Simulation of Thermal Instability in Non-Uniformities on the Surface of Cathodes of Vacuum Arcs

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Abstract- Instability stemming from the strong dependence of electron emission current on the local surface temperature plays an important role in current transfer to hot cathodes of arc discharges. In the case of vacuum arcs, this instability may lead to micro explosions on cathode surface even if the surface is planar. This work is concerned with numerical simulation of effect produced by surface non-uniformities. It is found that the effect is non-trivial: the presence of surface non-uniformities can not only accelerate the development of the instability, which is what one would expect intuitively, but also slow it down and even suppress.

I. INTRODUCTION

Instability stemming from the strong dependence of electron emission current on the local surface temperature plays an important role in current transfer to cathodes of vacuum arcs. The mechanism of the instability is the following: a local increase of the cathode temperature results in an increase of the local electron emission; the latter causes an increase in both the energy flux from the plasma to the cathode surface and in the Joule heat generation inside the cathode; the local surface temperature increases further etc.

If the arc electrical power supply keeps the arc current limited, say, not exceeding 100 A, then the development of the instability may lead to a formation of a steady-state cathode spot. If the arc power supply is capable of generating very high currents while the ballast resistance is not sufficiently high, no stable cathode spots are possible and potential outcomes of the development of the instability are quenching of the instability by heat conduction and micro explosions on the cathode surface even if the surface is planar [1].

In this work, effects produced by surface non-uniformities over development of the thermal instability leading to micro explosions on cathodes of vacuum arcs are studied by means of numerical modelling.

II. The model

Simulations reported in this work have been performed by means of the space-resolved numerical

model of vacuum arc-cathode attachments described in [2] and [3], and employed in [3] and [1]. Temporal evolutions of axially symmetric distributions of temperature and electrostatic potential in the cathode body are computed by means of solving in the cathode body the time-dependent axially symmetric heat conduction equation, written with account of Joule heat generation, and the axially symmetric current continuity equation supplemented with Ohm's law.

This work is concerned with qualitative investigation of the effect of surface non-uniformities on the development of thermal instability leading to micro explosions; convective heat transfer due to motion of the molten metal and deformation of the cathode surface are left beyond the scope of this work. A part of the cathode body that represents the computation domain is designated OABCD in Fig. 1. The near-cathode voltage drop U does not vary with time; an adequate approximation for conditions of high-power vacuum circuit breakers, where hundreds of cathode spots operate in parallel and the near-cathode voltage drop is not affected appreciably by ignition or extinction of an individual spot.



Fig. 1. Geometry of the problem

The computation domain OABCD may be viewed as consisting of a planar cathode matrix (domain I) into which a spherical particle (union of domains II and III) is partially imbedded. The particle can be maid of the same, or different, metal that the matrix. The position of

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the center of the sphere with respect to the matrix can be conveniently characterized by the fraction χ of the volume of the sphere embedded in the matrix: $\chi = V_{III} / (V_{II} + V_{III})$, where V_{II} and V_{III} are volumes of the protruding and imbedded parts of the sphere (domains II and III in Fig. 1).

Trial simulations have shown that the development of thermal instability is usually not significantly affected by the initial state; it is only essential that the initial state be such that the instability is not quenched by heat conduction. For example, the initial state employed in the modelling [1] was obtained by perturbing the steady state corresponding to the value of U being considered. Let us designate by $T_0 = T_0(\mathbf{r})$ and $\varphi_0 = \varphi_0(\mathbf{r})$ the stationary distributions of temperature and potential in the cathode corresponding to U being considered. The initial conditions for the nonstationary modelling have been formulated as $T = T_0 + \beta(T_0 - T_c)$, $\varphi = \varphi_0$, where and β is a given parameter and T_c is the average temperature of the cathode. It was found in [1] that simulations with small positive values of β lead to micro explosions, while simulations with small negative values lead to the steady state with the temperature and potential in the cathode body being constant and equal to T_c and 0, respectively, meaning that the arc attachment has been destroyed by heat conduction. Results reported below have been obtained with the use of this initial approximation with $\beta = 0.01$ unless indicated otherwise.

III. RESULTS

A. Hemispherical protrusions of different sizes

Let us start with the case of a particle and the matrix being made of the same material. The effect of the size of surface non-uniformities over the development of the thermal instability is illustrated by Fig. 2. Here temporal evolution of the maximum temperature inside the cathode body is shown for several values of the protrusion radius *R* for the case of hemispherical protrusions, $\chi = 1/2$, the material being copper, and U =20 V. Note that the maximum temperature is attained inside the protruding part of the cathode, except in cases of small *R*. Also shown for comparison is the case of a planar homogeneous cathode.



Fig. 2. Temporal evolution of maximum temperature of hemispherical copper protrusions. U = 20 V.

One can see that the thermal instability develops and

leads to (thermal) explosion in all the cases shown in Fig. 2a. Since particular cases $R \to 0$ and $R \to \infty$ are equivalent to the case of planar cathode, the explosion time for both very small and very large protrusions should be close to that for planar cathode, which is about 10 µs. Simulations have shown that this happens when R is below approximately 3 μ m, as exemplified by the line $R = 2 \mu m$ in Fig. 2a, or exceeds approximately 200 µm. One could intuitively expect that for intermediate R the explosion develops faster than in the case of planar cathode. The simulations have shown that this is true in some cases, as exemplified by the line R =8 µm in Fig. 2a, but, somewhat surprisingly, not in all cases, as exemplified by the line $R = 5.6 \ \mu m$. Furthermore, there is a narrow range of R values for which the explosion does not develop at all. The latter is exemplified by the solid line 1 in Fig. 2b: the simulations lead to a steady state, which, however, is different from the state $T_0 = T_0(\mathbf{r}), \ \phi_0 = \phi_0(\mathbf{r})$ in the vicinity of which the simulations have started. One can conclude that there are at least two possible steady states for $R = 5.4 \,\mu\text{m}$.

Detailed simulations with the use of a steady-state solver confirmed this conclusion, as seen from Fig. 3 (here I is current per attachment): three different steady-state solutions exist for each value of the protrusion radius in the range 5.37 $\mu m \leq R \leq 5.55 \mu m$. Stability of the steady states described by these solutions is illustrated by Fig. 2b, where temporal development is shown of perturbations of the three steady states existing for $R = 5.4 \,\mu\text{m}$. The lines marked 1, 2, and 3 in this figure refer to, respectively, the low-current state, the intermediate-current state, and the high-current state; solid lines refer to perturbations with $\beta = 0.01$ and dotted lines refer to perturbations with $\beta =$ -0.01. One can see that both positive ($\beta > 0$) and negative (β <0) perturbations of the intermediate-current state decay, i.e., this state is stable. The low- and high-current states are unstable: negative perturbations of the low-current state (dotted line 1) cause a destruction of the attachment by heat conduction and positive perturbations of the high-current state (solid line 3) cause thermal explosion, which are known situations [1]. On the other hand, positive perturbations of the low-current state (solid line 1) and negative perturbations of the high-current state (dotted line 3) lead to the intermediate-current state, situation not encountered previously.

One can say that the transition of steady states from low to high values of R, shown in Fig. 3, is accompanied by hysteresis, which is a rather frequent situation. However, the pattern of stability of this transition is not quite usual: normally states belonging to the section between the turning points (the retrograde section), which are depicted in Fig. 3 by open circles, are unstable and all the other states are stable; it is the other way round in this case. One can assume that states corresponding to the turning points are neutrally stable, which is consistent with the explosion time for $5.6 \ \mu m$ (value of *R* slightly above the upper limit of the range where hysteresis occurs) being quite large, around 0.2 ms.

Distributions of temperature in the cathode body in steady states corresponding to different values of the protrusion radius R are shown in Fig. 4. One can see that the above-discussed multiple steady states, one of which is stable, occur when the steady-state attachment area coincides with the surface of the protrusion.



Fig. 3. Characteristics of steady-state arc attachment to hemispherical on a copper cathode as functions of the protrusion radius. Full circles: unstable steady states. Open circles: stable steady states. U = 20 V.



Fig. 4. Distributions of temperature in the cathode body in steady states corresponding to different values of the protrusion radius. Hemispherical protrusions on a copper cathode. U = 20 V. The bar in kelvin.

B. Non-uniformities of different shapes

In this section, results of simulations with variable χ and fixed *R* are reported in order to illustrate development of the thermal instability in surface non-uniformities of different shapes. The radius *R* is set equal to 5.5 µm, which is close to the range of *R* values where hysteresis occurs in the case of hemispherical protrusions treated above. Three cases are considered: Cu particle and Cu matrix; Cr particle and Cu matrix, and Cu particle and Cr matrix.

The current per stationary attachment as function of χ , computed by means of a steady-state solver, is plotted in Fig. 5. The horizontal dashed and dotted lines depict currents to attachments on a planar cathode made of Cu

or Cr, respectively.



Fig. 5. Current to steady-state attachment of vacuum arc to a partially embedded spherical particle. 1: Cu particle, Cu matrix. 2: Cr particle, Cu matrix. 3: Cu particle, Cr matrix. Full circles: unstable steady states. Open circles: stable steady states. U = 20 V.

For χ close to unity, line 1 coincides with the dashed line as it should; the liming case of Cu cathode with a flat surface. As χ decreases, which corresponds to an increase of the height of the protrusion, the current decreases. This can be understood as follows: an increase of the height of a protrusion amounts to an increase of the thermal resistance of the part of the cathode surface to which the arc is attached; this is equivalent to a decrease of thermal conductivity and the latter causes a decrease of the attachment area and current [2], [3]. Furthermore, there is a range of χ values where hysteresis occurs: $0.49 \leq \chi \leq 0.51$.

Line 2 is qualitatively similar to line 1, although the range of χ values where hysteresis occurs is shifted to higher values, around $\chi \approx 0.96$. The current per attachment to a Cu particle embedded in Cr matrix (line 3) in the range $\chi \lesssim 0.9$ does not differ qualitatively from current in the case of Cr particle embedded in Cu matrix (line 2), however, there is no dramatic increase of current in the range $\chi \gtrsim 0.9$ and no hysteresis, which are present in the latter case.

Distributions of temperature in the cathode body are shown in Fig. 6. One can see that the hysteresis revealed by lines 1 and 2 in Fig. 5 occurs when the area of steady-state attachment coincides with the surface of the protruding part of the particle, similarly to what was found in the previous section. The absence of hysteresis in the case of Cu particle in a Cr matrix, depicted by line 3 Fig. 5, can be attributed to the fact that the attachment radius does not significantly exceed the particle radius even for $\chi = 1$ in this case.

Non-stationary modelling has shown that in the cases of Cu and Cr particles in Cu matrices the thermal instability develops and leads to thermal explosion for all values of χ beyond the range of hysteresis, the explosion time being of the order of 1 to 10 µs. The pattern of stability in the range of hysteresis is exactly the same as the one described in the preceding section: the intermediate-current state is stable; the low- and high-current states are unstable; negative perturbations of the low-current state cause a destruction of the attachment by heat conduction and positive perturbations lead to the intermediate-current state; positive perturbations of the high-current state cause thermal explosion and negative perturbations lead to the intermediate-current state; the explosion time for R values close to the turning points is quite large, which is consistent with states corresponding to the turning points being presumably neutrally stable.

In the case of Cu particle in Cr matrix, where hysteresis does not occur, the thermal instability develops and leads to thermal explosion for all values of χ . The explosion time is smaller than those for planar homogeneous cathodes made of both Cu or Cr and varies between approximately 0.7 µs for $\chi = 0.1$ and 3 µs for $\chi = 1$.



Fig. 6. Distributions of temperature in the cathode body.
Partially imbedded spherical particles. U = 20 V. The bar is the same as in Fig. 4. (a) Cu particle, cu matrix.
(b) Cr particle, Cu matrix. (c) Cu particle, Cr matrix.

IV. CONCLUSIONS

The effect of non-uniformities on the surface of vacuum arc cathodes over the development of the thermal instability is non-trivial. For example, in the case of hemispherical protrusions of different radii *R*, thermal instability develops faster than in planar cathodes if the radius of the protrusion exceeds 6 µm or is below 5 µm, although the difference is not very significant, up to about the factor of 4. In the intermediate range 5 µm $\leq R \leq 6$ µm, the instability develops slower than in planar cathodes; and if the attachment area coincides with the surface of the protrusion, which happens for $R \approx 5.5$ µm, then the instability may even not develop, or lead to a stable steady state rather than to thermal explosion.

In all the cases, the explosion time is of the order of 1 μ s or higher. One can conclude that the development of the thermal instability is affected by effects originating in motion of the molten metal.

The conclusion that surface non-uniformities similar in size to steady-state arc attachments can suppress the development of thermal instability and thus prevent thermal explosions, while being theoretically interesting, does not mean that arc attachments to these non-uniformities may exist for a long time, since the non-uniformities will anyway be destroyed by melting. However, this conclusion may be helpful for analysis of results of simulations of temporal evolution of cathode attachments of vacuum arcs with account of motion of the molten metal.

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