Phenomenological Approach to Simulation of Propagation of Spots over Cathodes of High-Power Vacuum Circuit Breakers

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***Abstract*- A phenomenological description of an ensemble of a large number of spots on negative contacts of high-power vacuum circuit breakers is developed by means of generalization of the concept of random walk of a single cathode spot in low-current vacuum arcs. The model is formulated in terms of a convection-diffusion equation governing the evolution of the number of spots per unit area, taking into account the variation of the number of spots with the arc current and the "retrograde repulsion" between spots. The approach is applied to description of the distribution of cathode spots during the initial expansion process after arc ignition in conditions of two independent experiments simulating high-power switches. A reasonably good agreement between the theory and the experiment is found. The developed model can be used as a module of global numerical models of the interruption process in high-power vacuum circuit breakers.**

I. INTRODUCTION

A significant amount of observations of current spots on cathodes of vacuum discharges has been made in laboratory experiments; e.g., [1]. A spot exists only for a short period of time, of the order of microsecond, after which the spot is extinguished and a new spot appears at some distance from the first one. At the macroscopic level, this phenomenon can be described as a random walk of a spot along the cathode surface.

If an external magnetic field tangential to the surface of the cathode is present, then an ordered motion is superimposed over the random walk. This motion is directed against the Lorentz force exerted by the magnetic field over the spot current and is usually referred to as retrograde. If two spots exist at the same moment in close positions, then they tend to move in opposite directions.

There are many works analyzing motion of a spot on cathodes of vacuum arc discharges under conditions of laboratory experiments. In [2]-[5], results of experiments were analyzed in terms of isotropic random walk of a free burning cathode spot. In [6], a convection-diffusion equation was postulated in order to describe the motion of cathode spots of vacuum arcs in the presence of external transverse and axial magnetic fields. Statistical methods are developed in order to interpret the random walk and retrograde motion of a cathode spot; see, e.g., [7]-[9].

In [10], the Monte Carlo method is used to simulate the random walk of a cathode spot with the aim to describe variations in velocity and trail width of arc spots initiated on nanostructured tungsten cathode with different thicknesses of the nanostructured layer, observed on the boundary of the thick and thin nanostructured layer regions.

All these approaches are phenomenological in the sense that they rely not on analysis of physical processes governing the spot motion (such as processes responsible for the retrograde motion) but rather on experimental observations.

With the exception of the work [4], all the above-mentioned works refer to conditions of laboratory experiments where the arc current is limited (does not exceed 100 A) and there is just one or a few spots on the cathode surface at any given moment.

Cathode attachment of an arc in a high-power vacuum current breaker represents an ensemble consisting of a large amount of individual spots operating simultaneously; e.g., [11]. In order to get a representative value, one can assume an arc current of 10 kA and a current per spot of 45 A [11], then the number of spots operating simultaneously is around 220. In this work, the above-described phenomenological approach is modified for the case of a large number of spots operating simultaneously and applied to conditions of high-power circuit breakers.

II. THEORETICAL

The motion of a cathode spot on a two-dimensional surface can be described by a random walk assuming that the motion of the spot is a sequence of displacements in x and y direction with a characteristic step length s and a characteristic time interval T. In conditions of isotropic diffusion and constant drift

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velocity, if distance s and time step T are small enough, the following equation governing the evolution of the probability density function $f(x,y,t)$ for the spot to be at position (x,y) at time t can be obtained (e.g., [6])

$$(\partial f/\partial t) + \nabla\cdot\Gamma = 0, \quad \Gamma = -D\nabla f + f\boldsymbol{v}, \tag{1}$$

where the diffusion coefficient D and drift velocity $\boldsymbol{v}$ depend on s and T, and on the probabilities that the spot during each time interval T travel a distance s in same x and y direction.

Assuming that there is no interaction between individual spots and multiplying (1) by the number N of spots, we obtain an equation governing the evolution of the surface density Ω of spots

$$(\partial\Omega/\partial t) + \nabla\cdot\Gamma = 0, \quad \Gamma = -D\nabla\Omega + \Omega\boldsymbol{v}, \tag{2}$$

where $\Omega = \Omega(x,y,t) = Nf(x,y,t)$. D and $\boldsymbol{v}$ have the meaning of a diffusion coefficient and of a convective (drift) velocity.

We stress that (1) applies to an individual spot and the unknown function f is the probability density function of a spot at a given time being localized at a given position. On the other hand, (2) applies to an ensemble of a large number of spots and the unknown function Ω is the surface density of spots at a given time at a given position. A further difference will appear when we introduce an account of a variation of the total number of spots on the contact with the temporal variation of the total arc current, which is essential in order to apply (2) to contacts of high-power circuit breakers. This amounts to introducing S the net local rate of creation of new spots into (2).

At this stage, we assume that the spots cannot go outside the boundary of the contact. We will also assume that the initial distribution $\Omega(x,y,t)$ is known and the net local rate of creation of new spots is proportional to the local density of those already existing: $S = \beta\Omega$, where $\beta = \beta(t)$ is a coefficient which depends on time but not on position on the cathode surface. Substituting this approximation into (2) and integrating over the cathode surface, one obtains

$$(dN/\mathrm{d}t) = \beta N, \tag{3}$$

where $N = \int\Omega ds$ is the total number of spots on the contact. Since $N = N(t) = I(t)/I_{\mathrm{spot}}$, being I_{spot} the current per spot, one can employ (3) in order to determine β in terms of the current wave.

The drift velocity $\boldsymbol{v}$ in (2) is represented as $\boldsymbol{v} = \boldsymbol{v}_1 + \boldsymbol{v}_2$, where $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are drift velocities associated with the retrograde motion in an external tangential magnetic field and with the "repulsion" of spots. The magnitude of drift velocity $\boldsymbol{v}_1$ in an external tangential magnetic field $\boldsymbol{B}_{\mathrm{t}}$ may be approximated as $v_1 = kB_{\mathrm{t}}$ [12], being k a proportionality constant.

The "repulsion" of spots is simulated as follows. The smaller the local scale of spatial variation in the density of spots, the greater the degree of anisotropy of the random walk, and therefore the greater the speed of ordered motion. If the anisotropy is still small, then the relation between the inverse local scale of spatial variation in the density of spots, $|\nabla(\ln\Omega)|$, and v_2 should be linear. The ordered motion is directed to where the density of spots decreases fastest, so $\boldsymbol{v}_2$ has the direction of $-\nabla\Omega$. Thus, one can set $\boldsymbol{v}_2 = -\delta D\nabla(\ln\Omega)$, where δ is a dimensionless constant parameter. In practical terms, this approach is equivalent to replacing the diffusion coefficient D in (2) by an effective diffusion coefficient $D_{\mathrm{eff}} = (1 + \delta)\ D.$ In other words, the "repulsion" transport enhances the diffusion (random-walk) transport by the factor of $1 + \delta$.

III. Analysis of Experiments

A. Description of cathode spot motion during sinusoidal current wave in the presence of external magnetic fields

As the first example, let us consider the distribution of cathode spots in a vacuum arc that was burning between axial magnetic field (AMF) contacts during a sinusoidal current half cycle in the presence of a superimposed (external) transversal magnetic field (TMF).

The measurements have been made in a synthetic test circuit similar to the setup described in [14]-[15]. The current half cycle had a peak amplitude of 3.68 kA (2.60 kA rms) and a duration of 9.50 ms (approx. 53 Hz). The arc was drawn by separating the movable anode from the fixed cathode immediately, i.e. 30 µs after the current onset $t = 0$ (arcing time 9.47 ms) at an average speed of 0.54 m/s up to a gap distance of 4.1 mm. The contact disks of 40 mm in diameter had planar surfaces without any slots and were made of copper-chromium (arc-melted CuCr50). At fully opened contact gap, the AMF had a bell-shaped radial profile with a specific flux density of 11.1 µT/A near the center of the cathode surface corresponding to 40.8 mT at current peak. The AMF was oriented perpendicular to the disk surfaces and directed from the anode to the cathode.

The TMF superimposed over the arc was produced by a nearby current return conductor, the current therein flowing anti-parallel to and in phase with the current in the arc. Thus the TMF was in phase with the arc current and was oriented parallel to the disk surfaces. The field was uni-directional in the contact gap and had a specific flux density of 1.7 µT/A near the center of the cathode surface corresponding to 6.25 mT at current peak. The Lorentz force associated with the TMF was "repulsing" the arc in a direction opposite to the return conductor.

The contact system was mounted inside a continuously pumped demountable ultrahigh vacuum chamber. A quartz window allowed a side-on view on the cathode surface at an angle of approximately 80 degree (1.4 rad) to the surface normal. A high-speed black-and-white CCD video camera (Phantom V7) was used to observe the cathode spots at a rate of 33,057 frames/s (interval 30.25 μs between two frames) and an exposure time as low as 2 μs. The image area of the camera chip covered 290×29 pixels corresponding to a recorded object area of 40×4 mm.

Due to the characteristics of the drive system opening the contacts, the arc was drawn near the edge of the contact disks before it diffused over the gap. The first spots were ignited in the half plane facing the return conductor. The spots then moved across the cathode disk in the direction of the (external) TMF field vector and perpendicular to it – with a tendency to drift parallel to the TMF Lorentz force.

Using the high-speed video recordings, the spatial and temporal evolution of the cathode spots was identified during the current half-cycle and the spot motion was analyzed by means of the Rayleigh distribution, which is an inherent characteristic of random walk.

The spot motion parameters were deduced from the experiments according to the following procedure.

First, the spot locations ($x(t)$, $y(t)$) recorded at time t after current onset $t = 0$ and related to the instantaneous arc current $I_{arc}(t)$ were analyzed by means of a commercial software (ImageJ) which allows the quantitative spatial identification of spots and tracks in video frames. Then the distance $r(t)$ of each spot location measured from the location (x_0, y_0) of the first ignited spot, occurring at t_0 = 30 μs, was evaluated according the relation

$$r(t) = [(x(t) - x_0)^2 + (y(t) - y_0)^2]^{1/2}. \qquad (4)$$

Finally, the spatial distribution of the spots was analyzed by means of the Rayleigh distribution $f(r)$,

$$f(r) = (r/(2 D_{eff} (t - t_0))) \exp(-r^2/(4D_{eff} (t - t_0))). \qquad (5)$$

The quantity $f(r)dr$ describes the probability that the spot displacement during the interval [t_0, t] is in the range [r, $r+dr$]. The diffusion coefficient D_{eff} was then determined for a given time interval [t_0, t] by plotting the logarithm of the probability $p(r > R)$ that r exceeds a given value R as a function of R^2/t (see for example [2]) according to

$$p(r>R) = \exp(-R^2/(4D_{eff} (t - t_0))). \qquad (6)$$

Table 1 summarizes the values retrieved from the experimental data: the observation time t with respect to current onset $t = 0$ (approx. equal to ignition time t_0), the total arc current I_{arc}, the number N_{spot} of recorded spots, the average current I_{spot} of a single spot, and the effective diffusion coefficient D_{eff}.

The spot currents in table 1 are average values not taking into account that different spots may carry different currents. It should be noted that the luminosity of individual spots noticeably varies; some small spots may even be too dim to be distinguished from noise and hence are not taken into account either. Note that the average current per spot during the whole half-cycle is approximately 24 A.

TABLE 1. ANALYSIS RESULTS

t (ms)	I_{arc} (kA)	N_{spot}	I_{spot} (A)	D_{eff} (10^{-3} m^2/s)
0.9	1.1	34	33	7.5
1.4	1.6	47	35	8.4
1.8	2.1	71	30	14.4
2.3	2.5	100	25	11.2
2.7	2.9	122	24	10.4
3.2	3.2	159	20	7.8
3.6	3.4	162	21	9.1
4.1	3.6	154	23	10.4
4.6	3.7	160	23	7.8
5.0	3.7	177	21	7.2
5.5	3.6	164	22	5.8
5.9	3.4	153	22	7.6
6.4	3.2	129	24	8.6
6.8	2.8	112	25	4.4
7.6	2.2	80	27	3.1
8.0	1.7	65	26	5.6
8.5	1.2	51	24	3.3
8.9	0.7	35	19	3.4
9.4	0.2	15	10	5.4

One can see that the effective diffusion coefficient in Table 1 maintains the same order of magnitude for all t; the average value is 7.5×10^{-3} m^2/s. This value exceeds by a factor of 75 the value of 10^{-4} m^2/s, reported for a single spot on a CuCr cathode [4].

Thus, it has been shown that the motion of spots in an ensemble of a large number of spots on negative contacts of high-power vacuum circuit breakers reveals the same Rayleigh distribution as the random walk of a single cathode spot in low-current vacuum arcs, however with a significantly higher value of the effective diffusion coefficient, in agreement with the theoretical considerations of Section II.

B. Description of initial cathode spot expansion after arc ignition in absence of external magnetic field

As the second example, the model was applied to analysis of the distribution of cathode spots during the initial expansion process after arc ignition in the absence of external magnetic field, reported in [13]. The contacts in these experiments had diameter of 40 mm and were made of copper, the sinusoidal current wave had frequency of 50 Hz and current peak of 7 kA. In order to simulate these experiments, the initial condition for surface density Ω of spots was set for the instant t = 0.407 ms, for which an arc root radius of about R_g = 1.9 mm has been estimated by means of analysis of Fig. 11 of the work [13]. Assuming a current per spot of 50 A, we set $\Omega = 1.6\times10^6$ m^{-2}. The value $D = 5\times10^{-4}$ m^{2}s^{-1} was taken as a representative value of the diffusion coefficient for copper [3].

The arc root radius R_g was deduced from the computed radial distribution of Ω as a value of r for which Ω equals 10% of the value $\Omega(r = 0)$. In Fig. 1, R_g is plotted as function of time for different values of δ. One can see that a reasonably good agreement between simulation results and experimental data is achieved by setting $\delta = 40$.

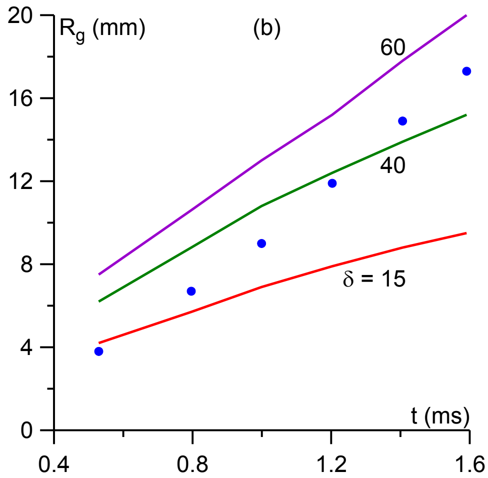

Fig. 1. Time dependence of cathode arc root radius, calculated with variable δ, in conditions of Fig. 11 of the work [13]. Circles: experiment.

The effect produced by the retrograde repulsion dominates over the one produced by diffusion due to random walk. In conditions of Fig. 1, the "repulsion" transport (given by the term $\delta D \nabla \Omega$) exceeds the diffusion transport due to random-walk ($D \nabla \Omega$) by the factor of about 40, which is of the same order of magnitude as the similar factor derived in the preceding section.

IV. CONCLUSIONS

A phenomenological description of an ensemble of a large number of spots in conditions of operation of contacts of high-power circuit breakers has been developed with the use of the concept of random walk of a single cathode spot, observed in the experiments with low-current arcs, modified to take into account the "retrograde repulsion" of spots. The evolution of the number of spots per unit area is governed by a convection-diffusion equation. This equation is similar to the convection-diffusion equation governing the random walk of a single cathode spot in low-current vacuum arcs, however with a different value of the diffusion coefficient. The latter difference is due to the retrograde repulsion. The developed approach is in a reasonable agreement with the experiment, and the effect produced by the retrograde repulsion exceeds the one produced by diffusion due to random walk by a factor of 75 in one experiment and of about 40 in another.

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