TCR} (S)	Base waveguide of the reactor controlled by the thyristor
B_{SVC} (S)	SVC admittance
K_{θ}	Magnification of the firing angle controller
u_{θ} (V)	Control signal quantity
y	Output of the system

line between the bus voltage of the power system and the node voltage of the power system.

Thus, the single-machine power system bus with SVC can be described by the following second-order incremental non-affine nonlinear equation:

$$\Sigma:\begin{cases}\Delta\dot{\delta}(t)=\Delta\omega(t)\\ \Delta\dot{\omega}(t)=\dfrac{\omega_0}{H}(P_m-N)-\dfrac{D}{\omega_0}\Delta\omega(t)\\ \Delta(\dot{\theta})=\dfrac{1}{T_\theta}(-\Delta\theta+K_\theta u_\theta)\\ y=\Delta\delta(t)\end{cases} \tag{14}$$

where

$$N=\frac{E_q' V_s\sin\delta}{x_1+x_2+x_1x_2\dfrac{B_\sigma\left[B_c+\dfrac{B_L}{\pi}(2\pi-\Psi(\theta))\right]}{B_\sigma+B_c+\dfrac{B_L}{\pi}[2\pi-\Psi(\theta)]}} \tag{15}$$

The increments are defined as: $\Delta\delta(t)=\delta(t)-\delta_0,\Delta\omega(t)=\omega(t)-\omega_0,\Delta\theta(t)=\theta(t)-\theta_0$, and the point $(\delta_0,\omega_0,\theta_0)$ is the balance point when $u_\theta=0$.

The model uses directly the thyristor firing angle θ as a control quantity, which has stronger operability.

4 Controller Design

As the response speed of the firing angle controller is much faster than that of the nonlinear system represented by Eqs. (14) and (15), that is, T_θ in Eqs. (14) and (15) is sufficiently small, the inertial link can be expressed as $\Delta\theta=K_\theta u_\theta$, then $\Delta\dot{\theta}=0$. Therefore, a non-affine nonlinear system reduced to a second-order is obtained.

In designing the controller, the time derivatives of y should be obtained.

$$\dot{y}=\Delta\dot{\delta}(t)=\Delta\omega(t) \tag{16}$$

$$\ddot{y}=\Delta\dot{\omega}(t)=\frac{\omega_0}{H}\left(P_m-\frac{E_q' V_s\sin\delta}{x_1+x_2+x_1x_2\dfrac{B_\sigma\left[B_c+\dfrac{B_L}{\pi}[2\pi-\Psi(\theta_0+K_\theta u_\theta)]\right]}{B_\sigma+B_c+\dfrac{B_L}{\pi}[2\pi-\Psi(\theta_0+K_\theta u_\theta)]}}-\frac{D}{\omega_0}\Delta\omega(t)\right) \tag{17}$$

Clearly, u_a is explicitly included in the definition of $\ddot{y}$, so the inverse system is a second-order system. Solve the inverse system equation from Eqs. (14) and (15), and replace the variable with $\eta=\ddot{y}$, then get the feedback control law of system composed of the output equation of the second-order integral inverse system,

$$\hat{\Sigma}_\theta:\begin{cases}u_\theta=\dfrac{\Psi^{-1}(\beta)-\theta_0}{K_\theta}\\ \beta=2\pi-\dfrac{\pi B_{\mathrm{TCR}}}{B_L}\\ B_{\mathrm{TCR}}=\dfrac{B_\sigma(B_c-B_{\mathrm{SVC}})-B_{cn}B_{\mathrm{SVC}}}{B_{\mathrm{SVC}}-B_\sigma}\\ B_{\mathrm{SVC}}=\dfrac{E_q' V_s\sin\delta}{x_1x_2\left[P_m-\dfrac{H}{\omega_0}\eta-\dfrac{D}{\omega_0}\Delta\omega(t)\right]}-\dfrac{x_1+x_2}{x_1x_2}\end{cases} \tag{18}$$

Although the inverse system method may encounter singularity, the system described in this article will not be 0 under normal operating conditions (i.e., the denominator in Eq. (18) will not be 0. By definition, K_θ is not 0, B_L is not 0, and $B_{\mathrm{SVC}}-B_\sigma$ is not 0. Then, x_1 is the sum of the generator's transient reactance, which is not 0. x_2 is the reactance of the transmission and is not 0. P_m in Eq. (18) is the generator's mechanical power, which is not 0 and is far from 0. In $H\eta/\omega_0$ and $D\Delta\omega(t)/\omega_0$, η and $\omega(t)$ are decimals approaching 0. Therefore, under normal operating conditions, $H\eta/\omega_0$ and $D\Delta\omega(t)/\omega_0$ must operate in the neighborhood of 0. Therefore, the value of $(P_m-H\eta/\omega_0-D\Delta\omega(t)/\omega_0)$ is still far away from 0.

The control law is fed back to the input of the original system in Eqs. (14) and (15), and the obtained second-order pseudo-linear system satisfies $\eta=\ddot{y}$. For this second-order pseudolinear system, the design can be further completed by the principle of linear control. For this type of second-order system, the expected transfer function can be directly set as

$$\frac{y(s)}{\gamma(s)}=\frac{1}{s^2+2\xi\omega s+\omega^2} \tag{19}$$

where γ is the reference input, ξ is the system damping coefficient, and ω is the system oscillation frequency. Thus, the required linear control law is

$$\eta=\gamma-(\omega^2 y+2\xi\omega y') \tag{20}$$

In the design problem of the second-order system in this work, let $\omega=5$, $\xi=0.707$, considering the adjustment problem of the system, let $\gamma=0$. The result of the pole configuration can then be transformed into a pair of complex numbers based on

$$s_{1,2}=\frac{1}{2}\left[-2\xi\omega\pm\sqrt{4\xi^2\omega^2-4\omega^2}\right]=\omega\left[-\xi\pm\sqrt{\xi^2-1}\right] \tag{21}$$

Thus $s_{1,2}=-3.535\pm3.535\mathrm{j}$.

5 Design of Control System Based on Simulink

Based on MATLAB, the dynamic simulation environment and its S-Function programming technology construct a non-affine nonlinear control system, based on differential algebraic equations, is shown in Fig. 5.

The parameters of the generator, line, and SVC used in the simulation can be selected according to Refs. [8, 9], shown in Table 4, where x'_d is the d-axis transient reactance of the generator, x_T is the transformer reactance, and x_q is the q-axis reactance of the generator.

Figures 6–9 show the response of the system with consideration of the initial value disturbance, short circuit, and their combined effect. Each group of four plots (a), (b), (c), and (d) represents the change in the generator's rotor operating angle δ, the SVC's access point voltage V_m (per-unit value), the generator's output active power P_e (per-unit value), and the SVC's adjustable susceptance B_{SVC} (per-unit value) under the same disturbance, respectively.

Fgiure 6a shows the meaning of each curve, so do Fig. 7a, Fig. 8a, and Fig. 9a. To facilitate the comparison, this study introduces approximate linearization optimal control as a comparison scheme, it is shown as a dotted line. To reflect the actual effect of control in the simulation process, this study introduces the simulation curves in (b), (c), and (d) in Figs. 6–9 into the limiting link. A large disturbance is selected for simulation.

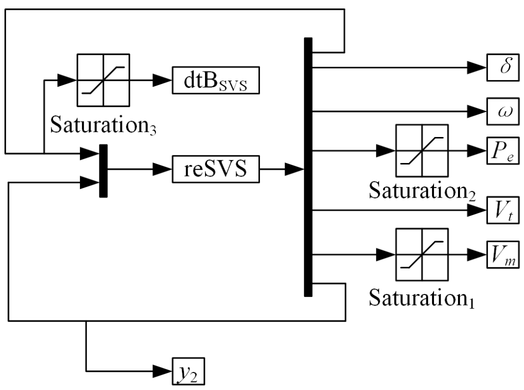

Fig. 5 Block diagram of Simulink simulation system on MATLAB.

Table 4 Simulation parameter and value.

Parameter	Value	Parameter	Value
B_C	0.65 S	x_1	0.4 Ω
B_L	−0.6 S	x_2	0.4 Ω
x'_d	0.32 Ω	x_q	0.48 Ω
D	1	V_s	1 V
H	5 kg·m^2	K_θ	5
x_T	0.12 Ω	B_σ	−6 S

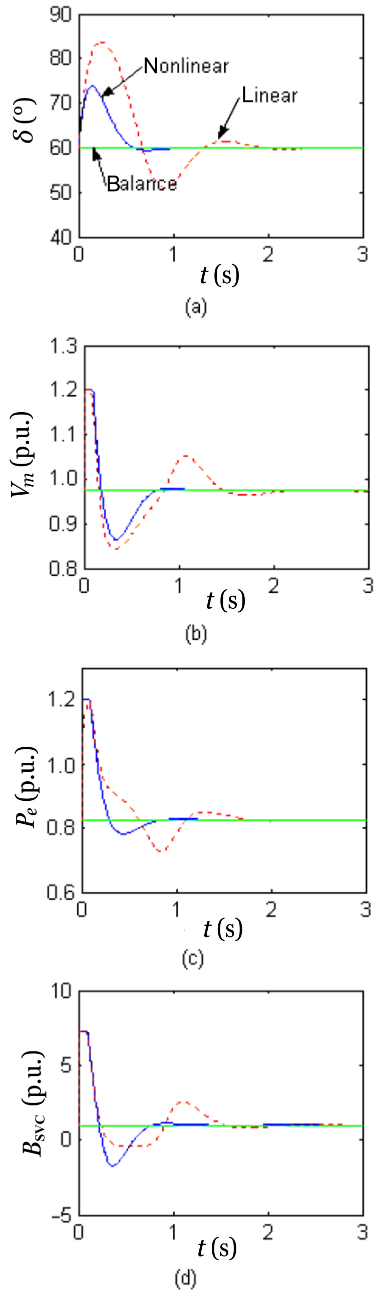

Fig. 6 Response of the system when the rotor angular velocity change is Δω=4.

The comparison of the simulation curves in Figs. 6–9 shows that when the power system is equipped with an SVC with the control law proposed in this work, the anti-interference stability of the system is greatly improved, and the control effect is clearly better than that of the linear control strategy. This result is especially true when the system is subjected to large and comprehensive

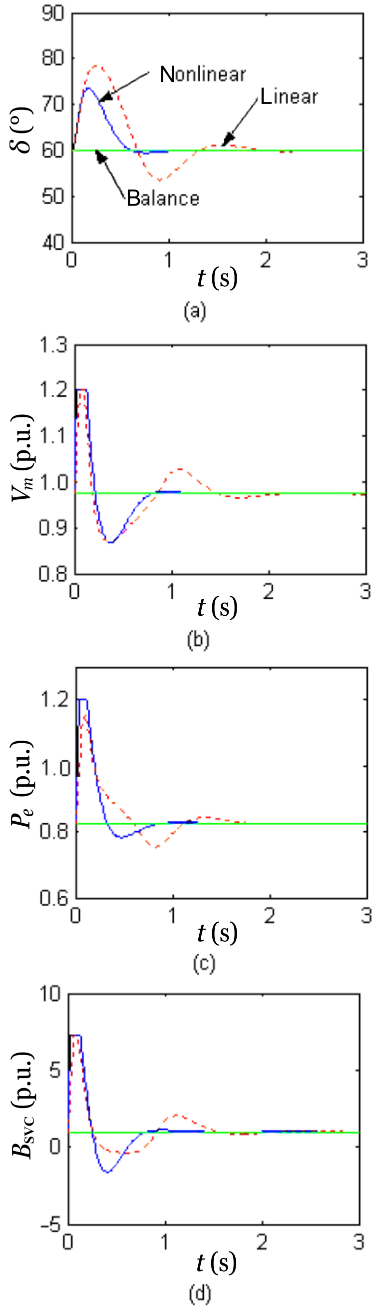

Fig. 7 Three-phase-to-ground short circuit starts from 0 s, and the system response for troubleshooting after 0.06 s.

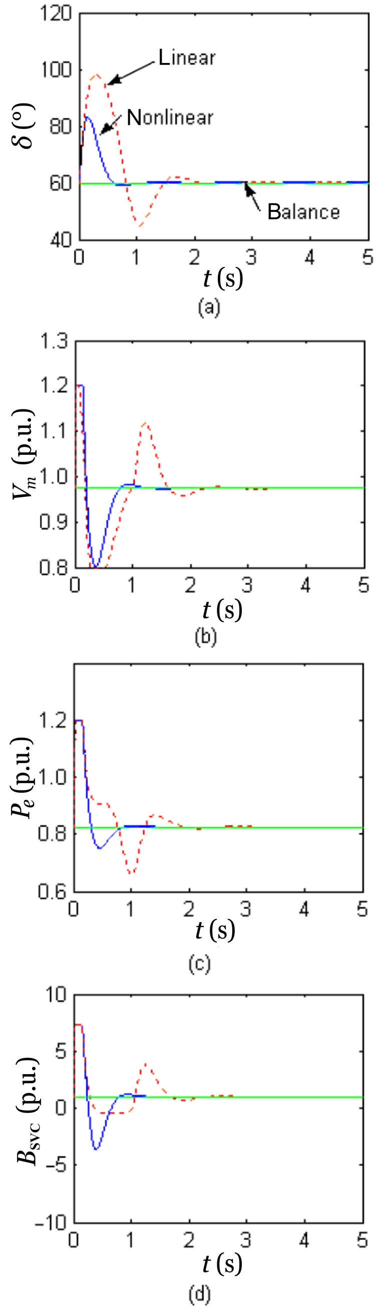

Fig. 8 Response of the system when the rotor angular velocity change is $\Delta\omega$=3 and the short circuit time is 0.04 s.

disturbances, e.g., rotor's angular velocity change $\Delta\omega = 4$ and short circuit time of 0.06 s. In this case, the linear solution cannot restore the system's normal operation, whereas the nonlinear solution is not affected significantly. Meanwhile, the control law designed in this work can effectively improve the dynamic performance of the power system. When the system is disturbed, the SVC can provide appropriate damping, so that the system achieves a good dynamic response, but it can also quickly produce active and reactive power. The power compensation quickly stabilizes the system back to the normal operating point, and thus keeping the voltage constant.

6 Conclusion

In the case of two disturbances acting simultaneously, the linear control scheme for small disturbance and medium disturbance can restore balance after a long time (compared to the nonlinear control scheme). However, in the case of large disturbances, the linear control scheme

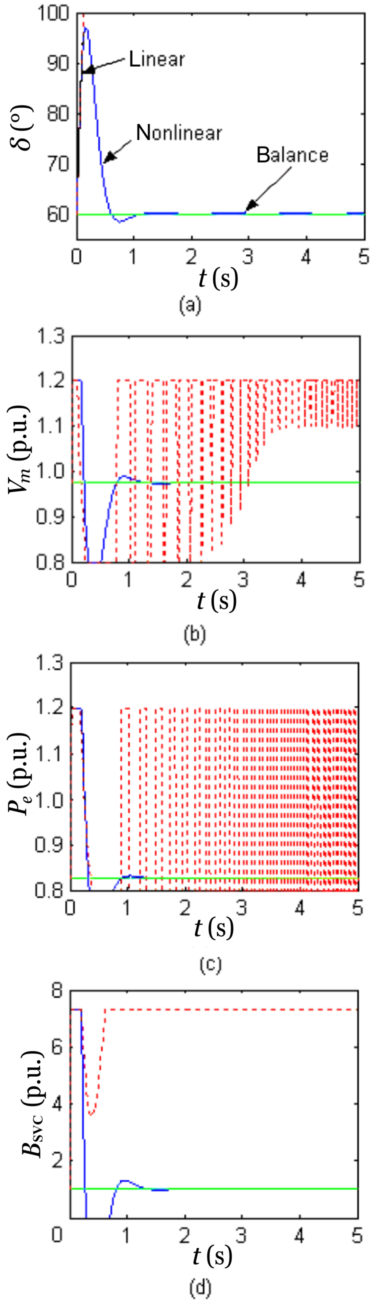

Fig. 9 System response when the rotor angular velocity change is $\Delta\omega$=4 and the short-circuit time is 0.06 s.

curve diverges after 0.1 s, while the nonlinear control scheme curve restores balance after 2 s. This comparison shows the advantages of nonlinear control.

In this paper, based on the Simulink dynamic simulation platform of Matlab, in combination with the S-Function programming technology, a non-affine nonlinear model based on differential-algebraic equations is constructed for a single-machine UAV microgrid power system with SVC. As a result, a combined controller based on the inverse system method and a quadratic optimization is designed.

Simulations are carried out under three conditions of initial value disturbance: short circuit, initial value disturbance, and short circuit integrated disturbance. The results show that the controller designed in this paper can significantly improve the dynamic performance of the power system and also the anti-interference ability of the system.

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